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Research Article

Generalized Derivations on Infinite Distributive Lattices

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Abstract: Our goal in this study is to determine the structure of generalized derivations on an infinitely distributive lattice. Our analysis reveals that the redefined mapping acts as a generalized biderivation in a particular setting. Moreover, we investigate some interesting properties of generalized derivation and generalized biderivation on a distributive lattice.

Keywords: infinitely distributive lattice, generalized derivation, generalized biderivation

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1. Introduction and preliminaries

Throughout its development, the employment of binary operations has evolved into a fundamental idea in algebraic structures, aiding in the explanation of abstract algebraic notions like rings, groups, semigroups, monoids, etc. [1]. As seen in [2, 3], binary operations have become crucial tools in the study of lattice theory and its many applications. Many ideas and characteristics, including the concept of the lattice itself, can be understood through binary operations.

The derivation theory in rings, near rings, and on BCI-algebras has been the subject of numerous studies. Bi-derivations, tri-derivations, and n-derivations in general are examples of multi-derivations that have been examined in prime and semi-prime rings. Numerous scholars have examined this idea, including authors in [4] who obtained the structure of (σ, ρ)-n-derivations on rings, where σ, ρ are automorphisms of a ring R.

Recently, Da Motta Ferreira and Macedo Ferreira [5] established some remarkable results about the additivity of n-multiplicative maps on alternative rings. Some interesting results related to the derivations on lattice obtained in [6, 7]. The authors in [8] developed the concept of symmetric f-derivations on a lattice and used it to investigate additional features in the lattice context. Furthermore, symmetric f-derivations are used to characterize the distributive and modular lattices in the domain of f-biderivations and explore the features of the kernel as seen in [9].

The concept of generalized derivations in rings was first presented by Bresar in [10]. More precisely, if there is a derivation d of R such that, for any $\mathfrak{s}$, $y \in R$, $G(\mathfrak{s}y) = G(\mathfrak{s})y + \mathfrak{s}d(y)$ (d is termed the derivation associated with G), then an additive map $G : R \to R$ is said to be a generalized derivation. A map of the form $G(\mathfrak{s}) = a\mathfrak{s} + \mathfrak{s}b$, for some a, $b \in R$, is a significant example; these generalized derivations are referred to as inner. Some remarkable results on generalized derivations on the ring are found in [11, 12]. Further generalization of derivations with involution obtained in [13].

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The idea of derivation and generalized derivation on lattices is described as follows [14, 15]: A function $\mathfrak{h} : \mathfrak{L} \longrightarrow \mathfrak{L}$ is known as a derivation on $\mathfrak{L}$, if $\mathfrak{h}$ fulfills

$$\mathfrak{h}(k \wedge w) = \big(\mathfrak{h}(k) \wedge w\big) \vee \big(k \wedge \mathfrak{h}(w)\big),$$

$$\mathfrak{h}(k \vee w) = \mathfrak{h}(k) \vee \mathfrak{h}(w),$$

for every $k,\, w \in \mathfrak{L}$. In similar way, a function $\mathfrak{F} : \mathfrak{L} \longrightarrow \mathfrak{L}$ is known as a generalized derivation on $\mathfrak{L}$ possessing associated derivation $\mathfrak{h}$, if $\mathfrak{F}$ fulfills

$$\mathfrak{F}(k \wedge w) = \big(\mathfrak{F}(k) \wedge w\big) \vee \big(k \wedge \mathfrak{h}(w)\big),$$

$$\mathfrak{F}(k \vee w) = \mathfrak{F}(k) \vee \mathfrak{F}(w),$$

for every $k,\, w \in \mathfrak{L}$.

Example 1 Let $\mathfrak{L} = \{0,\, b,\, a,\, 1\}$ be a given lattice described as below:

$$0 \longrightarrow b \longrightarrow a \longrightarrow 1.$$

Define $\mathfrak{F} : \mathfrak{L} \longrightarrow \mathfrak{L}$ and $\mathfrak{h} : \mathfrak{L} \longrightarrow \mathfrak{L}$ as

$$\mathfrak{h}(\kappa) = \begin{cases} 0, & \text{if } \kappa = 0,\, 1, \\ b, & \text{if } \kappa = a,\, b, \end{cases}$$

and

$$\mathfrak{F}(\kappa) = \begin{cases} \kappa, & \text{if } \kappa = 0,\, a,\, b, \\ a, & \text{if } \kappa = 1. \end{cases}$$

In such setting $\mathfrak{F}$ is a generalized derivation on $\mathfrak{L}$ associated with $\mathfrak{h}$ for each $\kappa \in \mathfrak{L}$.

The generalizations of derivations on lattices—such as $(f,\, g)$-derivations on lattices introduced in [16, 17], symmetric bi-derivations on lattices studied in [18], symmetric f-biderivations on lattices examined in [9], and generalized symmetric f-biderivations on lattices developed in [19, 20] naturally lead to the consideration of derivations on bounded hyperlattices. In addition, Yazrali and Öztürk [21] provide a further significant generalization by introducing permuting tri-f-derivations on lattices.

In the article [22], the author gave some equivalent conditions about lattice implication algebras by using generalized f-derivations. This idea is examined in detail using MV-algebra in [23]. Researchers in [24] introduced the concept of the trace of permuting n-$(f,\, g)$-derivation of a lattice and some related properties. Further, Bedda et al. [25] investigated the generalized permuting f-n-derivations on the lattice and characterized the distributive and modular lattices. An interesting study on derivations on differential distributive bilattices was found in [26, 27]. Recently, a structural development of demi-complementation was obtained on nearly distributive lattices in [28].

Inspired by all of the foregoing facts, we define generalized biderivation and generalized derivation on an infinite distributive lattice and highlight some order with related derivations.

2. Results on infinitely distributive lattice

Let $\mathfrak{L}$ be a distributive lattice with minimum 0. We say that $\mathfrak{L}$ is infinitely distributive when, for every a, $\mathfrak{s}_i \in \mathfrak{L}$, the following equalities hold provided that the above inf and sups exist:

$$a \wedge \left(\bigvee_i \mathfrak{s}_i \right) = \bigvee_i \left(a \wedge \mathfrak{s}_i \right),$$

and

$$a \vee \left(\bigwedge_i \mathfrak{s}_i \right) = \bigwedge_i \left(a \vee \mathfrak{s}_i \right).$$

The first identity is called the Join-infinite Identity (JID), and the second identity is called Meet-infinite Identity (MID). We frequently use the above two identities without reference. An atom namely $0 \neq e \in \mathfrak{L}_\mathfrak{e}$ is an element that covers the least element $0 \in \mathfrak{L}_\mathfrak{e}$. It means that there is nothing exists between them. The atom ensures that there is no element $\mathfrak{s}$ such that $0 < \mathfrak{s} < e$. For example, in the power set lattice of any set, the singleton sets are atoms.

Define the mappings as follows:

$$F : \mathfrak{L}_a \longrightarrow \mathfrak{L}_a$$

$$F(\mathfrak{s}) = \bigvee_{\substack{a \leq \mathfrak{s} \\ a \text{ is atom}}} a,$$

and

$$d : \mathfrak{L}_a \longrightarrow \mathfrak{L}_a$$

$$d(\mathfrak{s}) = \bigvee_{\substack{a \leq \mathfrak{s} \\ a \text{ is atom}}} a.$$

Theorem 1 Let $\mathfrak{L}_a$ be an infinitely distributive lattice with minimum 0. Then F is a generalized derivation with respect to the derivation d.

Proof. By definition, we have

$$F(\mathfrak{s} \vee y) = \bigvee_{\substack{\mathfrak{j} \leq (\mathfrak{s} \vee y) \\ \mathfrak{j} \text{ is atom}}} \mathfrak{j}.$$

If $\mathfrak{j} \leq \mathfrak{s} \vee y$, and $\mathfrak{j}$ is an atom, then

$$\begin{aligned} \mathfrak{j} \leq \mathfrak{s} \vee y &\Leftrightarrow \mathfrak{j} \wedge (\mathfrak{s} \vee y) = \mathfrak{j} \\ &\Leftrightarrow (\mathfrak{j} \wedge \mathfrak{s}) \vee (\mathfrak{j} \wedge y) = \mathfrak{j} \\ &\Leftrightarrow \mathfrak{j} \wedge \mathfrak{s} = \mathfrak{j} \ \vee \ \mathfrak{j} \wedge y = \mathfrak{j} \\ &\Leftrightarrow \mathfrak{j} \leq \mathfrak{s} \ \vee \ \mathfrak{j} \leq y. \end{aligned}$$

Thus

$$\begin{aligned} F(\mathfrak{s} \vee y) &= \bigvee_{\substack{\mathfrak{j} \leq (\mathfrak{s} \vee y) \\ \mathfrak{j} \text{ is atom}}} \mathfrak{j} \\ &= \left(\bigvee_{\substack{a_1 \leq \mathfrak{s} \\ a_1 \text{ is atom}}} a_1 \right) \ \vee \ \left(\bigvee_{\substack{a_2 \leq y \\ a_2 \text{ is atom}}} a_2 \right) \\ &= F(\mathfrak{s}) \vee F(y). \end{aligned}$$

Hence $F(\mathfrak{s} \vee y) = F(\mathfrak{s}) \vee F(y)$. Now, we aim to prove that

$$F(\mathfrak{s} \wedge y) = \Big(F(\mathfrak{s}) \wedge y\Big) \vee \Big(\mathfrak{s} \wedge d(y)\Big).$$

Note that:

$$\begin{aligned} F(\mathfrak{s}) \wedge y &= \left(\bigvee_{\substack{a \leq \mathfrak{s} \\ a \text{ is atom}}} a \right) \wedge y \\ &= \bigvee_{\substack{a \leq \mathfrak{s} \\ a \text{ is atom}}} (a \wedge y). \end{aligned}$$

Now, if $a \leq y$, then

$$F(\mathfrak{s}) \wedge y = \bigvee_{\substack{a \leq (\mathfrak{s} \wedge y) \\ a \text{ is atom}}} a$$

$$= F(\mathfrak{s} \wedge y).$$

If $a \nleq y$, then $a \wedge y = 0$ as a is an atom. Thus

$$F(\mathfrak{s}) \wedge y = \bigvee_{\substack{a \leq \mathfrak{s} \\ a \text{ is atom}}} 0$$

$$= 0.$$

Computing $\mathfrak{s} \wedge d(y)$, we obtain:

$$\mathfrak{s} \wedge d(y) = \mathfrak{s} \wedge \left(\bigvee_{\substack{a \leq y \\ a \text{ is atom}}} a \right)$$

$$= \bigvee_{\substack{a \leq y \\ a \text{ is atom}}} (\mathfrak{s} \wedge a).$$

Now, if $a \leq \mathfrak{s}$, then

$$\mathfrak{s} \wedge d(y) = \bigvee_{\substack{a \leq (\mathfrak{s} \wedge y) \\ a \text{ is atom}}} a$$

$$= F(\mathfrak{s} \wedge y).$$

If $a \nleq \mathfrak{s}$, then $a \wedge \mathfrak{s} = 0$ as a is an atom. Thus

$$\mathfrak{s} \wedge d(y) = \bigvee_{\substack{a \leq y \\ a \text{ is atom}}} 0$$

$$= 0.$$

Hence $F(\mathfrak{s} \wedge y) = \big(F(\mathfrak{s}) \wedge y\big) \vee \big(\mathfrak{s} \wedge d(y)\big)$. Therefore, F is a generalized derivation with respect to the derivation d. $\square$

Theorem 2 Let $\mathfrak{L}_\kappa$ be an infinitely distributive lattice with minimum 0. If F is a generalized derivation with respect to the derivation d, then

(1) $d(\mathfrak{s}) \leq F(\mathfrak{s})$;

(2) $F(\mathfrak{s}) \leq \mathfrak{s}$ for every $\mathfrak{s} \in \mathfrak{L}_\kappa$.

Proof. Let's start with the condition for each $\mathfrak{s} \in \mathfrak{L}_\kappa$

$$F(\mathfrak{s}) \wedge d(\mathfrak{s}) = F(\mathfrak{s} \wedge \mathfrak{s}) \wedge d(\mathfrak{s})$$

$$= \left(\bigvee_{\substack{\kappa \leq (\mathfrak{s} \wedge \mathfrak{s} = \mathfrak{s}) \\ \kappa \text{ is atom}}} \kappa \right) \wedge \left(\bigvee_{\substack{\kappa \leq \mathfrak{s} \\ \kappa \text{ is atom}}} \kappa \right)$$

$$= \bigvee_{\substack{\kappa \leq \mathfrak{s} \\ \kappa \text{ is atom}}} \kappa$$

$$= d(\mathfrak{s}).$$

This indicates that

$$d(\mathfrak{s}) \leq F(\mathfrak{s}) \quad \text{for each} \quad \mathfrak{s} \in \mathfrak{L}_\kappa. \tag{1}$$

In order to prove another inequality, we assume the case as below

$$F(\mathfrak{s}) \vee \mathfrak{s} = F(\mathfrak{s} \wedge \mathfrak{s}) \vee \big(d(\mathfrak{s}) \wedge \mathfrak{s}\big)$$

$$= \left(\left(\bigvee_{\substack{\kappa \leq \mathfrak{s} \\ \kappa \text{ is atom}}} \kappa \right) \wedge \mathfrak{s} \right) \vee \left(\bigvee_{\substack{\kappa \leq \mathfrak{s} \\ \kappa \text{ is atom}}} \kappa \wedge \mathfrak{s} \right)$$

$$= \kappa \vee \mathfrak{s}$$

$$= \mathfrak{s} \text{ from definition of } \kappa.$$

The above manipulation enables us to find

$$F(\mathfrak{s}) \leq \mathfrak{s} \quad \text{for each} \quad \mathfrak{s} \in \mathfrak{L}_\kappa. \tag{2}$$

Combining equations (1) and (2), we obtain $d(\mathfrak{s}) \leq \mathfrak{s}$ and $F(\mathfrak{s}) \leq \mathfrak{s}$ for every $\mathfrak{s} \in \mathfrak{L}_\kappa$. $\square$

3. Generalized biderivation on distributive lattice

Let's redefine the mappings $\mathfrak{B}$, $\delta : \mathfrak{L}_\mathfrak{a} \to \mathfrak{L}_\mathfrak{a}$ such that for any a, $\mathfrak{s}$, $\mathfrak{b} \in \mathfrak{L}_\mathfrak{a}$

$$\mathfrak{B}(\mathfrak{s}, \mathfrak{b}) = \bigvee_{a \leq (\mathfrak{s} \vee \mathfrak{b})} a \quad \text{and} \quad \delta(\mathfrak{s}, \mathfrak{b}) = \bigvee_{a \leq (\mathfrak{s} \wedge \mathfrak{b})} a.$$

In this setting, $\mathfrak{B}$ is a generalized biderivation with associted biderivation δ.

Example 2 Let $\mathfrak{L}$ be a lattice that has a least element 0. The function $\delta(x, y) = 0$ defines a symmetric biderivation on $\mathfrak{L}$. Now define a mapping $\mathfrak{B}$ on $\mathfrak{L}$ by $\mathfrak{B}(x, y) = x \wedge y$ for all x, $y \in \mathfrak{L}$. It follows that $\mathfrak{B}$ is a generalized symmetric biderivation on L associated with δ. Furthermore, for any fixed $z \in \mathfrak{L}$, the mapping $\mathfrak{B}$ on $\mathfrak{L}$ given by

$$\mathfrak{B}(x, y) = ((x \wedge y) \wedge z),$$

is also a generalized symmetric biderivation corresponding to $\delta(x, y) = 0$ on $\mathfrak{L}$ for any x, $y \in \mathfrak{L}$.

Theorem 3 Let $\mathfrak{L}_a$ be an infinitely distributive lattice with minimum 0. If $\mathfrak{B}$ is a generalized biderivation with respect to the biderivation δ, then for each $\mathfrak{s} \in \mathfrak{L}_a$

$$\delta(\mathfrak{s}, \mathfrak{s}) \leq \mathfrak{B}(\mathfrak{s}, \mathfrak{s}) \leq \mathfrak{s}.$$

Proof. Consider the condition for each $\mathfrak{s} \in \mathfrak{L}_a$

$$\begin{aligned}
\mathfrak{B}(\mathfrak{s}, \mathfrak{s}) \wedge \delta(\mathfrak{s}, \mathfrak{s}) &= \left(\bigvee_{a \leq (\mathfrak{s} \vee \mathfrak{s})} a \right) \wedge \left(\bigvee_{a \leq \mathfrak{s} \wedge \mathfrak{s}} a \right) \\
&= \left(\bigvee_{a \leq \mathfrak{s}} a \right) \wedge \left(\bigvee_{a \leq \mathfrak{s}} a \right) \\
&= \bigvee_{a \leq \mathfrak{s}} a \\
&= \bigvee_{a \leq (\mathfrak{s} \wedge \omega)} a \quad \text{for any} \quad \omega \in \mathfrak{L}_a \\
&= \delta(\mathfrak{s}, \omega) \\
&= \delta(\mathfrak{s}, \mathfrak{s}).
\end{aligned}$$

Which in turn to

$$\delta(\mathfrak{s},\,\mathfrak{s}) \le \mathfrak{B}(\mathfrak{s},\,\mathfrak{s}) \quad \text{for each} \;\; \mathfrak{s} \in \mathfrak{L}_a. \tag{3}$$

Next, evaluate the other part as

$$\begin{aligned}\mathfrak{B}(\mathfrak{s},\,\mathfrak{s}) &= \mathfrak{B}(\mathfrak{s} \vee \mathfrak{s},\,\mathfrak{s})\\ &= \mathfrak{B}(\mathfrak{s},\,\mathfrak{s}) \vee \mathfrak{s} \wedge \big(\mathfrak{s} \vee \delta(\mathfrak{s},\,\mathfrak{s})\big)\\ &= \mathfrak{B}(\mathfrak{s},\,\mathfrak{s}) \vee \mathfrak{s}.\end{aligned}$$

This implies that

$$\mathfrak{B}(\mathfrak{s},\,\mathfrak{s}) \le \mathfrak{s} \quad \text{for each} \;\; \mathfrak{s} \in \mathfrak{L}_a. \tag{4}$$

Simplify the both expression (3) and (4) to obtain

$$\delta(\mathfrak{s},\,\mathfrak{s}) \le \mathfrak{B}(\mathfrak{s},\,\mathfrak{s}) \le \mathfrak{s} \quad \text{for each} \;\; \mathfrak{s} \in \mathfrak{L}_a.$$

□

From the above discussion, we can derive the following results:

Theorem 4 Let $\mathfrak{L}_a$ be an infinitely distributive lattice with minimum 0. Then $\delta : \mathfrak{L}_a \to \mathfrak{L}_a$ such that $\delta(\mathfrak{s},\,\mathfrak{b}) = \bigvee_{a \le (\mathfrak{s} \wedge \mathfrak{b})} a$ is a biderivation on $\mathfrak{L}_a$ for each $\mathfrak{s},\,\mathfrak{b} \in \mathfrak{L}_a$.

Proof. We are given that

$$\delta(\mathfrak{s},\,\mathfrak{b}) = \bigvee_{\substack{a \le (\mathfrak{s} \wedge \mathfrak{b})\\ a \text{ atom}}} a.$$

We prove that

$$\delta(\mathfrak{s} \wedge c,\,\mathfrak{b}) = (\delta(s,\,\mathfrak{b}) \wedge c) \vee (\mathfrak{s} \wedge \delta(c,\,\mathfrak{b})).$$

By definition, we have

$$\delta(\mathfrak{s} \wedge c,\,\mathfrak{b}) = \bigvee_{\substack{a \le ((\mathfrak{s} \wedge c) \wedge \mathfrak{b})\\ a \text{ atom}}} a.$$

On the other hand, we have

$$(\delta(\mathfrak{s}, \mathfrak{b}) \wedge c) \vee (\mathfrak{s} \wedge \delta(c, \mathfrak{b})) = \left(\left(\bigvee_{a \leq (\mathfrak{s} \wedge \mathfrak{b})} a\right) \wedge c\right) \vee \left(\mathfrak{s} \wedge \left(\bigvee_{a \leq (c \wedge \mathfrak{b})} a\right)\right)$$

$$= \left(\bigvee_{a \leq (\mathfrak{s} \wedge \mathfrak{b})} (a \wedge c)\right) \vee \left(\bigvee_{a \leq (c \wedge \mathfrak{b})} (\mathfrak{s} \wedge a)\right).$$

Now, if $a \leq c$ and $a \leq \mathfrak{s}$, then

$$a \wedge c = a, \qquad \mathfrak{s} \wedge a = a,$$

as a is an atom. Hence,

$$(\delta(\mathfrak{s}, \mathfrak{b}) \wedge c) \vee (\mathfrak{s} \wedge \delta(c, \mathfrak{b})) = \left(\bigvee_{a \leq (\mathfrak{s} \wedge \mathfrak{b})} a\right) \vee \left(\bigvee_{a \leq (c \wedge \mathfrak{b})} a\right)$$

$$= \bigvee_{a \leq (\mathfrak{s} \wedge \mathfrak{b}) \vee (c \wedge \mathfrak{b})} a.$$

Using distributivity,

$$(\mathfrak{s} \wedge \mathfrak{b}) \vee (c \wedge \mathfrak{b}) = (\mathfrak{s} \vee c) \wedge \mathfrak{b}.$$

Therefore,

$$(\delta(\mathfrak{s}, \mathfrak{b}) \wedge c) \vee (\mathfrak{s} \wedge \delta(c, \mathfrak{b})) = \bigvee_{a \leq ((\mathfrak{s} \vee c) \wedge \mathfrak{b})} a. \tag{5}$$

Since $(\mathfrak{s} \wedge c) \leq (\mathfrak{s} \vee c)$, we have $((\mathfrak{s} \wedge c) \wedge \mathfrak{b}) \leq ((\mathfrak{s} \vee c) \wedge \mathfrak{b})$. Then we conclude from equation (5) that

$$(\delta(\mathfrak{s}, \mathfrak{b}) \wedge c) \vee (\mathfrak{s} \wedge \delta(c, \mathfrak{b})) = \bigvee_{a \leq ((\mathfrak{s} \wedge c) \wedge \mathfrak{b})} a$$

$$= \delta(\mathfrak{s} \wedge c, \mathfrak{b}).$$

Otherwise,

$$a \wedge c = 0, \qquad \mathfrak{s} \wedge a = 0,$$

and the result is proved. $\square$

Proposition 1 Let $\mathfrak{L}_a$ be an infinitely distributive lattice with minimum 0. If δ is a biderivation on $\mathfrak{L}_a$, then $\delta(\zeta, \zeta) \leq \zeta$ for each $\zeta \in \mathfrak{L}_a$.

Proof.

$$\delta(\zeta, \zeta) \wedge \zeta = \left(\bigvee_{\substack{a \leq (\zeta \wedge \zeta) \\ a \text{ atom}}} a \right) \wedge \zeta$$

$$= \bigvee_{a \leq (\zeta \wedge \zeta)} (a \wedge \zeta)$$

$$= \bigvee_{\substack{a \leq \zeta \\ a \text{ atom}}} a$$

$$= \delta(\zeta, \zeta).$$

Thus,

$$\delta(\zeta, \zeta) \leq \zeta.$$

$\square$

4. Discussion

Lattices have proved important in many domains, such as information theory, information retrieval, and information access control, as demonstrated in [29]. Durfee [30] addressed several cryptanalysis problems using techniques from number geometry. Algebraic techniques are used to cryptanalyze a number of public key cryptosystems, and tools from the theory of integer lattices are used to obtain the solution of some real-life problems.

Let the two complex Hausdorff topological linear spaces be $\mathfrak{V}_1$ and $\mathfrak{V}_2$. The algebra of all continuous linear mappings from $\mathfrak{V}_1$ into $\mathfrak{V}_2$ is represented by $L(\mathfrak{V}_1, \mathfrak{V}_2)$. The property of reflexivity is defined as for a subset $\mathcal{S}$ in $L(\mathfrak{V}_1, \mathfrak{V}_2)$, $\mathfrak{T} \in \mathcal{S}$ whenever $\mathfrak{T} \in L(\mathfrak{V}_1, \mathfrak{V}_2)$ and $\mathfrak{T}v \in (\bar{\mathcal{S}}v)$ for every $v \in \mathfrak{V}_1$, where $\bar{\mathcal{S}}v$ is the topological closure of $\mathcal{S}v$. The collection $\mathfrak{I}$ of closed subspaces of $\mathfrak{V}_1$ containing $\{0\}$ and $\mathfrak{V}_1$ such that, for each family $\mathcal{I}_\zeta$ containing elements from $\mathfrak{I}$. The sets both $\bigcap \mathcal{I}_\zeta$ and $\bigvee \mathcal{I}_\zeta$ are members of $\mathcal{I}$, where $\bigvee$ stands for the closed linear span of the family $\mathcal{I}_\zeta$. The algebra of all operators on $\mathfrak{V}_1$ that leave each element of $\mathfrak{I}$ invariant is denoted by Alg$\mathfrak{I}$ if $\mathfrak{I}$ is a subspace lattice. A nest is a totally ordered subspace lattice, let's say N, and the corresponding reflexive algebra alg$N$ is referred to as a nest algebra.

By this construction, the two structures are transferable. It would be fascinating to employ the derivations and generalized derivations defined in Section 2, and extend them to the nest N and reflexive algebra alg$N$. This is an exciting but technically challenging approach.

These frameworks are pivotal across various applications in computer science, engineering, and beyond. They captivate researchers' attention, paving the way for progress in organizing, contrasting, and optimizing data, thus providing essential resources for tackling complex problems in numerous fields.

5. Conclusion

The fundamental characteristics and noteworthy qualities at the core of this investigation are highlighted. Our method entails examining the properties of generalized derivations on infinite lattices from a purely theoretical perspective. Our theoretical analysis ideally extends and completes the changes made in the definition of F and d, exposing interesting facets of generalized biderivation. However, the idea raises important research challenges, particularly when it comes to allied fields like discrete mathematics, computer science, combinatorics, and more general algebraic structures.

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Author contributions

Abu Zaid Ansari developed the idea for this article, and Faiza Shujat enhanced it by analysis and computation of examples. Further Areej Almuhaimeed performed the computation and finalized the writing. Noorbhasha Rafi validate the findings, editing and review of all the results. All authors reviewed the article and gave their approval for the final version.

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Data availability

All data generated or analyzed during this study are included in this published article.

Conflict of interest

The authors declare no competing financial interest.

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