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Research Article

The Impact of Media Coverage on Obesity

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Abstract: In this paper, we develop a deterministic compartmental model for the obesity dynamics. Contrary to other contributions on this subject we explore the impact of the media on the spreading of this phenomenon in a constant population. Stability analysis shows that the disease-free equilibrium point is globally asymptotically stable when the number of reproduction is less than unity. We also show that the endemic equilibrium point when it exists is globally asymptotically stable under some conditions.

Keywords**:** overweight and obesity, equilibrium, stability

MSC: 92B05

1. Introduction

Overweight and Obesity are growing health problems worldwide. In general individuals are classified according to their weight by using the Body Mass Index (BMI) which is obtained by dividing the body mass over the square of the body height. If the BMI is less than 18.5 the person is classified underweight. If this measure is between 18.5 and 24.9, the weight is normal, and if it is between 25 and 30 the person is overweight. Once the BMI score is greater than 30 the person is considered obese [1]. Over the last decade, the incidence of obesity increased dramatically [2, 3]. Such phenomenon is particularly serious in the US but is also observed globally [4]. In France more than 15 percent of the population is obese and almost one in two women is in the situation of abdominal obesity.

Different factors may contribute to overweight and obesity such as an excessive intake of calories over the daily expenditure of energy, culture, genes, etc. Obesity entails substantial health costs. For example, obese individuals are more likely to suffer from diseases such as type 2 diabetes, cardiovascular diseases, hypertension and stroke, metabolic syndrome, cancer, osteoarthritis, and many more [5-7]. Besides health issues, obesity has a notable impact on economy [8] and [9].

Nowadays in many countries, obesity has become a prevalent problem which is mostly overlooked at the beginning. In some countries obesity captured more than 70 percent of the total population [10]. Obesity is now considered as the ‘‘Pandemia of the 21st Century’’. We are in an epidemic situation and some preventive actions are needed. Obesity has been identified as a contagious problem which is spreading with contacts over social networks [11-15] and was studied theoretically using epidemic models [13, 16, 17]. In [15] the authors claimed that prevention strategies are more

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effective by analyzing a variant of SIS epidemic model numerically. In [18] the authors experimented a system of SIS difference equation model and drew a similar conclusion. In [19] a new model is introduced for taking into account both social and nonsocial contagion risks of obesity. A way to complete the actions is to consider the role of the media that could be important. This importance has been underlined in [20] and [21] for the AIDS epidemic and in [22] and the references therein for the SARS coronavirus. To precise the intervention of media on diseases, the authors use in their model an incidence function with a key role in the qualitative description.

The paper is organized as follows. In Section 2 we introduce a new model for the spread of obesity taking in account the importance of the coverage media in order to make prevention. The media is characterized in this context with a reasonable incidence function. In Section 3 we study the existence of the equilibrium points and show that the existence of an endemic point for obesity depends on the properties of the incidence functions and the reproduction number $\mathcal{R}'_0$. In Section 4 we shall study the local and global stability of the disease-free equilibrium point and the unique endemic-point when it exists. We finish the paper with some numerical illustrations and some comments.

2. The model

We choose to build a model for which the population has a normalized constant size. Throughout the paper, the population at time t is divided into three separate classes of individuals: the proportion of normal weight individuals, $S(t)$; the proportion of overweight individuals, $O_w(t)$; the proportion of obese individuals, $O_b(t)$. We assume that the changes from the normal weight compartment to the others are essentially by contact, social pressure, or unhealthy lifestyle. Contrary to the model introduced in [17], here the arrival in the compartment of the obese is attenuated thanks to the media coverage as it has been done in [22, 23] for epidemic models.

Let us denote by β_1 (resp. β_{21}) the transmission rate by social pressure to adopt an unhealthy lifestyle and contact with the O_w group (resp. the O_b) group. β_{22} is the maximum reduced contact rate due to the presence of the media. γ is the rate at which overweight individuals become obese individuals due to an unhealthy lifestyle. δ denotes the rate at which overweight individuals move to the S group. η is the rate at which obese individuals with a healthy lifestyle move to the group of overweight individuals. The Figure 1 describes the transmission.

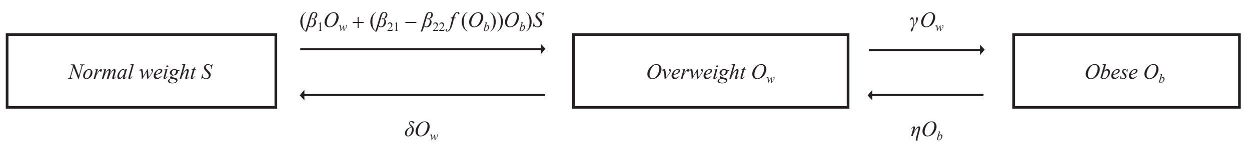

Figure 1. Diagram of the model

And we obtain the following system:

$$\begin{cases} \dfrac{dS}{dt} = \delta O_w - \beta_1 O_w S - (\beta_{21} - \beta_{22} f(O_b)) O_b S, \\[2ex] \dfrac{dO_w}{dt} = \beta_1 O_w S + (\beta_{21} - \beta_{22} f(O_b)) O_b S - (\gamma + \delta) O_w + \eta O_b, \\[2ex] \dfrac{dO_b}{dt} = \gamma O_w - \eta O_b, \end{cases} \tag{1}$$

with the normalization condition $S + O_w + O_b = 1$.

In what follows, we suppose that the incidence function $f : [0, 1] \rightarrow [0, 1]$ satisfies

$$f(0)=0,\ \ f'(O_b)\geq 0,\ \ f(O_b)\leq 1 \tag{2}$$

We also suppose in the sequel that $\beta_{21} > \beta_{22}$.

The case where $\beta_{22} = 0$ has been studied in [17]. In this case the dynamical system has two equilibrium points. The first one ($S_0^* = 1$, $O_{w0}^* = 0$, $O_{b0}^* = 0$) corresponds to a population with no obese people. This disease-free equilibrium point is locally asymptotically stable if $\beta_1 + \beta_2 \dfrac{\gamma}{\eta} < \delta$ and unstable otherwise (see [17]). The second equilibrium point (endemic-equilibrium point)

$$\left(S_1^* = \frac{\delta(\gamma+\eta)}{\eta\left(1+\frac{\gamma}{\eta}\right)\left(\beta_1+\beta_2\frac{\gamma}{\eta}\right)},\ O_{w1}^* = \frac{\left(\beta_1+\beta_2\frac{\gamma}{\eta}\right)-\delta}{\left(1+\frac{\gamma}{\eta}\right)\left(\beta_1+\beta_2\frac{\gamma}{\eta}\right)},\ O_{b1}^* = \frac{\gamma}{\eta}\frac{\left(\beta_1+\beta_2\frac{\gamma}{\eta}\right)-\delta}{\left(1+\frac{\gamma}{\eta}\right)\left(\beta_1+\beta_2\frac{\gamma}{\eta}\right)} \right)$$

corresponds to the case where there is a significant group of obeses. When $-\beta_2 \dfrac{\gamma}{\eta} < \beta_1 - \delta < \gamma + \eta$, this endemic equilibrium point is locally asymptotically stable (see [17]).

The basic reproduction number $\mathcal{R}_0$ (was first introduced by MacDonald [24], and appears in a large class of situations in epidemiology, see [25] for a suitable definition from Diekmann and Heersterbeek and the monograph [26] from Brauer and Castillo-Chavez.

In [17] $\mathcal{R}_0 = \dfrac{\beta_1\eta + \beta_2\gamma}{\eta\delta}$ and it is easy to see that the endemic equilibrium point can be rewritten

$$\left(S_1^* = \frac{\delta(\gamma+\eta)}{\eta\left(1+\frac{\gamma}{\eta}\right)\left(\beta_1+\beta_2\frac{\gamma}{\eta}\right)},\ O_{w1}^* = \frac{\delta(\mathcal{R}_0-1)}{\left(1+\frac{\gamma}{\eta}\right)\left(\beta_1+\beta_2\frac{\gamma}{\eta}\right)},\ O_{b1}^* = \frac{\gamma}{\eta}\frac{\delta(\mathcal{R}_0-1)}{\left(1+\frac{\gamma}{\eta}\right)\left(\beta_1+\beta_2\frac{\gamma}{\eta}\right)} \right).$$

3. Equilibrium points

In this section, we study the existence of equilibrium points for the new model introduced in this paper. Remark that since $S(t) = 1 - O_b(t) - O_w(t)$ the study of system (1) could be reduced to

$$\begin{cases} \dfrac{dO_w}{dt} = -\beta_1 O_w^2 + \left(\beta_1 - (\gamma+\delta) - \left(\beta_1 + \beta_{21} - \beta_{22} f(O_b)\right)O_b\right)O_w + \\ \left(\beta_{21} - \beta_{22} f(O_b) + \eta\right)O_b - \left(\beta_{21} - \beta_{22} f(O_b)\right)O_b^2, \\ \\ \dfrac{dO_b}{dt} = \gamma O_w - \eta O_b. \end{cases} \tag{3}$$

It is easy to see that an equilibrium point (O_w^*, O_b^*) must check the equality $O_w^* = \dfrac{\eta}{\gamma} O_b^*$. Thus O_b^* is a solution of the equation

$$\left(\beta_{21} + \beta_1\frac{\eta}{\gamma} + \beta_{21}\frac{\eta}{\gamma} + \beta_1\frac{\eta^2}{\gamma^2} - \beta_{22}\left(1+\frac{\eta}{\gamma}\right)f(O_b)\right)O_b^2 - \left(\left(\beta_1 - (\gamma+\delta)\right)\frac{\eta}{\gamma} + \beta_{21} - \beta_{22} f(O_b) + \eta\right)O_b = 0. \tag{4}$$

We obtain the disease-free equilibrium point $E_0 = (0, 0)$ but the others are not easy to be obtained in presence of the function f. In the sequel, we will give some conditions in order to obtain existence and uniqueness of an endemic equilibrium point.

The new basic reproduction number has been calculated using the next generation method (see [27-29]). So we obtain $\mathcal{R}_0' = \dfrac{\beta_1\eta + \beta_{21}\gamma}{\eta\delta}$ for the new model.

Theorem 1 We suppose that the function f satisfies (2) and $\sup_{x\in[0,1]} f'(x) \le \dfrac{\beta_1}{\beta_{22}}\left(1+\dfrac{\eta}{\gamma}\right)$. If $\mathcal{R}_0' \le 1$ then the model (1) has no endemic equilibria and there exists a unique endemic equilibrium point if $\mathcal{R}_0' > 1$.

Remark An example of functions satisfied the conditions of the theorem is the one taken for the numerical experiments.

Proof. The equilibrium system (4) inspires us to consider the function Ψ defined by

$$\Psi(O_b) = \left(\beta_{21} + (\beta_1 + \beta_{21})\frac{\eta}{\gamma} + \beta_1\frac{\eta^2}{\gamma^2} - \beta_{22}\left(1+\frac{\eta}{\gamma}\right)f(O_b)\right)O_b - \left((\beta_1 - \delta)\frac{\eta}{\gamma} + \beta_{21} - \beta_{22}f(O_b)\right). \tag{5}$$

Since $f(0) = 0$ we have

$$\Psi(0) = -\left(\beta_1\frac{\eta}{\gamma} + \beta_{21} - \delta\frac{\eta}{\gamma}\right) = -\left(1 - \frac{1}{\mathcal{R}_0'}\right)\left(\beta_1\frac{\eta}{\gamma} + \beta_{21}\right). \tag{6}$$

If $\mathcal{R}_0' > 1$ it is clear that $\Psi(0) < 0$.

Note that

$$\Psi(1) = \beta_{21} + \beta_1\frac{\eta}{\gamma} + \beta_{21}\frac{\eta}{\gamma} + \beta_1\frac{\eta^2}{\gamma^2} - \beta_{22}\left(1+\frac{\eta}{\gamma}\right)f(1) - \left((\beta_1 - \delta)\frac{\eta}{\gamma} + \beta_{21} - \beta_{22}f(1)\right) \tag{7}$$

After simplifications

$$\Psi(1) = (\beta_{21} - \beta_{22}f(1))\frac{\eta}{\gamma} + \beta_1\frac{\eta^2}{\gamma^2} + \delta\frac{\eta}{\gamma}. \tag{8}$$

Assumption (2) and the fact that $\beta_{21} - \beta_{22} > 0$ implies that $\Psi(1) > 0$.

The function Ψ is derivable, we have

$$\Psi'(O_b) = -\beta_{22}f'(O_b)\left(1+\frac{\eta}{\gamma}\right)O_b + \beta_{22}f'(O_b) + \beta_{21} + (\beta_1 + \beta_{21})\frac{\eta}{\gamma} + \beta_1\frac{\eta^2}{\gamma^2} - \beta_{22}\left(1+\frac{\eta}{\gamma}\right)f(O_b) \tag{9}$$

and the assumption $\sup_{x\in[0,1]} f'(x) \le \dfrac{\beta_1}{\beta_{22}}\left(1+\dfrac{\eta}{\gamma}\right)$ implies the positivity of Ψ'.

We have proved that $\Psi(1) > 0$, $\Psi(0) < 0$ if $\mathcal{R}_0' > 1$ and in addition Ψ is monotonically increasing. Thus Ψ has an unique positive root in the interval (0, 1).

In what follow, we denote by O_b^* this unique root, by O_w^* the associated value given by $O_w^* = \dfrac{\eta}{\gamma}O_b^*$ and by $E^* = (O_w^*, O_b^*)$ the associated equilibrium point.

4. Stability analysis

In this section, we study in a first time the local stability of the disease-free equilibrium point E_0 and the endemic equilibrium point E^* then we study the global stability of both points.

4.1 *Local stability*

For the moment we have this stability result for E_0.

Theorem 2 The disease-free equilibrium point E_0 is locally asymptotically stable if $\mathcal{R}_0' < 1$ and unstable if $\mathcal{R}_0' > 1$.

Proof. The Jacobian matrix $J = (J_{ij})$ of system (3) is a 2 × 2 matrix whose components are

$$J_{11} = -2\beta_1 O_w + \beta_1 - (\gamma + \delta) - \left(\beta_1 + \beta_{21} - \beta_{22} f(O_b)\right) O_b,$$

$$J_{12} = -\left(\beta_1 + \beta_{21} - \beta_{22} f(O_b)\right) O_w - \beta_{22} f'(O_b) O_b (1 - O_w - O_b)$$

$$+\beta_{21} - \beta_{22} f(O_b) + \eta - 2\left(\beta_{21} - \beta_{22} f(O_b)\right) O_b,$$

$$J_{21} = \gamma \text{ and } J_{22} = -\eta.$$

Since $f(0) = 0$, the characteristic polynomial of J evaluated at $E_0 = (0, 0)$ is given by

$$\lambda^2 + (\eta - \beta_1 + \gamma + \delta)\lambda + \eta(\delta - \beta_1) - \gamma\beta_{21}.$$

Via the Routh-Hurwitz criterion, this polynomial is stable if and only if

$$\eta - \beta_1 + \gamma + \delta > 0 \text{ and } \eta\left(\delta - \left(\beta_1 + \beta_{21}\frac{\gamma}{\eta}\right)\right) > 0.$$

These two conditions reduce to $\beta_1 + \beta_{21}\dfrac{\gamma}{\eta} < \delta$ which is equivalent to $\mathcal{R}_0' < 1$.

The stability of the characteristic polynomial is equivalent to the fact the two eigenvalues of $J(E_0)$ have negative real parts, which means that E_0 is locally asymptotically stable and achieves the proof.

Now we study the local asymptotical stability of the endemic equilibrium point when $\mathcal{R}_0' > 1$. When the point is explicitly given the simple application of Routh-Hurwitz criterion allows to conclude. Here, the situation is different, the endemic-point is not available and we only have a result on existence and uniqueness.

Theorem 3 We suppose that $\mathcal{R}_0' > 1$, $\gamma \le \eta$, $\delta < \beta_1$ and $\beta_1 < \gamma + \eta + \delta$. Then the endemic-equilibrium point E^* is locally asymptotically stable.

Proof. In order for E^* to be locally asymptotically stable, the following must hold:

$$Tr(J(E^*)) < 0 \text{ and } det(J(E^*)) > 0$$

where $J(E^*)$ is the Jacobian matrix of the system (3) evaluated at E^*.

We have

$$Tr(J(E^*)) = -2\beta_1 O_w^* + \beta_1 - (\gamma + \delta) - \left(\beta_1 + \beta_{21} - \beta_{22} f(O_b^*)\right) O_b^* - \eta.$$

This quantity is strictly negative when $\beta_1 < \gamma + \eta + \delta$.
We also have

$$det(J(E^*)) = \eta\left(2\beta_1 O_w^* - \beta_1 + (\gamma+\delta) + \left(\beta_1 + \beta_{21} - \beta_{22} f(O_b^*)\right)O_b^*\right)$$

$$+\gamma\left(\left(\beta_1 + \beta_{21} - \beta_{22} f(O_b^*)\right)O_w^* + \beta_{22} f'(O_b^*)O_b(1 - O_w^* - O_b^*)\right)$$

$$-\beta_{21} + \beta_{22} f(O_b^*) - \eta + 2\left(\beta_{21} - \beta_{22} f(O_b^*)\right)O_b^*\Big). \quad (10)$$

The fact that $E^* \neq (0, 0)$ and $O_w^* = \dfrac{\eta}{\gamma} O_b^*$ yields

$$det(J(E^*)) = \eta\left(\delta - \left(\beta_1 + \beta_{21}\frac{\gamma}{\eta}\right)\right) + \eta\left(\beta_1 + \beta_{21} - \beta_{22} f(O_b^*)\right)O_b^*$$

$$+\eta\left(\beta_1 + \beta_{21} - \beta_{22} f(O_b^*)\right)O_b^* + 2\gamma(\beta_{21} - \beta_{22} f(O_b^*))O_b^*$$

$$+2\beta_1\frac{\eta^2}{\gamma}O_b^* + \gamma\beta_{22} f(O_b^*) + \gamma\beta_{22} f'(O_b^*)O_b^*(1 - O_b^* - O_w^*). \quad (11)$$

Since O_b^* is a root of Ψ, we have

$$\left(1 + \frac{\eta}{\gamma}\right)(\beta_{21} - \beta_{22} f(O_b^*))O_b^* = (\beta_1 - \delta)\frac{\eta}{\gamma} + \beta_{21} - \beta_{22} f(O_b^*) - \beta_1\frac{\eta}{\gamma}\left(1 + \frac{\eta}{\gamma}\right)O_b^*$$

and thus

$$det(J(E^*)) = \eta\left(\delta - \beta_1 - \beta_{21}\frac{\gamma}{\eta}\right) + 2\beta_1\frac{\eta^2}{\gamma}O_b^* + 2\eta\beta_1 O_b^*$$

$$+\frac{2\eta\gamma}{\gamma+\eta}\left((\beta_1 - \delta)\frac{\eta}{\gamma} + \beta_{21} - \beta_{22} f(O_b^*) - \beta_1\frac{\eta}{\gamma}\left(1 + \frac{\eta}{\gamma}\right)O_b^*\right) + \gamma\beta_{22} f(O_b^*)$$

$$+2\gamma\left(\beta_{21} - \beta_{22} f(O_b^*)\right)O_b^* + \gamma\beta_{22} f'(O_b^*)O_b^*(1 - O_b^* - O_w^*) \quad (12)$$

which can be rewritten

$$det(J(E^*)) = \eta\left(\delta - \beta_1 - \beta_{21}\frac{\gamma}{\eta}\right) + 2\eta\beta_1 O_b^*$$

$$+\frac{2\eta^2}{\gamma+\eta}(\beta_1 - \delta) + \frac{2\eta\gamma}{\gamma+\eta}\beta_{21} + \gamma\beta_{22} f(O_b^*)\left(1 - \frac{2\eta}{\gamma+\eta}\right)$$

$$+2\gamma\left(\beta_{21}-\beta_{22}f(O_b^*)\right)O_b^*+\gamma\beta_{22}f'(O_b^*)O_b^*(1-O_b^*-O_w^*). \tag{13}$$

The assumptions (2) and $\beta_{21} > \beta_{22}$ imply that

$$det(J(E^*)) > \eta\left(\delta-\beta_1-\beta_{21}\frac{\gamma}{\eta}\right)+\frac{2\eta^2}{\gamma+\eta}(\beta_1-\delta)$$

$$+\frac{2\eta\gamma}{\gamma+\eta}\beta_{21}+\gamma\beta_{22}f(O_b^*)\left(1-\frac{2\eta}{\gamma+\eta}\right) \tag{14}$$

and

$$det(J(E^*)) > \eta\left[\delta\left(1-\frac{2\eta}{\gamma+\eta}\right)+\beta_1\left(\frac{2\eta}{\gamma+\eta}-1\right)\right.$$

$$\left.+\beta_{21}\left(\frac{2\gamma}{\gamma+\eta}-\frac{\gamma}{\eta}\right)+\beta_{22}\frac{\gamma}{\eta}f(O_b^*)\left(1-\frac{2\eta}{\gamma+\eta}\right)\right] \tag{15}$$

which can be rewritten as

$$det(J(E^*)) > \eta\left(1-\frac{2\eta}{\gamma+\eta}\right)\left[\delta-\beta_1-\beta_{21}\frac{\gamma}{\eta}+\beta_{22}f(O_b^*)\frac{\gamma}{\eta}\right]. \tag{16}$$

The fact that $\gamma \leq \eta$, $\delta < \beta_1$ and assumption (2) imply that the right side of the previous inequalities is positive, which completes the proof.

4.2 *Global stability*

In this subsection, we study the global behavior of the equilibria point for the system (3). The following theorem gives the global stability property of the disease-free equilibrium point E_0.

Theorem 4 The disease-free equilibrium point E_0 is globally asymptotically stable whenever $\beta_1 + \gamma - \delta < 0$ and $\beta_{21} - \eta < 0$.

Proof. To establish the global stability, we consider the function L defined by $L(t) = O_w(t) + 2O_b(t)$. We have

$$\dot{L} = -\beta_1 O_w^2+\left(\beta_1+\gamma-\delta-\left(\beta_1+\beta_{21}-\beta_{22}f(O_b)\right)O_b\right)O_w$$

$$+\left(\beta_{21}-\beta_{22}f(O_b)-\eta\right)O_b-\left(\beta_{21}-\beta_{22}f(O_b)\right)O_b^2. \tag{17}$$

Assumption (2), the fact that $\beta_{21} - \beta_{22} > 0$ and the assumption taken in the theorem imply that $\dot{L} \leq 0$. With these assumptions, we remark that $\dot{L} = 0$ if and only if $O_w = O_b = 0$ and then at the point E_0. We have proved that L is a Lyapunov function.

We also remark that when $\beta_1 + \gamma - \delta < 0$ and $\beta_{21} - \eta < 0$ then $\mathcal{R}_0' < 1$ thus E_0 is the only equilibrium point and the LaSalle's invariance principle implies that E_0 is globally asymptotically stable.

The next theorem gives an important result without additional conditions concerning the global stability of the endemic-equilibrium point.

Theorem 5 Under the assumptions of Theorem 3, the endemic-equilibrium point E^* is globally asymptotically stable.

Proof. If we denote by W and U the right side of the first and second line of the system (3) respectively, that is

$$W = -\beta_1 O_w^2 + \left(\beta_1 - (\gamma+\delta) - \left(\beta_1 + \beta_{21} - \beta_{22} f(O_b)\right) O_b\right) O_w$$

$$+\left(\beta_{21} - \beta_{22} f(O_b) + \eta\right) O_b - \left(\beta_{21} - \beta_{22} f(O_b)\right) O_b^2,$$

$$U = \gamma O_W - \eta O_b, \tag{18}$$

and we have to choose a Dulac function G and show that

$$\frac{\partial(GW)}{\partial O_w} + \frac{\partial(GU)}{\partial O_b} < 0.$$

With this, the Dulac criterion can be applied [30] which implies the nonexistence of close orbits and gives the desired conclusion.

Let us consider $G = \dfrac{1}{O_b . O_w}$. We have

$$\frac{\partial(GW)}{\partial O_w} = -\frac{\beta_1}{O_b} - \frac{1}{O_w^2}\left(\beta_{21} - \beta_{22} f(O_b) + \eta\right) + \frac{O_b}{O_w^2}\left(\beta_{21} - \beta_{22} f(O_b)\right) \tag{19}$$

and $\dfrac{\partial(GU)}{\partial O_b} = -\dfrac{\gamma}{O_b^2}$.

After simplifications, we obtain

$$\frac{\partial(GW)}{\partial O_w} + \frac{\partial(GU)}{\partial O_b} = -\frac{\beta_1}{O_b} - \frac{\gamma}{O_b^2} - \frac{\eta}{O_w^2} + \frac{O_b - 1}{O_w^2}\left(\beta_{21} - \beta_{22} f(O_b)\right) \tag{20}$$

then

$$\frac{\partial(GW)}{\partial O_w} + \frac{\partial(GU)}{\partial O_b} < 0$$

and the proof is complete.

5. Numerical experiments

In this section, our algorithm is tested with different values of the coefficients. All the experiments are implemented in Mapple 2021.2 and performed on Apple McBook Air 10.1 with Apple M1 chip (8-core CPU with 4 high-performance cores and 4 energy-efficient cores, GPU up to 8 cores, Neural Engine 16 cores) and RAM 8.00 GB.

For the variable at the origin, we take

$$S(0) = 0.8, \;\; O_w(0) = 0.15, \;\; O_b(0) = 0.05,$$

and for the incidence function, a good choice inspired from [5] is $f(O_b) = \dfrac{O_b}{0.5 + O_b}$.

In all the following figures, S is in red, O_w in blue and O_b in green. For the two first Figures 2, and 3, we take $\beta_1 = 0.001$, $\beta_{21} = 0.007$, $\delta = 0.002$, $\gamma = 0.0015$ and $\eta = 0.1$ in order to be in the case $\mathcal{R}_0' < 1$. We know that in this case $\mathcal{R}_0' < 1$, there is only one equilibrium state for which $S = 1$, $O_w = O_b = 0$ and this is what we obtain with Figures 2 and 3, however the value of β_{22}.

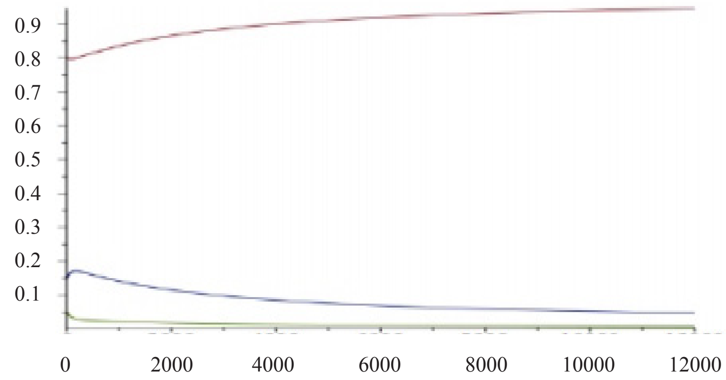

Figure 2. $\mathcal{R}_0' < 1$ and $\beta_{22} = 0$

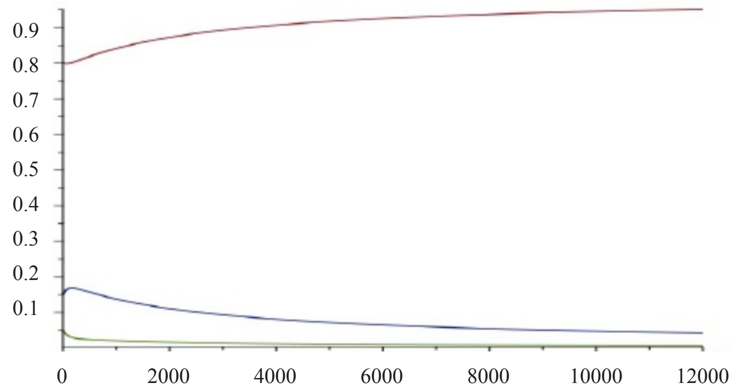

Figure 3. $\mathcal{R}_0' < 1$ and $\beta_{22} = 0.006$

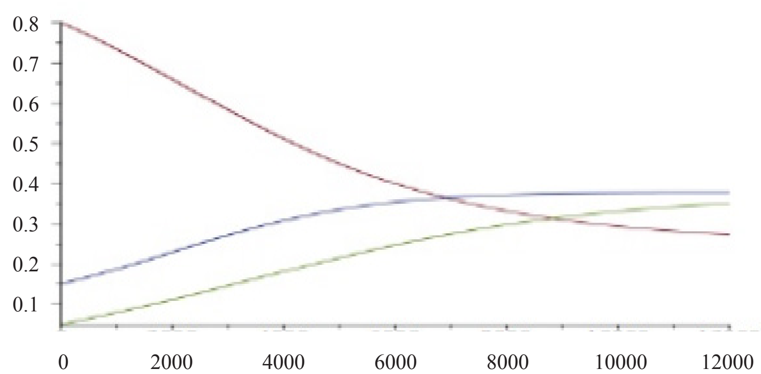

Figure 4. $\mathcal{R}_0' > 1$ and $\beta_{22} = 0$

In the three last Figures 4, 5, and 6, we take $\beta_1 = \beta_{21} = 0.0007$, $\delta = 0.00035$ and $\gamma = \eta = 0.00028$ and have $\mathcal{R}_0' > 1$. By comparing the three Figures 4, 5 and 6, we can notice that when β_{22} increases, when you have more media coverage, the number of normal-weight individuals, S decreases less that is the number of overweight and obese individuals increases less. Therefore these numerical experiments boost our theoretical study and highligt the influence of media coverage on the dynamical of our model.

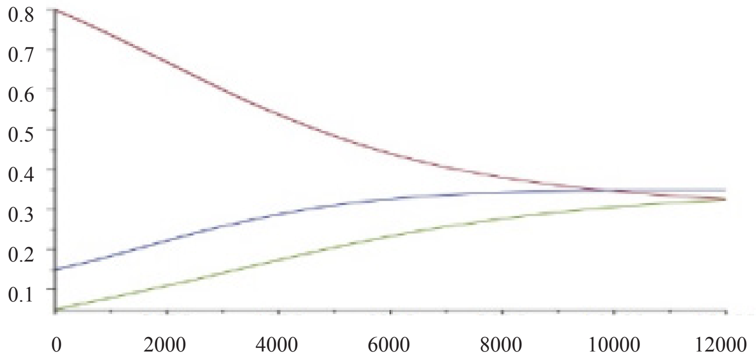

Figure 5. $\mathcal{R}_0' > 1$ and $\beta_{22} = 0.0006$

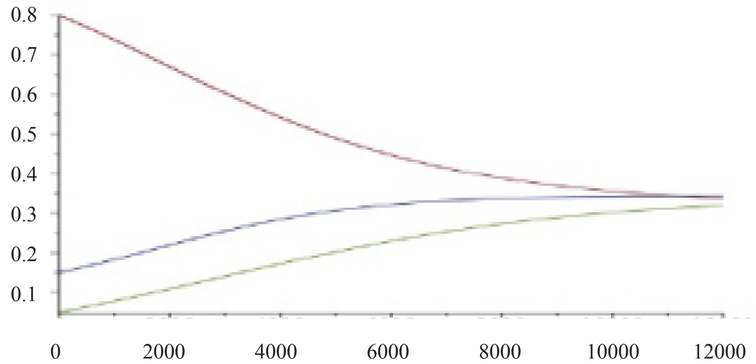

Figure 6. $\mathcal{R}_0' > 1$ and $\beta_{22} = 0.0006999$

6. Conclusion and comments

In this paper, we proposed a new mathematical model of epidemiological type in adult obesity taking into account the presence of a media. We then study equilibrium points and their different kind of stability. In the numerical section, different values of the parameters allow seeing the positive influence of the media, especially when the basic reproduction number is greater than 1. For future work, it might be interesting to use real data for simulations. In this paper, we consider only the actions of the media on obese individuals. We think that it is possible to consider actions only on overweight individuals or on both groups, this will be the aim of future works. This work could be a help to the authorities and convince them to intensify the use of media to reduce obesity.

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Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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