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Research Article

Structural Identifiability of Feedback Systems with Nonlinear Adulterating

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Abstract: We consider the structural identifiability estimation problem of Nonlinear Feedback Systems (NFS) with nonlinear adulterating. The problem of NFS Structural Identifiability (NFSI) has not been studied. Studying the NFSI problem guarantees the possibility of nonlinearity identification under uncertainty. Two cases of the adulterate influence are analyzed: (i) additive effect of feedback nonlinearity on the nonlinearity in the straight chain (ii) nonlinearity argument nonlinear adulterating in straight chain of the system. The basis for the identifiability estimation is: (a) the Geometric Frameworks (GF) analysis method reflecting properties of the nonlinear system; (b) structural frequency diagrams, and (c) the hierarchical immersion method. We obtain conditions of identifiability, unidentifiability and local identifiability for NFS. The influence of the nonlinear argument is analyzed on the system identifiability. We propose conditions for system unidentifiability verifying with a nonlinear argument of the function. The influence of the nonlinear argument is analyzed for estimating the system identifiability. Results are applicable in the synthesis of nonlinear control systems.

Keywords**:** structure, identifiability, nonlinear system of the second class, nonlinear adulterating, framework

MSC: 93C10, 93C95

1. Introduction

The identifiability problem of dynamic systems is the relevant areas of study. Foundational results are obtained on the Parametric Identification (PI) of systems [1-3]. The main direction PI is the analysis of a priori identifiability. Parametric identification methods are the basis of a priori Identifiability (AI). As a rule, the model structure sets a priori. Therefore, identification AI and Structural Identifiability (SI), which are often used in publications, are incorrect. AI methods are transferred to the estimation problem of the nonlinear systems structural identifiability. This approach eliminates the estimation problem of the nonlinearity structure.

Approximation methods are applied for identifiability problem solution [4-10]. In [4], the approach based on the analysis of the system output sensitivity is used to study identifiability. The effectiveness of this approach shows when studying the identifiability of the system parameter combination. In [10], local Parametric Identifiability (IPI) condition is obtained for various types of experimental data. A critical analysis of approaches used to estimate the

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identifiability of biological models is given in [8]. Methods for identifiability estimating nonlinear systems are based on the AI approaches. The requirements on data used for the PI task solution are given in [6]. The study of various types of identifiability (global, local, structural, and practical) are described in many works [4, 6, etc.]. Two main approaches (see [11]) are applied to estimate the structural identifiability of nonlinear dynamical systems. A priori methods (the first approach) only use the model definition (authors understand the definition as the structure choice). Posterior procedures (the second approach) apply the available data to find unidentifiable parameters. The basis of many a priori algorithms is the Lie group theory. A posteriori method uses the available data to perform identifiability analysis. They infer structural identifiability based on the adequacy of the model to experimental data (this is the stage of parametric estimation).

The method proposed to analyze the identifiability of closed-loop systems with nonlinear feedback in [12]. The method is based on the harmonic balance and the double Fourier Series (FS). FS approximates the nonlinearity. It shows that a multiplicative periodic reference signal enhances the identifiability of the system. Parametric identifiability is the main direction in the analysis of linear closed-loop systems (see for example [13-15]). PI guarantees the identifiability of the system. The structural identifiability problem is not solved by parametric methods under uncertainty. The parametric approach can only be applied to estimating nonlinearity in a specific area. The properties changing of the input may give different identification results. This is the main problem of the parametric approach. Works on PI problems do not consider the estimation problem of the system structure. Therefore, the concept of parametric identifiability does not reflect the essence of the structural identifiability problem. However, this terminology is actively using in the task of assessing identifiability. The NFS structural identifiability analysis requires the use of new approaches.

So, the system identifiability understands as the possibility to estimate its parameters (parametric identification). PI methods based on the information matrix nondegeneracy estimation used. Similar results are obtained in the parametric estimation theory. They are based on checking the Constant Excitation (CE) condition for the input and output of the system. As a rule, the model structure sets prior, and the essence of the structural identifiability is not always clear. The identifiability of nonlinear system is reduced to the parametric problem. PI paradigm is not always applicable to a nonlinear systems class. These systems work under uncertainty. The task becomes more complicated if the system has several nonlinearities [16]. Therefore, the structural identification problem solving (structural identifiability) is an urgent task. Parametric identifiability methods do not provide a solution to the SI problem. This is fairly for evaluating the NFS structural identifiability. In [16, 17], the SI methodology proposes for nonlinear dynamical systems. It is based on the GF analysis under uncertainty. The problem relates to the structural identification task closely. The structural identification problem solution is the basis for the SI of nonlinear systems under uncertainty. The approach is based on the GF properties analysis. Using GF allows you to solve the NFS structural identifiability problem. The GF approach is applicable for NFS with nonlinear adulteration. Nonlinear adulterating is a NFS special case.

Next, we give the SI problem solution for the feedback systems with nonlinear adulterating. We do not consider the identifiability of the system linear part. This problem has been studied well (see for example [3]). We consider two classes of NFS. The first class is NFS with nonlinearity in the return loop and forward contour. The second class is NFS with a nonlinear argument of the system main nonlinearity. We give the main results for the GF analysis. The analysis allows you to decide about the NFS structural identifiability.

2. Problem statement

Consider the system

$$\begin{aligned} \dot{X} &= AX + B_{\varphi}\Phi(Y) + B_u u, \\ Y &= C^T X, \end{aligned} \tag{1}$$

where $u \in \mathbb{R}$, $Y \in \mathbb{R}^g$ are input and output, $X \in \mathbb{R}^q$ is the state vector, $A \in \mathbb{R}^{q\times q}$, $B_u \in \mathbb{R}^q$, $B_{\varphi} \in \mathbb{R}^{q\times k}$, $C \in \mathbb{R}^{g\times q}$ are matrices of corresponding dimensions, $\Phi(X) \in \mathbb{R}^k$ is vector nonlinear function. A is Hurwitz matrix.

The nonlinear function $\varphi_i(\zeta) \in \Phi$ is smooth and satisfies the condition

$$\chi \in F_\varphi = \left\{\gamma_{1,i}\xi^2 \le \varphi_i(\xi)\xi \le \gamma_{2,i}\xi^2, \xi \ne 0, \ \varphi_i(0)=0, \gamma_{1,i} \ge 0, \gamma_{2,i} < \infty\right\}, \tag{2}$$

where $\zeta \in \mathbb{R}$ is the input of a nonlinear element. ζ is a linear combination of state variables. Information set

$$I_o = \left\{u(t), \ Y(t), \ t \in J = \left[t_0, t_k\right]\right\}. \tag{3}$$

We assume that some state variable x_i may depend on the difference in the outputs of multiple nonlinearities. Problem: analyze the set I_o and estimate the structural identifiability of the system (1).

3. Preliminary concepts

Let $\varphi(\cdot) = \Phi(X)$, $\varphi(\cdot) \in \mathbb{R}$, $y = Y$, $y \in \mathbb{R}$. Apply the approach to the GF construction [3, 17] and synthesize a framework S_{ey}. S_{ey} described by the function $f_{ey}: y \to e$, $e \in \mathbb{R}$. e reflects nonlinear processes in the system.

Assumption B1. The input $u(t)$ is constantly excited at the interval J. B2. The input $u(t)$ ensures an informative framework S_{ey}. B3. Let the framework S_{ey} be closed, and the area S_{ey} is not zero. Denote height S_{ey} as $h(S_{ey})$, where height is the distance between two points opposite sides of the framework S_{ey}. If $u(t)$ satisfies B1 and B2 conditions, then the input $u(t)$ is representative.

If the framework S_{ey} satisfies the assumption B3, then S_{ey} calls h-identifiable [17]. Consider the framework S_{ey}. Introduce notations: $D_y = \mathrm{dom}(S_{ey})$ is definitional domain S_{ey}, $D_y = D_y(D_y) = \max_t y(t) - \min_t y(t)$ is diameter D_y. Let $u(t) \in U$, where U is the acceptable set of inputs for the system (1). The set U contains representative inputs.

Definition 1 The input $u(t) \in U_s \subseteq U$ is the S-synchronizing system (1) if the domain D_y has a maximum diameter D_y on set $\{y(t), t \in J\}$.

Consider a reference framework S_{ey}^{ref}. S_{ey}^{ref} is the framework S_{ey} reflecting all properties of the function $\varphi(y)$. Designate by the diameter $D_y(S_{ey}^{ref})$ as D_y^{ref}. D_y^{ref} exists if the input the system (1) is S-synchronizing.

Definitions 1 show if $S_{ey} \cong S_{ey}^{ref}$, then $|D_y - D_y^{ref}| \le \varepsilon_y$, where $\varepsilon_y \ge 0$, $\cong$ is the proximity sign. Properties U_S

$$\left|D_y\left(S_{ey}\left(u(t)\big|_{u \in U_S}\right)\right) - D_y^{ref}\right| \le \varepsilon_y. \tag{4}$$

The fulfillment of condition $d_{h,y} = \max_{u_h} D_y$ guarantees h-identifiability of the system. The conditions for h-identifiability have the form

$$\left|D_y\left(S_{ey}\left(u(t)\big|_{u \in U_S}\right)\right) - d_{h,y}\right| \le \varepsilon_y. \tag{5}$$

If $u_h(t) \notin U$, then the condition of the unidentifiable or insignificant framework $NS_{ey} \ne S_{ey}$ is

$$\left|D_y\left(S_{ey}\left(u(t)\big|_{u \in U \setminus U_S}\right)\right) - d_{h,y}\right| > \varepsilon_y. \tag{6}$$

Let the input $u_h(t)$ synchronize the set D_y. If $u(t)$ is S-synchronizing, then we write $u_h(t) \in S$. Note that the finite set $\{u_h(t)\} \in S$ exists for system (1). The choice of optimal $u_h(t)$ depends on $d_{h,y}$ and (5). Provision of condition (5) is one of the SI conditions for the system (1).

Let $S_{ey} = F_{S_{ey}}^l \cup F_{S_{ey}}^r$, where $F_{S_{ey}}^l$, $F_{S_{ey}}^r$ are the left and right fragments S_{ey}. Secants for $F_{S_{ey}}^l$, $F_{S_{ey}}^r$

$$\gamma_S^r = a^r y + b^r,\ \gamma_S^l = a^l y + b^l. \quad (7)$$

Definition 2 If S_{ey} is h-identified and conditions $\left\| a^r \right| - \left| a^r \right\| \le \delta_h$, (4) satisfies, then the S_{ey} or the system (1) is structurally identified or h_{δ_h}-identifiable.

Let c_S is the centre of the framework S_{ey} on the set $J_y = \{y(t)\}$, c_{D_y} is the centre of the area D_y.

Theorem 1 [3, 17]. Let the set U_S of synchronizing inputs $u(t)$ consider for the system (1) and (i) $\varepsilon \ge 0$ exists such that $|c_S - c_{D_y}| \le \varepsilon$; (ii) $\left\| a^l \right| - \left| a^r \right\| \le \delta_h$, where a^l, a^r are coefficients of secants (7) for $(F_{S_{ey}}^l, F_{S_{ey}}^r) \subset S_{ey}$. Then the system (1) is h_{δ_h}-identifiable, and the input $u_h(t) \in S$.

We do not consider the method of constructing S_{ey}. It is described in [3, 17]. Examples of the framework S_{ey} obtain is given in section 4. Let the hypothetical framework S_{ey} (the framework S_{ey}^{ref}) have the diameter $d_{h,\Sigma}$.

Definition 3 The framework $S_{ey,i}$ has $d_{h,\Sigma}$-optimality property on the set U_h if $\varepsilon_\Sigma > 0$ exists such that $|d_{h,\Sigma} - D_{y,i}| \le \varepsilon_\Sigma\ \forall i = \overline{1,\#U_h}$. The $S_{ey,i}(u_{h,i})$-framework is locally structurally identifiable on the set U_h if it has the $d_{h,\Sigma}$-optimality property.

There may be some subset $\{u_{h,i}(t)\} \subset U_S \subseteq U$ $(i \ge 1)$ whose elements have the S-synchronizability property. Each $u_{h,i}(t)$ corresponds to the framework $S_{ey,i}(u_{h,i})$ with the diameter $D_{y,i}$ of the definition area $D_{y,i}$.

4. Additive adulterate

Consider the system (1) with feedback (self-oscillation system)

$$S_\varphi : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = c_1\left(\varphi_1(u) - \varphi_2(x_4)\right) + c_2 x_4, \\ \dot{x}_4 = \varphi_1(u) - \varphi_2(x_4), \\ y = x_4, \end{cases} \quad (8)$$

where $u = -k_1 x_2 - k_2 x_1$ is input, $y \in \mathbb{R}$ is the output, c_1, c_2, k_1, k_2 are system parameters, $\varphi_1(x) = c\ \mathrm{sign}(x)$ is sign function, $c > 0$,

$$\varphi_2(x_4) = \begin{cases} c, \text{if } x_4 > b, \\ 0, \text{if } x_4 \in [-b, b], \quad b > 0. \\ -c, \text{if } x_4 < -b, \end{cases}$$

We see the output of function $\varphi_1(u)$ adulterates by function $\varphi_2(x_4)$. (8) is the nonlinear system of the second class [18]. Harmonic linearization [18] does not apply to $\varphi_1(u)$, $\varphi_2(x_4)$. Find structural identifiability conditions for (8) using the results of section 3.

Determine the system S_φ solution under the following initial conditions and parameters: $x_1(0) = 2$, $x_2(0) = 1$, $x_3(0) = 0$, $x_4(0) = 1$, $c = 2$, $b = 1.5$, $c_1 = 2$, $c_2 = 2$, $k_1 = 1$, $k_2 = 1.5$. Form the set (3). The system (8) phase portrait shows in Figure 1.

We see (Figure 1) that oscillations (variable x_2) emerge at the system input, and limited oscillations (variable x_4) occur at the output. They are the result of the action of nonlinearity.

Presented frameworks show processes do not have distortions (the left part of the portrait corresponds to the right). Therefore, the system input is S-synchronizing (see section 3). The S-synchronizability condition is a prerequisite for the analysis of the system structural identifiability. We see (Figure 1) that the conclusion cannot make about the structural identifiability of the system based on the mapping $\gamma_{x_1,x_2} : x_1 \to x_2$.

Apply the hierarchical immersion method [17] for further analysis. It gives mappings that reflect the nonlinear system (8) state.

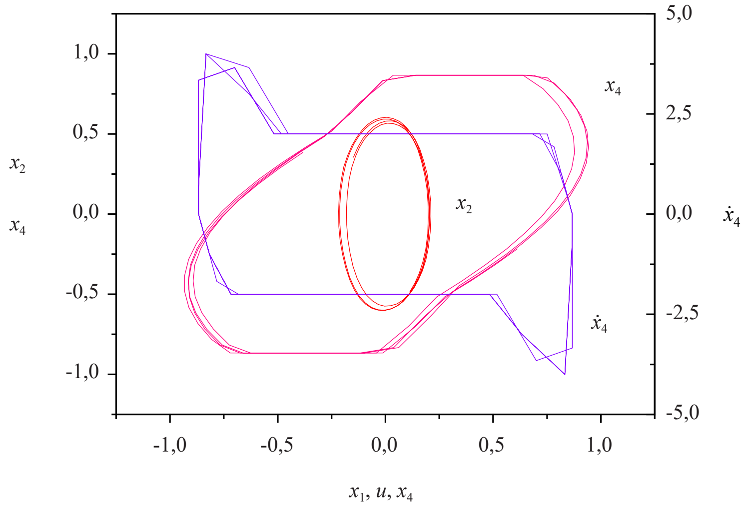


Figure 1. Phase portrait of system

Definition 4 System (8) is locally identifiable (LI) on the set $\{x_1, x_2\}$ if the framework $S_{x_2x_1}$ described by the mapping $\gamma_{x_1,x_2} : x_1 \rightarrow x_2$ has the maximum diameter of the definition domain.

Figure 1 shows that the framework $S_{x_2x_1}$ is LI. The analysis $S_{x_1\dot{x}_1}$ does not give the nonlinearity Structural Parameters (SP) estimate from adulteration. Go to a specific structural space. The mathematical model design is based on the relationship between variables.

Synthesize models on sets $\{u, \dot{x}_4\}$ and $\{x_1, \dot{x}_4\}$

$$\hat{\dot{x}}_{4,u} = 2.234u - 0.004,\ \dot{x}_{4,x_1} = -0.0712x_1 + 0.003. \tag{9}$$

Determine residuals $\varepsilon_{4,x_1} = \dot{x}_4 - \hat{\dot{x}}_{4,x_1}$, $\varepsilon_{4,u} = \dot{x}_4 - \hat{\dot{x}}_{4,u}$ and introduce frameworks $S_{\dot{x}_4,\varepsilon_{4,x_1}}$, $S_{\dot{x}_4,\varepsilon_{4,u}}$ which are described by functions $\gamma_{\varepsilon_{4,x_1},\dot{x}_4} : \varepsilon_{4,x_1} \rightarrow \dot{x}_4$ and $\gamma_{\varepsilon_{4,x_1},u} : \varepsilon_{4,u} \rightarrow \dot{x}_4$ (Figure 2).

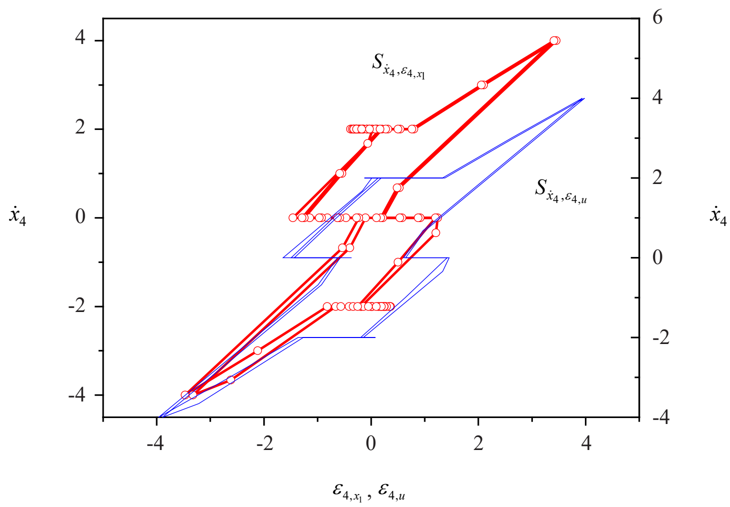


Figure 2. System frameworks for estimating identifiability based on analysis u, x_1

Frameworks $S_{\dot{x}_4,\varepsilon_{4,x_1}}$, $S_{\dot{x}_4,\varepsilon_{4,u}}$ reflect the influence of nonlinearities φ_1 and φ_2. We see (Figure 2) that frameworks have saturation equalling to 2 and -2, which coincides with the initial data of the system (8). Switching points of nonlinear functions shifted because the adulterating effect.

Construct secants for $S_{\dot{x}_4,\varepsilon_{4,x_1}}$, $S_{\dot{x}_4,\varepsilon_{4,u}}$

$$\gamma_{\dot{x}_4,\varepsilon_{4,x_1}} = a_{4,x_1}\varepsilon_{4,x_1},\ \gamma_{\dot{x}_4,\varepsilon_{4,u}} = a_{4,u}\varepsilon_{4,u} + b_{4,u}, \tag{10}$$

where $a_{4,x_1} = 1$, $a_{4,u} = 1.027$, $b_{4,u} = 0.22$.

Definition 5 The system (8) is structurally unidentifiable on the set $\{u(t), x_1(t)\}$ if the input is S-synchronizing, and secant (10) coefficients satisfy the condition $|a_{4,x_1} - a_{4,u}| \leq \delta$, where $\delta \geq 0$.

Definition 5 is applied to nonlinearities φ_1, φ_2 with $\delta = 0.005$. This conclusion is confirmed by Figure 3 (centered diagram), which shows the variables ε_{4,x_1} (grey) and $\varepsilon_{4,u}$ (white).

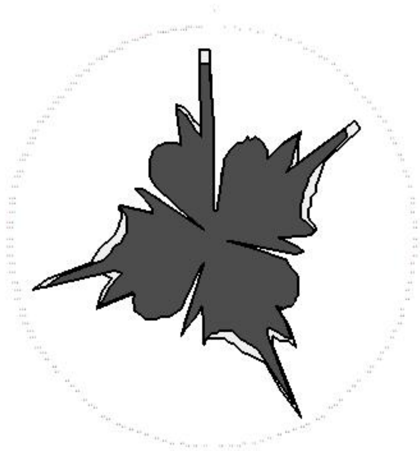


Figure 3. Comparison of variables ε_{4,x_1}, $\varepsilon_{4,u}$

The nonlinearity φ_1 dominates over φ_2, but its switching occurs at $x_1 \approx \pm 1.5$, which corresponds to parameters φ_2. SI is the confirmation of structural identification. Therefore, we conclude that adulteration complicates the identifiability of the system (8) structure.

Consider the space $\{\varepsilon_{4,x_1}, \varepsilon_{4,x_4}\}$, where ε_{4,x_4} forms similarly to ε_{4,x_1}, and decides on φ_1. Obtained results show that the system (8) structural identification possible on the nonlinearity $\chi(\varphi_1, \varphi_2)$ depended on φ_1, φ_2.

Statement 1 The system (8) nonlinear part $\chi(\varphi_1, \varphi_2)$ is locally structurally identified on the set $\{u(t), x_2(t), x_4(t)\}$ if the input $u(t) \in S$.

Proof statement 1 It follows from (8) and Figure 1 as the input is S-synchronizing. S-synchronizability guarantees the maximum definitional domain diameter of frameworks $S_{\dot{x}_4,\varepsilon_{4,x_1}}$, $S_{\dot{x}_4,\varepsilon_{4,u}}$. Therefore, $\chi(\varphi_1, \varphi_2)$ is h-identifiable. The system output depends on the difference $\varphi_1(u)$, $\varphi_2(x_4)$. The system (8) structure depends on structural parameters of the final function $\chi(\varphi_1, \varphi_2)$. ■

Corollary of statement 1 Functions $\varphi_1(\cdot)$, $\varphi_2(\cdot)$ are structurally unidentifiable on the set I_o.

Figure 4 gives indirect confirmation of statement 1, where the framework presents reflecting the function $\chi(\varphi_1, \varphi_2)$ influence, $\hat{\chi}(\varphi_1, \varphi_2)$ is function $\chi(\varphi_1, \varphi_2)$ estimation based on the analysis the set I_o.

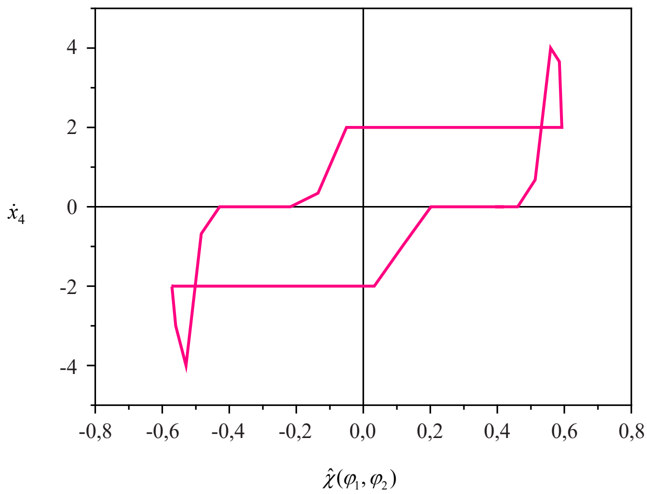


Figure 4. Evaluation of effect $\chi(\varphi_1, \varphi_2)$ on the system (8) structure

Figure 4 shows that the system has the saturation region, which corresponds to parameters of initial nonlinearities. The further analysis clarifies some parameters of initial nonlinearities.

Introduce the variable $v = u - x_2$ and synthesis the model for $\dot{x}_4$

$$\hat{\dot{x}}_4 = a_{4,0} + a_{4,1}v, \tag{11}$$

where $a_{4,0} = 0.125$, $a_{4,1} = 1.45$. Find the residual $\vartheta_4 = \dot{x}_4 - \hat{\dot{x}}_4$ and construct the framework $S_{\dot{x}_4,\vartheta_4}$ described by function $\gamma_{\vartheta_{41},\dot{x}_4} : \vartheta_{41} \to \dot{x}_4$. Determine for $S_{\dot{x}_4,\vartheta_4}$, $S_{\dot{x}_4,\varepsilon_{4,x_1}}$ secants

$$\gamma_{\vartheta_4,\dot{x}_4} = \alpha_{4,0}\vartheta_4 + \alpha_{4,1},\ \gamma_{\varepsilon_{4,x_1},\dot{x}_4} = \mu_{4,0}\varepsilon_{4,x_1} + \mu_{4,1}, \tag{12}$$

where $\alpha_{4,0} = 0.996$, $\alpha_{4,1} = -0.06$, $\mu_{4,0} = 1.028$, $\mu_{4,1} = -0.219$.

Theorem 2 If the input $u \in S$, and coefficients $\alpha_{4,0}$, $\mu_{4,0}$ of secants (12) for frameworks $S_{\dot{x}_4,\vartheta_4}$, $S_{\dot{x}_4,\varepsilon_{4,x_1}}$ satisfy the condition

$$\left|\alpha_{4,0} - \mu_{4,0}\right| \le \delta_{\alpha,\mu}, \tag{13}$$

where $\delta_{\alpha,\mu} \ge 0$ is specified value, then the function $\chi(\varphi_1, \varphi_2)$ is structurally identifiable on the set $\{\vartheta_4(t), \dot{x}_4(t)\}$.

Proof theorem 2 The input of the system (8) is S-synchronizing. According to statement 1, the framework $S_{\dot{x}_4,\varepsilon_{4,x_1}}$ has the maximum diameter of the definition domain. Therefore, the function $\chi(\varphi_1, \varphi_2)$ is locally structurally identifiable. The framework $S_{\dot{x}_4,\vartheta_4}$ has the maximum diameter of the definition domain also. We construct for $S_{\dot{x}_4,\vartheta_4}$, $S_{\dot{x}_4,\varepsilon_{4,x_1}}$ secants (12). The condition (13) satisfied with $\delta_{\alpha,\mu} = 0.04$. Hence, the function $\chi(\varphi_1, \varphi_2)$ is locally structurally identifiable on the set $\{\vartheta_4(t), \dot{x}_4(t)\}$. ■

The theorem 2 application simplifies the identifiability estimation of the system (8).

Figure 5 confirms the validity of theorem 2, where we present frameworks $S_{\dot{x}_4,\vartheta_4}$, $S_{\dot{x}_4,\varepsilon_{4,x_1}}$ and the adequacy of considered frameworks in space $(\vartheta_{41}, \varepsilon_{4,x_1})$. The coefficient of determination is 99% between ϑ_{41} and ε_{4,x_1}.

So, we show that system (8) is structurally unidentifiable on set I_o. Go into space $(\vartheta_4(t), \dot{x}_4(t))$ and obtain estimates for functions φ_1, φ_2.

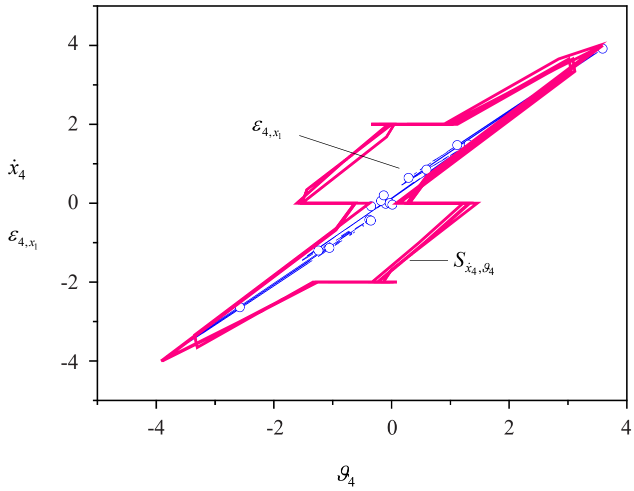


Figure 5. Theorem 2 verification

Apply the variable permutation procedure. Let $\dot{x}_4$ be the input, and x_1, x_2 outputs. This approach almost eliminates the adulteration influence. Construct frameworks $S_{x_1,\dot{x}_4}$, $S_{x_2,\dot{x}_4}$. Since, $u(t) \in S$, frameworks $S_{x_1,\dot{x}_4}$, $S_{x_2,\dot{x}_4}$ are significant [3, 17] and have the maximum diameter of the definitional domain. Apply theorem 2 and confirm by SI (h_{δ_h}-identifiability) of the system (8). Corresponded frameworks show in Figure 6. They give to evaluate properties for nonlinearities on the set of function switching intervals. Figure 6 shows that two nonlinearities are in the system. One nonlinearity (framework $S_{x_2,\dot{x}_4}$) is saturation, and the framework $S_{x_1,\dot{x}_4}$ is the saturation function with Dead Space (DS). DS does not coincide with the original area from the specific of the system. The saturation level is 2.

The relationship between u and x_2 (the determination coefficient is 0.98) effects the nonlinear properties of the system (8). Consequently, the system (8) and the nonlinearity $\chi(\varphi_1, \varphi_2)$ are structurally identifiable. Parameters $\chi(\varphi_1, \varphi_2)$ depend on the connection φ_1, φ_2 into the system.

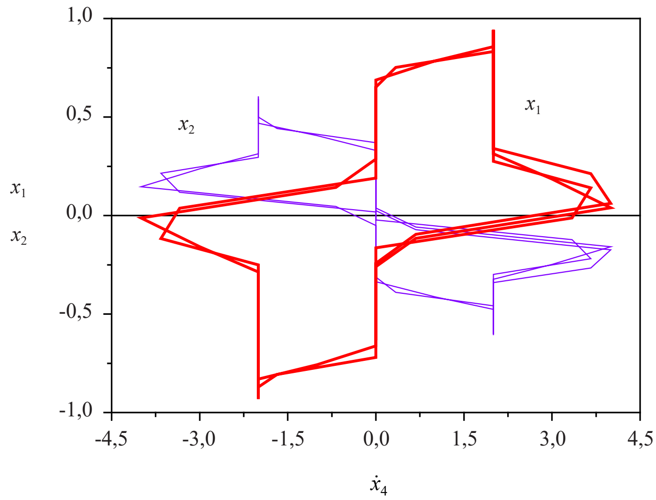


Figure 6. Frameworks $S_{x_1,\dot{x}_4}$, $S_{x_2,\dot{x}_4}$

So, we have

Statement 2 If: (i) $u \in S$ and $\dot{x}_4 \in S$; (ii) frameworks S_{x_1,x_4}, S_{x_2,x_4} described by functions $\gamma_{\dot{x}_4,x_1} : \dot{x}_4 \rightarrow x_1$ and

$\gamma_{\dot{x}_4, x_2} : \dot{x}_4 \to x_2$ have $d_{h,\Sigma}$-optimal properties, then system (8) and function $\chi(\varphi_1, \varphi_2)$ are structurally identifiable.

5. Nonlinear adulterate of nonlinearity argument

Consider the self-oscillation generation system with the Nonlinear Feedback (NF). NF adulterates the nonlinear function argument in a straight chain. The system contains the actuator with parameters k_4, T_2 and output y; gain element with parameters k_3, T_1; nonlinear feedback on x_2 with the function $f_2(x_2)$; linear feedback on x_2 with parameters k_1, k_5:

$$S_f : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = -\dfrac{1}{T_1 T_2} x_2 - \dfrac{T_1 + T_2}{T_1 T_2} x_3 + \dfrac{k_3 k_4}{T_1 T_2} f_1(u, b, c), \\ \dot{x}_4 = -k_1 k_5 x_2, \\ y = x_1, \end{cases} \tag{14}$$

where y is the rotation angle of the actuator shaft, $u = x_4 - k_{os} f_2(x_2)$ is function f_1 argument, $f_2(x_2) = (x_2)^2 \operatorname{sign}(x_2)$,

$$f_1(x_4) \begin{cases} c, \text{ if } x > b \\ 0, \text{ if } x \in [-b, b], \quad b > 0. \\ -c, \text{ if } x < -b \end{cases} \tag{15}$$

We have the adulterated argument of the function f_1. Hence, itself argument $u(x_2, x_4)$ is the nonlinear function. It complicates the SI assessment of the system (14).

Consider the system S_f with two sets of parameters

$$P_i = \left\{ k_{1(i)}, k_{3(i)}, k_{4(i)}, k_{5(i)}, k_{os(i)}, T_{1(i)}, T_{2(i)}, b_{(i)}, c_{(i)} \right\}, \; i = 1, 2, \tag{16}$$

where

$$P_1 = \{0.35, 10, 1, 1.25, 0.005, 2.5, 0.4, 0.45, 0.75\}, \; P_2 = \{0.35, 10, 1, 0.55, 0.005, 2.5, 0.4, 0.25, 2\}. \tag{17}$$

These sets guarantee self-oscillation in the system. Therefore, the input is constantly excited for P_i, and frameworks $S_{ey(i)}$ must be h-identifiable. However, the adulterating effect can give false h-identifiable frameworks $S_{ey(i)}$ (denote these frameworks and the system as $LS_{ey(i)}$ and $LS_{f(i)}$). Extend the frameworks $S_{ey(i)}$ set and their derivatives (frameworks $KS_{ey(i)}$) to select the reference information set I_0^r for analysis SI.

Consider the variable $u(t)$ and construct the structurally frequency diagram H_u for it. H_u reflects the distribution u according to the change in the system output.

Definition 6 If H_u contains areas conforming to system (14) features, then the variable $u(t)$ is structurally significant or $u \in H(H_u)$.

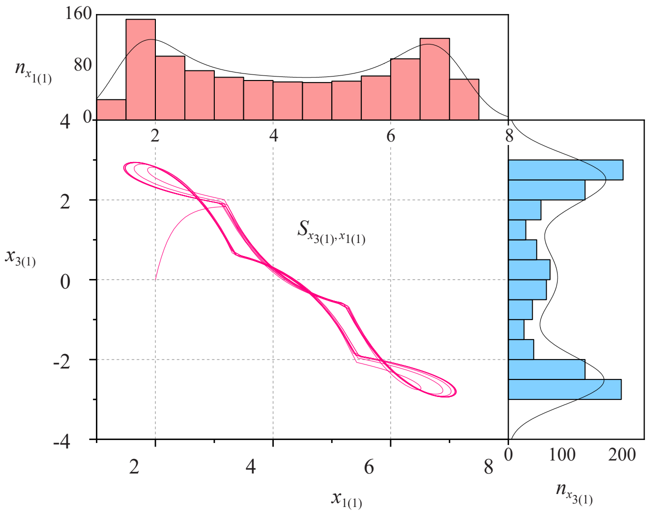
Definition 7 $S_{ey(i)}$-framework based on $u \in H(H_u)$ is structurally significant or $S_{ey(i)} \in SS_{ey(i)}$. The system $S_{f(i)}$ with $u \in H(H_u)$ is called structurally significant or $S_{f(i)} \in SS_{f(i)}(H(H_u))$. If $u \notin (H(H_u))$, then $S_{f(i)} \in SIS_{f(i)}(H(H_u))$.

Consider frameworks $S_{x3(i),x1(i)}$, $S_{u(i),x4(i)}$ is described by functions

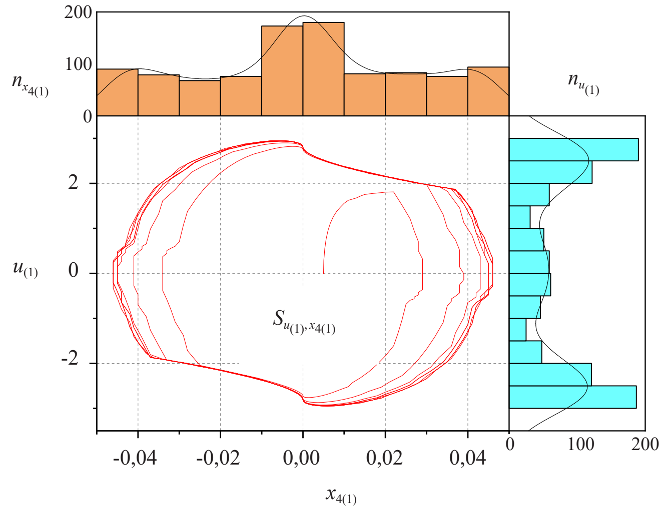
$$\eta_{3,1(i)} : x_{1(i)} \to x_{3(i)}, \; \mu_{4,u(i)} : u_{(i)} \to x_{4(i)}. \tag{18}$$

Present frameworks and their structural-frequency analysis [19] on sets (17) in Figures 7 and 8.

Theorem 3 Consider the system S_f with sets P_1, P_2 and (i) input $u_{(i)}$ is constantly excited; (ii) the system state reflects by the frameworks $S_{x_{3(i)},x_{1(i)}}$, $S_{u_{(i)},x_{4(i)}}$ and the structural-frequency diagrams $H_{(i)}(i = 1, 2)$.Then frameworks defined on the set P_1 are structurally significant, i.e. they are applied for system identifiability analysis.



(a) framework $S_{x_{3(1)},x_{1(1)}}$

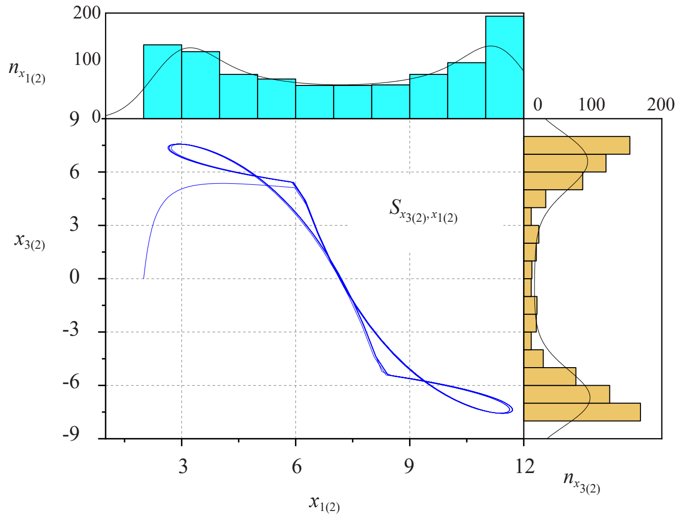


(b) framework $S_{u_{(1)},x_{4(1)}}$

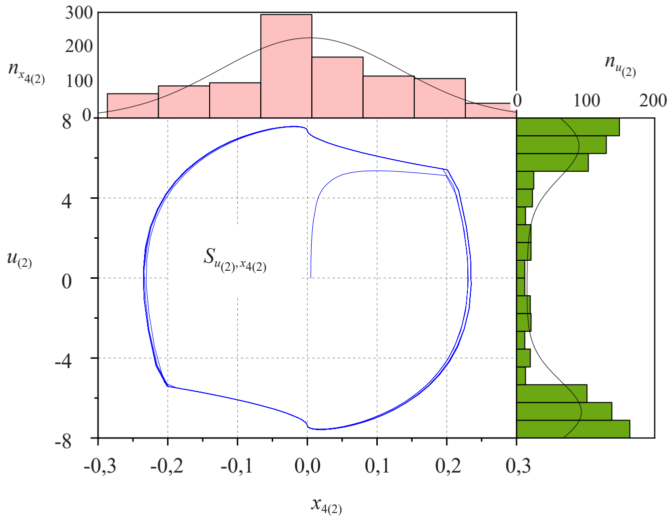
Figure 7. System (14) phase portraits with parameters P_1

Proof theorem 3 The system (14) is designed to generate self-oscillations. Therefore, the input $u_{(i)}$ satisfies the condition of constant excitation for the given P_i. Apply results of section 3 and obtain SI of the frameworks under consideration. Consider the Structural-frequency Diagrams (SDS) presented in Figures 7 and 8. The system S_f contains two nonlinearities. Therefore, SDS reflects their features. Consider frameworks $S_{u_{(i)},x_{4(i)}}$. We see that the framework $S_{u_{(1)},x_{4(1)}}$ contains fragments (see the distribution $n_{u_{1(i)}}$ in Figure 7), which not represented in $S_{u_{(2)},x_{4(2)}}$. Compare distributions $n_{u_{1(i)}}$. We see the input $u_{(1)}$ is more structurally significant than $u_{(2)}$. Consequently, $u_{(1)} \in H(H_{u_{(1)}})$, and

$u_{(2)} \notin H(H_{u_{(2)}})$. Thus, the system (14) with the set P_1 is the candidate for the analysis of the system S_f identifiability. Confirm these conclusions through the analysis frameworks $S_{x_{3(i)},x_{1(i)}}$ (Figure 8). Consequently, further SI analysis is based on the use of the information set for the system $S_f(P_1)$. Then $S_f(P_1) \in SS_{f(1)}(H(H_u))$, and $S_f(P_2) \in SIS_{f(2)}(H(H_u))$. ■



(a) framework $S_{x_{3(2)},x_{1(2)}}$



(b) framework $S_{u_{(2)},x_{4(2)}}$

Figure 8. System (14) phase portraits with parameters P_2

The framework $S_{x_{3(2)},x_{1(2)}}$ does not reflect the system (14) specifics. Explain it by the nonlinear feedback action. Consider the system (14) defined on the set of parameters

$$P_3 = \{0.35, 12, 1, 0.55, \{k_{os,i}\}, 3, 0.4, 0.5, 1.5\} \tag{19}$$

with different values k_{os}.

Statement 3 If the system (14) is defined on the set (19), then increasing the parameter k_{os} changes the domains of the function f_1 in the coordinate origin.

Proof of statement 3 Consider a neighborhood O_{f_1} of the function f_1 in coordinate origin. Let $k_{os}^* = 0.0025$ be the reference value for k_{os}. Increasing $k_{os,j} \in \{k_{os,i}\}$ gives a value increase in the function f_2. Growing f_2 gives change to the argument $u(k_{os,j})$ and the function f_1 compares to values of $f_1(u(k_{os}^*))$ at the same t. Therefore, the domain of values $O_{f_1(u)}\,|_{u=u(\{k_{os,i}\})}$ will not coincide with the domain $O_{f_1(u)}\,|_{u=u(k_{os}^*)}$ of the same function f_1. ■

Denote the values domain and the diameter of neighborhood $O_{f_1(u)}\,|_{u=u(\{k_{os,i}\})}$ as $\mathrm{Im}(O_{f_1}(u \neq u)(k_{os}^*))$ and $D_{O_{f_1(u)}}$.

Definition 8 Call the input $u(t)$ of the system (14) U_h-identifying if it minimizes $\mathrm{Im}(O_{f_1}(u \neq u)(k_{os}^*))$

$$u^* = \min_u D_{O_{f_1(u)}}.$$

Denote the set of U_h-identifying inputs as U_{U_h}.

Theorem 4 If the input $u(t)$ is constantly exciting and U_h-identifying, then the system (14) and the function f_1 are h_{δ_h}-identifiable.

The proof of theorem 4 is based on the construction of the framework S_{x_3,x_1} and the section 3 results application.

The domain $O_{f_1(u)}$ at $u \notin U_{U_h}$ can determine by the histogram or the framework S_{x_3,x_1}. The region $O_{f_1(u)}$ changes the interval $[-b, b]$ of the function f_1. It leads to the change in the definition area and the domain of values f_1. According to theorem 4, this violates the h_{δ_h}-identifiability property of the system (14).

Consider input satisfying conditions of theorem 4.

Theorem 5 If the input $u \in U_{U_h}$ is constantly exciting, then the function $f_2(x_2)$ of the system S_f is h_{δ_h}-identifiable.

The proof of theorem 5 follows from theorem 3 conditions fulfilment. Show the function $f_2(\cdot)$ estimation in Figure 9.

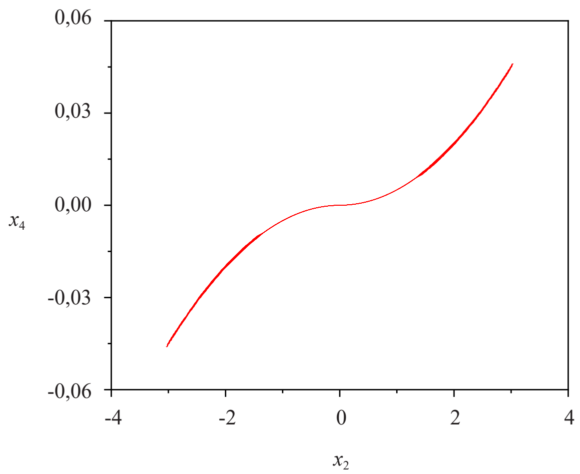


Figure 9. Change estimation of function f_2

6. Conclusion

The structural identifiability problem of nonlinear feedback systems considers with adulterating nonlinearity. We apply the analysis of geometric frameworks and hierarchical immersion method for identifiability estimation study under uncertainty. The nonlinear adulterating of the nonlinear function argument considered. Structural identifiability conditions are obtained for this case. We study the input influence on the nonlinearity structural identifiability in the system straight chain. We define a class of inputs that gives the solution to the SI problem. The adulteration influence of nonlinear argument shows on the nonlinearity parameters estimation. Structurally-frequency diagrams are the basis for the analysis of these systems. Conditions for unidentifiability verification are proposed at the nonlinear mixing of the argument. We consider the additive effect of feedback nonlinearity on the nonlinearity in the straight chain. It shows that the synchronizing input provides the solution to the structural identifiability problem. We have shown that this system is structurally unidentifiable according to obtained experimental data. A subset of the system states on which the system is locally structurally identifiable obtained.

Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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