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Research Article

Characterization of Linear Operators on Specific Classes of Algebras

Abu Zaid Ansari[1], Faiza Shujat[2*], Vishal K. Yadav[3]

[1]Department of Mathematics, Faculty of Science, Islamic University of Madinah, Madinah, Saudi Arabia
[2]Department of Mathematics, College of Science, Taibah University, Madinah, Saudi Arabia
[3]Department of Mathematics, D S College, Affiliated to R M PS University, Aligarh, 202001, India
E-mail: fullahkhan@taibahu.edu.sa

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Abstract: Let $s, t, u \geq 1$ be fixed integers and $\mathscr{K}$ be a Hilbert space. If $\mathscr{G}, \partial : \mathscr{A} \to \mathscr{A}$ are linear mappings that fulfill the following algebraic identity $2\mathscr{G}(X^{s+t+u}) = \mathscr{G}(X^s)\psi(X^{t+u}) + \gamma(X^s)\partial(X^t)\psi(X^u) + \gamma(X^{s+t})\partial(X^u) + \mathscr{G}(X^u)\psi(X^{t+s}) + \gamma(X^u)\partial(X^t)\psi(X^s) + \gamma(X^{u+t})\partial(X^s)$. Then $\mathscr{G}$ will be a generalized (ψ, γ)-derivation having (ψ, γ)-derivation ∂ on $\mathscr{A}$, where ψ and γ are automorphisms of $\mathscr{A}$, a Commutant Subspace Lattice (CSL) subalgebra of a von Neumann algebra on $\mathscr{K}$.

Keywords: algebra, semiprime rings, automorphisms, generalized (ψ, γ)-derivation

MSC: 16N60, 16W25, 46L10, 47L40

1. Introduction

Within this work, $\mathscr{R}$ denotes an associative ring and $\mathscr{K}$ is a complex Hilbert space that serves as the framework for the mathematical developments. We represent identity operator of $\mathscr{K}$ by I, and we employ the notation $\mathscr{B}(\mathscr{K})$ for the algebra consisting of all linear, bounded operators of $\mathscr{K}$. A ring $\mathscr{R}$ is characterized as q-torsion free, if $qr = 0$ leads to $r = 0$ for all $r \in \mathscr{R}$, where $q > 1$ denotes an integer, and $\mathscr{R}$ is defined as prime if $s\mathscr{R}r = \{0\}$ leads to $s = 0$ or $r = 0$. Moreover, it is characterized as semiprime if $s\mathscr{R}s = \{0\}$ results in $s = 0$. In what follows, the term 'subspace' will always refer to a norm-closed linear manifold, and the term 'projection' will always denote an orthogonal projection. For notational convenience, we shall, whenever appropriate, identify a subspace with the orthogonal projection onto it. The subspace lattice $\mathscr{L}$ is located strictly between the trivial subspace $\{0\}$ and the whole space $\mathscr{K}$, and is assumed to be closed under the usual lattice operations. Moreover, every subspace in $\mathscr{L}$ is invariant under the algebra $Alg\ \mathscr{L}$. For a comprehensive exposition of these fundamental concepts, the reader is referred to [1].

The text defines several types of derivations on ring $\mathscr{R}$ and algebra $\mathscr{A}$. For given two automorphisms γ and ψ, a map $\partial : \mathscr{A} \to \mathscr{A}$ is a (ψ, γ)-derivation if $\partial(X_1 + X_2) = \partial(X_1) + \partial(X_2)$ and $\partial(X_1X_2) = \partial(X_1)\psi(X_2) + \gamma(X_1)\partial(X_2)$ for all $X_1, X_2 \in \mathscr{A}$, and a Jordan (ψ, γ)-derivation, if $\partial(X_1^2) = \partial(X_1)\psi(X_1) + \gamma(X_1)\partial(X_1)$ for all $X_1 \in \mathscr{A}$. Moreover, generalized (ψ, γ)-derivation is an extension of a derivation which is defined as an additive map $\mathscr{G} : \mathscr{A} \to \mathscr{A}$ with an (ψ, γ)-derivation ∂ that holds $\mathscr{G}(X_1X_2) = \mathscr{G}(X_1)\psi(X_2) + \gamma(X_1)\partial(X_2)$. Similarly, a generalized Jordan (ψ, γ)-derivation is defined using a Jordan (ψ, γ)-derivation ∂ and the identity $\mathscr{G}(X_1^2) = \mathscr{G}(X_1)\psi(X_1) + \gamma(X_1)\partial(X_1)$ for all $X_1 \in \mathscr{A}$.

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2. Preliminaries

The investigation of Jordan (ψ, γ)-derivations on algebras and rings encompasses a extensive and developed area. Clearly, every generalized (ψ, γ)-derivation is, by definition, a generalized Jordan (ψ, γ)-derivation. However, Ashraf et al. [2] and Lanski [3] have provided instructive counterexamples that convincingly show the converse implication is, in general, not valid. In [2], it was demonstrated that every generalized Jordan (ψ, γ)-derivation defined on a Lie ideal of a prime ring is actually a generalized (ψ, γ)-derivation. A result is subsequently employed to establish, in a more general framework, that in [3], it is established that any additive self-mapping of a semiprime ring that behaves as a generalized (ψ, γ)-derivation on the subset of n^{th} powers is, in fact, a generalized (ψ, γ)-derivation on ring.

The fundamental results of Brešar [4, 5] and Vukman [6] are the inspiration to carry this work further. Brešar and Vukman [7] show that every Jordan (ψ, γ)-derivation acts as an (ψ, γ)-derivation in a prime ring that is 2-torsion-free. Further generalizations were obtained in [8–10]. Recent developments of the presented concept are discussed in [11] for nonlinear maps preserving mixed products on factors. Subsequently, a significant extension of a derivation was established on factor von Neumann algebras and on quotient rings in [12, 13]. The advances presented in [14, 15] provided researchers with a more complete perspective and new motivation. Building on these connections found in [16, 17] for mixed derivations on algebras, in [18] for Lie triple derivations, and in [19] for derivations in rings and algebras, some extensions of triple derivations on von Neumann algebras are presented in [20].

In [21], the researchers proved that if M is a factor von Neumann algebra, then every unital nonlinear mixed $*$-Jordan-type derivation is an additive $*$-derivation. Another remarkable extension obtained by the authors in [22] investigated the structure of mixed nonlinear skew Lie (Jordan) products on von Neumann algebras. Further developments concerning bi-skew Lie (Jordan) products on factor von Neumann algebras were found in [23], and additional results on certain functional equations related to derivations are presented in [24, 25]. Motivated by the study of nonlinear maps and local derivations in [26, 27] and previous literature review, we carry out the idea of the presented work.

Next, if we consider this type of result in algebra $\mathscr{A}$, we see that Lu [28] proved in 2009 that every Jordan derivation is a derivation on a CSL algebra. This pivotal finding was further expanded in [29], establishing that a Jordan derivation on a CSL subalgebra of the von Neumann algebra indeed acts as a derivation.

Inspired by the above line of investigation, in [1], the authors prove that every Jordan (ψ, γ)-derivation is an (ψ, γ)-derivation on a CSL subalgebra of a von Neumann algebra $\mathscr{A}$. Further, they proved that two linear mappings $\mathscr{G}$, $\partial : \mathscr{A} \to \mathscr{A}$ satisfying certain identity are generalized (ψ, γ)-derivation and (ψ, γ)-derivation respectively. Motivated by this research direction, recently Ansari et al. [30] have proved the following:

Suppose that two linear mappings $\mathscr{G}$, $\partial : \mathscr{A} \to \mathscr{A}$ that meet the conditions of the algebraic identity

$$2\mathscr{G}(X^{m+n}) = \mathscr{G}(X^m)\psi(X^n) + \gamma(X^m)\partial(X^n)$$

$$+\mathscr{G}(X^n)\psi(X^m) + \gamma(X^n)\partial(X^m),$$

and

$$\mathscr{G}(X^{m+n+p}) = \mathscr{G}(X^m)\psi(X^{n+p}) + \gamma(X^m)\partial(X^n)\psi(X^p) + \gamma(X^{m+n})\partial(X^p),$$

for each $X \in \mathscr{A}$, where $X^0 = I$, and γ and ψ are already defined in the beginning. Consequently, $\mathscr{G}$ constitutes a generalized (ψ, γ)-derivation with (ψ, γ)-derivation ∂ on $\mathscr{A}$ under some conditions on fixed integers m, n, p. Motivated by the above results, we extend the above results by considering the following weaker algebraic identity on $\mathscr{A}$.

$$2\mathscr{G}(X^{s+t+u}) = \mathscr{G}(X^s)\,\psi(X^{t+u}) + \gamma(X^s)\,\partial(X^t)\,\psi(X^u) + \gamma(X^{s+t})\,\partial(X^u)$$

$$+\mathscr{G}(X^u)\,\psi(X^{t+s}) + \gamma(X^u)\,\partial(X^t)\,\psi(X^s) + \gamma(X^{u+t})\,\partial(X^s),$$

universally hold for all $X \in \mathscr{A}$. This noteworthy fact indicates that the above identity imposes weaker requirements than identity satisfied by a generalized $(\psi,\ \gamma)$-derivation (which, of course, is itself a generalized Jordan $(\psi,\ \gamma)$-derivation). Note that if $\mathscr{G}$ is generalized $(\psi,\ \gamma)$-derivation with an $(\psi,\ \gamma)$-derivation ∂, then the above satisfies. In this work, we undertake a detailed analysis of the necessary conditions on $\mathscr{A}$ under which $\mathscr{G}$ becomes a generalized $(\psi,\ \gamma)$-derivation with an $(\psi,\ \gamma)$-derivation ∂, assuming that it fulfills the subtle identity outlined above.

In order to establish the principal theorems of this work, we first require the following lemma:

Lemma 1 ([1]) Assume that $\mathscr{K}$ is a Hilbert space and V is a von Neumann algebra on $\mathscr{K}$, and $\mathscr{L}$ is a CSL whose projection operators lie in V. Define $\mathscr{A} = V \cap Alg\,\mathscr{L}$ as a CSL subalgebra of the von Neumann algebra V. If ψ and γ are arbitrary automorphisms of $\mathscr{A}$, then every Jordan $(\psi,\ \gamma)$-derivation on $\mathscr{A}$ is, in fact, an $(\psi,\ \gamma)$-derivation on $\mathscr{A}$.

3. The primary result

We begin our study by focusing on the following result:

Theorem 1 Let $s,\ t,\ u \geq 1$ be fixed integers and $\mathscr{K}$ be a Hilbert space. Suppose that $\mathscr{A}$ is a CSL subalgebra of a von Neumann algebra on $\mathscr{K}$. If $\mathscr{G},\ \partial : \mathscr{A} \to \mathscr{A}$ are linear mappings that fulfill the following algebraic identity:

$$2\mathscr{G}(X^{s+t+u}) = \mathscr{G}(X^s)\,\psi(X^{t+u}) + \gamma(X^s)\,\partial(X^t)\,\psi(X^u) + \gamma(X^{s+t})\,\partial(X^u)$$

$$+\mathscr{G}(X^u)\,\psi(X^{t+s}) + \gamma(X^u)\,\partial(X^t)\,\psi(X^s) + \gamma(X^{u+t})\,\partial(X^s),$$

for each $X \in \mathscr{A}$, where the identity operator $I = X^0$. Consequently, $\mathscr{G}$ is a generalized $(\psi,\ \gamma)$-derivation on $\mathscr{A}$ associated with the $(\psi,\ \gamma)$-derivation ∂, where ψ and γ represent automorphisms of $\mathscr{A}$.

Proof. Provided that

$$\begin{aligned} 2\mathscr{G}(X^{s+t+u}) &= \mathscr{G}(X^s)\,\psi(X^{t+u}) + \gamma(X^s)\,\partial(X^t)\,\psi(X^u) \\ &\quad + \gamma(X^{s+t})\,\partial(X^u) + \mathscr{G}(X^u)\,\psi(X^{t+s}) \\ &\quad + \gamma(X^u)\,\partial(X^t)\,\psi(X^s) + \gamma(X^{u+t})\,\partial(X^s) \ \text{ for all } X \in \mathscr{A}. \end{aligned} \tag{1}$$

After substituting I for X, we get

$$2\mathscr{G}(I) = \mathscr{G}(I) + \partial(I) + \partial(I) + \mathscr{G}(I) + \partial(I) + \partial(I),$$

provided $\psi(I) = \gamma(I) = I$. Hence,

$$2\mathscr{G}(I)=2\mathscr{G}(I)+4\partial(I).$$

Since $\mathscr{A}$ is a complex algebra, it follows that

$$\partial(I)=0.$$

We continue with the condition (1) and replace X by $X+qI$ to obtain $2\mathscr{G}\Big(q^{s+t+u}I+\binom{s+t+u}{1}(q^{s+t+u-1})X+\binom{s+t+u}{2}(q^{s+t+u-2})X^2+\cdots\Big)=\mathscr{G}\Big(q^sI+\binom{s}{1}q^{s-1}X+\binom{s}{2}q^{s-2}X^2+\cdots\Big)\cdot\psi\Big(q^{t+u}I+\binom{t+u}{1}q^{t+u-1}X+\binom{t+u}{2}q^{t+u-2}X^2+\cdots\Big)+\gamma\Big(q^sI+\binom{s}{1}q^{s-1}X+\binom{s}{2}q^{s-2}X^2+\cdots\Big)\partial\Big(q^tI+\binom{t}{1}q^{t-1}X+\binom{t}{2}q^{t-2}X^2+\cdots\Big)\psi\Big(q^uI+\binom{u}{1}q^{u-1}X+\binom{u}{2}q^{u-2}X^2+\cdots\Big)+\gamma\Big(q^{s+t}I+\binom{s+t}{1}(q^{s+t-1})X+\binom{s+t}{2}(q^{s+t-2})X^2+\cdots\Big)\partial\Big(q^uI+\binom{u}{1}q^{u-1}X+\binom{u}{2}q^{u-2}X^2+\cdots\Big)+\mathscr{G}\Big(q^uI+\binom{u}{1}q^{u-1}X+\binom{u}{2}q^{u-2}X^2+\cdots\Big)\psi\Big(q^{s+t}I+\binom{s+t}{1}q^{s+t-1}X+\binom{s+t}{2}q^{s+t-2}X^2+\cdots\Big)+\gamma\Big(q^uI+\binom{u}{1}q^{u-1}X+\binom{u}{2}q^{u-2}X^2+\cdots\Big)\ \partial\Big(q^tI+\binom{t}{1}q^{t-1}X+\binom{t}{2}q^{t-2}X^2+\cdots\Big)\psi\Big(q^sI+\binom{s}{1}q^{s-1}X+\binom{s}{2}q^{s-2}X^2+\cdots\Big)+\gamma\Big(q^{u+t}I+\binom{u+t}{1}(q^{u+t-1})X+\binom{u+t}{2}(q^{u+t-2})X^2+\cdots\Big)\partial\Big(q^sI+\binom{s}{1}q^{s-1}X+\binom{s}{2}q^{s-2}X^2+\cdots\Big)$, where q is any positive integer.

Rewrite the above expression by using (1) with $\psi(I)=I$ and $\gamma(I)=I$ to obtain

$$q\mathscr{F}_1(X,\,I)+q^2\mathscr{F}_2(X,\,I)+\cdots+q^{s+t+u-1}\mathscr{F}_{s+t+u-1}(X,\,I)=0,\ \text{ for all } X\in\mathscr{A},$$

where $\mathscr{F}_m(X,\,I)$ symbolizes the coefficients of q^m's for all $m=1,\,2,\,\cdots,\,(s+t+u-1)$.

Let $N=s+t+u$. For fixed $X\in\mathscr{A}$, the expression obtained after substituting $X+qI$ is a polynomial in the scalar variable q, with coefficients in the complex vector space $\mathscr{A}$. Since the identity holds for every positive integer q, this vector-valued polynomial vanishes for infinitely many values of q. Hence, all its coefficients are zero. Therefore,

$$\mathscr{F}_m(X,\,I)=0,\ \text{ for all } X\in\mathscr{A} \text{ and for } m=1,\,2,\,\cdots,\,N.$$

Particularly, take $\mathscr{F}_{N-1}(X,\,I)=0$ and $\partial(I)=0$. Taking the coefficient of q^{N-1} gives

$$2N\mathscr{G}(X)=(s+u)\mathscr{G}(X)+(s+2t+u)\mathscr{G}(I)\psi(X)+(s+2t+u)\partial(X).$$

Since

$$2N-(s+u)=s+2t+u,$$

one obtains

$$(s+2t+u)\mathscr{G}(X)=(s+2t+u)\mathscr{G}(I)\psi(X)+(s+2t+u)\partial(X).$$

As $s+2t+u>0$, this yields

$$\mathscr{G}(X)=\mathscr{G}(I)\psi(X)+\partial(X),\ \text{ for all } X\in\mathscr{A}. \tag{2}$$

Now considering $\mathscr{F}_{N-2}(X,\ I)=0$ and utilizing a fact $\partial(I)=0$ and a notation $C_r=\binom{r}{2}$. The coefficient of q^{N-2} should give

$$\begin{aligned}N(N-1)\mathscr{G}(X^2)=&\ (C_s+C_u)\mathscr{G}(X^2)\\ &+\big(s(t+u)+u(t+s)\big)\mathscr{G}(X)\psi(X)\\ &+(C_{t+u}+C_{t+s})\mathscr{G}(I)\psi(X^2)\\ &+(2C_t+C_s+C_u)\partial(X^2)\\ &+2(st+su+tu)\gamma(X)\partial(X)\\ &+t(s+u)\partial(X)\psi(X).\end{aligned}$$

Using

$$\mathscr{G}(X)=\mathscr{G}(I)\psi(X)+\partial(X)$$

and

$$\mathscr{G}(X^2)=\mathscr{G}(I)\psi(X^2)+\partial(X^2),$$

the terms involving $\mathscr{G}(I)\psi(X^2)$ cancel. The remaining identity reduces to

$$2(su+st+tu)\partial(X^2)=2(st+su+tu)\partial(X)\psi(X)+2(st+su+tu)\gamma(X)\partial(X).$$

Since $su+st+tu>0$, it follows that

$$\partial(X^2)=\partial(X)\psi(X)+\gamma(X)\partial(X),\ \text{ for every } X\in\mathscr{A}.$$

Thus ∂ is a Jordan $(\psi,\ \gamma)$-derivation. Lemma 1 implies that ∂ is an $(\psi,\ \gamma)$-derivation on $\mathscr{A}$. Therefore,

$$\partial(XY) = \partial(X)\psi(Y) + \gamma(X)\partial(Y), \text{ for every } X, Y \in \mathscr{A}.$$

Using

$$\mathscr{G}(X) = \mathscr{G}(I)\psi(X) + \partial(X),$$

one then obtains

$$\begin{aligned}
\mathscr{G}(XY) &= \mathscr{G}(I)\psi(XY) + \partial(XY) \\
&= \mathscr{G}(I)\psi(X)\psi(Y) + \partial(X)\psi(Y) + \gamma(X)\partial(Y) \\
&= (\mathscr{G}(I)\psi(X) + \partial(X))\psi(Y) + \gamma(X)\partial(Y) \\
&= \mathscr{G}(X)\psi(Y) + \gamma(X)\partial(Y).
\end{aligned}$$

Hence $\mathscr{G}$ is a generalized (ψ, γ)-derivation on $\mathscr{A}$ having (ψ, γ)-derivation ∂. Thus, we obtain the desired outcome. $\square$

4. Conclusion

In this work, we identify the conditions under which a given map becomes a generalized (ψ, γ)-derivation with an (ψ, γ)-derivation that fulfills a certain identity. In addition, we encourage the reader to examine various functional identities closely connected to particular derivation types, including generalized (ψ, γ)-derivations in semiprime rings and the advanced idea of generalized-(ψ, γ)-higher derivations.

5. Open problems

The following problem would naturally connect the present results with a broader algebraic framework and significantly enhance the conceptual scope of the work. More precisely, if we consider the same algebraic identity on a (semi)prime ring but not necessarily with unity, then what will be the algebraic structure of these mappings? To be more exact: Let $s, t, u \geq 1$ be fixed integers. Suppose that $\mathscr{R}$ is a (semi)prime ring (not necessarily with unity) under some torsion condition. If $\mathscr{G}, \partial : \mathscr{R} \to \mathscr{R}$ are additive mappings that fulfill the following:

$$\begin{aligned}
2\mathscr{G}(x^{s+t+u}) &= \mathscr{G}(x^s)\,\psi(x^{t+u}) + \gamma(x^s)\,\partial(x^t)\,\psi(x^u) + \gamma(x^{s+t})\,\partial(x^u) \\
&\quad + \mathscr{G}(x^u)\,\psi(x^{t+s}) + \gamma(x^u)\,\partial(x^t)\,\psi(x^s) + \gamma(x^{u+t})\,\partial(x^s),
\end{aligned}$$

for all $x \in \mathscr{R}$. Consequently, $\mathscr{G}$ is a generalized (ψ, γ)-derivation on $\mathscr{A}$ associated with the (ψ, γ)-derivation ∂ on $\mathscr{R}$.

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Author contributions

All authors contributed equally to the conception, development, and preparation of this manuscript.

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Data availability

All data generated or analyzed during this study are included in this published article.

Conflict of interest

The authors declare that they have no conflicts of interest to disclose. All authors have reviewed the present manuscript and approved it for publication.

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