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Research Article

Seven Hundreds of Exotic ${}_3F_2$-Series Evaluated in π, $\sqrt{2}$ and $\log(1+\sqrt{2})$

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Abstract: By means of the linearization method, a nonterminating exotic ${}_3F_2$-series with five free integer parameters is reduced to particular instances of a known $\Omega(x, y)$ function. With a help of the Kummer and Thomae transformations, ten large classes of ${}_3F_2$-series are evaluated in terms of π, $\sqrt{2}$ and $\log(1+\sqrt{2})$. A collection of 700 elegant summation formulae is presented.

Keywords: hypergeometric series, nonterminating exotic ${}_3F_2$-series, linearization method, Thomae transformation, Kummer transformation

MSC: 33C20, 33F10

1. Introduction

Let $\mathbb{N}$ and $\mathbb{Z}$ stand for the sets of natural numbers and integers with $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. The shifted factorial is given by $(x)_0 \equiv 1$ and

$$(x)_n = x(x+1)\cdots(x+n-1) \text{ for } n \in \mathbb{N}.$$

We can express it, even when $n \in \mathbb{Z}$, as the Γ-function quotients

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)}, \text{ where } \Gamma(x) = \int_0^\infty u^{x-1}e^{-u}\mathrm{d}u \text{ for } \Re(x) > 0.$$

Their multi-parameter forms will be abbreviated compactly as

$$\left[\begin{matrix}\alpha, \beta, \cdots, \gamma \\ A, B, \cdots, C\end{matrix}\right]_n = \frac{(\alpha)_n(\beta)_n\cdots(\gamma)_n}{(A)_n(B)_n\cdots(C)_n},$$

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$$\Gamma\left[\begin{matrix}\alpha,\beta,\cdots,\gamma\\ A,B,\cdots,C\end{matrix}\right]=\frac{\Gamma(\alpha)\Gamma(\beta)\cdots\Gamma(\gamma)}{\Gamma(A)\Gamma(B)\cdots\Gamma(C)}.$$

Following Bailey [1], define the generalized hypergeometric series by

$${}_{1+p}F_p\left[\begin{matrix}a_0,a_1,\cdots,a_p\\ b_1,\cdots,b_p\end{matrix}\middle| z\right]=\sum_{n=0}^{\infty}\frac{(a_0)_n(a_1)_n\cdots(a_p)_n}{n!(b_1)_n\cdots(b_p)_n}z^n.$$

For $z = 1$, this series converges when the "parameter excess" (i.e., the sum of the denominator parameters minus that of the numerator ones) is greater than zero.

In this paper, we shall investigate ten large classes of nonterminating ${}_3F_2(1)$-series represented by the following sample evaluations:

$$\text{[A20]. } {}_3F_2\left[\begin{matrix}\frac{1}{4},\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4}\end{matrix}\middle| 1\right]=\frac{1}{4}\left\{\pi+2\log(1+\sqrt{2})\right\}.$$

$$\text{[B52]. } {}_3F_2\left[\begin{matrix}\frac{5}{4},\frac{3}{4},\frac{3}{4}\\ \frac{3}{2},\frac{7}{4}\end{matrix}\middle| 1\right]=\frac{3}{2}\left\{\pi-2\log(1+\sqrt{2})\right\}.$$

$$\text{[C13]. } {}_3F_2\left[\begin{matrix}1,\frac{1}{2},\frac{5}{4}\\ \frac{7}{4},\frac{9}{4}\end{matrix}\middle| 1\right]=\frac{5}{2\sqrt{2}}\left\{4\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}.$$

$$\text{[D13]. } {}_3F_2\left[\begin{matrix}1,\frac{1}{2},\frac{3}{4}\\ \frac{7}{4},\frac{9}{4}\end{matrix}\middle| 1\right]=\frac{5}{4\sqrt{2}}\left\{3\pi-2\sqrt{2}-6\log(1+\sqrt{2})\right\}.$$

$$\text{[E25]. } {}_3F_2\left[\begin{matrix}1,2,\frac{1}{4}\\ \frac{11}{4},\frac{11}{4}\end{matrix}\middle| 1\right]=\frac{49}{128\sqrt{2}}\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}.$$

$$\text{[F14]. } {}_3F_2\left[\begin{matrix}1,1,\frac{7}{4}\\ \frac{5}{4},\frac{13}{4}\end{matrix}\middle| 1\right]=\frac{3}{10\sqrt{2}}\left\{5\pi-8\sqrt{2}+10\log(1+\sqrt{2})\right\}.$$

$$[\text{G33}].\ {}_3F_2\left[\begin{matrix}\frac{3}{2},\ \frac{3}{2},\ \frac{5}{4}\\ \frac{9}{4},\ \frac{13}{4}\end{matrix}\middle|1\right]=\frac{25\pi^2\{10\sqrt{2}-9\pi+18\log(1+\sqrt{2})\}}{64\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

$$[\text{H38}].\ {}_3F_2\left[\begin{matrix}\frac{3}{2},\ \frac{3}{2},\ \frac{3}{4}\\ \frac{7}{4},\ \frac{11}{4}\end{matrix}\middle|1\right]=\frac{63\pi^2\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\}}{4\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

$$[\text{I69}].\ {}_3F_2\left[\begin{matrix}\frac{3}{4},\ \frac{5}{4},\ \frac{5}{4}\\ \frac{5}{2},\ \frac{11}{4}\end{matrix}\middle|1\right]=\frac{21\sqrt{\pi}\{46\sqrt{2}-13\pi-26\log(1+\sqrt{2})\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$[\text{J73}].\ {}_3F_2\left[\begin{matrix}\frac{3}{4},\ \frac{7}{4},\ \frac{9}{4}\\ \frac{5}{2},\ \frac{13}{4}\end{matrix}\middle|1\right]=\frac{\sqrt{\pi}\{22\sqrt{2}-15\pi+30\log(1+\sqrt{2})\}}{4\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

For the nonterminating ${}_3F_2$-series, there are three classical summation theorems named after Dixon, Watson and Whipple (cf. Bailey [1, §3.1, §3.3 and §3.4]). However, neither of them can evaluate the above ten series in closed forms. Milgram [2] made a systematic work about ${}_3F_2$-series focusing mainly on three series named after Dixon, Watson and Whipple as well as their variants. However, the series treated in this paper are far beyond these series covered by [2]. Even though most of the ten classes represented by the above displayed series are in the contiguous orbits of Gaussian ${}_2F_1(1)$-series, they cannot be evaluated directly from contiguous relations and the closed formula for ${}_2F_1(1)$-series. Further different formulae for ${}_3F_2$-series can be found in recent papers by Chen [3] and Ebisu-Iwasaki [4] through recurrence relations (cf. [5]).

As a unification of the first two series (i.e., the above [A20] and [B52]), we shall examine the following nonterminating "exotic" ${}_3F_2$-series

$$\mathcal{F}_\delta(a,c,e;b,d):={}_3F_2\left[\begin{matrix}\frac{1}{4}+a,\ \frac{3}{4}+c,\ \frac{1+2\delta}{4}+e\\ \frac{1}{2}+b,\ \frac{5+2\delta}{4}+d\end{matrix}\middle|1\right],$$

where $\delta = 0$ or 1 and $a, b, c, d, e \in \mathbb{Z}$ subject to $\sigma = b + d - a - c - e \geq 0$ so that the series converges, since in this case the parameter excess $\Delta = \sigma + \frac{1}{2} > 0$. We assume also that $d \geq e$, because otherwise, the series will be reducible to the ${}_2F_1(1)$ series, which can easily be evaluated by the Gauss summation theorem (cf. Bailey [1, §1.3]). The $\mathcal{F}_\delta$-series will be said "exotic" for the denominator parameter minus the numerator one in the last column results in a natural number.

By means of the linearization method (cf. [6]), we shall reduce, in the next section, the $\mathcal{G}_\delta$-series (the shifted $\mathcal{F}_\delta$-series) to the $\Omega_{m,n}$-series treated recently by the author [7]. Then in Section 3, the conclusive theorem and closed formulae will be given for the $\mathcal{F}_\delta$-series. Finally, the paper will end with Section 4, where further eight classes of ${}_3F_2$-series will be evaluated by applying the Thomae and Kummer transformations (cf. Bailey [1, §3.2 and Page 98]) to the $\mathcal{F}_\delta$-series.

In spite of the fact that there exist already numerous summation formulae for the hypergeometric series (see [8-11] for example), the series treated in this paper don't seem to have appeared previously in the literature, especially they are not covered by the two useful compendium [12, §8.1.2] and [13, §7.4.4], where a large number of closed formulae are collected for the ${}_3F_2(1)$ series with numerical parameters. Taking into account this fact, we present a full coverage of seven hundreds summation formulae in order to make them accessible by the readers. In order to assure the accuracy, all the displayed formulae are experimentally checked by appropriately devised Mathematica commands.

2. Reduction formulae for the $\mathcal{G}_\delta$-Series

In order to proceed smoothly with the reduction procedure, we shall examine the following $\mathcal{G}_\delta$-series

$$\mathcal{G}_\delta(a,c,e;\lambda,b,d) := \sum_{n=0}^{\infty}\begin{bmatrix}\frac{1}{4}+a, & \frac{3}{4}+c, & \frac{1+2\delta}{4}+e\\ 1+\lambda, & \frac{1}{2}+b, & \frac{5+2\delta}{4}+d\end{bmatrix}_n$$

which is equivalent to the $\mathcal{F}_\delta$-series by shifting backward the summation index with $n \to n-\lambda$. As in the $\mathcal{F}_\delta$-series, we assume for the $\mathcal{G}_\delta$-series that $a, b, c, d, e, \lambda \in \mathbb{Z}$ subject to the conditions $\lambda \geq 0$ for the series being well-defined, $d \geq e$ for the series being exotic, and the parameter excess $\Delta = 12 + \lambda + b + d - a - c - e \geq 0$ so that the series is convergent.

By means of the linearization method, which is devised by the author [6] to express in principle parametric hypergeometric series in terms of known evaluable ones, the $\mathcal{G}_\delta$-series will be reduced to particular instances of a known $\Omega_{m,n}(x, y)$ function, which has recently been evaluated by the author [7].

2.1 $b \geq \lambda$

By making use of the summation formula (cf. Bailey [1, §4.3]) for the terminating well-poised series

$${}_5F_4\left[\begin{matrix}a, & 1+\frac{a}{2}, & b, & d, & -m\\ \frac{a}{2}, & 1+a-b, & 1+a-d, & 1+a+m\end{matrix}\middle| 1\right] = \begin{bmatrix}1+a, & 1+a-b-d\\ 1+a-b, & 1+1-d\end{bmatrix}_m,$$

it is not difficult to validate the following lemma.

Lemma 1 (Linear relation: $m \in \mathbb{N}_0$).

$$(A+n)_m = \sum_{k=0}^{m}(B+n)_k(C+n)_{m-k}X_k,$$

where

$$X_k = \frac{B-C-m+2k}{B-C}\binom{m}{k}\frac{(A-C)_k(A-B)_{m-k}}{(1+B-C)_k(1-B+C)_{m-k}}.$$

According to this lemma, we have the equality

$$(1+\lambda+n)_{b-\lambda}=\sum_{k=0}^{b-\lambda}(\tfrac{1}{4}+a+n)_k(\tfrac{3}{4}+c+n)_{b-k-\lambda}X_k(a,b,c,\lambda),$$

where the connection coeffients are given by

$$X_k(a,b,c,\lambda)=\frac{\lambda+a-b-c-\frac{1}{2}+2k}{a-c-\frac{1}{2}}\binom{b-\lambda}{k}\frac{(\frac{1}{4}-c+\lambda)_k(\frac{3}{4}-a+\lambda)_{b-k-\lambda}}{(\frac{1}{2}+a-c)_k(\frac{3}{2}-a+c)_{b-k-\lambda}}.$$

By putting this relation inside the $\mathcal{G}_\delta$-series, we can reformulate the double sum

$$\mathcal{G}_\delta(a,c,e;\lambda,b,d)=\sum_{n=0}^{\infty}\left[\begin{matrix}\frac{1}{4}+a, & \frac{3}{4}+c, & \frac{1+2\delta}{4}+e\\ 1+\lambda, & \frac{1}{2}+b, & \frac{5+2\delta}{4}+d\end{matrix}\right]_n\sum_{k=0}^{b-\lambda}\frac{(\frac{1}{4}+a+n)_k(\frac{3}{4}+c+n)_{b-k-\lambda}}{(1+\lambda+n)_{b-\lambda}}X_k(a,b,c,\lambda)$$

$$=\sum_{k=0}^{b-\lambda}\frac{(\frac{1}{4}+a)_k(\frac{3}{4}+c)_{b-k-\lambda}}{(1+\lambda)_{b-\lambda}}X_k(a,b,c,\lambda)\sum_{n=0}^{\infty}\left[\begin{matrix}\frac{1}{4}+a+k, & \frac{3}{4}+b+c-k-\lambda, & \frac{1+2\delta}{4}+e\\ 1+b, & \frac{1}{2}+b, & \frac{5+2\delta}{4}+d\end{matrix}\right]_n.$$

Writing the last sum in terms of the $\mathcal{G}_\delta$-series, we have established the following transformation formula.
Proposition 2 (Reduction formula when $b\geq\lambda$).

$$\mathcal{G}_\delta(a,c,e;\lambda,b,d)=\sum_{k=0}^{b-\lambda}X_k(a,b,c,\lambda)\frac{(\frac{1}{4}+a)_k(\frac{3}{4}+c)_{b-k-\lambda}}{(1+\lambda)_{b-\lambda}}\mathcal{G}_\delta(a+k,b+c-k-\lambda,e;b,b,d)$$

We remark that under this transformation, the parameter excess Δ for all the $\mathcal{G}_\delta$-series on the right remains the same as that for the $\mathcal{G}_\delta$-series on the left.

2.2 $b<\lambda$

According to Lemma 1, we have another equality

$$(\tfrac{1}{2}+b+n)_{\lambda-b}=\sum_{k=0}^{\lambda-b}(\tfrac{1}{4}+a+n)_k(\tfrac{3}{4}+c+n)_{\lambda-b-k}\mathcal{X}_k(a,b,c,\lambda),$$

where the connection coefficients read as

$$\mathcal{X}_k(a,b,c,\lambda)=\frac{a+b-c-\lambda-\frac{1}{2}+2k}{a-c-\frac{1}{2}}\binom{\lambda-b}{k}\frac{(b-c-\frac{1}{4})_k(\frac{1}{4}-a+b)_{\lambda-b-k}}{(\frac{1}{2}+a-c)_k(\frac{3}{2}-a+c)_{\lambda-b-k}}.$$

By substituting it into the $\mathcal{G}_\delta$-series, we can manipulate the double sum

$$\mathcal{G}_\delta(a,c,e;\lambda,b,d)=\sum_{n=0}^{\infty}\begin{bmatrix}\frac{1}{4}+a, & \frac{3}{4}+c, & \frac{1+2\delta}{4}+e\\ 1+\lambda, & \frac{1}{2}+b, & \frac{5+2\delta}{4}+d\end{bmatrix}_n \sum_{k=0}^{\lambda-b}\frac{(\frac{1}{4}+a+n)_k(\frac{3}{4}+c+n)_{\lambda-b-k}}{(\frac{1}{2}+b+n)_{\lambda-b}}\mathcal{X}_k(a,b,c,\lambda)$$

$$=\sum_{k=0}^{\lambda-b}\frac{(\frac{1}{4}+a)_k(\frac{3}{4}+c)_{\lambda-b-k}}{(\frac{1}{2}+b)_{\lambda-b}}\mathcal{X}_k(a,b,c,\lambda)\sum_{n=0}^{\infty}\begin{bmatrix}\frac{1}{4}+a+k, & \frac{3}{4}+\lambda-b+c-k, & \frac{1+2\delta}{4}+e\\ 1+\lambda, & \frac{1}{2}+\lambda, & \frac{5+2\delta}{4}+d\end{bmatrix}_n.$$

Writing the last sum with respect to n in terms of $\mathcal{G}_\delta$-series, we establish the following transformation formula.
Proposition 3 (Reduction formula when $b<\lambda$).

$$\mathcal{G}_\delta(a,c,e;\lambda,b,d)=\sum_{k=0}^{\lambda-b}\mathcal{X}_k(a,b,c,\lambda)\frac{(\frac{1}{4}+a)_k(\frac{3}{4}+c)_{\lambda-b-k}}{(\frac{1}{2}+b)_{\lambda-b}}\mathcal{G}_\delta(a+k,\lambda-b+c-k,e;\lambda,\lambda,d).$$

2.3 $a>c$

According to the Chu-Vandermonde convolution formula on binomial coefficients, we have immediately the following lemma.
Lemma 4 (Linear relation: $m\in\mathbb{N}_0$).

$$(A+n)_m=\sum_{k=0}^{m}\binom{m}{k}(B+n)_k(A-B)_{m-k}.$$

When $a\geq c$, we can deduce from Lemma 4 the equality

$$(\tfrac{1}{4}+c+n)_{a-c}=\sum_{k=0}^{a-c}\binom{a-c}{k}(\tfrac{1+2\delta}{4}+e+n)_k(c-e-\tfrac{\delta}{2})_{a-c-k}.$$

Substituting this into the $\mathcal{G}_\delta$-series, we have the double series

$$\mathcal{G}_\delta(a,c,e;\lambda,\lambda,d)=\sum_{n=0}^{\infty}\begin{bmatrix}\frac{1}{4}+a, & \frac{3}{4}+c, & \frac{1+2\delta}{4}+e\\ 1+\lambda, & \frac{1}{2}+\lambda, & \frac{5+2\delta}{4}+d\end{bmatrix}_n \sum_{k=0}^{a-c}\binom{a-c}{k}\frac{(\frac{1+2\delta}{4}+e+n)_k}{(\frac{1}{4}+c+n)_{a-c}}(c-e-\tfrac{\delta}{2})_{a-c-k}$$

$$=\sum_{k=0}^{a-c}\binom{a-c}{k}\frac{(\frac{1+2\delta}{4}+e)_k}{(\frac{1}{4}+c)_{a-c}}(c-e-\tfrac{\delta}{2})_{a-c-k}\sum_{n=0}^{\infty}\begin{bmatrix}\frac{1}{4}+c, & \frac{3}{4}+c, & \frac{1+2\delta}{4}+e+k\\ 1+\lambda, & \frac{1}{2}+\lambda, & \frac{5+2\delta}{4}+d\end{bmatrix}_n.$$

This leads us to the following reduction formula.
Proposition 5 (Reduction formula from $a>c$ to $a=c$).

$$\mathcal{G}_\delta(a,c,e;\lambda,\lambda,d)=\sum_{k=0}^{a-c}\frac{(\frac{1+2\delta}{4}+e)_k}{(\frac{1}{4}+c)_{a-c}}\binom{a-c}{k}(c-e-\tfrac{\delta}{2})_{a-c-k}\mathcal{G}_\delta(c,c,e+k;\lambda,\lambda,d).$$

2.4 $a < c$

Analogously from Lemma 4, we have, in this case, another equality

$$(\tfrac{3}{4}+a+n)_{c-a}=\sum_{k=0}^{c-a}\binom{c-a}{k}(\tfrac{1+2\delta}{4}+e+n)_k(a-e+\tfrac{1-\delta}{2})_{c-a-k}.$$

Putting this inside the $\mathcal{G}_\delta$-series, we have the double series

$$\mathcal{G}_\delta(a,c,e;\lambda,\lambda,d)=\sum_{n=0}^{\infty}\begin{bmatrix}\frac{1}{4}+a,\frac{3}{4}+c,\frac{1+2\delta}{4}+e\\ 1+\lambda,\frac{1}{2}+\lambda,\frac{5+2\delta}{4}+d\end{bmatrix}_n\sum_{k=0}^{c-a}\binom{c-a}{k}\frac{(\frac{1+2\delta}{4}+e+n)_k}{(\frac{3}{4}+a+n)_{c-a}}(a-e+\tfrac{1-\delta}{2})_{c-a-k}$$

$$=\sum_{k=0}^{c-a}\binom{c-a}{k}\frac{(\frac{1+2\delta}{4}+e)_k}{(\frac{3}{4}+a)_{c-a}}(a-e+\tfrac{1-\delta}{2})_{c-a-k}\sum_{n=0}^{\infty}\begin{bmatrix}\frac{1}{4}+a,\frac{3}{4}+a,\frac{1+2\delta}{4}+e+k\\ 1+\lambda,\frac{1}{2}+\lambda,\frac{5+2\delta}{4}+d\end{bmatrix}_n.$$

This leads us to the following reduction formula.
Proposition 6 (Reduction formula from $a < c$ to $a = c$).

$$\mathcal{G}_\delta(a,c,e;\lambda,\lambda,d)=\sum_{k=0}^{c-a}\frac{(\frac{1+2\delta}{4}+e)_k}{(\frac{3}{4}+a)_{c-a}}(a-e+\tfrac{1-\delta}{2})_{c-a-k}\binom{c-a}{k}\mathcal{G}_\delta(a,a,e+k;\lambda,\lambda,d).$$

2.5 $d < e$

This case can happen, because the parameter “e” can be raised up by positive integers in Propositions 5 and 6.
Lemma 7 (Linear relation: $m\in\mathbb{N}_0$).

$$(A+n)_m=\sum_{k=0}^{m}(B+2n)_kY_k \text{ where } Y_k=\sum_{i=0}^{k}\frac{(-1)^i}{k!}\binom{k}{i}(A-\tfrac{B+i}{2})_m.$$

Proof. By substitution, we have to show the double sum identity

$$\mathcal{S}:=\sum_{k=0}^{m}(B+2n)_k\sum_{i=0}^{k}\frac{(-1)^i}{k!}\binom{k}{i}(A-\tfrac{B+i}{2})_m=(A+n)_m.$$

Interchanging the summation order, we can manipulate it as

$$
\begin{aligned}
\mathcal{S} &= \sum_{i=0}^{m} \frac{(-1)^i}{i!}(B+2n)_i\,(A-\tfrac{B+i}{2})_m \sum_{k=i}^{m}\binom{B+2n+k-1}{k-i}\\
&= \sum_{i=0}^{m} \frac{(-1)^i}{i!}(B+2n)_i\,(A-\tfrac{B+i}{2})_m \binom{B+m+2n}{m-i}\\
&= \frac{(B+2n)_{m+1}}{m!}\sum_{i=0}^{m}(-1)^i\binom{m}{i}\frac{(A-\frac{B+i}{2})_m}{B+2n+i}\\
&= \frac{(B+2n)_{m+1}}{m!}\times\frac{m!(A+n)_m}{(B+2n)_{m+1}}\\
&= (A+n)_m,
\end{aligned}
$$

where the last step has been justified by finite difference calculus (cf. Chu [14]).

In particular, specify Lemma 7 to the equality

$$
(\tfrac{5+2\delta}{4}+d+n)_{e-d-1} = \sum_{k=0}^{e-d-1}(\tfrac{1}{2}+2a+2n)_k Y_k^{\delta}(a,d,e),
$$

where

$$
Y_k^{\delta}(a,d,e) = \sum_{i=0}^{k}\frac{(-1)^i}{k!}\binom{k}{i}(1-a+d+\tfrac{\delta-i}{2})_{e-d-1}.
$$

Substituting this into the $\mathcal{G}_\delta$-series, we have the double series

$$
\begin{aligned}
\mathcal{G}_\delta(a,a,e;\lambda,\lambda,d) &= \sum_{n=0}^{\infty}\left[\begin{matrix}\frac{1}{4}+a, & \frac{3}{4}+a, & \frac{1+2\delta}{4}+e\\ 1+\lambda, & \frac{1}{2}+\lambda, & \frac{5+2\delta}{4}+d\end{matrix}\right]_n \sum_{k=0}^{e-d-1}\frac{(\frac{1}{2}+2a+2n)_k Y_k^{\delta}(a,d,e)}{(\frac{5+2\delta}{4}+d+n)_{e-d-1}}\\
&= \sum_{k=0}^{e-d-1} Y_k^{\delta}(a,d,e)\frac{(\frac{1}{2}+2a)_k}{(\frac{5+2\delta}{4}+d)_{e-d-1}}\sum_{n=0}^{\infty}\left[\begin{matrix}\frac{1+2k}{4}+a, & \frac{3+2k}{4}+a\\ 1+\lambda, & \frac{1}{2}+\lambda\end{matrix}\right]_n.
\end{aligned}
$$

Expressing the last sum, by reindexing, as

$$\sum_{n=0}^{\infty}\begin{bmatrix}\frac{1+2k}{4}+a, & \frac{3+2k}{4}+a\\ 1+\lambda, & \frac{1}{2}+\lambda\end{bmatrix}_n=\sum_{n=-\lambda}^{\infty}\begin{bmatrix}\frac{1+2k}{4}+a, & \frac{3+2k}{4}+a\\ 1+\lambda, & \frac{1}{2}+\lambda\end{bmatrix}_n-\sum_{n=-\lambda}^{-1}\begin{bmatrix}\frac{1+2k}{4}+a, & \frac{3+2k}{4}+a\\ 1+\lambda, & \frac{1}{2}+\lambda\end{bmatrix}_n$$

$$=\frac{(2\lambda)!}{(\frac{1}{2}-2a-k)_{2\lambda}}\sum_{n=0}^{\infty}\frac{(\frac{1}{2}+k+2a-2\lambda)_{2n}}{(2n)!}-\sum_{n=1}^{\lambda}\frac{(-2\lambda)_{2n}}{(\frac{1}{2}-2a-k)_{2n}}$$

and then evaluating the first sum by the Gauss theorem (cf. Bailey [1, §1.3])

$${}_2F_1\left[\begin{matrix}\frac{1+2k}{4}+a-\lambda, & \frac{3+2k}{4}+a-\lambda\\ \frac{1}{2}\end{matrix}\middle|\,1\right]=2^{2\lambda-2a-k-\frac{3}{2}},$$

we derive the following summation formula.

Proposition 8 (Reduction formula when $d < e$).

$$\mathcal{G}_\delta(a,a,e;\lambda,\lambda,d)=\sum_{k=0}^{e-d-1}Y_k^\delta(a,d,e)\frac{(\frac{1}{2}+2a)_k}{(\frac{5+2\delta}{4}+d)_{e-d-1}}\left\{\frac{2^{2\lambda-2a-k-\frac{3}{2}}(2\lambda)!}{(\frac{1}{2}-2a-k)_{2\lambda}}-\sum_{n=1}^{\lambda}\frac{(-2\lambda)_{2n}}{(\frac{1}{2}-2a-k)_{2n}}\right\}.$$

2.6 $d \geq e$

By making use of the equivalent expressions

$$\frac{(\frac{1+2\delta}{4}+e)_n}{(\frac{5+2\delta}{4}+d)_n}=\frac{(\frac{1+2\delta}{4}+e)_{1+d-e}}{(\frac{1+2\delta}{4}+e+n)_{1+d-e}},$$

$$\frac{(\frac{1}{2}+2a)_{2n}}{(\frac{1}{2}+2a-d+e)_{2n}}=\frac{(\frac{1}{2}+2a-d+e+2n)_{d-e}}{(\frac{1}{2}+2a-d+e)_{d-e}};$$

we have the partial fraction decomposition

$$\frac{(\frac{1}{2}+2a-d+e+2n)_{d-e}}{(\frac{1+2\delta}{4}+e+n)_{1+d-e}}=\sum_{k=0}^{d-e}\frac{(-1)^k}{(d-e)!}\binom{d-e}{k}\frac{(2a-d-e-2k-\delta)_{d-e}}{\frac{1+2\delta}{4}+e+n+k}.$$

Substituting this into $\mathcal{G}_\delta(a, a, e; b, d)$, we derive the following reduction formula.

Proposition 9 (Reduction formula for $d \geq e$).

$$\mathcal{G}_\delta(a,a,e;\lambda,\lambda,d)=\frac{(\frac{1+2\delta}{4}+e)_{1+d-e}}{(d-e)!(\frac{1}{2}+2a-d+e)_{d-e}}\sum_{k=0}^{d-e}(-1)^k\binom{d-e}{k}\frac{(2a-d-e-2k-\delta)_{d-e}}{\frac{1+2\delta}{4}+e+k}$$

$$\times\mathcal{G}_\delta\Big(a-\frac{d-e}{2},\ a-\frac{d-e}{2},\ e+k;\ \lambda,\ \lambda,\ e+k\Big).$$

2.7 $\Omega_{m,n}(x,y)$

Keeping in mind that the precedent reductions don't decrease the parameter excess for the $\mathcal{G}_\delta$-series, we have therefore $\Delta=\frac{1}{2}-2a+2\lambda>0$, or equivalently $a\le\lambda$, for the series $\mathcal{G}_\delta(a,a,d;\lambda,\lambda,d)$ on the right of Proposition 9.

In order to evaluate this series, write it explicitly as a bisection series

$$\mathcal{G}_\delta(a,a,d;\lambda,\lambda,d)=\sum_{n=0}^{\infty}\left[\begin{matrix}\frac{1}{4}+a,\ \frac{3}{4}+a,\ \frac{1+2\delta}{4}+d\\ 1+\lambda,\ \frac{1}{2}+\lambda,\ \frac{5+2\delta}{4}+d\end{matrix}\right]_n$$

$$=\sum_{n=0}^{\infty}\left[\begin{matrix}\frac{1}{2}+2a,\ \frac{1}{2}+\delta+2d\\ 1+2\lambda,\ \frac{3}{2}+\delta+2d\end{matrix}\right]_{2n}$$

$$=\sum_{n=-\lambda}^{\infty}\left[\begin{matrix}\frac{1}{2}+2a,\ \frac{1}{2}+\delta+2d\\ 1+2\lambda,\ \frac{3}{2}+\delta+2d\end{matrix}\right]_{2n}-\sum_{n=-\lambda}^{-1}\left[\begin{matrix}\frac{1}{2}+2a,\ \frac{1}{2}+\delta+2d\\ 1+2\lambda,\ \frac{3}{2}+\delta+2d\end{matrix}\right]_{2n},$$

which can be reformulated, by reindexing, as follows:

$$\mathcal{G}_\delta(a,a,d;\lambda,\lambda,d)=-\sum_{k=1}^{\lambda}\frac{(-2\lambda)_{2k}(\frac{1}{2}+\delta+2d)}{(\frac{1}{2}-2a)_{2k}(\frac{1}{2}+\delta+2d-2k)}$$

$$+\frac{(2\lambda)!(\frac{1}{2}+\delta+2d)}{(\frac{1}{2}-2a)_{2\lambda}(\frac{1}{2}+\delta+2d-2\lambda)}\sum_{n=0}^{\infty}\left[\begin{matrix}\frac{1}{2}+2a-2\lambda,\ \frac{1}{2}+\delta+2d-2\lambda\\ 1,\ \frac{3}{2}+\delta+2d-2\lambda\end{matrix}\right]_{2n}.$$

The last series can further be expressed as

$$\sum_{n=0}^{\infty}\left[\begin{matrix}\frac{1}{2}+2a-2\lambda,\ \frac{1}{2}+\delta+2d-2\lambda\\ 1,\ \frac{3}{2}+\delta+2d-2\lambda\end{matrix}\right]_{2n}=\frac{1}{2}\,{}_2F_1\left[\begin{matrix}\frac{1}{2}+2a-2\lambda,\ \frac{1}{2}+\delta+2d-2\lambda\\ \frac{3}{2}+\delta+2d-2\lambda\end{matrix}\middle|1\right]$$

$$+\frac{1}{2}\,{}_2F_1\left[\begin{matrix}\frac{1}{2}+2a-2\lambda,\ \frac{1}{2}+\delta+2d-2\lambda\\ \frac{3}{2}+\delta+2d-2\lambda\end{matrix}\middle|-1\right]$$

which are particular cases of the series examined recently by the author [7]:

$$\Omega_{m,n}(x,y):={}_2F_1\left[\begin{matrix}x,\ m-x\\ n+\frac{1}{2}\end{matrix}\middle|y^2\right]\text{ where } m,n\in\mathbb{Z}. \tag{1}$$

This series can be evaluated recursively by the following formulae.

Lemma 10 (Chu [7, Theorems 2, 4 and 8]: Recurrence formula). For the two integers m and n subject to $m\le n$, the following formula holds

$$m\ge 0:\ \Omega_{m,n}(x,y)=\frac{(\frac{1}{2})_n}{y^{2n}}\sum_{i=0}^{n-m}\binom{n-m}{i}\frac{(x)_i(m-x)_{n-m-i}}{(2x-n+i)_i(m-2x-i)_{n-m-i}}$$

$$\times\sum_{j=0}^{n}(-1)^{n-j}\binom{n}{j}\frac{2x+2i-2j}{(2x+2i-n-j)_{n+1}}\Omega_{0,0}(x+i-j,y),$$

$$m<0:\ \Omega_{m,n}(x,y)=\sum_{k=0}^{-m}(-1)^k\binom{-m}{k}\frac{2x+2k}{2x-m+k}\Omega_{0,n-m}(x+k,y)$$

$$\times\frac{(\frac{1}{2}-n)_m(x)_k(\frac{1}{2}+x-m+n)_k(2x-m)_{m+k}}{(2x-m)_k(1+x-m)_{m+k}(\frac{1}{2}+x-n)_{m+k}};$$

where the series $\Omega_{0,0}$ is evaluated by

$$\Omega_{0,0}(x,y)={}_2F_1\left[\begin{matrix}x,\ -x\\ \frac{1}{2}\end{matrix}\middle|y^2\right]=\cos(2x\ \arcsin\ y).$$

Therefore, we get the expression of the $\mathcal{G}_\delta$-series in terms of the Ω-series.

Theorem 11 (Reduction formula).

$$\mathcal{G}_\delta(a,a,d;\lambda,\lambda,d)=\sum_{k=1}^{\lambda}\frac{(-2\lambda)_{2k}(\delta+2d+\frac{1}{2})}{(\frac{1}{2}-2a)_{2k}(2k-\delta-2d-\frac{1}{2})}+\frac{(2\lambda)!(\frac{1}{2}+\delta+2d)}{2(\frac{1}{2}-2a)_{2\lambda}(\frac{1}{2}+\delta+2d-2\lambda)}$$

$$\times\left\{\begin{array}{l}\Omega_{1+\delta+2a+2d-4\lambda,1+\delta+2d-2\lambda}(\frac{1}{2}+2a-2\lambda,1)\\+\Omega_{1+\delta+2a+2d-4\lambda,1+\delta+2d-2\lambda}(\frac{1}{2}+2a-2\lambda,\sqrt{-1})\end{array}\right\}.$$

In particular, the two special values are useful:

$$\Omega_{0,0}(x,1)=\cos(\pi x) \text{ and } \Omega_{0,0}(x,\sqrt{-1})=\cosh\big(2x\log(1+\sqrt{2})\big).$$

3. Conclusive theorem and summation formulae

Based on the reduction formulae established in the last section, we can evaluate, for integers a, b, c, d, e, $\lambda \in \mathbb{Z}$ subject to the conditions

$$\lambda\geq 0,\ d\geq e \text{ and } \lambda+b+d-a-c-e\geq 0,$$

the $\mathcal{G}$-series in terms of $\Omega_{0,0}$ series by carrying out the following procedure:

• Step-A: Apply Propositions 2 and 3 to express $\mathcal{G}_\delta(a, c, e; \lambda, b, d)$ in terms of $\mathcal{G}_\delta(a, c, e; \lambda, \lambda, d)$ and then go to Step-B.

• Step-B: Apply Propositions 5 and 6 to express $\mathcal{G}_\delta(a, c, e; \lambda, \lambda, d)$ in terms of $\mathcal{G}_\delta(a, a, e; \lambda, \lambda, d)$ and then go to Step-C.

• Step-C: For $d < e$, evaluate directly $\mathcal{G}_\delta(a, a, e; \lambda, \lambda, d)$ by Proposition 8. Otherwise, for $d \geq e$, apply Proposition 9 to express $\mathcal{G}_\delta(a, a, e; \lambda, \lambda, d)$ in terms of $\mathcal{G}_\delta(a, a, d; \lambda, \lambda, d)$ and then go to Step-D.

• Step-D: Finally, apply Lemma 10 and Theorem 11 to evaluate $\mathcal{G}_\delta(a, a, d; \lambda, \lambda, d)$ explicitly in terms of trigonometric and hyperbolic functions.

Observing that the $\mathcal{F}_\delta$-series results in the $\lambda = 0$ case of the $\mathcal{G}_\delta$-series

$$\mathcal{F}_\delta(a, c, e; b, d) = \mathcal{G}_\delta(a, c, e; 0, b, d).$$

we have consequently shown the following conclusive theorem.

Theorem 12 (Conclusion). For any quintuple integers

$$a,b,c,d,e\in\mathbb{Z} \text{ subject to } d\geq e \text{ and } \sigma=b+d-a-c-e\geq 0,$$

the $\mathcal{F}_\delta(a, c, e; b, d)$ series can always be evaluated by a finitely linear combination of trigonometric function $\cos(\pi x)$ and hyperbolic function $\cosh(2x\log(1+\sqrt{2}))$ (with $x \in \frac{1}{2}+\mathbb{Z}$ and the coefficients being rational numbers).

According to the procedure described at the beginning of this section, we have devised appropriately Mathematica commands to compute closed expressions for $\mathcal{F}_\delta(a, b, c, d, e)$. Our results suggest that, in general, for any quintuple integers

$$a,b,c,d,e\in\mathbb{Z} \text{ subject to } d\geq e \text{ and } \sigma=b+d-a-c-e\geq 0$$

the $\mathcal{F}_\delta(a, c, e; b, d)$ series is in $\mathbb{Q}[\pi, \sqrt{2}, \log(1+\sqrt{2})]$, just a linear combination of π, $\sqrt{2}$ and $\log(1+\sqrt{2})$ with coefficients being rational.

Now we are going to give two collections of closed formulae respectively for the $\mathcal{F}_0$ and $\mathcal{F}_1$ series. For the sake of brevity, the argument "1" will be suppressed from the notation of ${}_3F_2$-series.

3.1 *Class A: Summation formulae for $\mathcal{F}_0$-series*

$$\text{A1. } {}_3F_2\Big[-\frac{1}{4}, -\frac{3}{4}, -\frac{3}{4}; -\frac{1}{2}, \frac{1}{4}\Big] = \frac{2\sqrt{2}+3\pi+6\log(1+\sqrt{2})}{8}.$$

$$\text{A2. } {}_3F_2\Big[-\frac{1}{4}, -\frac{3}{4}, -\frac{3}{4}; -\frac{1}{2}, \frac{5}{4}\Big] = \frac{46\sqrt{2}+51\pi+102\log(1+\sqrt{2})}{256}.$$

$$\text{A3. } {}_3F_2\Big[-\frac{1}{4}, -\frac{3}{4}, -\frac{3}{4}; \frac{1}{2}, \frac{1}{4}\Big] = \frac{10\sqrt{2}-3\pi-6\log(1+\sqrt{2})}{4}.$$

$$\text{A4. } {}_3F_2\Big[-\frac{1}{4}, -\frac{3}{4}, -\frac{3}{4}; \frac{1}{2}, \frac{5}{4}\Big] = \frac{122\sqrt{2}-15\pi-30\log(1+\sqrt{2})}{128}.$$

$$\text{A5. } {}_3F_2\Big[-\frac{1}{4}, -\frac{3}{4}, -\frac{3}{4}; \frac{3}{2}, \frac{1}{4}\Big] = \frac{74\sqrt{2}-15\pi-30\log(1+\sqrt{2})}{50}.$$

$$\text{A6. } {}_3F_2\Big[-\frac{1}{4}, -\frac{3}{4}, -\frac{3}{4}; \frac{3}{2}, \frac{5}{4}\Big] = \frac{682\sqrt{2}+105\pi+210\log(1+\sqrt{2})}{1600}.$$

$$\text{A7. } {}_3F_2\Big[\frac{1}{4}, -\frac{1}{4}, -\frac{3}{4}; -\frac{1}{2}, \frac{5}{4}\Big] = \frac{10\sqrt{2}+9\pi+18\log(1+\sqrt{2})}{64}.$$

$$\text{A8. } {}_3F_2\Big[\frac{1}{4}, -\frac{1}{4}, -\frac{3}{4}; -\frac{1}{2}, \frac{9}{4}\Big] = \frac{15\{42\sqrt{2}+41\pi+82\log(1+\sqrt{2})\}}{4096}.$$

$$\text{A9. } {}_3F_2\Big[\frac{1}{4}, -\frac{1}{4}, -\frac{3}{4}; \frac{1}{2}, \frac{5}{4}\Big] = \frac{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})}{32}.$$

$$\text{A10. } {}_3F_2\Big[\frac{1}{4}, -\frac{1}{4}, -\frac{3}{4}; \frac{1}{2}, \frac{9}{4}\Big] = \frac{5\{146\sqrt{2}+45\pi+90\log(1+\sqrt{2})\}}{2048}.$$

$$\text{A11. } {}_3F_2\Big[\frac{1}{4}, -\frac{1}{4}, -\frac{3}{4}; \frac{3}{2}, \frac{5}{4}\Big] = \frac{3\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\}}{80}.$$

$$\text{A12. } {}_3F_2\Big[\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}; -\frac{1}{2}, \frac{5}{4}\Big] = \frac{2\sqrt{2}+3\pi+6\log(1+\sqrt{2})}{16}.$$

A13. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{46\sqrt{2}+51\pi+102\log(1+\sqrt{2})\right\}}{1536}.$$

A14. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{2\sqrt{2}+\pi+2\log(1+\sqrt{2})}{8}.$$

A15. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{34\sqrt{2}+21\pi+42\log(1+\sqrt{2})\right\}}{768}.$$

A16. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})}{12}.$$

A17. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\left\{75\pi-98\sqrt{2}+150\log(1+\sqrt{2})\right\}}{1152}.$$

A18. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{5}{4}\right]=\frac{105\pi-118\sqrt{2}+210\log(1+\sqrt{2})}{350}.$$

A19. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{256}.$$

A20. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\pi+2\log(1+\sqrt{2})}{4}.$$

A21. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{128}.$$

A22. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{\pi-2\sqrt{2}+2\log(1+\sqrt{2})}{2}.$$

A23. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{64}.$$

A24. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{5}{2},\ \frac{5}{4}\right]=\frac{15\pi-34\sqrt{2}+30\log(1+\sqrt{2})}{25}.$$

A25. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{195\pi-562\sqrt{2}+390\log(1+\sqrt{2})}{160}.$$

A26. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{7}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{11\pi-2\sqrt{2}+22\log(1+\sqrt{2})\right\}}{192}.$$

A27. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{\pi-\sqrt{2}+2\log(1+\sqrt{2})}{3}.$$

A28. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\left\{7\pi-10\sqrt{2}+14\log(1+\sqrt{2})\right\}}{96}.$$

A29. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{5}{4}\right]=\frac{2\left\{5\pi-8\sqrt{2}+10\log(1+\sqrt{2})\right\}}{25}.$$

A30. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{3\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{80}.$$

A31. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{2\sqrt{2}+9\pi+18\log(1+\sqrt{2})}{32}.$$

A32. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{58\sqrt{2}+81\pi+162\log(1+\sqrt{2})\right\}}{2048}.$$

A33. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})}{16}.$$

A34. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{39\pi-10\sqrt{2}+78\log(1+\sqrt{2})\right\}}{1024}.$$

A35. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{15\pi-26\sqrt{2}+30\log(1+\sqrt{2})}{40}.$$

A36. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{255\pi-538\sqrt{2}+510\log(1+\sqrt{2})}{512}.$$

A37. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{5}{4}\right]=\frac{135\pi-266\sqrt{2}+270\log(1+\sqrt{2})}{300}.$$

A38. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{32}.$$

A39. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\left\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{16}.$$

A40. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{58\sqrt{2}-15\pi-30\log(1+\sqrt{2})}{8}.$$

A41. $_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{2\sqrt{2}+21\pi+42\log(1+\sqrt{2})\right\}}{384}.$

A42. $_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{192}.$

A43. $_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\left\{50\sqrt{2}-3\pi-6\log(1+\sqrt{2})\right\}}{288}.$

A44. $_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{598\sqrt{2}-105\pi-210\log(1+\sqrt{2})}{336}.$

A45. $_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\left\{2\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{24}.$

A46. $_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{5\pi-14\sqrt{2}+10\log(1+\sqrt{2})}{4}.$

A47. $_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{2\sqrt{2}+69\pi+138\log(1+\sqrt{2})\right\}}{1024}.$

A48. $_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\pi-2\sqrt{2}+2\log(1+\sqrt{2})}{8}.$

A49. $_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{19\pi-18\sqrt{2}+38\log(1+\sqrt{2})\right\}}{512}.$

A50. $_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{5\pi-6\sqrt{2}+10\log(1+\sqrt{2})}{20}.$

A51. $_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{75\pi-98\sqrt{2}+150\log(1+\sqrt{2})}{256}.$

A52. $_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{5}{4}\right]=\frac{45\pi-62\sqrt{2}+90\log(1+\sqrt{2})}{150}.$

A53. $_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{495\pi-442\sqrt{2}+990\log(1+\sqrt{2})}{1920}.$

A54. $_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{1}{4}\right]=\frac{\sqrt{2}-3\pi-6\log(1+\sqrt{2})}{4}.$

A55. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})}{128}.$$

A56. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{1}{4}\right]=\frac{3\pi-4\sqrt{2}+6\log(1+\sqrt{2})}{2}.$$

A57. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{33\pi-38\sqrt{2}+66\log(1+\sqrt{2})}{64}.$$

A58. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{1}{4}\right]=\frac{15\pi-14\sqrt{2}+30\log(1+\sqrt{2})}{25}.$$

A59. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{345\pi-502\sqrt{2}+690\log(1+\sqrt{2})}{800}.$$

A60. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{1}{4}\right]=\frac{2\left\{45\pi-22\sqrt{2}+90\log(1+\sqrt{2})\right\}}{225}.$$

A61. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{5}{4}\right]=\frac{315\pi-514\sqrt{2}+630\log(1+\sqrt{2})}{720}.$$

A62. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{63\pi-26\sqrt{2}+126\log(1+\sqrt{2})\right\}}{512}.$$

A63. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{256}.$$

A64. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{122\sqrt{2}-15\pi-30\log(1+\sqrt{2})}{128}.$$

A65. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{586\sqrt{2}-135\pi-270\log(1+\sqrt{2})}{192}.$$

A66. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\left\{\pi-2\sqrt{2}+2\log(1+\sqrt{2})\right\}}{4}.$$

A67. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{5\left\{3\pi-10\sqrt{2}+6\log(1+\sqrt{2})\right\}}{2}.$$

A68. $${}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{34\sqrt{2}+21\pi+42\log(1+\sqrt{2})\right\}}{1024}.$$

A69. $_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{15\left\{26\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{512}.$

A70. $_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{634\sqrt{2}-15\pi-30\log(1+\sqrt{2})}{768}.$

A71. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{10\sqrt{2}-3\pi-6\log(1+\sqrt{2})\right\}}{48}.$

A72. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{72}.$

A73. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{5\left\{21\pi-62\sqrt{2}+42\log(1+\sqrt{2})\right\}}{84}.$

A74. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=5\left\{4\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}.$

A75. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{45\left\{46\sqrt{2}-13\pi-26\log(1+\sqrt{2})\right\}}{32}.$

A76. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{17}{4}\right]=\frac{585\left\{850\sqrt{2}-243\pi-486\log(1+\sqrt{2})\right\}}{4096}.$

A77. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{7}{2},\ \frac{13}{4}\right]=\frac{9\left\{574\sqrt{2}-165\pi-330\log(1+\sqrt{2})\right\}}{16}.$

A78. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{135\left\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{112}.$

A79. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{17}{4}\right]=\frac{585\left\{394\sqrt{2}-103\pi-206\log(1+\sqrt{2})\right\}}{14336}.$

A80. $_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{13}{4}\right]=\frac{9\left\{85\pi-286\sqrt{2}+170\log(1+\sqrt{2})\right\}}{56}.$

A81. $_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{-128}.$

A82. $_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})}{64}.$

A83. $_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{9}{4}\right] = \frac{5\left\{9\pi - 22\sqrt{2} + 18\log(1+\sqrt{2})\right\}}{96}.$

A84. $_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{13}{4}\right] = \frac{9\left\{5\pi - 14\sqrt{2} + 10\log(1+\sqrt{2})\right\}}{8}.$

A85. $_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{17}{4}\right] = \frac{117\left\{35\pi - 114\sqrt{2} + 70\log(1+\sqrt{2})\right\}}{512}.$

A86. $_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{13}{4}\right] = \frac{9\left\{25\pi - 86\sqrt{2} + 50\log(1+\sqrt{2})\right\}}{4}.$

A87. $_3F_2\left[\frac{5}{4},\ \frac{9}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{17}{4}\right] = \frac{117\left\{55\pi - 106\sqrt{2} + 110\log(1+\sqrt{2})\right\}}{1792}.$

A88. $_3F_2\left[\frac{5}{4},\ \frac{9}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{13}{4}\right] = \frac{9\left\{94\sqrt{2} - 25\pi - 50\log(1+\sqrt{2})\right\}}{14}.$

A89. $_3F_2\left[\frac{7}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{5}{4}\right] = \frac{5\left\{2\sqrt{2} - 3\pi - 6\log(1+\sqrt{2})\right\}}{64}.$

A90. $_3F_2\left[\frac{7}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{9}{4}\right] = \frac{5\left\{166\sqrt{2} - 33\pi - 66\log(1+\sqrt{2})\right\}}{8192}.$

A91. $_3F_2\left[\frac{7}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{1}{4}\right] = \frac{6\pi - 7\sqrt{2} + 12\log(1+\sqrt{2})}{2}.$

A92. $_3F_2\left[\frac{7}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{5}{4}\right] = \frac{27\pi - 34\sqrt{2} + 54\log(1+\sqrt{2})}{32}.$

A93. $_3F_2\left[\frac{7}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right] = \frac{5\left\{489\pi - 598\sqrt{2} + 978\log(1+\sqrt{2})\right\}}{4096}.$

A94. $_3F_2\left[\frac{7}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{1}{4}\right] = \frac{2\left\{15\pi - 19\sqrt{2} + 30\log(1+\sqrt{2})\right\}}{25}.$

A95. $_3F_2\left[\frac{7}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{5}{4}\right] = \frac{195\pi - 242\sqrt{2} + 390\log(1+\sqrt{2})}{400}.$

A96. $_3F_2\left[\frac{7}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{1}{4}\right] = \frac{4\left\{45\pi - 52\sqrt{2} + 90\log(1+\sqrt{2})\right\}}{225}.$

A97. $${}_3F_2\Big[\frac{7}{4},\ \frac{9}{4},\ \frac{9}{4};\ \frac{5}{2},\frac{17}{4}\Big]=\frac{117\big\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\big\}}{320}.$$

A98. $${}_3F_2\Big[\frac{7}{4},\ \frac{9}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{13}{4}\Big]=9\big\{18\sqrt{2}-5\pi-10\log(1+\sqrt{2})\big\}.$$

A99. $${}_3F_2\Big[\frac{7}{4},\ \frac{9}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{17}{4}\Big]=\frac{39\big\{574\sqrt{2}-165\pi-330\log(1+\sqrt{2})\big\}}{32}.$$

A100. $${}_3F_2\Big[\frac{9}{4},\ \frac{9}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{17}{4}\Big]=\frac{117\big\{85\pi-286\sqrt{2}+170\log(1+\sqrt{2})\big\}}{112}.$$

3.2 *Class B: Summation formulae for $\mathcal{F}_1$-series*

B1. $${}_3F_2\Big[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{3}{4}\Big]=\frac{10\sqrt{2}+3\pi-6\log(1+\sqrt{2})}{16}.$$

B2. $${}_3F_2\Big[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{7}{4}\Big]=\frac{298\sqrt{2}+87\pi-174\log(1+\sqrt{2})}{512}.$$

B3. $${}_3F_2\Big[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{3}{4}\Big]=\frac{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})}{8}.$$

B4. $${}_3F_2\Big[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\Big]=\frac{98\sqrt{2}+75\pi-150\log(1+\sqrt{2})}{256}.$$

B5. $${}_3F_2\Big[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{3}{4}\Big]=\frac{26\sqrt{2}+15\pi-30\log(1+\sqrt{2})}{60}.$$

B6. $${}_3F_2\Big[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\Big]=\frac{3\big\{114\sqrt{2}+35\pi-70\log(1+\sqrt{2})\big\}}{640}.$$

B7. $${}_3F_2\Big[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{3}{4}\Big]=\frac{706\sqrt{2}+315\pi-630\log(1+\sqrt{2})}{1470}.$$

B8. $${}_3F_2\Big[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{3}{4}\Big]=\frac{4\sqrt{2}+\pi-2\log(1+\sqrt{2})}{8}.$$

B9. $${}_3F_2\Big[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{7}{4}\Big]=\frac{3\big\{46\sqrt{2}+13\pi-26\log(1+\sqrt{2})\big\}}{256}.$$

B10. $${}_3F_2\Big[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{11}{4}\Big]=\frac{21\big\{850\sqrt{2}+243\pi-486\log(1+\sqrt{2})\big\}}{32768}.$$

B11. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{3}{4}\right] = \frac{2\sqrt{2}+\pi-2\log(1+\sqrt{2})}{4}.$

B12. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{7}{4}\right] = \frac{3\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{128}.$

B13. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{3}{4}\right] = \frac{10\sqrt{2}+3\pi-6\log(1+\sqrt{2})}{18}.$

B14. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{7}{4}\right] = \frac{122\sqrt{2}+15\pi-30\log(1+\sqrt{2})}{192}.$

B15. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{11}{4}\right] = \frac{35\left\{478\sqrt{2}+21\pi-42\log(1+\sqrt{2})\right\}}{24576}.$

B16. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{3}{4}\right] = \frac{422\sqrt{2}+105\pi-210\log(1+\sqrt{2})}{735}.$

B17. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{7}{4}\right] = \frac{5462\sqrt{2}+105\pi-210\log(1+\sqrt{2})}{7840}.$

B18. $_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{7}{4}\right] = \frac{3\left\{14\sqrt{2}+5\pi-10\log(1+\sqrt{2})\right\}}{64}.$

B19. $_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{11}{4}\right] = \frac{21\left\{114\sqrt{2}+35\pi-70\log(1+\sqrt{2})\right\}}{4096}.$

B20. $_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{7}{4}\right] = \frac{3\left\{6\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{32}.$

B21. $_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{11}{4}\right] = \frac{7\left\{166\sqrt{2}+33\pi-66\log(1+\sqrt{2})\right\}}{2048}.$

B22. $_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{7}{4}\right] = \frac{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})}{16}.$

B23. $_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{11}{4}\right] = \frac{7\left\{146\sqrt{2}-45\pi+90\log(1+\sqrt{2})\right\}}{1024}.$

B24. $_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{7}{4}\right] = \frac{298\sqrt{2}-105\pi+210\log(1+\sqrt{2})}{280}.$

B25. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ -\frac{1}{2},\ \frac{11}{4}\Big]=\frac{21\big\{2\sqrt{2}-5\pi+10\log(1+\sqrt{2})\big\}}{512}.$

B26. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\Big]=\frac{3\big\{2\sqrt{2}+\pi-2\log(1+\sqrt{2})\big\}}{8}.$

B27. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{1}{2},\ \frac{11}{4}\Big]=\frac{7\big\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\big\}}{256}.$

B28. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\Big]=\frac{3\big\{2\sqrt{2}-\pi+2\log(1+\sqrt{2})\big\}}{4}.$

B29. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{3}{2},\ \frac{11}{4}\Big]=\frac{7\big\{34\sqrt{2}-21\pi+42\log(1+\sqrt{2})\big\}}{128}.$

B30. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{5}{2},\ \frac{7}{4}\Big]=\frac{22\sqrt{2}-15\pi+30\log(1+\sqrt{2})}{10}.$

B31. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{5}{2},\ \frac{11}{4}\Big]=\frac{7\big\{194\sqrt{2}-165\pi+330\log(1+\sqrt{2})\big\}}{320}.$

B32. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{7}{4};\ \frac{1}{2},\ \frac{11}{4}\Big]=\frac{7\big\{26\sqrt{2}+15\pi-30\log(1+\sqrt{2})\big\}}{192}.$

B33. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{11}{4}\Big]=\frac{7\big\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\big\}}{96}.$

B34. $_3F_2\Big[\frac{1}{4},\ \frac{3}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{11}{4}\Big]=\frac{21\big\{-2\sqrt{2}+5\pi-10\log(1+\sqrt{2})\big\}}{80}.$

B35. $_3F_2\Big[\frac{1}{4},\ \frac{7}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{11}{4}\Big]=\frac{35\big\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\big\}}{1024}.$

B36. $_3F_2\Big[\frac{1}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{11}{4}\Big]=\frac{7\big\{122\sqrt{2}+15\pi-30\log(1+\sqrt{2})\big\}}{1536}.$

B37. $_3F_2\Big[\frac{1}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{11}{4}\Big]=\frac{7\big\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\big\}}{256}.$

B38. $_3F_2\Big[\frac{1}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{11}{4}\Big]=\frac{3\big\{114\sqrt{2}+35\pi-70\log(1+\sqrt{2})\big\}}{640}.$

B39. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\left\{10\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{72}.$$

B40. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{7\left\{22\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{60}.$$

B41. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{7}{4}\right]=\frac{58\sqrt{2}+15\pi-30\log(1+\sqrt{2})}{128}.$$

B42. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{50\sqrt{2}+3\pi-6\log(1+\sqrt{2})}{64}.$$

B43. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{134\sqrt{2}-15\pi+30\log(1+\sqrt{2})}{160}.$$

B44. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{1534\sqrt{2}-315\pi+630\log(1+\sqrt{2})}{1680}.$$

B45. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{7}{4}\right]=\frac{34\sqrt{2}+15\pi-30\log(1+\sqrt{2})}{32}.$$

B46. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{11}{4}\right]=\frac{7\left\{562\sqrt{2}+195\pi-390\log(1+\sqrt{2})\right\}}{5120}.$$

B47. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})}{16}.$$

B48. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{7\left\{98\sqrt{2}+75\pi-150\log(1+\sqrt{2})\right\}}{2560}.$$

B49. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{38\sqrt{2}-15\pi+30\log(1+\sqrt{2})}{40}.$$

B50. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{82\sqrt{2}-45\pi+90\log(1+\sqrt{2})}{60}.$$

B51. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{21\left\{10\sqrt{2}+7\pi-14\log(1+\sqrt{2})\right\}}{128}.$$

B52. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{3\left\{\pi-2\log(1+\sqrt{2})\right\}}{2}.$$

B53. $_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{21\left\{-2\sqrt{2}+5\pi-10\log(1+\sqrt{2})\right\}}{64}.$

B54. $_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=3\pi-2\sqrt{2}-6\log(1+\sqrt{2}).$

B55. $_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{7\left\{51\pi-46\sqrt{2}-102\log(1+\sqrt{2})\right\}}{32}.$

B56. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{11}{4}\right]=\frac{21\left\{106\sqrt{2}+55\pi-110\log(1+\sqrt{2})\right\}}{2048}.$

B57. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{3\left\{2\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{16}.$

B58. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{21\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{1024}.$

B59. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})}{8}.$

B60. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\left\{10\sqrt{2}+39\pi-78\log(1+\sqrt{2})\right\}}{512}.$

B61. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{21\pi-2\sqrt{2}-42\log(1+\sqrt{2})}{28}.$

B62. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{525\pi-338\sqrt{2}-1050\log(1+\sqrt{2})}{256}.$

B63. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\left\{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{16}.$

B64. $_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{7\left\{10\sqrt{2}-9\pi+18\log(1+\sqrt{2})\right\}}{8}.$

B65. $_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{11}{4}\right]=\frac{7\left\{266\sqrt{2}+135\pi-270\log(1+\sqrt{2})\right\}}{1280}.$

B66. $_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{21\left\{-2\sqrt{2}+5\pi-10\log(1+\sqrt{2})\right\}}{640}.$

B67. $ {}_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\left\{134\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{1600}. $

B68. $ {}_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{7\left\{74\sqrt{2}+135\pi-270\log(1+\sqrt{2})\right\}}{2400}. $

B69. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{7}{4}\right]=\frac{9\left\{6\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{128}. $

B70. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{11}{4}\right]=\frac{21\left\{394\sqrt{2}+103\pi-206\log(1+\sqrt{2})\right\}}{16384}. $

B71. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{2}+\pi-2\log(1+\sqrt{2})}{2}. $

B72. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{3\left\{10\sqrt{2}+7\pi-14\log(1+\sqrt{2})\right\}}{64}. $

B73. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{63\left\{62\sqrt{2}+37\pi-74\log(1+\sqrt{2})\right\}}{8192}. $

B74. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{3}{4}\right]=\frac{4\sqrt{2}+3\pi-6\log(1+\sqrt{2})}{9}. $

B75. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{38\sqrt{2}+33\pi-66\log(1+\sqrt{2})}{96}. $

B76. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{3}{4}\right]=\frac{2\left\{34\sqrt{2}+21\pi-42\log(1+\sqrt{2})\right\}}{147}. $

B77. $ {}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{3\left\{86\sqrt{2}+105\pi-210\log(1+\sqrt{2})\right\}}{784}. $

B78. $ {}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{21\left\{2\sqrt{2}-5\pi+10\log(1+\sqrt{2})\right\}}{256}. $

B79. $ {}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{384}. $

B80. $ {}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{3\left\{34\sqrt{2}-21\pi+42\log(1+\sqrt{2})\right\}}{64}. $

B81. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{7\left\{3\pi-2\sqrt{2}-6\log(1+\sqrt{2})\right\}}{2}.$$

B82. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{15}{4}\right]=\frac{77\left\{51\pi-46\sqrt{2}-102\log(1+\sqrt{2})\right\}}{192}.$$

B83. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{7}{4};\ \frac{7}{2},\ \frac{15}{4}\right]=\frac{77\left\{225\pi-218\sqrt{2}-450\log(1+\sqrt{2})\right\}}{96}.$$

B84. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{15}{4}\right]=\frac{11\left\{21\pi-2\sqrt{2}-42\log(1+\sqrt{2})\right\}}{48}.$$

B85. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{19}{4}\right]=\frac{11\left\{525\pi-338\sqrt{2}-1050\log(1+\sqrt{2})\right\}}{1024}.$$

B86. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{15}{4}\right]=\frac{11\left\{106\sqrt{2}-105\pi+210\log(1+\sqrt{2})\right\}}{24}.$$

B87. $${}_3F_2\left[\frac{5}{4},\ \frac{11}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{19}{4}\right]=\frac{33\left\{86\sqrt{2}+105\pi-210\log(1+\sqrt{2})\right\}}{896}.$$

B88. $${}_3F_2\left[\frac{5}{4},\ \frac{11}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{15}{4}\right]=\frac{11\left\{105\pi-82\sqrt{2}-210\log(1+\sqrt{2})\right\}}{42}.$$

B89. $${}_3F_2\left[\frac{7}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{11}{4}\right]=\frac{7\left\{586\sqrt{2}+135\pi-270\log(1+\sqrt{2})\right\}}{10240}.$$

B90. $${}_3F_2\left[\frac{7}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{7\left\{634\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{5120}.$$

B91. $${}_3F_2\left[\frac{7}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\left\{1414\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{12800}.$$

B92. $${}_3F_2\left[\frac{7}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{7\left\{30\log(1+\sqrt{2})-15\pi-74\sqrt{2}\right\}}{480}.$$

B93. $${}_3F_2\left[\frac{7}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\left\{26\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{1200}.$$

B94. $${}_3F_2\left[\frac{7}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{7\left\{82\sqrt{2}-45\pi+90\log(1+\sqrt{2})\right\}}{600}.$$

B95. $${}_3F_2\left[\frac{7}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{15}{4}\right]=\frac{77\left\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\right\}}{160}.$$

B96. $${}_3F_2\left[\frac{7}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{19}{4}\right]=\frac{77\left\{398\sqrt{2}-291\pi+582\log(1+\sqrt{2})\right\}}{4096}.$$

B97. $${}_3F_2\left[\frac{7}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{15}{4}\right]=\frac{77\left\{86\sqrt{2}-87\pi+174\log(1+\sqrt{2})\right\}}{48}.$$

B98. $${}_3F_2\left[\frac{7}{4},\ \frac{9}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{19}{4}\right]=\frac{11\left\{274\sqrt{2}+147\pi-294\log(1+\sqrt{2})\right\}}{512}.$$

B99. $${}_3F_2\left[\frac{7}{4},\ \frac{9}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{15}{4}\right]=\frac{33\left\{7\pi-6\sqrt{2}-14\log(1+\sqrt{2})\right\}}{4}.$$

B100. $${}_3F_2\left[\frac{9}{4},\ \frac{11}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{19}{4}\right]=\frac{55\left\{314\sqrt{2}-273\pi+546\log(1+\sqrt{2})\right\}}{224}.$$

4. Thomae and Kummer transformations

Recall the Thomae and Kummer transformations (cf. Bailey [1, §3.2 and Page 98])

$${}_3F_2\left[\begin{matrix}a,c,e\\b,d\end{matrix}\middle|1\right]={}_3F_2\left[\begin{matrix}\Delta,b-a,d-a\\c+\Delta,e+\Delta\end{matrix}\middle|1\right]\Gamma\left[\begin{matrix}\Delta,b,d\\a,c+\Delta,e+\Delta\end{matrix}\right],\tag{2}$$

$${}_3F_2\left[\begin{matrix}a,c,e\\b,d\end{matrix}\middle|1\right]={}_3F_2\left[\begin{matrix}a,b-c,b-e\\\Delta+a,b\end{matrix}\middle|1\right]\Gamma\left[\begin{matrix}\Delta,d\\\Delta+a,d-a\end{matrix}\right];\tag{3}$$

where $\Delta = b + d - a - c - e$ denotes the parameter excess.

Taking into account that the $\mathcal{F}_\delta$ series is symmetric with respect to its numerator parameters and to its denominator ones, we can make use of (2) and (3) to transform each of the $\mathcal{F}_0$ and $\mathcal{F}_1$ series into four new classes of ${}_3F_2(1)$-series. The related summation formulae will be recorded in this section.

4.1 *Class C*

According to the Thomae transformation (2), we can express the following "Class-C" series in terms of the $\mathcal{F}_0$-series ($\sigma = b + d - a - c - e$):

$${}_3F_2\left[\begin{matrix}a+\frac{1}{2},\ c+\frac{1}{4},\ e+1\\b+\frac{5}{4},\ d+\frac{3}{4}\end{matrix}\middle|1\right]=\frac{\Gamma(\sigma+\frac{1}{4})\Gamma(b+\frac{5}{4})\Gamma(d+\frac{3}{4})}{\Gamma(a+\frac{1}{2})\Gamma(\sigma+c+\frac{1}{2})\Gamma(\sigma+e+\frac{5}{4})}\,{}_3F_2\left[\begin{matrix}\sigma+\frac{1}{4},\ b-a+\frac{3}{4},\ d-a+\frac{1}{4}\\\sigma+c+\frac{1}{2},\ \sigma+e+\frac{5}{4}\end{matrix}\middle|1\right].$$

Then the closed formulae for this series (excluding the divergent series) can be obtained from those shown in "Class A".

$$\text{C1. } {}_3F_2\left[1,\ \frac{1}{2},\ -\frac{7}{4};\ \frac{1}{4},\ \frac{3}{4}\right] = \frac{2\sqrt{2} + 21\pi + 42\log(1+\sqrt{2})}{-96\sqrt{2}}.$$

$$\text{C2. } {}_3F_2\left[1,\ \frac{1}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{3}{4}\right] = \frac{2\sqrt{2} + 3\pi + 6\log(1+\sqrt{2})}{-4\sqrt{2}}.$$

$$\text{C3. } {}_3F_2\left[1,\ \frac{1}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{7}{4}\right] = \frac{10\sqrt{2} - 3\pi - 6\log(1+\sqrt{2})}{8\sqrt{2}}.$$

$$\text{C4. } {}_3F_2\left[1,\ \frac{1}{2},\ -\frac{3}{4};\ \frac{3}{4},\ \frac{5}{4}\right] = \frac{3\pi - 2\sqrt{2} + 6\log(1+\sqrt{2})}{16\sqrt{2}}.$$

$$\text{C5. } {}_3F_2\left[1,\ \frac{1}{2},\ -\frac{3}{4};\ \frac{5}{4},\ -\frac{1}{4}\right] = \frac{2\sqrt{2} + 9\pi + 18\log(1+\sqrt{2})}{8\sqrt{2}}.$$

$$\text{C6. } {}_3F_2\left[1,\ \frac{1}{2},\ -\frac{3}{4};\ \frac{9}{4},\ -\frac{1}{4}\right] = \frac{5\left\{14\sqrt{2} + 3\pi + 6\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$$

$$\text{C7. } {}_3F_2\left[1,\ \frac{1}{2},\ \frac{1}{4};\ \frac{3}{4},\ \frac{5}{4}\right] = \frac{\pi + 2\log(1+\sqrt{2})}{2\sqrt{2}}.$$

$$\text{C8. } {}_3F_2\left[1,\ \frac{1}{2},\ \frac{1}{4};\ \frac{3}{4},\ \frac{9}{4}\right] = \frac{5\left\{2\sqrt{2} + \pi + 2\log(1+\sqrt{2})\right\}}{24\sqrt{2}}.$$

$$\text{C9. } {}_3F_2\left[1,\ \frac{1}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{7}{4}\right] = \frac{3\left\{\pi - 2\sqrt{2} + 2\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$$

$$\text{C10. } {}_3F_2\left[1,\ \frac{1}{2},\ \frac{1}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{5\left\{5\pi - 14\sqrt{2} + 10\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

$$\text{C11. } {}_3F_2\left[1,\ \frac{1}{2},\ \frac{1}{4};\ \frac{9}{4},\ -\frac{1}{4}\right] = \frac{5\left\{2\sqrt{2} - \pi - 2\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$$

$$\text{C12. } {}_3F_2\left[1,\ \frac{1}{2},\ \frac{5}{4};\ \frac{3}{4},\ \frac{9}{4}\right] = \frac{5\left\{\pi - \sqrt{2} + 2\log(1+\sqrt{2})\right\}}{3\sqrt{2}}.$$

$$\text{C13. } {}_3F_2\left[1,\ \frac{1}{2},\ \frac{5}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{5\left\{4\sqrt{2} - \pi - 2\log(1+\sqrt{2})\right\}}{2\sqrt{2}}.$$

C14. $${}_3F_2\left[1,\ \frac{1}{2},\ \frac{5}{4};\ \frac{7}{4},\ \frac{13}{4}\right]=\frac{9\left\{94\sqrt{2}-25\pi-50\log(1+\sqrt{2})\right\}}{56\sqrt{2}}.$$

C15. $${}_3F_2\left[1,\ \frac{1}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{11}{4}\right]=\frac{7\left\{18\sqrt{2}-5\pi-10\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$$

C16. $${}_3F_2\left[1,\ \frac{3}{2},\ -\frac{7}{4};\ \frac{3}{4},\ \frac{5}{4}\right]=\frac{34\sqrt{2}+21\pi+42\log(1+\sqrt{2})}{-512\sqrt{2}}.$$

C17. $${}_3F_2\left[1,\ \frac{3}{2},\ -\frac{3}{4};\ \frac{3}{4},\ \frac{5}{4}\right]=\frac{10\sqrt{2}+9\pi+18\log(1+\sqrt{2})}{-32\sqrt{2}}.$$

C18. $${}_3F_2\left[1,\ \frac{3}{2},\ -\frac{3}{4};\ \frac{3}{4},\ \frac{9}{4}\right]=\frac{5\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{256\sqrt{2}}.$$

C19. $${}_3F_2\left[1,\ \frac{3}{2},\ -\frac{3}{4};\ \frac{5}{4},\ \frac{7}{4}\right]=\frac{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})}{64\sqrt{2}}.$$

C20. $${}_3F_2\left[1,\ \frac{3}{2},\ \frac{1}{4};\ \frac{3}{4},\ \frac{9}{4}\right]=\frac{5\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

C21. $${}_3F_2\left[1,\ \frac{3}{2},\ \frac{1}{4};\ \frac{3}{4},\ \frac{13}{4}\right]=\frac{9\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

C22. $${}_3F_2\left[1,\ \frac{3}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{7}{4}\right]=\frac{3\left\{2\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{8\sqrt{2}}.$$

C23. $${}_3F_2\left[1,\ \frac{3}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{11}{4}\right]=\frac{7\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{48\sqrt{2}}.$$

C24. $${}_3F_2\left[1,\ \frac{3}{2},\ \frac{1}{4};\ \frac{7}{4},\ \frac{9}{4}\right]=\frac{15\left\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$$

C25. $${}_3F_2\left[1,\ \frac{3}{2},\ \frac{5}{4};\ \frac{3}{4},\ \frac{13}{4}\right]=\frac{9\left\{5\pi-6\sqrt{2}+10\log(1+\sqrt{2})\right\}}{8\sqrt{2}}.$$

C26. $${}_3F_2\left[1,\ \frac{3}{2},\ \frac{5}{4};\ \frac{7}{4},\ \frac{9}{4}\right]=\frac{15\left\{\pi-2\sqrt{2}+2\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$$

C27. $${}_3F_2\left[1,\ \frac{3}{2},\ \frac{5}{4};\ \frac{7}{4},\ \frac{13}{4}\right]=\frac{9\left\{5\pi-14\sqrt{2}+10\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

C28. $$ {}_3F_2\left[1,\ \frac{3}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{11}{4}\right]=\frac{35\left\{3\pi-10\sqrt{2}+6\log(1+\sqrt{2})\right\}}{8\sqrt{2}}. $$

C29. $$ {}_3F_2\left[1,\ \frac{3}{2},\ \frac{5}{4};\ \frac{11}{4},\ \frac{13}{4}\right]=\frac{63\left\{25\pi-86\sqrt{2}+50\log(1+\sqrt{2})\right\}}{32\sqrt{2}}. $$

C30. $$ {}_3F_2\left[1,\ \frac{3}{2},\ \frac{9}{4};\ \frac{7}{4},\ \frac{13}{4}\right]=\frac{9\left\{5\pi-8\sqrt{2}+10\log(1+\sqrt{2})\right\}}{10\sqrt{2}}. $$

C31. $$ {}_3F_2\left[1,\ \frac{5}{2},\ \frac{1}{4};\ \frac{7}{4},\ \frac{9}{4}\right]=\frac{5\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{64\sqrt{2}}. $$

C32. $$ {}_3F_2\left[1,\ \frac{5}{2},\ \frac{5}{4};\ \frac{7}{4},\ \frac{13}{4}\right]=\frac{9\left\{15\pi-26\sqrt{2}+30\log(1+\sqrt{2})\right\}}{32\sqrt{2}}. $$

C33. $$ {}_3F_2\left[1,\ \frac{5}{2},\ \frac{5}{4};\ \frac{7}{4},\ \frac{17}{4}\right]=\frac{65\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{256\sqrt{2}}. $$

C34. $$ {}_3F_2\left[1,\ \frac{5}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{11}{4}\right]=\frac{35\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{48\sqrt{2}}. $$

C35. $$ {}_3F_2\left[1,\ \frac{5}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{15}{4}\right]=\frac{55\left\{21\pi-62\sqrt{2}+42\log(1+\sqrt{2})\right\}}{288\sqrt{2}}. $$

C36. $$ {}_3F_2\left[1,\ \frac{5}{2},\ \frac{5}{4};\ \frac{11}{4},\ \frac{13}{4}\right]=\frac{21\left\{58\sqrt{2}-15\pi-30\log(1+\sqrt{2})\right\}}{64\sqrt{2}}. $$

C37. $$ {}_3F_2\left[1,\ \frac{5}{2},\ \frac{9}{4};\ \frac{7}{4},\ \frac{17}{4}\right]=\frac{13\left\{45\pi-62\sqrt{2}+90\log(1+\sqrt{2})\right\}}{80\sqrt{2}}. $$

C38. $$ {}_3F_2\left[1,\ \frac{5}{2},\ \frac{9}{4};\ \frac{11}{4},\ \frac{13}{4}\right]=\frac{21\left\{15\pi-34\sqrt{2}+30\log(1+\sqrt{2})\right\}}{40\sqrt{2}}. $$

C39. $$ {}_3F_2\left[2,\ \frac{1}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{7}{4}\right]=\frac{2\sqrt{2}+21\pi+42\log(1+\sqrt{2})}{-32\sqrt{2}}. $$

C40. $$ {}_3F_2\left[2,\ \frac{1}{2},\ -\frac{3}{4};\ \frac{3}{4},\ \frac{5}{4}\right]=\frac{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})}{-64\sqrt{2}}. $$

C41. $$ {}_3F_2\left[2,\ \frac{1}{2},\ \frac{1}{4};\ \frac{3}{4},\ \frac{9}{4}\right]=\frac{5\left\{11\pi-2\sqrt{2}+22\log(1+\sqrt{2})\right\}}{96\sqrt{2}}. $$

$$\text{C42. } {}_3F_2\left[2,\ \frac{1}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{7}{4}\right] = \frac{3\left\{3\pi - 2\sqrt{2} + 6\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

$$\text{C43. } {}_3F_2\left[2,\ \frac{1}{2},\ \frac{1}{4};\ \frac{9}{4},\ \frac{11}{4}\right] = \frac{7\left\{2\sqrt{2} + 5\pi + 10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

$$\text{C44. } {}_3F_2\left[2,\ \frac{1}{2},\ \frac{5}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{5\left\{2\sqrt{2} + \pi + 2\log(1+\sqrt{2})\right\}}{8\sqrt{2}}.$$

$$\text{C45. } {}_3F_2\left[2,\ \frac{1}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{11}{4}\right] = \frac{7\left\{5\pi - 14\sqrt{2} + 10\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

$$\text{C46. } {}_3F_2\left[2,\ \frac{1}{2},\ \frac{9}{4};\ \frac{11}{4},\ \frac{13}{4}\right] = \frac{9\left\{94\sqrt{2} - 25\pi - 50\log(1+\sqrt{2})\right\}}{40\sqrt{2}}.$$

$$\text{C47. } {}_3F_2\left[2,\ \frac{3}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{11}{4}\right] = \frac{7\left\{14\sqrt{2} + 3\pi + 6\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$$

$$\text{C48. } {}_3F_2\left[2,\ \frac{3}{2},\ \frac{1}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{15\left\{2\sqrt{2} + 5\pi + 10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

$$\text{C49. } {}_3F_2\left[2,\ \frac{3}{2},\ \frac{5}{4};\ \frac{7}{4},\ \frac{13}{4}\right] = \frac{45\left\{7\pi - 10\sqrt{2} + 14\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$$

$$\text{C50. } {}_3F_2\left[2,\ \frac{3}{2},\ \frac{9}{4};\ \frac{11}{4},\ \frac{13}{4}\right] = \frac{63\left\{5\pi - 14\sqrt{2} + 10\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

4.2 *Class D*

By means of the Thomae transformation (2), we have the following "Class-D" series in terms of the $\mathcal{F}_1$-series ($\sigma = b + d - a - c - e$):

$${}_3F_2\left[\begin{matrix} a+\frac{1}{2},\ c-\frac{1}{4},\ e+1 \\ b+\frac{3}{4},\ d+\frac{5}{4} \end{matrix}\middle|\, 1\right] = \frac{\Gamma(\sigma+\frac{3}{4})\Gamma(b+\frac{3}{4})\Gamma(d+\frac{5}{4})}{\Gamma(a+\frac{1}{2})\Gamma(\sigma+c+\frac{1}{2})\Gamma(\sigma+e+\frac{7}{4})}\,{}_3F_2\left[\begin{matrix} \sigma+\frac{3}{4},\ b-a+\frac{1}{4},\ d-a+\frac{3}{4} \\ \sigma+c+\frac{1}{2},\ \sigma+e+\frac{7}{4} \end{matrix}\middle|\, 1\right].$$

Then the closed formulae for this series (excluding the divergent series) can be deduced from those shown in "Class B".

$$\text{D1. } {}_3F_2\left[1,\ \frac{1}{2},\ -\frac{9}{4};\ \frac{1}{4},\ \frac{3}{4}\right] = \frac{5\left\{18\log(1+\sqrt{2}) - 9\pi - 22\sqrt{2}\right\}}{256\sqrt{2}}.$$

D2. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{5}{4};\ \frac{1}{4},\ \frac{3}{4}\right] = \frac{10\log(1+\sqrt{2}) - 5\pi - 14\sqrt{2}}{16\sqrt{2}}.$$

D3. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{5}{4};\ \frac{1}{4},\ \frac{7}{4}\right] = \frac{9\left\{2\sqrt{2} - 5\pi + 10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

D4. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{5}{4}\right] = \frac{26\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})}{96\sqrt{2}}.$$

D5. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{5}{4};\ \frac{5}{4},\ -\frac{1}{4}\right] = \frac{34\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})}{24\sqrt{2}}.$$

D6. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{5}{4};\ \frac{9}{4},\ -\frac{1}{4}\right] = \frac{74\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})}{48\sqrt{2}}.$$

D7. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{1}{4};\ \frac{1}{4},\ \frac{7}{4}\right] = \frac{3\left\{2\sqrt{2} - \pi + 2\log(1+\sqrt{2})\right\}}{8\sqrt{2}}.$$

D8. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{5}{4}\right] = \frac{2\sqrt{2} + \pi - 2\log(1+\sqrt{2})}{4\sqrt{2}}.$$

D9. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{9}{4}\right] = \frac{5\left\{10\sqrt{2} + 3\pi - 6\log(1+\sqrt{2})\right\}}{72\sqrt{2}}.$$

D10. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{1}{4};\ \frac{5}{4},\ \frac{7}{4}\right] = \frac{3\left\{2\sqrt{2} + 3\pi - 6\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

D11. $${}_3F_2\left[1,\ \frac{1}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{5\left\{21\pi - 2\sqrt{2} - 42\log(1+\sqrt{2})\right\}}{96\sqrt{2}}.$$

D12. $${}_3F_2\left[1,\ \frac{1}{2},\ \frac{3}{4};\ \frac{5}{4},\ \frac{7}{4}\right] = \frac{3\left\{\pi - 2\log(1+\sqrt{2})\right\}}{2\sqrt{2}}.$$

D13. $${}_3F_2\left[1,\ \frac{1}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{5\left\{3\pi - 2\sqrt{2} - 6\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$$

D14. $${}_3F_2\left[1,\ \frac{1}{2},\ \frac{3}{4};\ \frac{9}{4},\ \frac{11}{4}\right] = \frac{21\left\{7\pi - 6\sqrt{2} - 14\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

D15. $${}_3F_2\left[1,\ \frac{3}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{5}{4}\right] = \frac{30\log(1+\sqrt{2}) - 15\pi - 58\sqrt{2}}{192\sqrt{2}}.$$

D16. $_3F_2\left[1,\ \frac{3}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{9}{4}\right] = \frac{3\left\{5\pi - 2\sqrt{2} - 10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$

D17. $_3F_2\left[1,\ \frac{3}{2},\ -\frac{5}{4};\ \frac{5}{4},\ \frac{7}{4}\right] = \frac{122\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})}{512\sqrt{2}}.$

D18. $_3F_2\left[1,\ \frac{3}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{9}{4}\right] = \frac{5\left\{2\sqrt{2} + 3\pi - 6\log(1+\sqrt{2})\right\}}{48\sqrt{2}}.$

D19. $_3F_2\left[1,\ \frac{3}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{13}{4}\right] = \frac{3\left\{26\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})\right\}}{160\sqrt{2}}.$

D20. $_3F_2\left[1,\ \frac{3}{2},\ -\frac{1}{4};\ \frac{5}{4},\ \frac{7}{4}\right] = \frac{3\left\{6\sqrt{2} + \pi - 2\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$

D21. $_3F_2\left[1,\ \frac{3}{2},\ -\frac{1}{4};\ \frac{5}{4},\ \frac{11}{4}\right] = \frac{7\left\{22\sqrt{2} + 9\pi - 18\log(1+\sqrt{2})\right\}}{256\sqrt{2}}.$

D22. $_3F_2\left[1,\ \frac{3}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{5\left\{14\sqrt{2} - 3\pi + 6\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$

D23. $_3F_2\left[1,\ \frac{3}{2},\ \frac{3}{4};\ \frac{5}{4},\ \frac{11}{4}\right] = \frac{7\left\{2\sqrt{2} + 3\pi - 6\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$

D24. $_3F_2\left[1,\ \frac{3}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{15\left\{2\sqrt{2} - \pi + 2\log(1+\sqrt{2})\right\}}{8\sqrt{2}}.$

D25. $_3F_2\left[1,\ \frac{3}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{13}{4}\right] = \frac{3\left\{22\sqrt{2} - 15\pi + 30\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$

D26. $_3F_2\left[1,\ \frac{3}{2},\ \frac{3}{4};\ \frac{9}{4},\ \frac{11}{4}\right] = \frac{35\left\{10\sqrt{2} - 9\pi + 18\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$

D27. $_3F_2\left[1,\ \frac{3}{2},\ \frac{7}{4};\ \frac{9}{4},\ \frac{11}{4}\right] = \frac{35\left\{3\pi - 2\sqrt{2} - 6\log(1+\sqrt{2})\right\}}{12\sqrt{2}}.$

D28. $_3F_2\left[1,\ \frac{5}{2},\ -\frac{5}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{634\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})}{2048\sqrt{2}}.$

D29. $_3F_2\left[1,\ \frac{5}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{9}{4}\right] = \frac{5\left\{50\sqrt{2} + 3\pi - 6\log(1+\sqrt{2})\right\}}{384\sqrt{2}}.$

D30. $${}_3F_2\left[1,\ \frac{5}{2},\ -\frac{1}{4};\ \frac{9}{4},\ \frac{11}{4}\right]=\frac{35\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{1024\sqrt{2}}.$$

D31. $${}_3F_2\left[1,\ \frac{5}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{13}{4}\right]=\frac{3\left\{38\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$$

D32. $${}_3F_2\left[1,\ \frac{5}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{17}{4}\right]=\frac{13\left\{82\sqrt{2}-45\pi+90\log(1+\sqrt{2})\right\}}{320\sqrt{2}}.$$

D33. $${}_3F_2\left[1,\ \frac{5}{2},\ \frac{3}{4};\ \frac{9}{4},\ \frac{11}{4}\right]=\frac{35\left\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\right\}}{192\sqrt{2}}.$$

D34. $${}_3F_2\left[1,\ \frac{5}{2},\ \frac{3}{4};\ \frac{9}{4},\ \frac{15}{4}\right]=\frac{55\left\{34\sqrt{2}-21\pi+42\log(1+\sqrt{2})\right\}}{512\sqrt{2}}.$$

D35. $${}_3F_2\left[1,\ \frac{5}{2},\ \frac{3}{4};\ \frac{11}{4},\ \frac{13}{4}\right]=\frac{63\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

D36. $${}_3F_2\left[1,\ \frac{5}{2},\ \frac{7}{4};\ \frac{9}{4},\ \frac{15}{4}\right]=\frac{55\left\{21\pi-2\sqrt{2}-42\log(1+\sqrt{2})\right\}}{288\sqrt{2}}.$$

D37. $${}_3F_2\left[1,\ \frac{5}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{13}{4}\right]=\frac{7\left\{22\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

D38. $${}_3F_2\left[2,\ \frac{1}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{5}{4}\right]=\frac{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})}{128\sqrt{2}}.$$

D39. $${}_3F_2\left[2,\ \frac{1}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{9}{4}\right]=\frac{5\left\{26\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{288\sqrt{2}}.$$

D40. $${}_3F_2\left[2,\ \frac{1}{2},\ -\frac{1}{4};\ \frac{5}{4},\ \frac{7}{4}\right]=\frac{3\left\{10\sqrt{2}+7\pi-14\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$$

D41. $${}_3F_2\left[2,\ \frac{1}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{9}{4}\right]=\frac{5\left\{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

D42. $${}_3F_2\left[2,\ \frac{1}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{13}{4}\right]=\frac{15\left\{21\pi-2\sqrt{2}-42\log(1+\sqrt{2})\right\}}{224\sqrt{2}}.$$

D43. $${}_3F_2\left[2,\ \frac{1}{2},\ \frac{3}{4};\ \frac{9}{4},\ \frac{11}{4}\right]=\frac{7\left\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$$

D44. $_3F_2\left[2,\ \frac{1}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{13}{4}\right]=\frac{27\left\{7\pi-6\sqrt{2}-14\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$

D45. $_3F_2\left[2,\ \frac{3}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{13}{4}\right]=\frac{9\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$

D46. $_3F_2\left[2,\ \frac{3}{2},\ -\frac{1}{4};\ \frac{5}{4},\ \frac{11}{4}\right]=\frac{21\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{1024\sqrt{2}}.$

D47. $_3F_2\left[2,\ \frac{3}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{9}{4}\right]=\frac{5\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{256\sqrt{2}}.$

D48. $_3F_2\left[2,\ \frac{3}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{13}{4}\right]=\frac{15\left\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$

D49. $_3F_2\left[2,\ \frac{3}{2},\ \frac{3}{4};\ \frac{9}{4},\ \frac{11}{4}\right]=\frac{105\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$

D50. $_3F_2\left[2,\ \frac{3}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{13}{4}\right]=\frac{105\left\{10\sqrt{2}-9\pi+18\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$

4.3 *Class E*

In view of the Kummer transformation (3), we get the following "Class-E" series in terms of the $\mathcal{F}_1$-series ($\sigma = b + d - a - c - e$):

$$ {}_3F_2\left[\left.\begin{matrix} a+\frac{1}{4},\ c+1,\ e+1 \\ b+\frac{7}{4},\ d+\frac{3}{4} \end{matrix}\right| 1\right]=\frac{\Gamma(\sigma+\frac{1}{4})\Gamma(d+\frac{3}{4})}{\Gamma(\sigma+a+\frac{1}{2})\Gamma(d-a+\frac{1}{2})}\,{}_3F_2\left[\left.\begin{matrix} a+\frac{1}{4},\ b-c+\frac{3}{4},\ b-e+\frac{3}{4} \\ \sigma+\frac{7}{4},\ \sigma+a+\frac{1}{2} \end{matrix}\right| 1\right]. $$

Then the closed formulae for this series (excluding the divergent series) can be deduced from those shown in "Class B".

E1. $_3F_2\left[1,\ 1,\ -\frac{3}{4};\ \frac{3}{4},\ \frac{3}{4}\right]=\frac{6\log(1+\sqrt{2})-3\pi-10\sqrt{2}}{8\sqrt{2}}.$

E2. $_3F_2\left[1,\ 1,\ -\frac{3}{4};\ \frac{3}{4},\ \frac{7}{4}\right]=\frac{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})}{16\sqrt{2}}.$

E3. $_3F_2\left[1,\ 1,\ -\frac{3}{4};\ \frac{3}{4},\ \frac{11}{4}\right]=\frac{7\left\{26\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{480\sqrt{2}}.$

E4. ${}_3F_2\left[1,\ 1,\ -\frac{3}{4};\ \frac{7}{4},\ -\frac{1}{4}\right]=\frac{34\sqrt{2}+15\pi-30\log(1+\sqrt{2})}{8\sqrt{2}}.$

E5. ${}_3F_2\left[1,\ 1,\ -\frac{3}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{38\sqrt{2}-15\pi+30\log(1+\sqrt{2})}{32\sqrt{2}}.$

E6. ${}_3F_2\left[1,\ 1,\ -\frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{7\left\{82\sqrt{2}-45\pi+90\log(1+\sqrt{2})\right\}}{320\sqrt{2}}.$

E7. ${}_3F_2\left[1,\ 1,\ -\frac{3}{4};\ \frac{11}{4},\ -\frac{1}{4}\right]=\frac{7\left\{74\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{240\sqrt{2}}.$

E8. ${}_3F_2\left[1,\ 1,\ \frac{1}{4};\ \frac{3}{4},\ \frac{7}{4}\right]=\frac{3\left\{2\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$

E9. ${}_3F_2\left[1,\ 1,\ \frac{1}{4};\ \frac{3}{4},\ \frac{11}{4}\right]=\frac{7\left\{10\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{72\sqrt{2}}.$

E10. ${}_3F_2\left[1,\ 1,\ \frac{1}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{9\left\{2\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{8\sqrt{2}}.$

E11. ${}_3F_2\left[1,\ 1,\ \frac{1}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{7\left\{22\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{48\sqrt{2}}.$

E12. ${}_3F_2\left[1,\ 1,\ \frac{5}{4};\ \frac{3}{4},\ \frac{11}{4}\right]=\frac{7\left\{4\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{6\sqrt{2}}.$

E13. ${}_3F_2\left[1,\ 1,\ \frac{5}{4};\ \frac{3}{4},\ \frac{15}{4}\right]=\frac{11\left\{34\sqrt{2}+21\pi-42\log(1+\sqrt{2})\right\}}{252\sqrt{2}}.$

E14. ${}_3F_2\left[1,\ 1,\ \frac{5}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{9\left\{\pi-2\log(1+\sqrt{2})\right\}}{2\sqrt{2}}.$

E15. ${}_3F_2\left[1,\ 1,\ \frac{5}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{7\left\{3\pi-2\sqrt{2}-6\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$

E16. ${}_3F_2\left[1,\ 1,\ \frac{5}{4};\ \frac{7}{4},\ \frac{15}{4}\right]=\frac{11\left\{105\pi-82\sqrt{2}-210\log(1+\sqrt{2})\right\}}{168\sqrt{2}}.$

E17. ${}_3F_2\left[1,\ 2,\ -\frac{3}{4};\ \frac{3}{4},\ \frac{7}{4}\right]=\frac{30\log(1+\sqrt{2})-15\pi-58\sqrt{2}}{64\sqrt{2}}.$

E18. $${}_3F_2\left[1,\ 2,\ -\frac{3}{4};\ \frac{3}{4},\ \frac{11}{4}\right] = \frac{21\left\{5\pi - 2\sqrt{2} - 10\log(1+\sqrt{2})\right\}}{640\sqrt{2}}.$$

E19. $${}_3F_2\left[1,\ 2,\ -\frac{3}{4};\ \frac{7}{4},\ \frac{7}{4}\right] = \frac{50\sqrt{2} + 3\pi - 6\log(1+\sqrt{2})}{128\sqrt{2}}.$$

E20. $${}_3F_2\left[1,\ 2,\ -\frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\right] = \frac{7\left\{134\sqrt{2} - 15\pi + 30\log(1+\sqrt{2})\right\}}{1280\sqrt{2}}.$$

E21. $${}_3F_2\left[1,\ 2,\ -\frac{3}{4};\ \frac{11}{4},\ -\frac{1}{4}\right] = \frac{7\left\{266\sqrt{2} + 135\pi - 270\log(1+\sqrt{2})\right\}}{320\sqrt{2}}.$$

E22. $${}_3F_2\left[1,\ 2,\ \frac{1}{4};\ \frac{3}{4},\ \frac{11}{4}\right] = \frac{7\left\{26\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})\right\}}{96\sqrt{2}}.$$

E23. $${}_3F_2\left[1,\ 2,\ \frac{1}{4};\ \frac{7}{4},\ \frac{7}{4}\right] = \frac{9\left\{6\sqrt{2} + \pi - 2\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$$

E24. $${}_3F_2\left[1,\ 2,\ \frac{1}{4};\ \frac{7}{4},\ \frac{11}{4}\right] = \frac{7\left\{14\sqrt{2} - 3\pi + 6\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$$

E25. $${}_3F_2\left[1,\ 2,\ \frac{1}{4};\ \frac{11}{4},\ \frac{11}{4}\right] = \frac{49\left\{5\pi - 2\sqrt{2} - 10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

E26. $${}_3F_2\left[1,\ 2,\ \frac{5}{4};\ \frac{7}{4},\ \frac{11}{4}\right] = \frac{21\left\{2\sqrt{2} + 3\pi - 6\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

E27. $${}_3F_2\left[1,\ 2,\ \frac{5}{4};\ \frac{7}{4},\ \frac{15}{4}\right] = \frac{11\left\{21\pi - 2\sqrt{2} - 42\log(1+\sqrt{2})\right\}}{96\sqrt{2}}.$$

E28. $${}_3F_2\left[1,\ 2,\ \frac{5}{4};\ \frac{11}{4},\ \frac{11}{4}\right] = \frac{49\left\{10\sqrt{2} - 9\pi + 18\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$$

E29. $${}_3F_2\left[1,\ 2,\ \frac{5}{4};\ \frac{11}{4},\ \frac{15}{4}\right] = \frac{77\left\{106\sqrt{2} - 105\pi + 210\log(1+\sqrt{2})\right\}}{192\sqrt{2}}.$$

E30. $${}_3F_2\left[1,\ 2,\ \frac{9}{4};\ \frac{11}{4},\ \frac{15}{4}\right] = \frac{231\left\{7\pi - 6\sqrt{2} - 14\log(1+\sqrt{2})\right\}}{80\sqrt{2}}.$$

E31. $${}_3F_2\left[1,\ 3,\ -\frac{3}{4};\ \frac{3}{4},\ \frac{11}{4}\right] = \frac{7\left\{270\log(1+\sqrt{2}) - 135\pi - 586\sqrt{2}\right\}}{5120\sqrt{2}}.$$

E32. $${}_3F_2\left[1,\ 3,\ -\frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{7\left\{634\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{10240\sqrt{2}}.$$

E33. $${}_3F_2\left[1,\ 3,\ \frac{1}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{7\left\{122\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{512\sqrt{2}}.$$

E34. $${}_3F_2\left[1,\ 3,\ \frac{1}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{49\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{1024\sqrt{2}}.$$

E35. $${}_3F_2\left[1,\ 3,\ \frac{5}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{49\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{256\sqrt{2}}.$$

E36. $${}_3F_2\left[1,\ 3,\ \frac{5}{4};\ \frac{11}{4},\ \frac{15}{4}\right]=\frac{77\left\{34\sqrt{2}-21\pi+42\log(1+\sqrt{2})\right\}}{512\sqrt{2}}.$$

E37. $${}_3F_2\left[2,\ 2,\ -\frac{3}{4};\ \frac{3}{4},\ \frac{11}{4}\right]=\frac{7\left\{390\log(1+\sqrt{2})-195\pi-562\sqrt{2}\right\}}{2560\sqrt{2}}.$$

E38. $${}_3F_2\left[2,\ 2,\ -\frac{3}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{174\log(1+\sqrt{2})-87\pi-298\sqrt{2}}{512\sqrt{2}}.$$

E39. $${}_3F_2\left[2,\ 2,\ -\frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{7\left\{98\sqrt{2}+75\pi-150\log(1+\sqrt{2})\right\}}{5120\sqrt{2}}.$$

E40. $${}_3F_2\left[2,\ 2,\ \frac{1}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{21\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{256\sqrt{2}}.$$

E41. $${}_3F_2\left[2,\ 2,\ \frac{1}{4};\ \frac{7}{4},\ \frac{15}{4}\right]=\frac{77\left\{122\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{7680\sqrt{2}}.$$

E42. $${}_3F_2\left[2,\ 2,\ \frac{1}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{49\left\{34\sqrt{2}-21\pi+42\log(1+\sqrt{2})\right\}}{512\sqrt{2}}.$$

E43. $${}_3F_2\left[2,\ 2,\ \frac{5}{4};\ \frac{7}{4},\ \frac{15}{4}\right]=\frac{77\left\{38\sqrt{2}+33\pi-66\log(1+\sqrt{2})\right\}}{384\sqrt{2}}.$$

E44. $${}_3F_2\left[2,\ 2,\ \frac{5}{4};\ \frac{7}{4},\ \frac{19}{4}\right]=\frac{33\left\{86\sqrt{2}+105\pi-210\log(1+\sqrt{2})\right\}}{1792\sqrt{2}}.$$

E45. $${}_3F_2\left[2,\ 2,\ \frac{5}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{441\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

E46. ${}_3F_2\left[2,\ 2,\ \frac{5}{4};\ \frac{11}{4},\ \frac{15}{4}\right] = \frac{539\left\{51\pi - 46\sqrt{2} - 102\log(1+\sqrt{2})\right\}}{768\sqrt{2}}.$

E47. ${}_3F_2\left[2,\ 2,\ \frac{9}{4};\ \frac{11}{4},\ \frac{15}{4}\right] = \frac{539\left\{14\sqrt{2} - 3\pi + 6\log(1+\sqrt{2})\right\}}{320\sqrt{2}}.$

E48. ${}_3F_2\left[2,\ 2,\ \frac{9}{4};\ \frac{11}{4},\ \frac{19}{4}\right] = \frac{11\left\{314\sqrt{2} - 273\pi + 546\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$

E49. ${}_3F_2\left[2,\ 3,\ \frac{1}{4};\ \frac{11}{4},\ \frac{11}{4}\right] = \frac{49\left\{166\sqrt{2} + 33\pi - 66\log(1+\sqrt{2})\right\}}{4096\sqrt{2}}.$

E50. ${}_3F_2\left[2,\ 3,\ \frac{5}{4};\ \frac{11}{4},\ \frac{15}{4}\right] = \frac{539\left\{10\sqrt{2} + 39\pi - 78\log(1+\sqrt{2})\right\}}{2048\sqrt{2}}.$

4.4 *Class F*

By making use of the Kummer transformation (3), we find the following "Class-F" series in terms of the $\mathcal{F}_0$-series ($\sigma = b + d - a - c - e$):

$${}_3F_2\left[\left.\begin{matrix} a+\frac{3}{4},\ c+1,\ e+1 \\ b+\frac{5}{4},\ d+\frac{5}{4} \end{matrix}\right| 1\right] = \frac{\Gamma(\sigma-\frac{1}{4})\Gamma(d+\frac{5}{4})}{\Gamma(\sigma+a+\frac{1}{2})\Gamma(d-a+\frac{1}{2})}\,{}_3F_2\left[\left.\begin{matrix} a+\frac{3}{4},\ b-c+\frac{1}{4},\ b-e+\frac{1}{4} \\ \sigma+\frac{5}{4},\ \sigma+a+\frac{1}{2} \end{matrix}\right| 1\right].$$

Then the closed formulae for this series (excluding the divergent series) can be derived from those shown in "Class A".

F1. ${}_3F_2\left[1,\ 1,\ -\frac{1}{4};\ \frac{1}{4},\ \frac{9}{4}\right] = \frac{5\left\{10\sqrt{2} - 3\pi - 6\log(1+\sqrt{2})\right\}}{24\sqrt{2}}.$

F2. ${}_3F_2\left[1,\ 1,\ -\frac{1}{4};\ \frac{1}{4},\ \frac{9}{4}\right] = \frac{5\left\{10\sqrt{2} - 3\pi - 6\log(1+\sqrt{2})\right\}}{24\sqrt{2}}.$

F3. ${}_3F_2\left[1,\ 1,\ -\frac{1}{4};\ \frac{1}{4},\ \frac{13}{4}\right] = \frac{9\left\{74\sqrt{2} - 15\pi - 30\log(1+\sqrt{2})\right\}}{400\sqrt{2}}.$

F4. ${}_3F_2\left[1,\ 1,\ -\frac{1}{4};\ \frac{5}{4},\ \frac{5}{4}\right] = \frac{2\sqrt{2} + \pi + 2\log(1+\sqrt{2})}{8\sqrt{2}}.$

F5. ${}_3F_2\left[1,\ 1,\ -\frac{1}{4};\ \frac{5}{4},\ \frac{9}{4}\right] = \frac{5\left\{3\pi - 2\sqrt{2} + 6\log(1+\sqrt{2})\right\}}{48\sqrt{2}}.$

F6. $${}_3F_2\left[1,\ 1,\ -\frac{1}{4};\ \frac{5}{4},\ \frac{13}{4}\right]=\frac{3\left\{105\pi-118\sqrt{2}+210\log(1+\sqrt{2})\right\}}{800\sqrt{2}}.$$

F7. $${}_3F_2\left[1,\ 1,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\left\{21\pi-62\sqrt{2}+42\log(1+\sqrt{2})\right\}}{288\sqrt{2}}.$$

F8. $${}_3F_2\left[1,\ 1,\ \frac{3}{4};\ \frac{1}{4},\ \frac{13}{4}\right]=\frac{3\left\{15\pi-14\sqrt{2}+30\log(1+\sqrt{2})\right\}}{20\sqrt{2}}.$$

F9. $${}_3F_2\left[1,\ 1,\ \frac{3}{4};\ \frac{1}{4},\ \frac{17}{4}\right]=\frac{13\left\{45\pi-22\sqrt{2}+90\log(1+\sqrt{2})\right\}}{600\sqrt{2}}.$$

F10. $${}_3F_2\left[1,\ 1,\ \frac{3}{4};\ \frac{5}{4},\ \frac{9}{4}\right]=\frac{5\left\{\pi-2\sqrt{2}+2\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$$

F11. $${}_3F_2\left[1,\ 1,\ \frac{3}{4};\ \frac{5}{4},\ \frac{13}{4}\right]=\frac{3\left\{15\pi-34\sqrt{2}+30\log(1+\sqrt{2})\right\}}{40\sqrt{2}}.$$

F12. $${}_3F_2\left[1,\ 1,\ \frac{3}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\left\{3\pi-10\sqrt{2}+6\log(1+\sqrt{2})\right\}}{8\sqrt{2}}.$$

F13. $${}_3F_2\left[1,\ 1,\ \frac{7}{4};\ \frac{1}{4},\ \frac{17}{4}\right]=\frac{13\left\{45\pi-52\sqrt{2}+90\log(1+\sqrt{2})\right\}}{90\sqrt{2}}.$$

F14. $${}_3F_2\left[1,\ 1,\ \frac{7}{4};\ \frac{5}{4},\ \frac{13}{4}\right]=\frac{3\left\{5\pi-8\sqrt{2}+10\log(1+\sqrt{2})\right\}}{10\sqrt{2}}.$$

F15. $${}_3F_2\left[1,\ 1,\ \frac{7}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\left\{4\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{6\sqrt{2}}.$$

F16. $${}_3F_2\left[1,\ 1,\ \frac{7}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{9\left\{18\sqrt{2}-5\pi-10\log(1+\sqrt{2})\right\}}{4\sqrt{2}}.$$

F17. $${}_3F_2\left[1,\ 2,\ -\frac{1}{4};\ \frac{5}{4},\ \frac{9}{4}\right]=\frac{5\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{192\sqrt{2}}.$$

F18. $${}_3F_2\left[1,\ 2,\ -\frac{1}{4};\ \frac{5}{4},\ \frac{13}{4}\right]=\frac{27\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{640\sqrt{2}}.$$

F19. $${}_3F_2\left[1,\ 2,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\left\{50\sqrt{2}-3\pi-6\log(1+\sqrt{2})\right\}}{1152\sqrt{2}}.$$

F20. $${}_3F_2\left[1,\ 2,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{13}{4}\right] = \frac{598\sqrt{2} - 105\pi - 210\log(1+\sqrt{2})}{256\sqrt{2}}.$$

F21. $${}_3F_2\left[1,\ 2,\ \frac{3}{4};\ \frac{5}{4},\ \frac{13}{4}\right] = \frac{3\left\{15\pi - 26\sqrt{2} + 30\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$$

F22. $${}_3F_2\left[1,\ 2,\ \frac{3}{4};\ \frac{5}{4},\ \frac{17}{4}\right] = \frac{13\left\{135\pi - 266\sqrt{2} + 270\log(1+\sqrt{2})\right\}}{1600\sqrt{2}}.$$

F23. $${}_3F_2\left[1,\ 2,\ \frac{3}{4};\ \frac{9}{4},\ \frac{9}{4}\right] = \frac{25\left\{6\sqrt{2} - \pi - 2\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$$

F24. $${}_3F_2\left[1,\ 2,\ \frac{3}{4};\ \frac{9}{4},\ \frac{13}{4}\right] = \frac{15\left\{58\sqrt{2} - 15\pi - 30\log(1+\sqrt{2})\right\}}{64\sqrt{2}}.$$

F25. $${}_3F_2\left[1,\ 2,\ \frac{7}{4};\ \frac{5}{4},\ \frac{17}{4}\right] = \frac{13\left\{45\pi - 62\sqrt{2} + 90\log(1+\sqrt{2})\right\}}{240\sqrt{2}}.$$

F26. $${}_3F_2\left[1,\ 2,\ \frac{7}{4};\ \frac{9}{4},\frac{13}{4}\right] = \frac{15\left\{5\pi - 14\sqrt{2} + 10\log(1+\sqrt{2})\right\}}{16\sqrt{2}}.$$

F27. $${}_3F_2\left[1,\ 2,\ \frac{7}{4};\ \frac{13}{4},\ \frac{13}{4}\right] = \frac{81\left\{25\pi - 86\sqrt{2} + 50\log(1+\sqrt{2})\right\}}{32\sqrt{2}}.$$

F28. $${}_3F_2\left[1,\ 2,\ \frac{11}{4};\ \frac{13}{4},\ \frac{13}{4}\right] = \frac{27\left\{94\sqrt{2} - 25\pi - 50\log(1+\sqrt{2})\right\}}{56\sqrt{2}}.$$

F29. $${}_3F_2\left[1,\ 3,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{9}{4}\right] = \frac{25\left\{26\sqrt{2} + \pi + 2\log(1+\sqrt{2})\right\}}{1024\sqrt{2}}.$$

F30. $${}_3F_2\left[1,\ 3,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{13}{4}\right] = \frac{3\left\{634\sqrt{2} - 15\pi - 30\log(1+\sqrt{2})\right\}}{2048\sqrt{2}}.$$

F31. $${}_3F_2\left[1,\ 3,\ \frac{3}{4};\ \frac{9}{4},\ \frac{13}{4}\right] = \frac{15\left\{122\sqrt{2} - 15\pi - 30\log(1+\sqrt{2})\right\}}{512\sqrt{2}}.$$

F32. $${}_3F_2\left[1,\ 3,\ \frac{3}{4};\ \frac{9}{4},\ \frac{17}{4}\right] = \frac{13\left\{586\sqrt{2} - 135\pi - 270\log(1+\sqrt{2})\right\}}{1024\sqrt{2}}.$$

F33. $${}_3F_2\left[1,\ 3,\ \frac{7}{4};\ \frac{9}{4},\ \frac{17}{4}\right] = \frac{325\left\{9\pi - 22\sqrt{2} + 18\log(1+\sqrt{2})\right\}}{768\sqrt{2}}.$$

F34. $${}_3F_2\left[2,\ 2,\ -\frac{1}{4};\ \frac{5}{4},\ \frac{13}{4}\right]=\frac{3\left\{122\sqrt{2}-15\pi-30\log(1+\sqrt{2})\right\}}{512\sqrt{2}}.$$

F35. $${}_3F_2\left[2,\ 2,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\left\{34\sqrt{2}+21\pi+42\log(1+\sqrt{2})\right\}}{4608\sqrt{2}}.$$

F36. $${}_3F_2\left[2,\ 2,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{5\left\{75\pi-98\sqrt{2}+150\log(1+\sqrt{2})\right\}}{1024\sqrt{2}}.$$

F37. $${}_3F_2\left[2,\ 2,\ \frac{3}{4};\ \frac{5}{4},\ \frac{17}{4}\right]=\frac{39\left\{345\pi-502\sqrt{2}+690\log(1+\sqrt{2})\right\}}{6400\sqrt{2}}.$$

F38. $${}_3F_2\left[2,\ 2,\ \frac{3}{4};\ \frac{5}{4},\ \frac{21}{4}\right]=\frac{221\left\{315\pi-514\sqrt{2}+630\log(1+\sqrt{2})\right\}}{53760\sqrt{2}}.$$

F39. $${}_3F_2\left[2,\ 2,\ \frac{3}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{75\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{256\sqrt{2}}.$$

F40. $${}_3F_2\left[2,\ 2,\ \frac{3}{4};\ \frac{9}{4},\ \frac{17}{4}\right]=\frac{39\left\{195\pi-562\sqrt{2}+390\log(1+\sqrt{2})\right\}}{2560\sqrt{2}}.$$

F41. $${}_3F_2\left[2,\ 2,\ \frac{7}{4};\ \frac{9}{4},\ \frac{17}{4}\right]=\frac{39\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

F42. $${}_3F_2\left[2,\ 2,\ \frac{7}{4};\ \frac{13}{4},\ \frac{13}{4}\right]=\frac{675\left\{46\sqrt{2}-13\pi-26\log(1+\sqrt{2})\right\}}{128\sqrt{2}}.$$

F43. $${}_3F_2\left[2,\ 2,\ \frac{7}{4};\ \frac{13}{4},\ \frac{17}{4}\right]=\frac{351\left\{574\sqrt{2}-165\pi-330\log(1+\sqrt{2})\right\}}{256\sqrt{2}}.$$

F44. $${}_3F_2\left[2,\ 2,\ \frac{11}{4};\ \frac{13}{4},\ \frac{17}{4}\right]=\frac{351\left\{85\pi-286\sqrt{2}+170\log(1+\sqrt{2})\right\}}{448\sqrt{2}}.$$

F45. $${}_3F_2\left[2,\ 3,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{15\left\{146\sqrt{2}+45\pi+90\log(1+\sqrt{2})\right\}}{8192\sqrt{2}}.$$

F46. $${}_3F_2\left[2,\ 3,\ \frac{3}{4};\ \frac{9}{4},\ \frac{17}{4}\right]=\frac{39\left\{255\pi-538\sqrt{2}+510\log(1+\sqrt{2})\right\}}{4096\sqrt{2}}.$$

F47. $${}_3F_2\left[2,\ 3,\ \frac{7}{4};\ \frac{9}{4},\ \frac{21}{4}\right]=\frac{221\left\{495\pi-442\sqrt{2}+990\log(1+\sqrt{2})\right\}}{30720\sqrt{2}}.$$

F48. $_3F_2\left[2,\ 3,\ \frac{7}{4};\ \frac{13}{4},\ \frac{17}{4}\right]=\frac{1755\left\{35\pi-114\sqrt{2}+70\log(1+\sqrt{2})\right\}}{2048\sqrt{2}}.$

F49. $_3F_2\left[3,\ 3,\ -\frac{1}{4};\ \frac{9}{4},\ \frac{17}{4}\right]=\frac{975\left\{478\sqrt{2}-21\pi-42\log(1+\sqrt{2})\right\}}{917504\sqrt{2}}.$

F50. $_3F_2\left[3,\ 3,\ \frac{7}{4};\ \frac{17}{4},\ \frac{17}{4}\right]=\frac{38025\left\{850\sqrt{2}-243\pi-486\log(1+\sqrt{2})\right\}}{32768\sqrt{2}}.$

4.5 *Class G*

According to the Thomae transformation (2), we obtain the following "Class-G" series in terms of the $\mathcal{F}_1$-series ($\sigma = b + d - a - c - e$):

$$_3F_2\left[\begin{matrix} a+\frac{1}{2},\ c+\frac{1}{4},\ e+\frac{3}{2} \\ b+\frac{5}{4},\ d+\frac{5}{4} \end{matrix}\middle| 1\right]=\frac{\Gamma(\sigma+\frac{1}{4})\Gamma(b+\frac{5}{4})\Gamma(d+\frac{5}{4})}{\Gamma(a+\frac{1}{2})\Gamma(\sigma+c+\frac{1}{2})\Gamma(\sigma+e+\frac{7}{4})}\cdot {_3F_2}\left[\begin{matrix} \sigma+\frac{1}{4},\ b-a+\frac{3}{4},\ d-a+\frac{3}{4} \\ \sigma+c+\frac{1}{2},\ \sigma+e+\frac{7}{4} \end{matrix}\middle| 1\right]$$

Then the closed formulae for this series (excluding the divergent series) can be obtained from those shown in "Class B".

G1. $_3F_2\left[\frac{1}{2},\ -\frac{1}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{1}{4}\right]=\frac{\pi^2\left\{\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G2. $_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{7}{4};\ \frac{1}{4},\ \frac{1}{4}\right]=\frac{3\pi^2\left\{2\log(1+\sqrt{2})-\pi-6\sqrt{2}\right\}}{64\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G3. $_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{1}{4}\right]=\frac{\pi^2\left\{2\log(1+\sqrt{2})-\pi-4\sqrt{2}\right\}}{4\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G4. $_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{2\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G5. $_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{\pi-2\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G6. $_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{7}{4};\ \frac{1}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{110\log(1+\sqrt{2})-55\pi-106\sqrt{2}\right\}}{1024\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G7. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{10\log(1+\sqrt{2})-5\pi-14\sqrt{2}\right\}}{32\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G8. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{9}{4}\right]=\frac{5\pi^2\left\{2\sqrt{2}-5\pi+10\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G9. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{3}{4};\ \frac{5}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{10\sqrt{2}+7\pi-14\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G10. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{2\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G11. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{9}{4}\right]=\frac{5\pi^2\left\{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{96\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G12. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ \frac{1}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\right\}}{1152\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G13. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{3\pi-2\sqrt{2}-6\log(1+\sqrt{2})\right\}}{72\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G14. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{75\pi^2\left\{7\pi-6\sqrt{2}-14\log(1+\sqrt{2})\right\}}{224\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G15. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ -\frac{3}{4};\ \frac{1}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{18\log(1+\sqrt{2})-9\pi-22\sqrt{2}\right\}}{1536\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G16. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ -\frac{3}{4};\ \frac{5}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G17. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{9}{4}\right]=\frac{5\pi^2\left\{26\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{576\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G18. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{1}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{38\sqrt{2}+33\pi-66\log(1+\sqrt{2})\right\}}{6912\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G19. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{1}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{25\pi^2\left\{274\sqrt{2}+147\pi-294\log(1+\sqrt{2})\right\}}{43008\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G20. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{10\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{432\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G21. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{25\pi^2\left\{21\pi-2\sqrt{2}-42\log(1+\sqrt{2})\right\}}{1344\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G22. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{9}{4};\ \frac{13}{4},\ \frac{13}{4}\right]=\frac{15\pi^2\left\{105\pi-82\sqrt{2}-210\log(1+\sqrt{2})\right\}}{784\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G23. $${}_3F_2\left[\frac{1}{2},\ \frac{7}{2},\ \frac{5}{4};\ \frac{13}{4},\ \frac{13}{4}\right]=\frac{45\pi^2\left\{86\sqrt{2}+105\pi-210\log(1+\sqrt{2})\right\}}{25088\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G24. $${}_3F_2\left[\frac{1}{2},\ \frac{7}{2},\ \frac{9}{4};\ \frac{13}{4},\ \frac{13}{4}\right]=\frac{3\pi^2\left\{422\sqrt{2}+105\pi-210\log(1+\sqrt{2})\right\}}{1568\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G25. $${}_3F_2\left[\frac{3}{2},\ -\frac{1}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{4\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{36\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G26. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ -\frac{7}{4};\ \frac{5}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{206\log(1+\sqrt{2})-103\pi-394\sqrt{2}\right\}}{16384\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G27. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ -\frac{3}{4};\ \frac{5}{4},\ \frac{5}{4}\right]=\frac{\pi^2\left\{26\log(1+\sqrt{2})-13\pi-46\sqrt{2}\right\}}{256\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G28. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ -\frac{3}{4};\ \frac{5}{4},\ \frac{9}{4}\right]=\frac{5\pi^2\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{2048\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G29. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{9}{4}\right]=\frac{5\pi^2\left\{6\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G30. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{13}{4}\right]=\frac{5\pi^2\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{512\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G31. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{1}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G32. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{2\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G33. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{25\pi^2\left\{10\sqrt{2}-9\pi+18\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G34. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{5}{4};\ \frac{13}{4},\ \frac{13}{4}\right]=\frac{75\pi^2\left\{86\sqrt{2}-87\pi+174\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G35. $${}_3F_2\left[\frac{3}{2},\ \frac{5}{2},\ -\frac{3}{4};\ \frac{5}{4},\ \frac{9}{4}\right]=\frac{5\pi^2\left\{70\log(1+\sqrt{2})-35\pi-114\sqrt{2}\right\}}{4096\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G36. $${}_3F_2\left[\frac{3}{2},\ \frac{5}{2},\ -\frac{3}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{62\sqrt{2}+37\pi-74\log(1+\sqrt{2})\right\}}{32768\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G37. $${}_3F_2\left[\frac{3}{2},\ \frac{5}{2},\ \frac{1}{4};\ \frac{5}{4},\ \frac{13}{4}\right]=\frac{5\pi^2\left\{122\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{1024\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G38. $${}_3F_2\left[\frac{3}{2},\ \frac{5}{2},\ \frac{1}{4};\ \frac{9}{4},\ \frac{9}{4}\right]=\frac{25\pi^2\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{1536\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G39. $${}_3F_2\left[\frac{3}{2},\ \frac{5}{2},\ \frac{1}{4};\ \frac{9}{4},\ \frac{13}{4}\right]=\frac{25\pi^2\left\{10\sqrt{2}+39\pi-78\log(1+\sqrt{2})\right\}}{4096\sqrt{2}\Gamma^4(\frac{3}{4})}.$$

G40. $_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{13}{4}\Big] = \frac{25\pi^2\left\{14\sqrt{2} - 3\pi + 6\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G41. $_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{17}{4}\Big] = \frac{325\pi^2\left\{34\sqrt{2} - 21\pi + 42\log(1+\sqrt{2})\right\}}{7168\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G42. $_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ \frac{5}{4};\ \frac{13}{4},\ \frac{13}{4}\Big] = \frac{75\pi^2\left\{51\pi - 46\sqrt{2} - 102\log(1+\sqrt{2})\right\}}{512\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G43. $_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ \frac{9}{4};\ \frac{13}{4},\ \frac{13}{4}\Big] = \frac{15\pi^2\left\{22\sqrt{2} - 15\pi + 30\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G44. $_3F_2\Big[\frac{3}{2},\ \frac{7}{2},\ \frac{5}{4};\ \frac{13}{4},\ \frac{13}{4}\Big] = \frac{15\pi^2\left\{122\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})\right\}}{1024\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G45. $_3F_2\Big[\frac{5}{2},\ -\frac{1}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{9}{4}\Big] = \frac{25\pi^2\left\{34\sqrt{2} + 21\pi - 42\log(1+\sqrt{2})\right\}}{10584\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G46. $_3F_2\Big[\frac{5}{2},\ \frac{5}{2},\ \frac{1}{4};\ \frac{9}{4},\ \frac{13}{4}\Big] = \frac{25\pi^2\left\{166\sqrt{2} + 33\pi - 66\log(1+\sqrt{2})\right\}}{8192\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G47. $_3F_2\Big[\frac{5}{2},\ \frac{5}{2},\ \frac{5}{4};\ \frac{9}{4},\ \frac{17}{4}\Big] = \frac{325\pi^2\left\{26\sqrt{2} - \pi + 2\log(1+\sqrt{2})\right\}}{2048\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G48. $_3F_2\Big[\frac{5}{2},\ \frac{5}{2},\ \frac{5}{4};\ \frac{13}{4},\ \frac{13}{4}\Big] = \frac{225\pi^2\left\{34\sqrt{2} - 21\pi + 42\log(1+\sqrt{2})\right\}}{1024\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G49. $_3F_2\Big[\frac{5}{2},\ \frac{5}{2},\ \frac{9}{4};\ \frac{13}{4},\ \frac{17}{4}\Big] = \frac{585\pi^2\left\{5\pi - 2\sqrt{2} - 10\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^4(\frac{3}{4})}.$

G50. $_3F_2\Big[\frac{5}{2},\ \frac{7}{2},\ \frac{5}{4};\ \frac{13}{4},\ \frac{17}{4}\Big] = \frac{585\pi^2\left\{146\sqrt{2} - 45\pi + 90\log(1+\sqrt{2})\right\}}{16384\sqrt{2}\Gamma^4(\frac{3}{4})}.$

4.6 *Class H*

Applying the Thomae transformation (2), we can express the following "Class-H" series in terms of the $\mathcal{F}_0$-series ($\sigma = b + d - a - c - e$):

$$ {}_3F_2\left[\begin{matrix} a+\frac{1}{2},\ c-\frac{1}{4},\ e+\frac{1}{2} \\ b+\frac{3}{4},\ d+\frac{3}{4} \end{matrix}\middle|\, 1\right] = \frac{\Gamma(\sigma+\frac{3}{4})\Gamma(b+\frac{3}{4})\Gamma(d+\frac{3}{4})}{\Gamma(a+\frac{1}{2})\Gamma(\sigma+c+\frac{1}{2})\Gamma(\sigma+e+\frac{5}{4})}\, {}_3F_2\left[\begin{matrix} \sigma+\frac{3}{4},\ b-a+\frac{1}{4},\ d-a+\frac{1}{4} \\ \sigma+c+\frac{1}{2},\ \sigma+e+\frac{5}{4} \end{matrix}\middle|\, 1\right] $$

Then the closed formulae for this series (excluding the divergent series) can be deduced from those shown in "Class A".

H1. $${}_3F_2\left[\frac{1}{2},\ -\frac{3}{2},\ -\frac{5}{4};\ -\frac{1}{4},\ -\frac{1}{4}\right] = \frac{24\pi^2\left\{6\pi - 7\sqrt{2} + 12\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H2. $${}_3F_2\left[\frac{1}{2},\ -\frac{1}{2},\ -\frac{9}{4};\ -\frac{1}{4},\ -\frac{1}{4}\right] = \frac{15\pi^2\left\{3\pi - 2\sqrt{2} + 6\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H3. $${}_3F_2\left[\frac{1}{2},\ -\frac{1}{2},\ -\frac{5}{4};\ -\frac{1}{4},\ -\frac{1}{4}\right] = \frac{8\pi^2\left\{3\pi - \sqrt{2} + 6\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H4. $${}_3F_2\left[\frac{1}{2},\ -\frac{1}{2},\ -\frac{5}{4};\ \frac{3}{4},\ -\frac{1}{4}\right] = \frac{6\pi^2\left\{2\sqrt{2} - \pi - 2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H5. $${}_3F_2\left[\frac{1}{2},\ -\frac{1}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{3}{4}\right] = \frac{8\pi^2\left\{\pi - \sqrt{2} + 2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H6. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{9}{4};\ \frac{3}{4},\ -\frac{1}{4}\right] = \frac{3\pi^2\left\{2\sqrt{2} + 69\pi + 138\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H7. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{5}{4};\ \frac{3}{4},\ -\frac{1}{4}\right] = \frac{\pi^2\left\{2\sqrt{2} + 9\pi + 18\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H8. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{3}{4}\right] = \frac{\pi^2\left\{11\pi - 2\sqrt{2} + 22\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H9. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{5}{4};\ \frac{7}{4},\ -\frac{1}{4}\right]=\frac{9\pi^2\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H10. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{3}{4}\right]=\frac{4\pi^2\left\{\pi+2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H11. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{7}{4}\right]=\frac{3\pi^2\left\{2\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H12. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{27\pi^2\left\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H13. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{36\pi^2\left\{4\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H14. $${}_3F_2\left[\frac{1}{2},\ \frac{1}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{63\pi^2\left\{94\sqrt{2}-25\pi-50\log(1+\sqrt{2})\right\}}{25\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H15. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{9}{4};\ \frac{3}{4},\ \frac{3}{4}\right]=\frac{3\pi^2\left\{33\pi-166\sqrt{2}+66\log(1+\sqrt{2})\right\}}{512\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H16. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{3}{4}\right]=\frac{\pi^2\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{-8\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H17. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{7}{4}\right]=\frac{9\pi^2\left\{19\pi-18\sqrt{2}+38\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H18. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{5}{4};\ \frac{7}{4},\ -\frac{1}{4}\right]=\frac{3\pi^2\left\{63\pi-26\sqrt{2}+126\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H19. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{7}{4}\right]=\frac{3\pi^2\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H20. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{11}{4}\right]=\frac{63\pi^2\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{80\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H21. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{9\pi^2\left\{7\pi-10\sqrt{2}+14\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H22. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{18\pi^2\left\{\pi-2\sqrt{2}+2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H23. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{63\pi^2\left\{5\pi-14\sqrt{2}+10\log(1+\sqrt{2})\right\}}{10\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H24. $${}_3F_2\left[\frac{1}{2},\ \frac{3}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{882\pi^2\left\{18\sqrt{2}-5\pi-10\log(1+\sqrt{2})\right\}}{25\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H25. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{3\pi^2\left\{33\pi-38\sqrt{2}+66\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H26. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{63\pi^2\left\{75\pi-98\sqrt{2}+150\log(1+\sqrt{2})\right\}}{640\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H27. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{21\pi^2\left\{15\pi-26\sqrt{2}+30\log(1+\sqrt{2})\right\}}{20\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H28. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{15}{4}\right]=\frac{77\pi^2\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H29. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{3}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{441\pi^2\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{400\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H30. $${}_3F_2\left[\frac{1}{2},\ \frac{5}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{49\pi^2\left\{15\pi-34\sqrt{2}+30\log(1+\sqrt{2})\right\}}{25\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H31. $${}_3F_2\left[\frac{3}{2},\ -\frac{1}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{3}{4}\right]=\frac{3\pi^2\left\{27\pi-34\sqrt{2}+54\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H32. $${}_3F_2\left[\frac{3}{2},\ -\frac{1}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{3}{4}\right]=\frac{4\pi^2\left\{3\pi-4\sqrt{2}+6\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H33. $${}_3F_2\left[\frac{3}{2},\ -\frac{1}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{7}{4}\right]=\frac{9\pi^2\left\{5\pi-6\sqrt{2}+10\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H34. $${}_3F_2\left[\frac{3}{2},\ -\frac{1}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{36\pi^2\left\{5\pi-8\sqrt{2}+10\log(1+\sqrt{2})\right\}}{25\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H35. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ -\frac{5}{4};\ \frac{3}{4},\ \frac{7}{4}\right]=\frac{3\pi^2\left\{58\sqrt{2}+81\pi+162\log(1+\sqrt{2})\right\}}{-128\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H36. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ -\frac{1}{4};\ \frac{3}{4},\ \frac{11}{4}\right]=\frac{21\pi^2\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H37. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{7}{4}\right]=\frac{9\pi^2\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H38. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\right]=\frac{63\pi^2\left\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H39. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{3}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{441\pi^2\left\{46\sqrt{2}-13\pi-26\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H40. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{11}{4}\right]=\frac{147\pi^2\left\{3\pi-10\sqrt{2}+6\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H41. $${}_3F_2\left[\frac{3}{2},\ \frac{3}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{15}{4}\right]=\frac{4851\pi^2\left\{25\pi-86\sqrt{2}+50\log(1+\sqrt{2})\right\}}{100\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H42. $${}_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ -\frac{1}{4};\ \frac{7}{4},\ \frac{11}{4}\Big]=\frac{21\pi^2\big\{39\pi-10\sqrt{2}+78\log(1+\sqrt{2})\big\}}{256\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H43. $${}_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{15}{4}\Big]=\frac{231\pi^2\big\{122\sqrt{2}-15\pi-30\log(1+\sqrt{2})\big\}}{320\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H44. $${}_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ \frac{3}{4};\ \frac{11}{4},\ \frac{11}{4}\Big]=\frac{147\pi^2\big\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\big\}}{32\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H45. $${}_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ \frac{3}{4};\ \frac{11}{4},\ \frac{15}{4}\Big]=\frac{4851\pi^2\big\{35\pi-114\sqrt{2}+70\log(1+\sqrt{2})\big\}}{1280\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H46. $${}_3F_2\Big[\frac{3}{2},\ \frac{5}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{15}{4}\Big]=\frac{539\pi^2\big\{58\sqrt{2}-15\pi-30\log(1+\sqrt{2})\big\}}{40\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H47. $${}_3F_2\Big[\frac{5}{2},\ -\frac{3}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{7}{4}\Big]=\frac{4\pi^2\big\{45\pi-52\sqrt{2}+90\log(1+\sqrt{2})\big\}}{75\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H48. $${}_3F_2\Big[\frac{5}{2},\ -\frac{1}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{7}{4}\Big]=\frac{6\pi^2\big\{15\pi-14\sqrt{2}+30\log(1+\sqrt{2})\big\}}{25\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H49. $${}_3F_2\Big[\frac{5}{2},\ -\frac{1}{2},\ \frac{3}{4};\ \frac{7}{4},\ \frac{11}{4}\Big]=\frac{7\pi^2\big\{45\pi-62\sqrt{2}+90\log(1+\sqrt{2})\big\}}{50\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

H50. $${}_3F_2\Big[\frac{7}{2},\ -\frac{1}{2},\ \frac{7}{4};\ \frac{11}{4},\ \frac{11}{4}\Big]=\frac{49\pi^2\big\{45\pi-22\sqrt{2}+90\log(1+\sqrt{2})\big\}}{1125\sqrt{2}\Gamma^4(\frac{1}{4})}.$$

4.7 *Class I*

By means of the Kummer transformation (3), we derive the following "Class-I" series in terms of $\mathcal{F}_0$-series ($\sigma = b + d - a - c - e$):

$${}_3F_2\left[\left.\begin{matrix} a+\frac{1}{4},\ c-\frac{1}{4},\ e+\frac{1}{4} \\ b+\frac{1}{2},\ d+\frac{3}{4}\end{matrix}\right|1\right]=\frac{\Gamma(\sigma+1)\Gamma(d+\frac{3}{4})}{\Gamma(\sigma+a+\frac{5}{4})\Gamma(d-a+\frac{1}{2})}\,{}_3F_2\left[\left.\begin{matrix} a+\frac{1}{4},\ b-c+\frac{3}{4},\ b-e+\frac{1}{4} \\ b+\frac{1}{2},\ \sigma+a+\frac{5}{4}\end{matrix}\right|1\right].$$

Then the closed formulae for this series (excluding the divergent series) can be obtained from those shown in "Class A".

$$\text{I1. } {}_3F_2\left[-\frac{3}{4},\ -\frac{5}{4},\ -\frac{7}{4};\ -\frac{1}{2},\ -\frac{1}{4}\right]=\frac{\sqrt{\pi}\left\{26\sqrt{2}-63\pi-126\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I2. } {}_3F_2\left[-\frac{3}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{58\sqrt{2}+81\pi+162\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I3. } {}_3F_2\left[-\frac{3}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ \frac{1}{2},\ -\frac{1}{4}\right]=\frac{2\sqrt{\pi}\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I4. } {}_3F_2\left[-\frac{1}{4},\ -\frac{3}{4},\ -\frac{7}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{34\sqrt{2}+21\pi+42\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I5. } {}_3F_2\left[-\frac{1}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{10\sqrt{2}+9\pi+18\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I6. } {}_3F_2\left[-\frac{1}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{42\sqrt{2}+41\pi+82\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I7. } {}_3F_2\left[-\frac{1}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I8. } {}_3F_2\left[\frac{1}{4},\ -\frac{3}{4},\ -\frac{9}{4};\ -\frac{1}{2},\ -\frac{1}{4}\right]=\frac{5\sqrt{\pi}\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I9. } {}_3F_2\left[\frac{1}{4},\ -\frac{3}{4},\ -\frac{9}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{166\sqrt{2}-33\pi-66\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

$$\text{I10. } {}_3F_2\left[\frac{1}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ -\frac{1}{4}\right]=\frac{2\sqrt{\pi}\left\{3\pi-\sqrt{2}+6\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I11. $${}_3F_2\left[\frac{1}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I12. $${}_3F_2\left[\frac{1}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ \frac{1}{2},\ -\frac{1}{4}\right]=\frac{4\sqrt{\pi}\left\{2\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I13. $${}_3F_2\left[\frac{1}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{19\pi-18\sqrt{2}+38\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I14. $${}_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{7}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}+21\pi+42\log(1+\sqrt{2})\right\}}{12\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I15. $${}_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I16. $${}_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{7}{4}\right]=\frac{\sqrt{\pi}\left\{46\sqrt{2}+51\pi+102\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I17. $${}_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I18. $${}_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{\sqrt{\pi}\left\{39\pi-10\sqrt{2}+78\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I19. $${}_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{3}{4}\right]=\frac{2\sqrt{\pi}\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I20. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{9}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}+69\pi+138\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I21. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}+9\pi+18\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I22. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{58\sqrt{2}+81\pi+162\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I23. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{11\pi-2\sqrt{2}+22\log(1+\sqrt{2})\right\}}{6\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I24. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{10\sqrt{2}+9\pi+18\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I25. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{42\sqrt{2}+41\pi+82\log(1+\sqrt{2})\right\}}{512\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I26. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{3}{4}\right]=\frac{2\sqrt{\pi}\left\{\pi+2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I27. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I28. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{3}{4}\right]=\frac{4\sqrt{\pi}\left\{2\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{3\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I29. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{2\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I30. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{34\sqrt{2}+21\pi+42\log(1+\sqrt{2})\right\}}{96\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I31. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I32. $${}_3F_2\left[\frac{1}{4},\ \frac{1}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{50\sqrt{2}-3\pi-6\log(1+\sqrt{2})\right\}}{36\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I33. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{\sqrt{\pi}\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I34. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{146\sqrt{2}+45\pi+90\log(1+\sqrt{2})\right\}}{320\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I35. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{\sqrt{\pi}\left\{122\sqrt{2}-15\pi-30\log(1+\sqrt{2})\right\}}{10\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I36. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{14\sqrt{2}+3\pi+6\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I37. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{6\sqrt{\pi}\left\{\pi-2\sqrt{2}+2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I38. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I39. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{12\sqrt{\pi}\left\{5\pi-14\sqrt{2}+10\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I40. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{35\pi-114\sqrt{2}+70\log(1+\sqrt{2})\right\}}{20\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I41. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{9}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{15\pi-26\sqrt{2}+30\log(1+\sqrt{2})\right\}}{20\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I42. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{24\sqrt{\pi}\left\{5\pi-8\sqrt{2}+10\log(1+\sqrt{2})\right\}}{25\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I43. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{50\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I44. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{13}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{45\pi-62\sqrt{2}+90\log(1+\sqrt{2})\right\}}{75\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I45. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{63\pi-26\sqrt{2}+126\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I46. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{3}{4}\right]=\frac{\sqrt{\pi}\left\{\pi-2\sqrt{2}+2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I47. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{19\pi-18\sqrt{2}+38\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I48. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{21\pi+34\sqrt{2}+42\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I49. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I50. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{39\pi-10\sqrt{2}+78\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I51. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{3}{4}\right]=\frac{8\sqrt{\pi}\left\{\pi-\sqrt{2}+2\log(1+\sqrt{2})\right\}}{3\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I52. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{\sqrt{\pi}\left\{7\pi-10\sqrt{2}+14\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I53. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{55\pi-106\sqrt{2}+110\log(1+\sqrt{2})\right\}}{35\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I54. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{3\pi-2\sqrt{2}+6\log(1+\sqrt{2})\right\}}{6\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I55. $${}_3F_2\left[\frac{1}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{58\sqrt{2}-15\pi-30\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I56. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{634\sqrt{2}-15\pi-30\log(1+\sqrt{2})\right\}}{600\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I57. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{3}{2},\ \frac{15}{4}\right]=\frac{231\sqrt{\pi}\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{400\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I58. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{14\sqrt{\pi}\left\{15\pi-34\sqrt{2}+30\log(1+\sqrt{2})\right\}}{25\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I59. $${}_3F_2\left[\frac{1}{4},\ \frac{9}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{5\pi-6\sqrt{2}+10\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I60. $${}_3F_2\left[\frac{1}{4},\ \frac{9}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{75\pi-98\sqrt{2}+150\log(1+\sqrt{2})\right\}}{160\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I61. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{3\sqrt{\pi}\left\{26\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I62. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\right]=\frac{\sqrt{\pi}\left\{10\sqrt{2}-3\pi-6\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I63. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{122\sqrt{2}-15\pi-30\log(1+\sqrt{2})\right\}}{160\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I64. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{\sqrt{\pi}\left\{15\pi-26\sqrt{2}+30\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I65. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{2\sqrt{\pi}\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{3\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I66. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{5}{4};\ \frac{1}{2},\ \frac{15}{4}\right]=\frac{231\sqrt{\pi}\left\{26\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I67. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I68. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{24\sqrt{\pi}\left\{4\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I69. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{46\sqrt{2}-13\pi-26\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I70. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{5}{4};\ \frac{7}{2},\ \frac{7}{4}\right]=\frac{48\sqrt{\pi}\left\{94\sqrt{2}-25\pi-50\log(1+\sqrt{2})\right\}}{35\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I71. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{5\pi-14\sqrt{2}+10\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I72. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{11}{4}\right]=\frac{6\sqrt{\pi}\left\{85\pi-286\sqrt{2}+170\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I73. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{13}{4};\ \frac{5}{2},\ \frac{15}{4}\right]=\frac{77\sqrt{\pi}\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{48\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I74. $${}_3F_2\left[\frac{3}{4},\ \frac{9}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\right]=\frac{2\sqrt{\pi}\left\{15\pi-14\sqrt{2}+30\log(1+\sqrt{2})\right\}}{25\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I75. $${}_3F_2\left[\frac{3}{4},\ \frac{9}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{7}{4}\right]=\frac{4\sqrt{\pi}\left\{45\pi-62\sqrt{2}+90\log(1+\sqrt{2})\right\}}{75\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I76. $${}_3F_2\left[\frac{3}{4},\ \frac{9}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{11}{4}\right]=\frac{12\sqrt{\pi}\left\{94\sqrt{2}-25\pi-50\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I77. $${}_3F_2\Big[\frac{3}{4},\ \frac{13}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{7}{4}\Big]=\frac{8\sqrt{\pi}\left\{45\pi-52\sqrt{2}+90\log(1+\sqrt{2})\right\}}{225\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I78. $${}_3F_2\Big[\frac{5}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ \frac{1}{2},\ -\frac{1}{4}\Big]=\frac{4\sqrt{\pi}\left\{7\sqrt{2}-6\pi-12\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I79. $${}_3F_2\Big[\frac{5}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{3}{4}\Big]=\frac{\sqrt{\pi}\left\{27\pi-34\sqrt{2}+54\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I80. $${}_3F_2\Big[\frac{5}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{7}{4}\Big]=\frac{\sqrt{\pi}\left\{2\sqrt{2}+21\pi+42\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I81. $${}_3F_2\Big[\frac{5}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{3}{4}\Big]=\frac{2\sqrt{\pi}\left\{3\pi-4\sqrt{2}+6\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I82. $${}_3F_2\Big[\frac{5}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{7}{4}\Big]=\frac{\sqrt{\pi}\left\{33\pi-38\sqrt{2}+66\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I83. $${}_3F_2\Big[\frac{5}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{3}{4}\Big]=\frac{4\sqrt{\pi}\left\{5\pi-6\sqrt{2}+10\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I84. $${}_3F_2\Big[\frac{5}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{7}{4}\Big]=\frac{\sqrt{\pi}\left\{75\pi-98\sqrt{2}+150\log(1+\sqrt{2})\right\}}{20\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I85. $${}_3F_2\Big[\frac{5}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{11}{4}\Big]=\frac{21\sqrt{\pi}\left\{9\pi-22\sqrt{2}+18\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I86. $${}_3F_2\Big[\frac{5}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{7}{4}\Big]=\frac{\sqrt{\pi}\left\{2\sqrt{2}+\pi+2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I87. $${}_3F_2\Big[\frac{5}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{7}{4}\Big]=\frac{18\sqrt{\pi}\left\{6\sqrt{2}-\pi-2\log(1+\sqrt{2})\right\}}{7\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I88. $${}_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{15}{4}\right]=\frac{77\sqrt{\pi}\left\{50\sqrt{2}-3\pi-6\log(1+\sqrt{2})\right\}}{144\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I89. $${}_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{42\sqrt{\pi}\left\{3\pi-10\sqrt{2}+6\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I90. $${}_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{7}{2},\ \frac{11}{4}\right]=\frac{84\sqrt{\pi}\left\{25\pi-86\sqrt{2}+50\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I91. $${}_3F_2\left[\frac{5}{4},\ \frac{5}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{15}{4}\right]=\frac{11\sqrt{\pi}\left\{21\pi-62\sqrt{2}+42\log(1+\sqrt{2})\right\}}{6\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I92. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{100\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I93. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{15}{4}\right]=\frac{77\sqrt{\pi}\left\{58\sqrt{2}-15\pi-30\log(1+\sqrt{2})\right\}}{20\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I94. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{11}{4}\right]=\frac{168\sqrt{\pi}\left\{18\sqrt{2}-5\pi-10\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I95. $${}_3F_2\left[\frac{5}{4},\ \frac{9}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{21\sqrt{\pi}\left\{2\sqrt{2}+5\pi+10\log(1+\sqrt{2})\right\}}{80\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I96. $${}_3F_2\left[\frac{5}{4},\ \frac{9}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{3\sqrt{\pi}\left\{55\pi-106\sqrt{2}+110\log(1+\sqrt{2})\right\}}{40\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I97. $${}_3F_2\left[\frac{7}{4},\ \frac{9}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{11}{4}\right]=\frac{7\sqrt{\pi}\left\{74\sqrt{2}-15\pi-30\log(1+\sqrt{2})\right\}}{250\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I98. $${}_3F_2\left[\frac{7}{4},\ \frac{9}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{15}{4}\right]=\frac{231\sqrt{\pi}\left\{25\pi-86\sqrt{2}+50\log(1+\sqrt{2})\right\}}{5\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I99. $${}_3F_2\left[\frac{7}{4},\ \frac{13}{4},\ -\frac{3}{4};\ \frac{5}{2},\ \frac{11}{4}\right]=\frac{14\sqrt{\pi}\left\{45\pi-22\sqrt{2}+90\log(1+\sqrt{2})\right\}}{1125\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

I100. $${}_3F_2\left[\frac{9}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{3}{2},\ \frac{3}{4}\right]=\frac{8\sqrt{\pi}\left\{15\pi-19\sqrt{2}+30\log(1+\sqrt{2})\right\}}{25\sqrt{2}\Gamma^2(\frac{1}{4})}.$$

4.8 *Class J*

By invoking the Kummer transformation (3), we have following "Class-J" series in terms of the $\mathcal{F}_1$-series ($\sigma = b + d - a - c - e$):

$${}_3F_2\left[\left.\begin{matrix} a+\frac{3}{4},\ c+\frac{1}{4},\ e-\frac{1}{4} \\ b+\frac{1}{2},\ d+\frac{5}{4}\end{matrix}\right|1\right]=\frac{\Gamma(\sigma+1)\Gamma(d+\frac{5}{4})}{\Gamma(\sigma+a+\frac{7}{4})\Gamma(d-a+\frac{1}{2})}\cdot{}_3F_2\left[\left.\begin{matrix} \sigma+\frac{3}{4},\ b-c+\frac{1}{4},\ b-e+\frac{3}{4} \\ b+\frac{1}{2},\ \sigma+a+\frac{7}{4}\end{matrix}\right|1\right]$$

Then the closed formulae for this series (excluding the divergent series) can be deduced from those shown in "Class B".

J1. $${}_3F_2\left[-\frac{1}{4},\ -\frac{5}{4},\ -\frac{7}{4};\ -\frac{1}{2},\ \frac{1}{4}\right]=\frac{\sqrt{\pi}\left\{106\sqrt{2}+55\pi-110\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J2. $${}_3F_2\left[-\frac{1}{4},\ -\frac{3}{4},\ -\frac{9}{4};\ -\frac{1}{2},\ \frac{1}{4}\right]=\frac{5\sqrt{\pi}\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{48\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J3. $${}_3F_2\left[-\frac{1}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{1}{4}\right]=\frac{\sqrt{\pi}\left\{14\sqrt{2}+5\pi-10\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J4. $${}_3F_2\left[-\frac{1}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{114\sqrt{2}+35\pi-70\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J5. $${}_3F_2\left[-\frac{1}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{1}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}-5\pi+10\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J6. $${}_3F_2\left[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{7}{4};\ -\frac{1}{2},\ \frac{1}{4}\right]=\frac{3\sqrt{\pi}\left\{6\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J7. $_3F_2\left[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{1}{4}\right]=\frac{\sqrt{\pi}\left\{4\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J8. $_3F_2\left[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{46\sqrt{2}+13\pi-26\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J9. $_3F_2\left[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{1}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{2\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J10. $_3F_2\left[-\frac{1}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J11. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{58\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{96\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J12. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{122\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{144\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J13. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{10\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J14. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{298\sqrt{2}+87\pi-174\log(1+\sqrt{2})\right\}}{2304\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J15. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{6\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J16. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{166\sqrt{2}+33\pi-66\log(1+\sqrt{2})\right\}}{1152\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J17. $_3F_2\left[\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{36\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J18. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{9}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{266\sqrt{2}+135\pi-270\log(1+\sqrt{2})\right\}}{480\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J19. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{34\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{48\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J20. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{26\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{72\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J21. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{58\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{384\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J22. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J23. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{192\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J24. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{6\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J25. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{10\sqrt{2}+39\pi-78\log(1+\sqrt{2})\right\}}{288\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J26. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{6\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J27. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{1}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{166\sqrt{2}+33\pi-66\log(1+\sqrt{2})\right\}}{1024\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J28. $ {}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{\pi-2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{3}{4})}. $

J29. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{3}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J30. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{7}{4};\ \frac{1}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{122\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{512\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J31. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{24\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J32. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{10\sqrt{2}+39\pi-78\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J33. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{\sqrt{\pi}\left\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\right\}}{12\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J34. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{\sqrt{\pi}\left\{398\sqrt{2}-291\pi+582\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J35. $${}_3F_2\left[\frac{1}{4},\ \frac{3}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{\sqrt{\pi}\left\{274\sqrt{2}+147\pi-294\log(1+\sqrt{2})\right\}}{448\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J36. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{\sqrt{\pi}\left\{266\sqrt{2}+135\pi-270\log(1+\sqrt{2})\right\}}{384\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J37. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{26\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{288\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J38. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{4\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{9\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J39. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{38\sqrt{2}+33\pi-66\log(1+\sqrt{2})\right\}}{432\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J40. $${}_3F_2\left[\frac{1}{4},\ \frac{7}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J41. $${}_3F_2\left[\frac{3}{4},\ -\frac{3}{4},\ -\frac{5}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}-5\pi+10\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J42. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{7}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{106\sqrt{2}+55\pi-110\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J43. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{14\sqrt{2}+5\pi-10\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J44. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ -\frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{114\sqrt{2}+35\pi-70\log(1+\sqrt{2})\right\}}{1024\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J45. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{1}{4}\right]=\frac{\sqrt{\pi}\left\{\sqrt{2}+\pi-2\log(1+\sqrt{2})\right\}}{\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J46. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{10\sqrt{2}+7\pi-14\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J47. $${}_3F_2\left[\frac{3}{4},\ -\frac{1}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{31\sqrt{2}+37\pi-74\log(1+\sqrt{2})\right\}}{512\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J48. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{5}{4}\right]=\frac{\sqrt{\pi}\left\{2\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J49. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J50. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{1}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{50\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J51. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{2\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J52. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{3}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{34\sqrt{2}-21\pi+42\log(1+\sqrt{2})\right\}}{64\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J53. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{5}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{10\sqrt{2}-9\pi+18\log(1+\sqrt{2})\right\}}{6\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J54. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{9}{4};\ \frac{3}{2},\ \frac{13}{4}\right]=\frac{\sqrt{\pi}\left\{38\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J55. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{3\sqrt{\pi}\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J56. $${}_3F_2\left[\frac{3}{4},\ \frac{3}{4},\ \frac{13}{4};\ \frac{5}{2},\ \frac{17}{4}\right]=\frac{13\sqrt{\pi}\left\{74\sqrt{2}+135\pi-270\log(1+\sqrt{2})\right\}}{2400\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J57. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{\sqrt{\pi}\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J58. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{2\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{48\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J59. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{13}{4}\right]=\frac{\sqrt{\pi}\left\{98\sqrt{2}+75\pi-150\log(1+\sqrt{2})\right\}}{256\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J60. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\right\}}{72\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J61. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{13}{4}\right]=\frac{\sqrt{\pi}\left\{73\sqrt{2}-45\pi+90\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J62. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{34\sqrt{2}-21\pi+42\log(1+\sqrt{2})\right\}}{84\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J63. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{1}{2},\ \frac{17}{4}\right]=\frac{13\sqrt{\pi}\left\{634\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{2048\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J64. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{14\sqrt{2}-3\pi+6\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J65. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{3}{2},\ \frac{17}{4}\right]=\frac{13\sqrt{\pi}\left\{146\sqrt{2}-45\pi+90\log(1+\sqrt{2})\right\}}{1024\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J66. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{3\pi-2\sqrt{2}-6\log(1+\sqrt{2})\right\}}{3\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J67. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{51\pi-46\sqrt{2}-102\log(1+\sqrt{2})\right\}}{16\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J68. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{7}{4};\ \frac{7}{2},\ \frac{9}{4}\right]=\frac{10\sqrt{\pi}\left\{7\pi-6\sqrt{2}-14\log(1+\sqrt{2})\right\}}{7\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J69. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{11}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{21\pi-2\sqrt{2}-42\log(1+\sqrt{2})\right\}}{56\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J70. $${}_3F_2\left[\frac{3}{4},\ \frac{5}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{314\sqrt{2}-273\pi+546\log(1+\sqrt{2})\right\}}{196\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J71. $${}_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ -\frac{3}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{2\sqrt{2}-5\pi+10\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J72. $${}_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{3}{2},\ \frac{17}{4}\right]=\frac{13\sqrt{\pi}\left\{134\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{640\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J73. $${}_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{\sqrt{\pi}\left\{22\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J74. $${}_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ \frac{9}{4};\ \frac{7}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{106\sqrt{2}-105\pi+210\log(1+\sqrt{2})\right\}}{14\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J75. $${}_3F_2\left[\frac{3}{4},\ \frac{7}{4},\ \frac{13}{4};\ \frac{5}{2},\ \frac{17}{4}\right]=\frac{13\sqrt{\pi}\left\{82\sqrt{2}-45\pi+90\log(1+\sqrt{2})\right\}}{240\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J76. $${}_3F_2\left[\frac{3}{4},\ \frac{9}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{13}{4}\right]=\frac{\sqrt{\pi}\left\{134\sqrt{2}-15\pi+30\log(1+\sqrt{2})\right\}}{160\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J77. $${}_3F_2\left[\frac{3}{4},\ \frac{9}{4},\ -\frac{1}{4};\ \frac{5}{2},\ \frac{13}{4}\right]=\frac{3\sqrt{\pi}\left\{114\sqrt{2}+35\pi-70\log(1+\sqrt{2})\right\}}{560\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J78. $${}_3F_2\left[\frac{3}{4},\ \frac{9}{4},\ \frac{11}{4};\ \frac{7}{2},\ \frac{13}{4}\right]=\frac{5\sqrt{\pi}\left\{-82\sqrt{2}+105\pi-210\log(1+\sqrt{2})\right\}}{49\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J79. $${}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{\sqrt{\pi}\left\{634\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{576\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J80. $${}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{50\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{288\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J81. $${}_3F_2\left[\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{4};\ \frac{3}{2},\ \frac{9}{4}\right]=\frac{5\sqrt{\pi}\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{144\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J82. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{5}{4};\ \frac{1}{2},\ \frac{9}{4}\right]=\frac{\sqrt{\pi}\left\{30\log(1+\sqrt{2})-15\pi-74\sqrt{2}\right\}}{144\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J83. $${}_3F_2\left[\frac{5}{4},\ \frac{7}{4},\ -\frac{1}{4};\ \frac{1}{2},\ \frac{13}{4}\right]=\frac{9\sqrt{\pi}\left\{5\pi-2\sqrt{2}-10\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J84. $_3F_2\left[\frac{5}{4}, \frac{7}{4}, -\frac{1}{4}; \frac{3}{2}, \frac{9}{4}\right] = \frac{5\sqrt{\pi}\left\{10\sqrt{2}+3\pi-6\log(1+\sqrt{2})\right\}}{108\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J85. $_3F_2\left[\frac{5}{4}, \frac{7}{4}, -\frac{1}{4}; \frac{3}{2}, \frac{13}{4}\right] = \frac{\sqrt{\pi}\left\{122\sqrt{2}+15\pi-30\log(1+\sqrt{2})\right\}}{192\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J86. $_3F_2\left[\frac{5}{4}, \frac{7}{4}, -\frac{1}{4}; \frac{5}{2}, \frac{9}{4}\right] = \frac{5\sqrt{\pi}\left\{21\pi-2\sqrt{2}-42\log(1+\sqrt{2})\right\}}{128\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J87. $_3F_2\left[\frac{5}{4}, \frac{7}{4}, \frac{7}{4}; \frac{3}{2}, \frac{17}{4}\right] = \frac{65\sqrt{\pi}\left\{26\sqrt{2}-\pi+2\log(1+\sqrt{2})\right\}}{512\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J88. $_3F_2\left[\frac{5}{4}, \frac{7}{4}, \frac{7}{4}; \frac{5}{2}, \frac{13}{4}\right] = \frac{15\sqrt{\pi}\left\{10\sqrt{2}-9\pi+18\log(1+\sqrt{2})\right\}}{8\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J89. $_3F_2\left[\frac{5}{4}, \frac{7}{4}, \frac{7}{4}; \frac{7}{2}, \frac{13}{4}\right] = \frac{5\sqrt{\pi}\left\{86\sqrt{2}-87\pi+174\log(1+\sqrt{2})\right\}}{4\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J90. $_3F_2\left[\frac{5}{4}, \frac{7}{4}, \frac{11}{4}; \frac{5}{2}, \frac{17}{4}\right] = \frac{195\sqrt{\pi}\left\{34\sqrt{2}-21\pi+42\log(1+\sqrt{2})\right\}}{896\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J91. $_3F_2\left[\frac{5}{4}, \frac{7}{4}, \frac{11}{4}; \frac{7}{2}, \frac{13}{4}\right] = \frac{45\sqrt{\pi}\left\{7\pi-6\sqrt{2}-14\log(1+\sqrt{2})\right\}}{14\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J92. $_3F_2\left[\frac{5}{4}, \frac{11}{4}, -\frac{1}{4}; \frac{5}{2}, \frac{9}{4}\right] = \frac{5\sqrt{\pi}\left\{34\sqrt{2}+21\pi-42\log(1+\sqrt{2})\right\}}{441\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J93. $_3F_2\left[\frac{5}{4}, \frac{11}{4}, -\frac{1}{4}; \frac{5}{2}, \frac{13}{4}\right] = \frac{3\sqrt{\pi}\left\{86\sqrt{2}+105\pi-210\log(1+\sqrt{2})\right\}}{784\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J94. $_3F_2\left[\frac{7}{4}, -\frac{1}{4}, -\frac{3}{4}; -\frac{1}{2}, \frac{9}{4}\right] = \frac{25\sqrt{\pi}\left\{22\sqrt{2}+9\pi-18\log(1+\sqrt{2})\right\}}{1536\sqrt{2}\Gamma^2(\frac{3}{4})}.$

J95. $${}_3F_2\Big[\frac{7}{4}, \frac{7}{4}, \frac{9}{4}; \frac{5}{2}, \frac{17}{4}\Big] = \frac{39\sqrt{\pi}\left\{5\pi - 2\sqrt{2} - 10\log(1+\sqrt{2})\right\}}{32\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J96. $${}_3F_2\Big[\frac{7}{4}, \frac{9}{4}, -\frac{1}{4}; \frac{3}{2}, \frac{13}{4}\Big] = \frac{\sqrt{\pi}\left\{26\sqrt{2} + 15\pi - 30\log(1+\sqrt{2})\right\}}{80\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J97. $${}_3F_2\Big[\frac{7}{4}, \frac{9}{4}, -\frac{1}{4}; \frac{5}{2}, \frac{13}{4}\Big] = \frac{\sqrt{\pi}\left\{298\sqrt{2} - 105\pi + 210\log(1+\sqrt{2})\right\}}{280\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J98. $${}_3F_2\Big[\frac{7}{4}, \frac{9}{4}, \frac{11}{4}; \frac{7}{2}, \frac{17}{4}\Big] = \frac{65\sqrt{\pi}\left\{106\sqrt{2} - 105\pi + 210\log(1+\sqrt{2})\right\}}{56\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J99. $${}_3F_2\Big[\frac{9}{4}, -\frac{1}{4}, -\frac{1}{4}; \frac{3}{2}, \frac{13}{4}\Big] = \frac{\sqrt{\pi}\left\{1414\sqrt{2} - 15\pi + 30\log(1+\sqrt{2})\right\}}{1600\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

J100. $${}_3F_2\Big[\frac{9}{4}, \frac{11}{4}, -\frac{1}{4}; \frac{5}{2}, \frac{13}{4}\Big] = \frac{\sqrt{\pi}\left\{422\sqrt{2} + 105\pi - 210\log(1+\sqrt{2})\right\}}{980\sqrt{2}\Gamma^2(\frac{3}{4})}.$$

5. Conclusions

As widely known, the hypergeometric series plays an important role in mathematics, physics and applied sciences. The ten classes of ${}_3F_2$-series formulae presented in this paper may serve as a useful reference documentary for mathematicians and applied scientists in their research works. Among the seven hundreds of the ${}_3F_2$-series evaluations recorded in this paper, we succeed in locating the only series labeled by "C26" in an unpublished draft by Campbell and Abrarov [15] (HAL-01897255, Corollary 7), who found it by specializing in a quotient expression of two ${}_3F_2$-series. Instead, we recovered it directly by applying transformation (2) to the $\mathcal{F}_0(0, 0, 0; 1, 0)$ series labeled by "A22".

Conflict of interest

The authors declare that they have no conflict of interest.

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