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Review

On *F*-Contractions: A Survey

Nicola Fabiano[1], Zoran Kadelburg[2], Nikola Mirkov[1*], Vesna Šešum Čavić[3], Stojan Radenović[4]

[1]"Vinča" Institute of Nuclear Sciences-National Institute of the Republic of Serbia, University of Belgrade, Mike Petrovića Alasa 12-14, 11351 Belgrade, Serbia
[2]Faculty of Mathematics, University of Belgrade, Studentski trg 16, 11000 Beograd, Serbia
[3]Faculty of Civil Engineering, University of Belgrade, Bulevar Kralja Aleksandra 73, 11000 Belgrade, Serbia
[4]Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11000 Belgrade, Serbia
E-mail: nmirkov@vin.bg.ac.rs

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Abstract: D. Wardowski proved in 2012 a generalization of Banach Contraction Principle by introducing F-contractions in metric spaces. In the next ten years, a great number of researchers used Wardowski's approach, or some of its modifications, to obtain new fixed point results for single- and multivalued mappings in various kinds of spaces. In this review article, we present a survey of these investigations, including some improvements, in particular concerning conditions imposed on function F entering the contractive condition.

Keywords: F-contraction, fixed point, generalized metric space, multivalued mapping

MSC: 47H10, 54H25

1. Introduction and preliminaries

The year 2022 marks the 100th anniversary of the publication (in [1]) of Banach Contraction Principle (or Banach Fixed Point Theorem), the milestone of Metric Fixed Point Theory. Let us recall the formulation of this fundamental theorem:

Theorem 1.1 [1] If T is a mapping from a complete metric space (X, d) into itself and if there is a constant $\lambda \in [0, 1)$ such that for every x, y from X,

$$d(Tx, Ty) \leq \lambda d(x, y) \tag{1}$$

holds, then there exists exactly one point $z \in X$ such that $T(z) = z$. Moreover, for each point $x_0 \in X$, the iterative sequence $\{x_n\}$ defined by $x_n = Tx_{n-1}$, $n \in \mathbb{N}$ converges to z.

The mapping $T : X \to X$ satisfying (1) is called a contraction. Among other things, Banach used his theorem to solve a special type of integral equations.

Remark 1.2 Actually, this result appeared two years earlier in Banach's Ph.D. Thesis (written in Polish) at L'vov University. However, it did not become widely known until its publication in the journal Fundamenta Mathematica.

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Starting from 1922, a large number of mathematicians tried to generalize this famous theorem. These generalizations went in two main directions:

(a) The known axioms of metric space (X, d) were modified and so a lot of new spaces, so-called generalized metric spaces, were introduced. We mention just some of them: b-metric spaces, partial metric spaces, metric-like spaces, cone metric spaces, G-metric spaces, rectangular metric spaces, etc.

(b) Condition (1) was replaced by various other conditions that generalize the condition of contraction.

There were also a lot of attempts which combined both directions.

In this review, we talk about a special generalization of Banach's result, which was introduced in 2012 by the Polish mathematician Darius Wardowski [2] (note that it is now the 10th anniversary of this result). This generalization is of direction (b).

Namely, Wardowski considered mappings $F : (0, +\infty) \to \mathbb{R}$ that satisfy the following conditions:

(F1) F is a strictly increasing function;

(F2) A sequence $t_n \in (0, +\infty)$ converges to zero if and only if $F(t_n) \to -\infty$ as $n \to +\infty$;

(F3) $\lim_{t\to 0^+} t^k F(t) = 0$ for some $k \in (0, 1)$.

Wardowski denoted by $\mathcal{F}$ the collection of mappings $F : (0, +\infty) \to \mathbb{R}$ that satisfy the conditions (F1), (F2) and (F3). Using such functions, he introduced a new type of contraction in a given metric space in the following way:

Definition 1.3 Let $F \in \mathcal{F}$ and let T be a mapping from a metric space (X, d) into itself. If there is a positive number τ such that for all $x, y \in X$ for which $d(Tx, Ty) > 0$,

$$\tau + F(d(Tx, Ty)) \le F(d(x, y)) \tag{2}$$

holds, then the mapping T is called an F-contraction.

The main result of D.Wardowski was the following.

Theorem 1.4 Each F-contraction T on a complete metric space (X, d) has a unique fixed point. Moreover, for each $x_0 \in X$, the corresponding Picard sequence $\{T^n x_0\}$ converges to that fixed point.

Obviously, taking $F(t) = \log t$ and $\tau = \log(1/\lambda)$, $\lambda \in (0, 1)$ the condition (2) reduces to (1), i.e., Theorem 1.4 is a generalization of Theorem 1.1. Moreover, by an example, Wardowski showed that this generalization was genuine (see further Example 2).

This nice result inspired dozens of mathematicians to try to obtain new results by:

1. applying similar idea in various other spaces (among them those mentioned under (a));
2. modifying the contractive condition (2) in various ways;
3. modifying conditions (F1)-(F3) for the function F.

In this survey, we present briefly some of these attempts, together with some modifications and improvements, in particular concerning properties (F1)-(F3). A review treating some other aspects of these problems can be found in [3].

2. A modification of Wardowski's theorem and its proof

First of all, we illustrate relationship between the properties (F1)-(F3).

Example 1 Consider the following functions that map $(0, +\infty)$ into $\mathbb{R}$: $F_1(x) = e^x$, $F_2 = -\frac{1}{x}$, $F_3(x) = \frac{x-1}{x}$, $F_4(x) = x^x$, $F_5(x) = \log x$, $F_6(x) = -\frac{1}{x^{1/n}}$, $n \in \mathbb{N}$, $F_7(t) = t + \log t$. Then:

1. F_1 satisfies (F1) and (F3) but not (F2);
2. F_2 and F_3 satisfy (F1) and (F2) but not (F3);
3. F_4 satisfies (F3) but not (F1) and (F2);
4. F_5, F_6 and F_7 satisfy all three properties.

Now we list some properties of a function F that follow just from property (F1):

1. F is continuous almost everywhere.
2. At each point $r \in (0, +\infty)$ there exist its left and right limits $\lim_{t\to r^-} F(t) = F(r^-)$ and $\lim_{t\to r^+} F(t) = F(r^+)$. Moreover,

for the function F one of the following two properties holds: $F(0^+) = m \in \mathbb{R}$ or $F(0^+) = -\infty$.

3. Property (F2) is equivalent to

(F2′) $F(0^+) = -\infty$, as well as to

(F2″) $\inf_{t \in (0,+\infty)} F(t) = -\infty$.

For more details see [4, 5].

We recall the following two properties of sequences in metric spaces that have often been used, sometimes implicitly, in proving fixed point results (see, e.g., [6-13] for the first property and [10-14] for the second one).

Lemma 2.1 Let $\{x_n\}$ be a Picard sequence of a self-map T in a metric space (X, d) (i.e., $x_n = Tx_{n-1}$, $n \in \mathbb{N}$). If

$$d(x_{n+1}, x_n) < d(x_n, x_{n-1}) \tag{3}$$

holds for each $n \in \mathbb{N}$, then $x_n \neq x_m$ whenever $n \neq m$.

Lemma 2.2 Let $\{x_n\}$ be a sequence in a metric space (X, d) such that $d(x_n, x_{n+1}) \to 0$ as $n \to +\infty$. If $\{x_n\}$ is not a Cauchy sequence then there exist $\varepsilon > 0$ and two sequences $\{m_k\}$ and $\{n_k\}$ of positive integers, satisfying $n_k > m_k > k$, such that the following sequences tend to ε^+ as $k \to +\infty$:

$$d(x_{m_k}, x_{n_k}),\ d(x_{m_k}, x_{n_k+1}),\ d(x_{m_k-1}, x_{n_k}),$$

$$d(x_{m_k-1}, x_{n_k+1}),\ d(x_{m_k+1}, x_{n_k+1}),\ \ldots$$

We now formulate an improved version of Wardowski's result that is given in [15, Corollary 2]. Our proof is shorter than the original one, since it uses Lemmas 2.1 and 2.2 (for the sake of brevity, we treat just the basic case $\alpha = 1$ of [15, Theorem 5]).

Theorem 2.3 Let (X, d) be a complete metric space and T be a self-mapping on X. Assume that there exist a strictly increasing function $F : (0, +\infty) \to \mathbb{R}$ and $\tau > 0$ such that (2) holds for all $x, y \in X$ with $Tx \neq Ty$. Then T has a unique fixed point in X.

Proof. Let $x_0 \in X$ be arbitrary, let $\{x_n\}$ be the corresponding Picard sequence defined by $x_n = Tx_{n-1}$, $n \in \mathbb{N}$, and denote $d_n = d(x_{n-1}, x_n)$. We can assume that $d_n > 0$ (i.e., $x_{n-1} \neq x_n$) for each $n \in \mathbb{N}$ (otherwise there is nothing to prove).

Putting $x = x_{n-1}$, $y = x_n$ in the condition (2), we get that

$$\tau + F(d_{n+1}) \leq F(d_n), \tag{4}$$

and hence (using (F1)) $d_{n+1} < d_n$ for each $n \in \mathbb{N}$. It follows by Lemma 2.1 that $x_n \neq x_m$ whenever $n \neq m$. Also, the sequence $\{d_n\}$ must converge to some $d \geq 0$. If $d > 0$, then, passing to the limit when $n \to +\infty$ in (4), it follows that $\tau + F(d^+) \leq F(d^+)$ which is in contradiction with $\tau > 0$. Hence,

$$\lim_{n \to +\infty} d_n = 0. \tag{5}$$

Suppose now that $\{x_n\}$ is not a Cauchy sequence and consider the sequences $\{m_k\}$ and $\{n_k\}$ that satisfy conditions as in Lemma 2.2. Since $n_k > m_k$, it follows that $x_{m_k} \neq x_{n_k}$, hence we can use contractive condition (2) with $x = x_{n_k}$ and $y = x_{m_k}$. We obtain that

$$\tau + F(d(x_{n_k+1}, x_{m_k+1})) \leq F(d(x_{n_k}, x_{m_k})).$$

Passing to the limit as $k \to +\infty$, using Lemma 2.2 and the mentioned property (2) of the increasing function F, we get that

$$\tau + F(\varepsilon^+) \le F(\varepsilon^+),$$

which is in contradiction with $\tau > 0$. Hence, $\{x_n\}$ is a Cauchy sequence and, since (X, d) is complete, it converges to some $x^* \in X$.

Observe now that the contractive condition (2) (where F satisfies the property (F1)) implies that the mapping T is continuous. Hence, it follows in a routine way that x^* is a unique fixed point of T. □

Remark 2.4 Note that the case (5) can only take place if $F(0^+) = -\infty$. Indeed, if $d = 0$, then it follows from (4) that $\tau + F(0^+) \le F(0^+)$, which is impossible if $F(0^+)$ is finite. It means that condition (F2) for the function F is implicitly contained in the formulation of Theorem 2.3. In other words, there is no mapping T which satisfies Wardowski's condition (2) with function F satisfying (F1) and not satisfying (F2). Hence, no essentially new results can be obtained by using functions F satisfying just (F1). Bearing this in mind, we will formulate our further results assuming both properties (F1) and (F2).

We recall here the original Wardowski's example that shows that his result is more general than the Banach's one.

Example 2 [2, Example 2.1] Consider the set $X = \{x_n \mid n \in \mathbb{N}\}$ where $x_n = \sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$, equipped with the standard metric given by $d(x, y) = |x - y|$, $x, y \in X$. Then, (X, d) is a complete metric space. Let $T : X \to X$ be defined by $T(x_1) = x_1$ and $T(x_n) = x_{n-1}$, $n > 1$. It is easy to see that Banach's condition (1) is not satisfied, but Wardowski's condition (2) holds with $\tau = 1$ and $F(t) = t + \ln t$. For details see [2].

The next example shows that Theorem 2.3 is a genuine generalization of Theorem 1.4.

Example 3 [15, Example 1] Let $X = \{a_n \mid n \in \mathbb{N}\} \cup \{b\}$ and $d : X \times X \to [0, +\infty)$ be given by $d(a_n, a_n) = d(b, b) = 0$, $n \in \mathbb{N}$ and $d(a_n, a_{n+p}) = d(a_{n+p}, a_n) = d(a_n, b) = d(b, a_n) = \dfrac{1}{n}$ for $n, p \in \mathbb{N}$. Obviously, (X, d) is a complete metric space. Let $T : X \to X$ be given by $Ta_n = a_{n+1}$, $n \in \mathbb{N}$ and $Tb = b$. It is shown in [15] that T cannot satisfy conditions of Theorem 1.4 with any function $F \in \mathcal{F}$, but it satisfies condition of Theorem 2.3 with $\tau \le 1$ and $F(t) = -\dfrac{1}{t}$ (note that this function does not satisfy the property (F3)). For details see [15].

Remark 2.5 The following result was proved in [15] as Theorem 6.

Let (X, d), T and F satisfy the conditions of Theorem 2.3, except that, instead of (F1), F is supposed to be continuous and satisfy condition (F2). Then the same conclusion holds.

By an example (see [15, Example 2]), it was shown that this result was essentially different from Theorem 2.3, i.e., there exists a mapping T in a metric space (X, d) which does not satisfy conditions of Theorem 2.3, but which satisfies conditions presented in this remark and which has a unique fixed point.

3. Some generalizations

3.1 *Common fixed points*

Let (X, d) be a metric space and T, S be two self-mappings on X. Recall that if $T(x) = S(x) = y$ then x is called a coincidence point of T and S, and y is said to be their point of coincidence. If, moreover, $y = x$ then x is called a common fixed point of T and S. In order to obtain a Wardowski-type version of the famous Jungck theorem [16], the authors of [17] called T an F-contraction with respect to S if there exists a function F and $\tau > 0$ such that

$$\tau + F(d(Tx, Ty)) \le F(d(Sx, Sy)) \tag{6}$$

holds for all $x, y \in X$ satisfying $Tx \ne Ty$. They proved some coincidence and common fixed point results for a pair of mappings acting in ordered metric spaces.

For the sake of simplicity, we formulate these results here without using ordering. Again, just conditions (F1) and (F2) are used and the proof is much shorter because Lemmas 2.1 and 2.2 are used.

Theorem 3.1 Let (X, d) be a complete metric space and T, S be two self-mappings on X, such that $T(X) \subseteq SX$, one of these subsets being closed. If T is an F-contraction with respect to S, where the function F satisfies conditions

(F1) and (F2), then they have a unique point of coincidence. Moreover, if T and S are weakly compatible (i.e., if they commute at their coincidence points), then they have a unique common fixed point.

Proof. The condition (6), together with property (F1), immediately imply that $d(Tx, Ty) < d(Sx, Sy)$ if $Tx \neq Ty$. Further, let $x_0 \in X$ be arbitrary and let the respective Picard-Jungck sequence be defined by

$$y_n = Tx_n = Sx_{n+1}, \text{ for } n \in \mathbb{N} \cup \{0\}.$$

Then, using property (F1), it follows that $d(y_n, y_{n+1}) < d(y_{n-1}, y_n)$ if $y_n \neq y_{n+1}$. Now, again, by using Lemmas 2.1, and 2.2 it easily follows that $\{y_n\}$ is a Cauchy sequence. The rest of the proof is standard. □

Results on coincidence and common fixed points under F-contractions were also obtained in [18-22].

3.2 *Property (P)*

Denote by $Fix(T)$ the set of fixed points of a self-map T in a metric space (X, d). Recall that it is said that T satisfies property (P) if $Fix(T^n) = Fix(T)$ holds for each $n \in \mathbb{N}$ (here T^n denotes the n-th iterate of the mapping T). It is easy to prove the following assertion.

Proposition 3.2 Let T be a self-map in a metric space (X, d) satisfying condition (2) with function F satisfying (F1) and (F2). Then T has the property (P).

Proof. Since T satisfies the condition (2), then so does T^n for each $n \in \mathbb{N}$. Indeed, for $x, y \in X$,

$$F(d(T^n x, T^n y)) \le F(d(T^{n-1}x, T^{n-1}y)) - \tau \le \cdots$$

$$\le F(d(x, y)) - n\tau \le F(d(x, y)) - \tau.$$

Hence, if $T^n x \neq T^n y$, then $T^{n-1}x \neq T^{n-1}y$, ..., and condition (2) holds for T^n. By Theorem 2.3, T^n has a unique fixed point, and since, obviously, $Fix(T) \subseteq Fix(T^n)$, it follows that property (P) is satisfied. □

A result about the property (P) for F-contractions was also obtained in [17].

3.3 *Generalized contraction*

The following result is an improvement of [23, Theorem 2.4]. Its proof can be also made shorter, similarly as in some earlier mentioned cases.

Theorem 3.3 Let (X, d) be a complete metric space and T be a self-map in X satisfying that

$$\tau + F(d(Tx, Ty)) \le F\left(\max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\right\}\right)$$

holds for some $\tau > 0$, some function $F : (0, +\infty) \to \mathbb{R}$ satisfying (F1) and (F2), and all $x, y \in X$ satisfying $d(Tx, Ty) > 0$. If T or F is continuous, then T has a unique fixed point.

Further types of generalized F-contractions are numerous. In [24] the authors are interested in Prešić type F-contractive condition in the setting of metric spaces. Ahmad et al. [25] prove some new fixed point results for singlevalued and multivalued mappings in complete metric spaces. In [26], the authors give two new generalized types of contractions, i.e. the Branciari F-contractions and multiplicative F-contractions. In [27], new fixed point results of Hardy-Rogers-type for α-η-GF-contraction are given in the setting of a complete metric space. The same setting is true for [28], where new fixed point theorems for generalized F-contractions regarding rational expressions are presented. Generalized F-contractions for complete metric spaces also appear in [29, 30]. Bashir et al. [31], extend Wardowski's idea of F-contraction by introducing the reversed generalized F-contraction mapping. Further generalizations are F_ω-contractions [32], F-g-contractions [33], F-contractive mappings of Hardy-Rogers-type [34, 35]. In [36] the notion of dynamic process for generalized F-contraction mappings is introduced. Further examples of generalizations are τ

-generalization on fixed point results for F-contractions [37]; l-F type Suzuki contraction [38]; p-hybrid Wardowski contractions [39]; two generalizations from [40]; Ćirić type generalized F-contractions [41]; extended interpolative Ćirić-Reich-Rus type F-contractions [42]; (ω-F)-contractions [43]; Geraghty-Wardowski type contractions [44]; F-Suzuki contractions [45]; ψF-contractions and ψF-quasicontractions [46, 47]; F_S-contractions and F_J-contractions [48]; generalized F-contractive mappings in the framework of G-metric spaces [49]; F-convex contraction via admissible mapping [50, 51]; F_c-contractions [52]; (a, F)-contractions [53]. Important improvements and extensions are given in [54-58]. Generalization and application to existence problem of some classes of integral equations are given in [59].

3.4 *Best proximity points*

In [60], the authors obtained some results concerning so-called best proximity points of non-self mappings in a complete metric space. We note here just that their Theorems 3.1, 3.2 and 3.3 remain true when just conditions (F1) and (F2) are imposed on the function F, and that the proofs of these results can be made much shorter by using Lemmas 2.1 and 2.2, as was done here in the case of Theorem 2.3.

Other results on best proximity points using F-contractions were obtained in [61].

3.5 *Results in b-metric spaces*

Recall that a triplet (X, d, s) is called a b-metric space if X is a non-empty set, $s \geq 1$ is a given real number and $d : X \times X \to [0, +\infty)$ satisfies axioms of a metric spaces, but with the triangular inequality replaced by the condition

$$d(x,z) \leq s[d(x,y)+d(y,z)]$$

for all $x, y, z \in X$.

Results about fixed points in b-metric spaces using Wardowski-type conditions were obtained in several papers. We note here that most of these results can be obtained with weaker assumptions on the function F. For example, it is the case for a common fixed point result [62, Theorem 1] for four mappings which was proved with weaker assumptions in [63].

We present also another kind of result, obtained by Suzuki in [64], where different assumptions are imposed on the function F (note that (F1) is here not used).

Theorem 3.4 [64, Theorem 23] Let (X, d, s) be a complete b-metric space and T be a self-map on X. Assume that there exist $\tau > 0$ and a function $F : (0, +\infty) \to \mathbb{R}$ satisfying (F2) such that (2) holds for all $x, y \in X$ with $Tx \neq Ty$. Then T has a unique fixed point.

There is a number of other papers where F-contractions in b-metric spaces were investigated. In [65], Alqahtani et al. study the existence and uniqueness of F-contractions in b-metric space and apply to differential equations with fractional derivatives; Alsulami et al. [66] study F-Suzuki type contraction in complete b-metric spaces and investigate the existence of fixed points of such mappings; Cosentino et al. [67] study integrodifferential problems via fixed point theory in b-metric spaces; in [68], the authors introduce the notions of extended F-contraction of Hardy-Rogers type, extended F-contraction of Suzuki-Hardy-Rogers type and generalized F-weak contraction of Hardy-Rogers type and establish some new fixed point results for such kind of mappings in the setting of complete b-metric spaces; in [69], the authors show that the results of applications of Wardowski's method of F-contractions in fixed point results for single and multivalued mappings in b-metric spaces can be obtained directly, i.e., without using most of the conditions set for the auxiliary F-function; in [70], the result of Cosentino et al. is generalized by introducing Feng and Liu type F-contractions; Lukacs and Kajanto [71] study F-contractions and their generalizations in the context of b-metric spaces. The classes of mappings called TAC-Suzuki-Berinde type F-contractions and TAC-Suzuki-Berinde type rational F-contractions are introduced in the framework of b-metric spaces in [72] and some fixed point results are proved. Nazam et al. [73] obtain a common fixed point result for four mappings under F-contraction in b-metric spaces. Parvaneh et al. [74] introduce the α-β-FG-contraction and generalize the Wardowski fixed point result in b-metric and ordered b-metric spaces; Piri and Kumam [75] present fixed point theorems for generalized F-Suzuki-contraction mappings in complete b-metric spaces.

3.6 *Multivalued mappings*

Investigation of fixed points of multivalued mappings started in 1969 with the work [76] of Nadler. After 2012, a lot of researchers attempted to apply Wardowski's approach to such problems in various spaces, e.g., metric spaces with a directed graph [77], or domain of sets endowed with directed graph [78]. Others are concerned with fixed point theorem for multivalued mappings with δ-distance using Wardowski's technique [79, 80], or generalized multivalued F-contraction mappings in complete metric space [81]. In [82], the authors introduce the notions of α-F-contractions, by combining the notions of α-Ψ-contraction and F-contraction and obtain fixed point theorems for multivalued mappings. The multivalued almost F-contractions are discussed in [83], while the multivalued F-contractions in [84], and multivalued nonlinear F-contractions in [85], all on complete metric spaces. Amini-Harandi [86] introduces a new concept of set-valued contraction and proves a fixed point theorem which generalizes some well-known results. Wardowski type α-F-contractive approach for nonself multivalued mappings is given in [87]. The multivaled mappings in partial symmetric spaces are treated in [88]. In [89], the authors give a partial answer to Reich's problem on multivalued contraction mappings and generalize Mizoguchi-Takahashi's fixed point theorem using a new approach of multivalued orthogonal (τ, F)-contraction mappings in the framework of orthogonal metric spaces. F-contractions on quasi metric spaces were treated in [90]. Debnath and Srivastava [91] introduce a new and proper extension of Kannan's fixed point theorem to the case of multivalued maps. The extended multivalued F-contraction in metric-like spaces is treated in [92]. The existence of best proximity points for multivalued Suzuki-Edelstein-Wardowski type α-proximal contractions in the setting of complete metric spaces and partially ordered metric spaces is shown in [93], while the existence of fixed points for multivalued modified F-contractions in the context of complete metric spaces is shown in [94]. Iqbal et al. [95] introduce the notion of α-type F-τ-contraction and establish related fixed point results in metric spaces and partially ordered metric spaces. Išik et al. [96] introduce generalized Wardowski type quasi-contractions called α-(ψ, Ω)-contractions for a pair of multivalued mappings and prove the existence of the common fixed point for such mappings. Jaradat et al. [97], introduce the concept of multivalued generalized ψ-F-contraction (weakly ψ-F-contraction) as a generalization of multivalued generalized ψ-F-contraction and also introduce the concept of multivalued generalized α-ψ-F-contractions and prove PPF-dependent fixed point results in the setting of a Banach space. In [98], the authors present some fixed point theorems by combining the contractions of Geraghty and Hardy-Rogers with F-contraction and α-admissible concept in the setting of set-valued mappings under weaker conditions. Klim and Wardowski [99] extend the concept of F-contractive mappings to the case of nonlinear F-contractions and prove a fixed point theorem via dynamic processes. Fixed point results for multivalued maps on complete metric spaces without using the Hausdorff metric are presented in [100, 101]. Some Wardowski-Feng-Liu type fixed point theorems for multivalued mappings in complete (ordered) metric spaces are presented in [102]. The set-valued $(\alpha - \varphi)$-F-contraction mappings in the setting of a partial metric space are introduced in [103]. Further, multivalued F-contractions in 0-complete partial metric spaces are studied in [104]; common fixed point theorems for a pair of multivalued mappings satisfying a new Ćirić-type rational F-contraction condition in complete dislocated metric spaces in [105]; some fixed point theorems, coincidence point theorems and common fixed point theorems for multivalued F-contractions involving a binary relation that is not necessarily a partial order, in the context of generalized metric spaces (in the sense of Jleli and Samet) in [106]; fixed point results for closed multivalued F-contractions or multivalued mappings which satisfy an F-contractive condition of Hardy-Rogers-type, in the setting of complete metric spaces or complete ordered metric spaces in [107]; set-valued G-Prešić type F-contractions on product spaces when the underlying space is a complete metric space endowed with a graph in [108]; generalized dynamic process for generalized multivalued F-contraction of Hardy-Rogers type in b-metric spaces is discussed in [109], while the set-valued Prešić type almost contractive mapping, Prešić type almost F-contractive mapping and set-valued Prešić type almost F-contractive mapping in metric space are introduced in [110].

We present here just one of the basic results of this kind.

Let (X, d) be a metric space, $CB(X)$ be the family of its nonempty, closed and bounded subsets, and $K(X)$ the family of its nonempty compact subsets. Recall that the Hausdorff-Pompeiu metric on $CB(X)$ is the function $H : CB(X) \times CB(X) \to [0, +\infty)$ defined for $A, B \in CB(X)$ by

$$H(A,B) = \max\left\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\right\}.$$

The following result was proved in [84].

Theorem 3.5 [84, Theorem 2.2] Let (X, d) be a complete metric space and $T : X \to K(X)$ be a (multivalued) mapping satisfying

$$\tau + F(H(Tx,Ty)) \le F(d(x,y))$$

for some $\tau > 0$ and $F : (0, +\infty) \to \mathbb{R}$ satisfying conditions (F1), (F2) and (F3), and for all $x, y \in X$ with $H(Tx, Ty) > 0$. Then T has a fixed point, i.e., there exists $x^* \in X$ such that $x^* \in Tx^*$.

An open question (which we state at the end of this article) is whether, like in a single-valued case, this result remains valid if just assumptions (F1) and (F2) are imposed on the function F.

3.7 *Some other areas of research*

The following is the list of other areas where F-contractions were investigated and respective articles.

Problems including spaces with additional structure. For example, the spaces with α-admissible mappings are treated in [111-115]. The ordered spaces are discussed in [116-119]. The relational-theoretic spaces are studied in [120, 121]. The spaces endowed with a graph were considered in [122, 123].

Spaces with alternate distance or two metrics and respective F-contractions were considered in [124-126].

Vector-valued spaces and Perov-type problems were treated in [127, 128].

F-contractions in metric-like and b-metric-like spaces were under investigation in [9, 129-135].

Partial metric spaces and F-contractions in them were treated in [136-141].

Modular and fuzzy-metric spaces and F-contractions in them were observed in [142-147].

Other generalized metric spaces and respective problems were treated in [148-152].

4. Some possibilities for further investigation

We present some open problems for further investigation.

Question 4.1 Does an F-version hold for Ćirić's quasicontraction (see [153] or [154]), i.e., does the following result hold true: If (X, d) is a complete metric space and if the self-map T on X satisfies

$$\tau + F(d(Tx,Ty)) \le F(\max\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\})$$

for all $x, y \in X$ with $Tx \ne Ty$, where $\tau > 0$ and F satisfies conditions (F1) and (F2), then T has a unique fixed point?

Question 4.2 Let (X, d) be a complete metric space and let the self-map T on X satisfy

$$\tau + F(d(T^n x, T^n y)) \le F(d(x,y))$$

for all $x, y \in X$ with $T^n x \ne T^n y$, where $\tau > 0$, F satisfies conditions (F1) and (F2), and $n = n(x, y)$. Does T have a unique fixed point? The positive answer would be a generalization of Sehgal's result [155].

Question 4.3 Can conditions for the function F be reduced to (F1) and (F2), and can the proof be made simpler in some results for multivalued mappings in the same way as it was presented in this survey for single-valued mappings?

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Conflict of interest

The authors declare no conflict of interest.

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