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Research Article

Adaptive Identification of System with Bouc-Wen Hysteresis Modifications

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Abstract: The adaptive identification method developed to evaluate the parameters of the Bouc-Wen hysteresis (BWH). The adaptive approach based on the use of adaptive observer. We synthesize adaptive identification algorithms using the second Lyapunov method. Requirements for the input of the system which guarantee the identification of parameters considered. We propose BWH modifications (BWHM). Adaptive algorithms for estimating BWHM parameters developed. The boundedness of adaptive system processes shown in coordinate and parametric spaces. We prove the exponential dissipativity of processes in an adaptive system by using the Lyapunov vector function method. Estimating method proposed for signaling uncertainty in the system.

***Keywords*:** hysteresis, Bouc-Wen model, adaptive observer, exponential dissipativity, Lyapunov vector function, uncertainty

MSC: 93B30, 34C55, 93C40

1. Introduction

Various models [1] apply to the description of the hysteresis. But the Bouc-Wen model (BWM) [2-3] has the widest application. Many BWM modifications (BWMM) [4-9] were proposed. Each model considered the features of the object [10-15]. The BWM successful application depends on the identification of its parameters [16-21]. Various algorithms are used for BWM identification [22-24]. An adaptive parameter identification method [14] was proposed for the BW hysteresis model. Identification of BW hysteresis parameters is based on time data consider in [15]. The algorithm is based on the least squares method and the sensitivity analysis of the output.

In [15, 17], adaptive algorithms propose for the BWM parameters estimation with the data forgetting [8]. Paper [18] presents an adaptive on-line identification methodology with a variable trace method to adjust the adaptation gain matrix. Most BWMM are based on the BWH approximation in some working areas of the object [19-24]. The approximation method choice depends on the requirements of the control and the workspace. Parametric identification procedures apply to obtain simplified BWH models.

Most proposed approaches use the derivative measurement by the output of the system. This possibility does not always exist in practical applications. There are studies [25] which estimates of BWM parameters do not coincide with the results obtained for other input data. Explain it with the fact that the BWM should be stable and ensure the adequacy

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of a physical process [4].

The conditions to be satisfied by the Bouc-Wen model are considered in [26]. The major difficulties of the BWM parameters estimation are (i) the ensuring model stability (ii) the input choice. The stability imposes restrictions on the ranges of changes in model parameters. The choice of parameters belonging to the stability domain does not always give an adequate BWM [26].

So, the set of algorithms and procedures proposed for the Bouc-Wen model parameters identification. The models reflect the features of the system under study. As a rule, the change area BWM parameters set a priori. This is also true for some system parameters. It is often assumed that all system derivatives are measured. This assumption is not always true, which makes the algorithms unrealizable. Most identification procedures are valid only in some areas. Therefore, the design of identification algorithms is an urgent problem for BWH under uncertainty.

Below, we propose the adaptive identification based on adaptive observer. It is used for the problem solution stability and identification for system (1)-(4). Method are based on the approach proposed in [27-28] and does not require measuring derivatives of the system output. We believe that only the input and output of the system can be measured. BWH modifications are considered. They reduce the use of the model and remove the stability problem.

The paper has the following structure. Section 2 contains the problem statement. Section 3 considers the proposed modifications of the BWH (BWHM). These modifications guarantee the stable solving of the system with BWHM and its identification. Section 4 contains requirements on an input that guarantee the structural identifiability of the system with BWHM. The adaptive observer and the analysis of its properties are considered in sections 5, 6. We present modeling and discussion of results in sections 7, 8. Appendixes contain the stability proof of the adaptive system.

2. Problem statement

Consider the system S_{BW}

$$m\ddot{x}+c\dot{x}+F(x,z,t)=f(t), \tag{1}$$

$$F(x,z,t)=\alpha kx(t)+(1-\alpha)kdz(t), \tag{2}$$

$$\dot{z}=d^{-1}\left(a\dot{x}-\beta|\dot{x}||z|^{n}\,sign(z)-\gamma\dot{x}|z|^{n}\right), \tag{3}$$

$$y(t)=x(t), \tag{4}$$

where $m > 0$ is mass, $c > 0$ is damping, $F(x, z, t)$ is the recovering force, $d > 0$, $n > 0$, $k > 0$, $\alpha \in (0, 1)$, $f(t)$ is exciting force, α, β, γ are some numbers. The system (1)-(4) are the basis for the classic BWH presentation. All further studies on BWH are based on the modification of this system. Equations (1)-(4) are used for the analysis of nonlinear mechanical systems. Adaptation of system S_{BW} to real objects requires BWH modification.

The system (1)-(4) are widely used for the processes analysis in construction mechanics, control of complex mechanical systems, modeling the work of damping devices and the like. Equation (1) describes an object that is affected by the restoring force $F(x, z, t)$ and the exciting force $f(t)$. In applications, various approximations $F(x, z, t)$ are used, reflecting the specific of the system.

The set of the experimental data

$$\mathrm{I}_o=\{f(t),\, y(t),\, t\in J\}, \tag{5}$$

where $J \subset R$ is the given time interval. Denote the system parameters vector as $A = [m, c, a, k, \alpha, \beta, \gamma, n]^T$.

Problem: design the adaptive observer for vector A estimation to

$$\lim_{t\to\infty}\left|\hat{y}(t)-y(t)\right|\leq \pi_y \tag{6}$$

where $\hat{y}\in R$ is the output of the adaptive observer, $\pi_y \geq 0$.

3. System S_{BW} modifications

Various modifications of BWH have been proposed (see, for example, [9, 11, 26]). They consider features and properties of the system. System (1)-(3) are the basis for modifications. The analysis shows that the last term in (3) guarantees "fine-tuning" the BW hysteresis in the saturation or switching areas. If this is not critical for the system, then by selecting parameters of the S1-system, this term in the equation (3) can be compensated. In addition, some modifications are simplified and increase the system (1)-(3) stability. The main purpose of making structural changes is to simplify the system and improve its properties. We propose the following modifications of the Bouc-Wen model (3) [28]

$$\mathcal{M}_{\rho\varpi\mu\upsilon\beta n}:\dot{z}=-\rho z\left|\dot{x}\right|^{\varpi}+\pi\left|\dot{x}\right|^{\mu} sign(\dot{x})-\beta\left|\dot{x}\right|^{\upsilon}\left|z\right|^{n} sign(z), \tag{7}$$

$$\mathcal{M}_{\mu\beta n}:\dot{z}=\pi\left|\dot{x}\right|^{\mu} sign\left(\dot{x}\right)-\beta\left|\dot{x}\right|\left|z\right|^{n} sign(z), \tag{8}$$

$$\mathcal{M}_{\mu\upsilon\beta n}:\dot{z}=\pi\left|\dot{x}\right|^{\mu} sign\left(\dot{x}\right)-\beta\left|\dot{x}\right|^{\upsilon}\left|z\right|^{n} sign(z). \tag{9}$$

The linear component on z in (7) increases the feasibility model, and stability of the system. As the system is nonlinear, the function $|\dot{x}(t)|^{\varpi}$ is introduced to ensure the required hysteresis state. It guarantees a change z in the specified boundaries. Parameters $\rho > 0$, $\omega > 0$ are some numbers.

We have not tried to due reproduce BWH (3) using modifications (7)-(9). A detailed analysis of the models (7)-(9) parameters effect of on hysteresis is given in [28].

4. About influence $f(t)$ on BWH parameters identifiability

The input choice is an important stage in the nonlinear systems identification. These issues are discussed in [28-29]. The input $f(t)$ of the system must be constantly excited and have the property of S-synchronizability. These conditions are the basis for the structural identifiability of the system (1)-(3). They guarantee the system parameters evaluation using adaptive algorithms.

5. Design of adaptive observer

5.1 *System S_{BW}*

Let $d = 1$, $a = 1$. Substitute $F(x, z, t)$ in (1) and write it as

$$\left(s^2+\bar{a}_1 s+\bar{a}_2\right)x+\bar{a}_3 z=bf, \tag{10}$$

where

$$s=\frac{d}{dt},\ \overline{a}_1=\frac{c}{m},\ \overline{a}_2=\frac{\alpha k}{m},\ \overline{a}_3=\frac{(1-\alpha)k}{m},\ b=\frac{1}{m}.$$

Reduce (10) to an identification form on $\dot{x}$. Divide the left and right parts (10) into $s+\mu$, where $\mu>0$ does not coincide with roots of the polynomial $s^2+a_1s+a_2$. Then (10)

$$\dot{x}=a_1x+a_2p_x+a_3p_z+bp_f, \tag{11}$$

$$\dot{p}_x=-\mu p_x+x,$$

$$\dot{p}_f=-\mu p_f+f,$$

$$\dot{p}_z=-\mu p_z+z, \tag{12}$$

where

$$a_1=-\frac{c-\mu m}{m},\ a_2=-\frac{\alpha k-\mu(c-\mu m)}{m},\ a_3=-\frac{(1-\alpha)k}{m}.$$

Variables p_i, $i=x, f, z$ obtained from equations $p_i=i/(s+\mu)$. Equations (11), (12) contain only measurable variables except z. It complicates the identification of the system S_{BW} parameters.

Remark 1. Simplifications $d=1$ and $a=1$ do not affect the parameters (11) identification. Consideration d, a increases the number of estimated parameters. The system (10)-(12) are used to guarantee the system (1)-(4) parameters identification on the set (5). It excludes the use of the non-measurable derivative $\dot{x}$ in parametric identification.

Apply the model for parameters estimate of equation (11)

$$\dot{\hat{x}}=-k_x(\hat{x}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{a}_3p_z+\hat{b}p_f, \tag{13}$$

where $k_x>0$ is specified number, $\hat{a}_i(t)$, $i=1, 2, 3$ and $\hat{b}(t)$ are adjusted parameters.

Designate $e=\hat{x}-x$ and obtain the equation for the identification error from (11), (13)

$$\dot{e}=-k_xe+\Delta a_1x+\Delta a_2p_x+\Delta a_3p_z+\Delta bp_f, \tag{14}$$

where $\Delta a_1=\hat{a}_1(t)-a_1$, $\Delta a_2=\hat{a}_2(t)-a_2$, $\Delta a_3=\hat{a}_3(t)-a_3$, $\Delta b=\hat{b}(t)-b.$

The (14) is not solvable as the variable z is unknown in (12). Receive the current estimate for z. Consider the model

$$\dot{\hat{x}}_{\overline{z}}=-k_x(\hat{x}_{\overline{z}}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{b}p_f. \tag{15}$$

Determine the residual $\varepsilon_z=x-\hat{x}_{\overline{z}}$ and use it for the variable z estimation. Apply the model

$$\dot{\hat{z}}=-k_z\left(\hat{z}-\varepsilon_z\right)+\tilde{\dot{x}}-\hat{\beta}\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z})-\hat{\gamma}\tilde{\dot{x}}\left|\hat{z}\right|^n, \tag{16}$$

where $\tilde{\dot{x}}=\left(x(t+\tau)-x(t)\right)/\tau$; $k_z>0$ is specified number; $\hat{\beta}$, $\hat{\gamma}$ are the hysteresis (3) parameters estimations; τ is the integration step.

Introduce the residual $\varepsilon = \hat{z} - \varepsilon_z$ and obtain the equation for ε

$$\dot{\varepsilon} = -k_z \varepsilon + \Delta\dot{x} + \Delta\beta \left|\tilde{\dot{x}}\right| \left|\hat{z}\right|^n sign\left(\hat{z}\right) + \beta\eta_\beta + \Delta\gamma \tilde{\dot{x}} \left|\hat{z}\right|^n + \gamma\eta_\gamma, \tag{17}$$

$$\eta_\beta = \left|\dot{x}\right| \left|z\right|^n sign\left(z\right) - \left|\tilde{\dot{x}}\right| \left|\hat{z}\right|^n sign\left(\hat{z}\right), \tag{18}$$

$$\eta_\gamma = \dot{x}\left|z\right|^n - \tilde{\dot{x}}\left|\hat{z}\right|^n, \tag{19}$$

where $\Delta\dot{x} = \tilde{\dot{x}} - \dot{x},\ \Delta\beta = \beta - \hat{\beta},\ \Delta\gamma = \gamma - \hat{\gamma}.$

Then the equation (13)

$$\dot{\hat{x}} = -k_x(\hat{x} - x) + \hat{a}_1 x + \hat{a}_2 p_x + \hat{a}_3 p_{\hat{z}} + \hat{b} p_f, \tag{20}$$

where

$$\dot{p}_{\hat{z}} = -\mu p_{\hat{z}} + \hat{z}. \tag{21}$$

Then (15)

$$\dot{e} = -k_x e + \Delta a_1 x + \Delta a_2 p_x + \Delta a_3 p_{\hat{z}} + \Delta b p_f. \tag{22}$$

Synthesize algorithms for tuning parameters of adaptive models. Consider the Lyapunov function (LF) $V_e(t)$ = $0.5e^2(t)$ and obtain for $\dot{V}_\varepsilon$

$$\dot{V}_e = -k_x e^2 + e\left(\Delta a_1 x + \Delta a_2 p_x + \Delta a_3 p_{\hat{z}} + \Delta b p_f\right). \tag{23}$$

Obtain adaptive algorithms from the condition $\dot{V}_e \leq 0$

$$\Delta\dot{a}_1 = -\gamma_1 e_x,$$

$$\Delta\dot{a}_2 = -\gamma_2 p_x,$$

$$\Delta\dot{a}_3 = -\gamma_3 e p_{\hat{z}},$$

$$\Delta b = -\gamma_b e p_f, \tag{24}$$

where $\gamma_i > 0$, $i = 1, 2, 3$; $\gamma_b > 0$.

Synthesize algorithms for tuning model (16) parameters. Consider $V_\varepsilon(t) = 0.5\varepsilon^2(t)$ and equation (17). Then $\dot{V}_\varepsilon$

$$\dot{V}_\varepsilon = \varepsilon\dot{\varepsilon} = -k_z \varepsilon^2 + \varepsilon\left(\Delta\dot{x} + \Delta\beta \left|\tilde{\dot{x}}\right| \left|\hat{z}\right|^n sign\left(\hat{z}\right) + \beta\eta_\beta + \Delta\gamma \tilde{\dot{x}} \left|\hat{z}\right|^n + \gamma\eta_\gamma\right), \tag{25}$$

where ε satisfies equation (17).

We receive from (25)

$$\Delta\dot{\beta} = -\chi_\beta \varepsilon \left|\tilde{x}\right| \left|\hat{z}\right|^n sign(\hat{z}),$$

$$\Delta\dot{\gamma} = -\chi_\gamma \varepsilon \tilde{x} \left|\hat{z}\right|^n, \tag{26}$$

where $\chi_\beta > 0$, $\chi_\gamma > 0$ are parameters that ensure the algorithms convergence.

Several algorithms are used to estimate the indicator n in (11). Their effectiveness depends on several factors. A simple algorithm has the form

$$\dot{\hat{n}} = \begin{cases} -\gamma_n \varepsilon \hat{\beta} \left|\hat{z}\right|^{\hat{n}-1} \hat{z}\tilde{x}, \text{ if } \left|\dfrac{\varepsilon}{\varepsilon_z}\right| \in [\upsilon_0, \upsilon_1], \\ \qquad 0, \qquad \text{if } \left|\dfrac{\varepsilon}{\varepsilon_z}\right| \notin [\upsilon_0, \upsilon_1], \end{cases} \tag{27}$$

where υ_0, υ_1 are set positive numbers, $\gamma_n > 0$.

So, equations (12), (17), (21), (22), (24), (26), (27) describe the adaptive identification system for the S_{BW}-system. Denote this system as AS_{BW}.

5.2 System (1), (2) with hysteresis $\mathcal{M}_{\rho\omega\mu\upsilon\beta n}$, $\mathcal{M}_{\mu\beta n}$, $\mathcal{M}_{\mu\upsilon\beta n}$

1. Model $\mathcal{M}_{\rho\omega\mu\upsilon\beta n}$. Equations (16)-(19) have the form in this case

$$\dot{\hat{z}} = -k_z(\hat{z} - \varepsilon_z) - \hat{\rho}\hat{z}\left|\dot{\tilde{x}}\right|^{\omega} + \hat{\pi}\left|\dot{x}\right|^{\mu} sign(\dot{\tilde{x}}) - \beta\left|\dot{\tilde{x}}\right|^{\upsilon}\left|\hat{z}\right|^n sign(\hat{z}), \tag{28}$$

$$\dot{\varepsilon} = -k_z\varepsilon - \Delta\rho\hat{z}\left|\dot{\tilde{x}}\right|^{\omega} + \Delta\pi\left|\dot{\tilde{x}}\right|^{\mu} sign(\dot{\tilde{x}}) - \Delta\beta\left|\dot{\tilde{x}}\right|^{\upsilon}\left|\hat{z}\right|^n sign(\hat{z}) + \rho\bar{\eta}_\rho + \pi\bar{\eta}_\pi + \beta\bar{\eta}_\beta, \tag{29}$$

$$\bar{\eta}_\rho = \left|\dot{x}\right|^{\omega} z - \left|\dot{\tilde{x}}\right|^{\omega}\hat{z}, \tag{30}$$

$$\bar{\eta}_\pi = \left|\dot{\tilde{x}}\right|^{\mu} sign(\dot{\tilde{x}}) - \left|\dot{x}\right|^{\mu} sign(\dot{x}), \tag{31}$$

$$\bar{\eta}_\beta = \left|\dot{x}\right|^{\upsilon}\left|z\right|^n sign(z) - \left|\dot{\tilde{x}}\right|^{\upsilon}\left|\hat{z}\right|^n sign(\hat{z}). \tag{32}$$

Consider $\dot{V}_\varepsilon$

$$\dot{V}_\varepsilon = -k_z\varepsilon^2 + \varepsilon\left(-\Delta\rho\hat{z}\left|\dot{\tilde{x}}\right|^{\omega} + \Delta\pi\left|\dot{\tilde{x}}\right|^{\mu} sign(\dot{\tilde{x}}) - \Delta\beta\left|\dot{\tilde{x}}\right|^{\upsilon}\left|\hat{z}\right|^n sign(\hat{z}) + \rho\bar{\eta}_\rho + \pi\bar{\eta}_\pi + \beta\bar{\eta}_\beta\right) \tag{33}$$

and obtain algorithms

$$\Delta\dot{\beta} = \chi_\beta \varepsilon \left|\tilde{x}\right| \left|\hat{z}\right|^n sign\left(\hat{z}\right),$$

$$\Delta\dot{\pi} = -\chi_\pi \varepsilon \left|\tilde{x}\right|^\mu sign\left(\tilde{x}\right),$$

$$\Delta\dot{\rho} = -\chi_\rho \varepsilon \hat{z} \left|\tilde{x}\right|^\omega, \tag{34}$$

where $\chi_\beta > 0$, $\chi_\rho > 0$, $\chi_\pi > 0$ are parameters guaranteed convergence of algorithms, $\Delta\rho = \hat{\rho}(t) - \rho$, $\Delta\pi = \hat{\pi}(t) - \pi$.

The structure of algorithms for estimating n, ω, μ coincides with (27).

2. Model $\mathcal{M}_{\mu\beta n}$. Equations (16)-(19) have the form

$$\dot{\hat{z}} = -k_z\left(\hat{z} - \varepsilon_z\right) + \hat{\pi}\left|\tilde{x}\right|^\mu sign\left(\tilde{x}\right) - \hat{\beta}\left|\tilde{x}\right|\left|\hat{z}\right|^n sign(\hat{z}), \tag{16.2}$$

$$\dot{\varepsilon} = -k_z\varepsilon - \Delta\beta\left|\tilde{x}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right) + \Delta\pi\left|\tilde{x}\right|^\mu sign\left(\tilde{x}\right) + \beta\tilde{\eta}_\beta + \pi\tilde{\eta}_\pi, \tag{17.2}$$

$$\tilde{\eta}_\beta = \left|\dot{x}\right|\left|z\right|^n sign\left(z\right) - \left|\tilde{x}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right), \tag{18.2}$$

$$\tilde{\eta}_\pi = \left|\dot{x}\right|^\mu sign\left(\dot{x}\right) - \left|\tilde{x}\right|^\mu sign\left(\tilde{x}\right). \tag{19.2}$$

3. Model $\mathcal{M}_{\mu\upsilon\beta n}$. Equations (16)-(19) have the form

$$\dot{\hat{z}} = -k_z\left(\hat{z} - \varepsilon_z\right) + \hat{\pi}\left|\tilde{x}\right|^\mu sign\left(\tilde{x}\right) - \hat{\beta}\left|\tilde{x}\right|^\upsilon \left|\hat{z}\right|^n sign(\hat{z}), \tag{16.3}$$

$$\dot{\varepsilon} = -k_z\varepsilon - \Delta\beta\left|\tilde{x}\right|^\upsilon \left|\hat{z}\right|^n sign\left(\hat{z}\right) + \Delta\pi\left|\tilde{x}\right|^\mu sign\left(\tilde{x}\right) + \beta\hat{\eta}_\beta + \pi\hat{\eta}_\pi, \tag{17.3}$$

$$\hat{\eta}_\beta = \left|\dot{x}\right|^\upsilon \left|z\right|^n sign\left(z\right) - \left|\tilde{x}\right|^\upsilon \left|\hat{z}\right|^n sign\left(\hat{z}\right), \tag{18.3}$$

$$\hat{\eta}_\pi = \left|\dot{x}\right|^\mu sign\left(\dot{x}\right) - \left|\tilde{x}\right|^\mu sign\left(\tilde{x}\right). \tag{19.3}$$

Algorithms structurally coincide with (34) for (16.2) and (16.3).

6. Properties AS_{BW}

Evaluate properties of the AS_{BW}-system. Consider the subsystem AS_X described by equations (22), (24). Let $\Delta K(t) \triangleq \left[\Delta a_1(t), \Delta a_2(t), \Delta a_3(t), \Delta b(t)\right]^T$,

$$V_K(t) \overset{\Delta}{=} 0,5\Delta K^T(t)\Gamma^{-1}\Delta K(t), \tag{35}$$

$$V(t) = V_e(t) + V_K(t), \tag{36}$$

where $\Gamma = diag(\gamma_1, \gamma_2, \gamma_3, \gamma_b)$.

Assumption 1. The input of the system (1)-(3) is constantly excited and bounded, i.e. the condition

$$\mathcal{PE}_\eta^f : f^2(t) \geq \eta \tag{37}$$

is valid for $\exists \eta > 0$ and $\forall t \geq t_0$ on some interval [0, T].

Theorem 1. Let (i) functions $V_e(t) = 0.5e^2(t)$, $V_K(t)$ are positive definite and satisfy conditions $\inf_{|e| \to \infty} V_e(e) \to \infty$, $\inf_{\|\Delta K\| \to \infty} V_K(\Delta K) \to \infty$; (ii) assumption 1 for $f(t)$ satisfied. Then (i) all trajectories of the system AS_X bounded, (ii) belong area

$$\mathrm{G}_t = \{(e, \Delta K) : V(t) \leq V(t_0)\},$$

(a3) the estimation

$$\int_{t_0}^{t} 2k_x V_e(\tau) d\tau \leq V(t_0) - V(t) \tag{38}$$

is fair.

We give the proof of Theorem 1 in Appendix A.

Theorem 1 shows the restriction of adaptive system AS_X trajectories. Ensuring of asymptotic stability in the system demands to impose additional conditions. Consider these conditions. Let $P(t) \overset{\Delta}{=} \left[x(t)\ p_x(t)\ p_{\dot{z}}(t) p_f(t)\right]^T$.

Definition 1. The vector P is constantly excited with a level v or have property $\mathcal{PE}_v$ if

$$\mathcal{PE}_v : P(t)P^T(t) \geq vI_4 \tag{39}$$

fairly for $\exists v > 0$ and $\forall t \geq t_0$ on some interval $T > 0$, where $I_4 \in R^4$ is the unity matrix.

If the vector $P(t)$ has property $\mathcal{PE}_v$, then we will write $P(t) \in \mathcal{PE}_v$.

The system S_{BW} is stable, and the input $f(t)$ is restricted. Therefore, present the property $\mathcal{PE}_v$ for the matrix $B_P(t) = P(t)P^T(t)$ as

$$\mathcal{PE}_{v,\overline{v}} : vI_l \leq B_P(t) \leq \overline{v}I_l \quad \forall t \geq t_0, \tag{40}$$

where $\overline{v} > 0$ is some number.

Let the estimate for $V_K(t)$ be fair

$$0.5\,\beta_l^{-1}(\Gamma)\|\Delta K(t)\|^2 \leq V_K(t) \leq 0.5\,\beta_1^{-1}(\Gamma)\|\Delta K(t)\|^2, \tag{41}$$

where $\beta_1(\Gamma)$, $\beta_l(\Gamma)$ are minimal and maximum eigenvalues of the matrix Γ.

Apply (40), (41) and get estimations for $\dot{V}_e$, $\dot{V}_K$

$$\dot{V}_e \leq -k_x V_e + \frac{\overline{\nu}\beta_l(\Gamma)}{k_x} V_K, \tag{42}$$

$$\dot{V}_K \leq -\frac{3}{4}\vartheta\nu\beta_1(\Gamma)V_K + \frac{8}{3}\vartheta V_e, \tag{43}$$

where $\vartheta > 0$ is some number. We describe the method of obtaining estimates (42), (43) in [30].

Theorem 2. Let conditions be satisfied (i) positive definite Lyapunov functions $V_e(t) = 0.5e^2(t)$ and $V_K(t) = 0.5\Delta K^T(t)$ $\Gamma^{-1}\Delta K(t)$ allow the indefinitely small highest limit at $|e(t)| \to 0$, $\|\Delta K(t)\| \to 0$; (ii) $P(t) \in \mathcal{PE}_{\nu,\overline{\nu}}$; (iii) equality $e\Delta K^T P = \vartheta\left(\Delta K^T B \Delta K + e^2\right)$ is fair in the area $O_v(O)$ with $0 < \vartheta$, where $O = \{0, 0^{3m}\} \subset R \times R^{3m} \times J_{0,\infty}$, O_v is some neighborhood of the point O; (iv) the function $V_K(t)$ satisfies (41); (v) $\dot{V}_\varepsilon$, $\dot{V}_K$ satisfy the system of inequalities

$$\begin{bmatrix} \dot{V}_e \\ \dot{V}_K \end{bmatrix} \leq \underbrace{\begin{bmatrix} -k_x & \dfrac{\overline{\nu}\,\beta_l(\Gamma)}{k_x} \\ \dfrac{8}{3}\vartheta & -\dfrac{3\nu\vartheta\beta_1(\Gamma)}{4} \end{bmatrix}}_{A_V} \begin{bmatrix} V_e \\ V_K \end{bmatrix}; \tag{44}$$

(vi) the upper solution for $V_{e,K}(t) = \left[V_e(t)\ V_K(t)\right]^T$ satisfies to the comparison equation $\dot{S} = A_V S$ if

$$V_\rho(t) \leq s_\rho(t) \quad \forall\left(t \geq t_0\right) \& \left(V_\rho\left(t_0\right) \leq s_\rho\left(t_0\right)\right), \tag{45}$$

where $\rho = e, K$, $S = [s_e\, s_K]^T$, $A_V \in R^{2\times 2}$ is M-matrix [31]. Then the system AS_X is exponentially stable with the estimation

$$V_{e,K}(t) \leq e^{A_V(t-t_0)} S(t_0),\ V_{e,K} = \left[V_e\ V_K\right]^T, \tag{46}$$

if

$$k_x > 0,\ k_x \geq \frac{4}{3}\sqrt{\frac{2\overline{\nu}\beta_l(\Gamma)}{\nu\beta_1(\Gamma)}}. \tag{47}$$

Theorem 2 shows if $P(t) \in \mathcal{PE}_{\nu,\overline{\nu}}$, then the adaptive system AS_X gives accurate estimates of system (11) parameters. The system parameters satisfy condition (47). We suppose that the variable $p_{\bar{z}}$ bounded.

The boundedness of the variable $\hat{x}_{\bar{z}}$ follows from the system stability.

Consider subsystem AS_Z described by equations (17), (25) and (26). Introduce Lyapunov functions

$$V_{\varepsilon\beta\gamma}(t) = V_\varepsilon(t) + V_{\beta,\gamma}(t), \tag{48}$$

$$V_{\beta,\gamma}(t) = 0.5\chi_\beta^{-1}\left(\Delta\beta(t)\right)^2 + 0.5\chi_\gamma^{-1}\left(\Delta\gamma(t)\right)^2. \tag{49}$$

Theorem 3. Let (1) functions $V_\varepsilon(t) = 0.5\varepsilon^2(t)$, $V_{\beta,\gamma}(t)$ are positive definite and satisfy condition

$$\inf_{|\varepsilon|\to\infty} V_\varepsilon(\varepsilon) \to \infty, \quad \inf_{\|[\Delta\beta,\Delta\gamma]\|\to\infty} V_{\beta,\gamma}(\Delta\beta,\Delta\gamma) \to \infty;$$

(2) the function $V_{\varepsilon\beta\gamma}(t)$ has the form (88); (3) the function

$$\tilde{g}_1(t) = \sup_{\varepsilon\in\Omega} \frac{|\varepsilon|^{n+1}(t)}{V_\varepsilon(t,\varepsilon)}, \; g_1 = \sup_{\varepsilon\in\Omega} \tilde{g}_1(t), \tag{50}$$

exists, where Ω is the definition range of the subsystem AS_Z; (4) $|\Delta\dot{x}| \le \delta_\Delta$, $\delta_\Delta \ge 0$, $|\dot{x}| \le \upsilon$, $\upsilon > 0$; (5) the assumption 1 holds for the system (1)-(3). Then (i) all trajectories of the system AS_Z bounded, (ii) trajectories belong in the area

$$\mathrm{G}_\varepsilon = \left\{(\varepsilon,\Delta\beta,\Delta\gamma) : V_{\varepsilon\beta\gamma}(t) \le V_{\varepsilon\beta\gamma}(t_0)\right\},$$

(iii) the estimation

$$\int_{t_o}^{t} \left(k_z - \upsilon(\beta+\gamma)g_1\right)V_\varepsilon(\tau)d\tau + \frac{1}{2\left(k_z - \upsilon(\beta+\gamma)g_1\right)\left(t-t_0\right)}\left(\delta_\Delta\right)^2 \le V_{\varepsilon\beta\gamma}(t_0) - V_{\varepsilon\beta\gamma}(t), \tag{51}$$

is fair if

$$k_z > \upsilon(\beta+\gamma)g_1. \tag{52}$$

We give the proof of Theorem 1 in Appendix B.

So, the boundedness of trajectories in the adaptive system AS_{BW} was proved. The trajectories limitation of the subsystem AS_Z is a more complex problem in the parametric and output spaces. The estimation (51) shows that the quality of AS_Z-system processes depends on the output derivative of the S_{BW}-system. The guarantee of the AS_Z-system stability is the fulfillment of the condition (52). This conclusion explains problems in implementing various procedures for BWM identifying. The following result gives more exact estimations for AS_Z-system.

Theorem 4. Let (i) positive definite Lyapunov functions

$$V_\varepsilon(t) = 0.5\varepsilon^2(t), \; V_{\beta,\gamma}(t) = 0.5\chi_\beta^{-1}\left(\Delta\beta\right)^2 + 0.5\chi_\gamma^{-1}\left(\Delta\gamma\right)^2 \tag{53}$$

allow the indefinitely small highest limit at $|\varepsilon(t)| \to 0$, $\|[\Delta\beta(t), \Delta\gamma(t)]\| \to 0$; (ii) $P(t) \in \mathcal{PE}_{\nu,\overline{\nu}}$; (iii) $c_1 > 0$, $c_2 > 0$ exist such that

$$\varepsilon\Delta\gamma\tilde{\dot{x}}|\hat{z}|^n = c_2\left[\left(\Delta\gamma\right)^2\left(\tilde{\dot{x}}|\hat{z}|^n\right)^2 + \varepsilon^2\right],$$

$$\varepsilon\Delta\beta\left|\tilde{\dot{x}}\right||\hat{z}|^n\, sign\left(\hat{z}\right) = c_1\left[\left(\Delta\beta\right)^2\left(\left|\tilde{\dot{x}}\right||\hat{z}|^n\right)^2 + \varepsilon^2\right] \tag{54}$$

in area $O_v(O)$, where $O = \{0, 0^2\} \subset R \times R^2 \times J_{0,\infty}$, O_v is some neighborhood of a point O; (iv) the inequality $(\varepsilon - \varepsilon_z)^{2n} \ge$

c_z holds for almost all t, where $c_z \geq 0$; (v) $\pi_x \geq 0$ u $\omega > 0$ exist such that $(\tilde{x})^2 \geq \pi_x$ and $|\varepsilon - \varepsilon_z| \leq \omega|\varepsilon|$; (vi) the function

$$g_2(t) = \sup_{\varepsilon \in \Omega} \frac{|\varepsilon|^{2(n+1)}(t)}{V_\varepsilon(t,\varepsilon)},\ g_2 = \sup_{\varepsilon \in \Omega} \tilde{g}_2(t), \tag{55}$$

exists, where Ω the definition range of the subsystem; (vii) $\dot{V}_\varepsilon$, $\dot{V}_{\beta,\gamma}$ satisfy the system of inequalities

$$\begin{bmatrix} \dot{V}_\varepsilon \\ \dot{V}_{\beta,\gamma} \end{bmatrix} \leq \underbrace{\begin{bmatrix} -\left(k_z - 2\tilde{\upsilon} g_1 - \omega \upsilon g_2\right) & \lambda \chi \omega \upsilon \\ c & -\dfrac{d_s}{2} \end{bmatrix}}_{A_\varepsilon} \begin{bmatrix} V_\varepsilon \\ V_{\beta,\gamma} \end{bmatrix} + \underbrace{\begin{bmatrix} \dfrac{1}{2k_z} \\ 0 \end{bmatrix}}_{B_\varepsilon} \left(\delta_\Delta\right)^2. \tag{56}$$

(viii) the upper solution for $V_{\varepsilon,\beta,\gamma} = [V_\varepsilon(t) V_{\beta,\gamma}(t)]^T$ satisfies to the equation

$$\dot{\tilde{S}} = A_\varepsilon \tilde{S} + B_\varepsilon \left(\delta_\Delta\right)^2, \tag{57}$$

if

$$V_{\tilde{\rho}}(t) \leq \tilde{s}_{\tilde{\rho}}(t) \quad \forall \left(t \geq t_0\right) \& \left(V_{\tilde{\rho}}\left(t_0\right) \leq \tilde{s}_{\tilde{\rho}}\left(t_0\right)\right), \tag{58}$$

where $\tilde{S} = \left[\tilde{s}_\varepsilon\ \tilde{s}_{\beta,\gamma}\right]^T$, $\tilde{\rho} = \varepsilon, (\beta,\gamma)$, $A_\varepsilon \in R^{2\times 2}$ is M-matrix. Then the system AS_Z is exponentially dissipative with the estimate

$$V_{\varepsilon,\beta,\gamma}(t) \leq e^{A_\varepsilon\left(t-t_0\right)} \tilde{S}(t_0) + \left(\delta_\Delta\right)^2 \int_{t_0}^{T} e^{A_\varepsilon(t-\tau)} B_\varepsilon d\tau, \tag{59}$$

if

$$k_z > 2\tilde{\upsilon} g_1 - \omega \upsilon g_2,\ \left(k_z - 2\tilde{\upsilon} g_1 - \omega \upsilon g_2\right) d_s > 2c\lambda\chi\omega\upsilon,\ d_s > 0,$$

where

$$\overline{\chi} = \min\left(\chi_\beta, \chi_\gamma\right),\ \overline{c} = \min\left(c_1, c_2\right),\ \chi = \max\left(\chi_\beta, \chi_\gamma\right),\ d_s = \chi \pi_x \overline{c} c_z.$$

So, the system AS_Z is exponentially dissipative. The dissipativity area depends on the informational set I_o of the S_{BW}-system.

Get results that show the possibility of using adaptive observers to parameters identification of the S_{BW}-system. Properties of system (1) with BWHM supervene from the presented theorems.

7. Simulation results

Consider the engine control system (1)-(3) with parameters: $n = 1.5$, $c = 2$, $m = 1$, $\beta = 0.5$, $\gamma = 0.2$, $\alpha = 0.7$, $k = 0.6$. Let $d = a = 1$. Exciting force $f(t) = 2 - 2\sin(0.15\pi t)$. The system is modeled with initial conditions $x(0) = 1$, $\dot{x}(0) = 0$, $z(0) = 1$. Form the set I_o. The system phase portrait and output of the hysteresis shown in Figure 1.

Estimate the structural identifiability of the system (1)-(3). Construct the structure $S_{\tilde{e}y}$ (Figure 2) using the method [32]. A variable $\tilde{e} \in R$ is $\tilde{e} = \dot{x} - \hat{\dot{x}}_h$. $\hat{\dot{x}}_h$ is an estimation of the steady state (process) in the S_{BW}-system for $\forall t \geq 9.85$s, and $\tilde{e}$ is the nonlinearity estimation in the corresponding space.

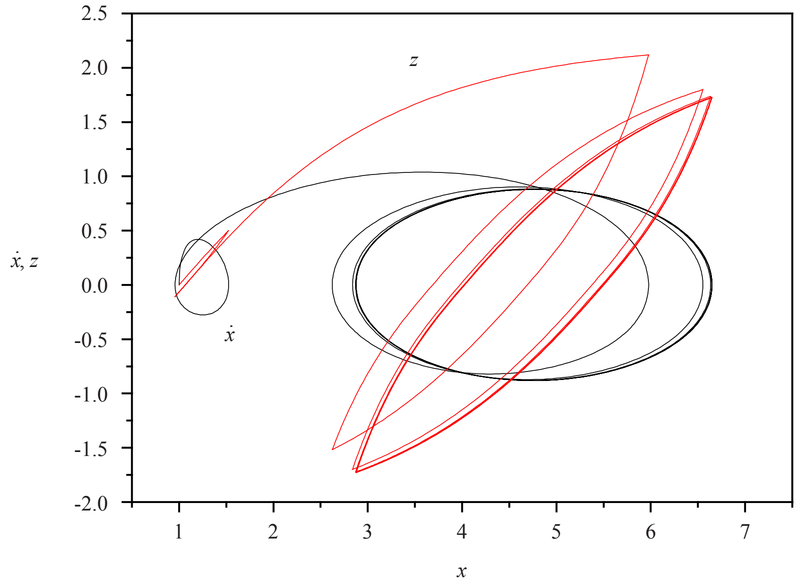

Figure 1. System phase portrait and hysteresis change

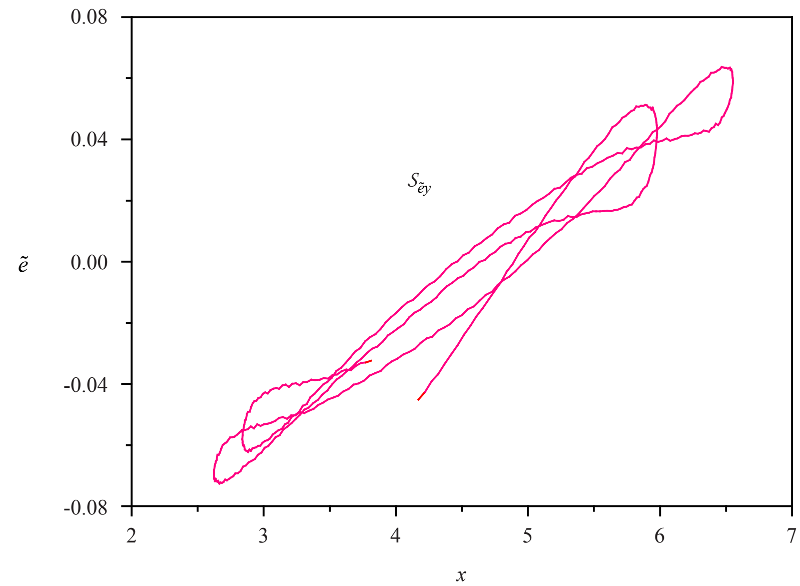

Figure 2. Structure $S_{\tilde{e}y}$ for assessing possibility of solving identification problem

As follows from Figures 1, 2, definition areas z and $\tilde{e}$ coincide. Analysis $S_{\tilde{e}y}$ shows that the system S_{BW} is structurally identifiable, and input $f(t)$ is S-stabilizing.

Consider the system parameters identification. Determine the parameter μ of the system (13) using the transient process analysis for $\tilde{e}$ and $t < 9.85$s. Calculate Lyapunov exponents (LE) [33]. The estimation for the maximum LE is -0.9. Therefore, we set $\mu = 0.8$. Initial conditions in (12) are equal to zero.

Adaptive system work results are presented in Figures 3-5. Parameters k_x, k_z equal to 2.5 and 0.75. The tuning process of $\mathcal{AS}_X$-systems parameters (the model (12)) is shown in Figure 3. Figure 4 showed the model (16) parameters tuning.

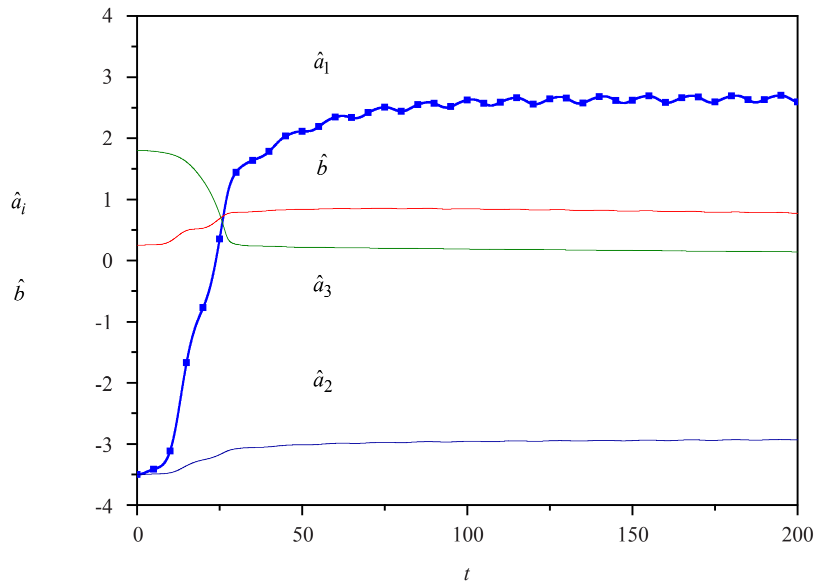

Figure 3. Tuning of model (13) parameters

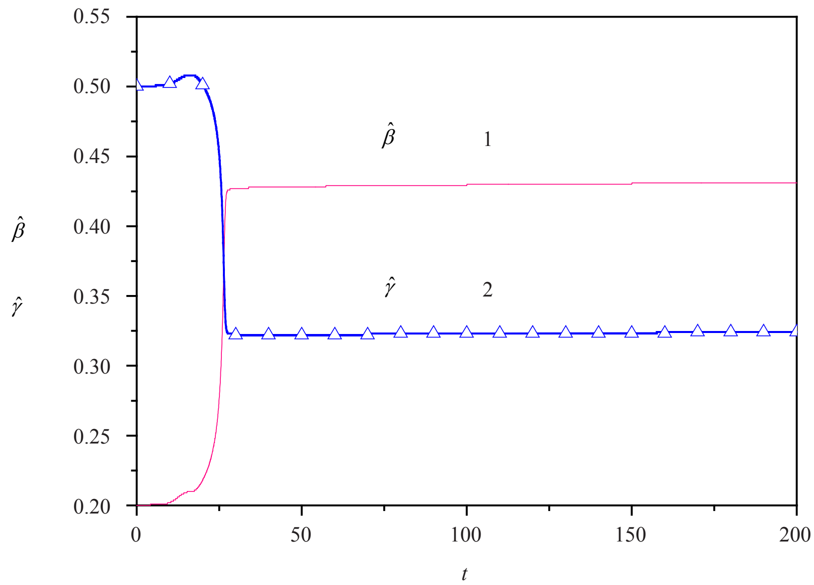

Figure 4. Tuning of model (16) parameters: 1 is tuning $\hat{\beta}$, 2 is tuning $\hat{\gamma}$

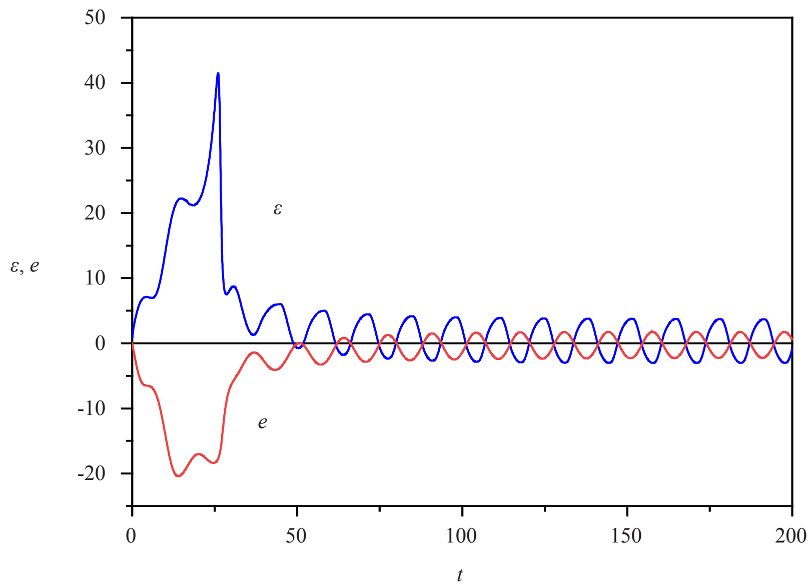

Figure 5. Outputs modification of systems AS_X, AS_Z

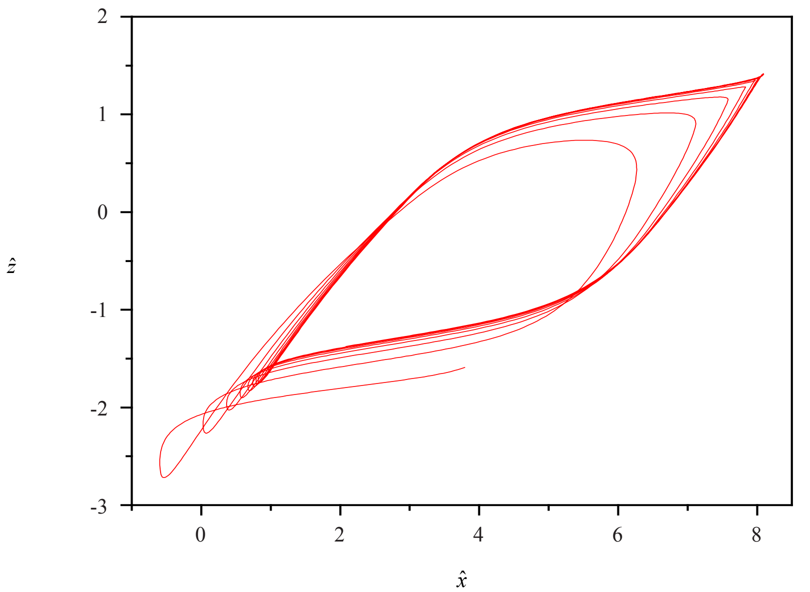

Figure 6. Hysteresis estimation at adaptation of AS_{BW}-system

Show the modification of identification errors e, ε in Figure 5. We see that the accuracy of obtained estimations depends on the numbers of tuned parameters, and the level $\dot{x}$ and properties $f(t)$. Obtained results confirm statements of theorems 3 and 4. The AS_Z-system work results influence the tuning processes in the AS_X-system. Gain coefficients in (25), (26) and (27) are $\chi_\beta = 0.0000002$, $\chi_\gamma = 0.0000002$, $\gamma_4 = 0.00005$, $\gamma_1 = 0.0002$, $\gamma_2 = 0.00001$, $\gamma_3 = 0.00002$. The parameter n is 1.5 in (16).

Remark 2. Modeling results of the system AS_{BW} with the algorithm (27) showed that the algorithm is sensitive to various perturbations, increases the adaptation time and requires further study.

The hysteresis output estimation is shown in Figure 6. Comparison of determination coefficients $r_{xz} = 0.864$ for the reference BWH in (Figure 1) and the resulting BWH (Figure 6) $r_{\hat{x}\hat{z}} = 0.764$ confirms the effectiveness of the proposed approach.

Figure 7 presents comparing results estimates $\hat{z}$ and ε_z, obtaining in subsystems AS_X and AS_Z on the interval [25; 70]s. We analyze the dependence $\hat{\varepsilon}_z(\hat{z})$ and show the approach effectiveness as the coefficient of determination is $r_{\hat{z},\varepsilon_z} = 0.91$. In Figure 7, we represent the secant $\hat{\varepsilon}_z(\hat{z})$. Results confirm the adequacy of the obtained estimate $\hat{z}$.

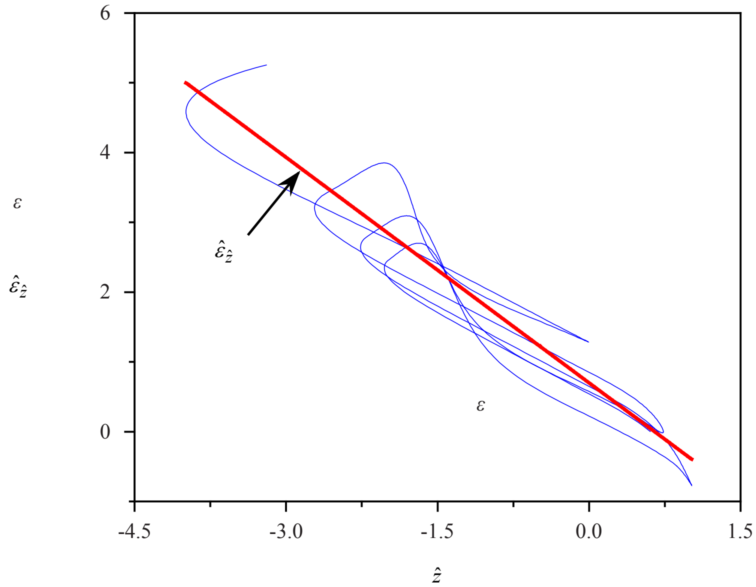

Figure 7. Comparison of estimates $\hat{z}$ and uncertainty ε_z

Figures 8-12 represent the work of the adaptive system with (8), π = 1. Tuning of models (20) and (16.2) parameters shows in Figures 8, 9.

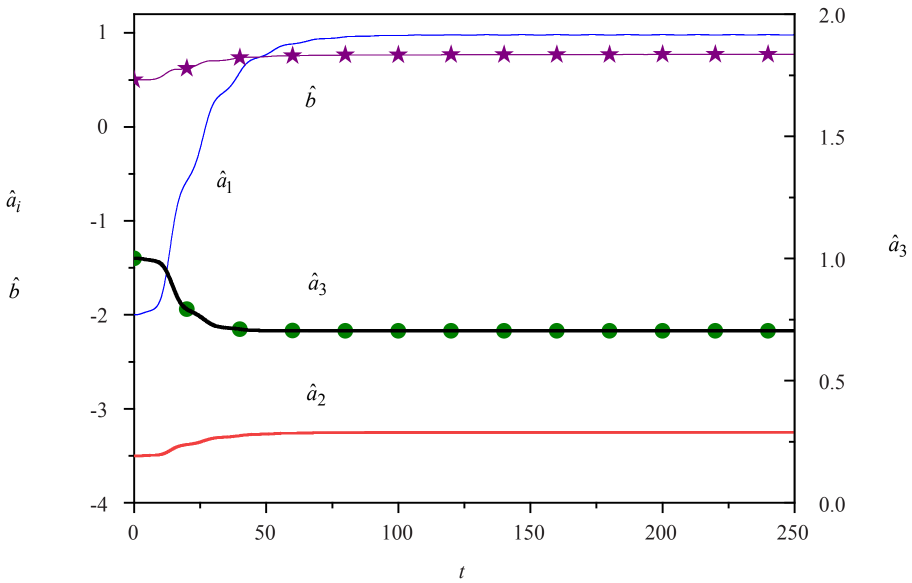

Figure 8. Tuning of model (13) parameters

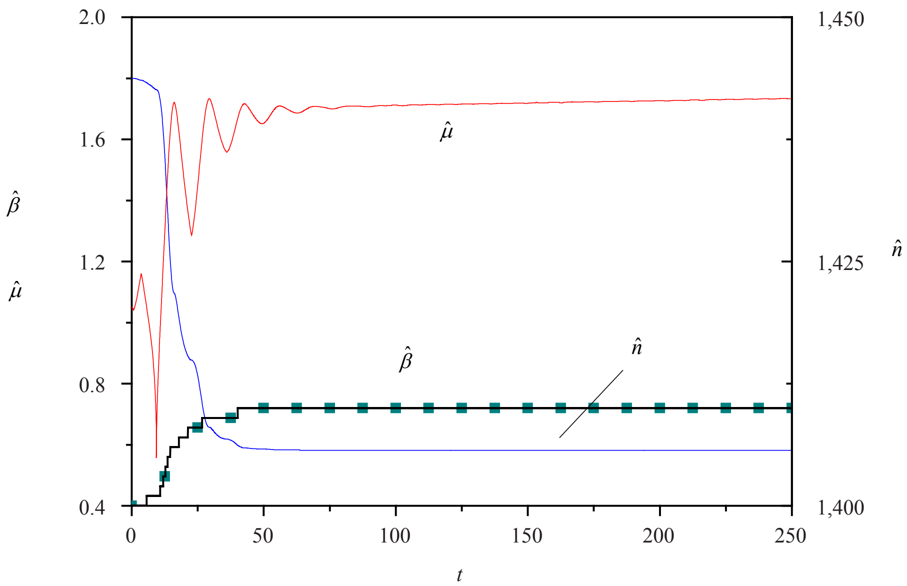

Figure 9. Tuning of model (16) parameters

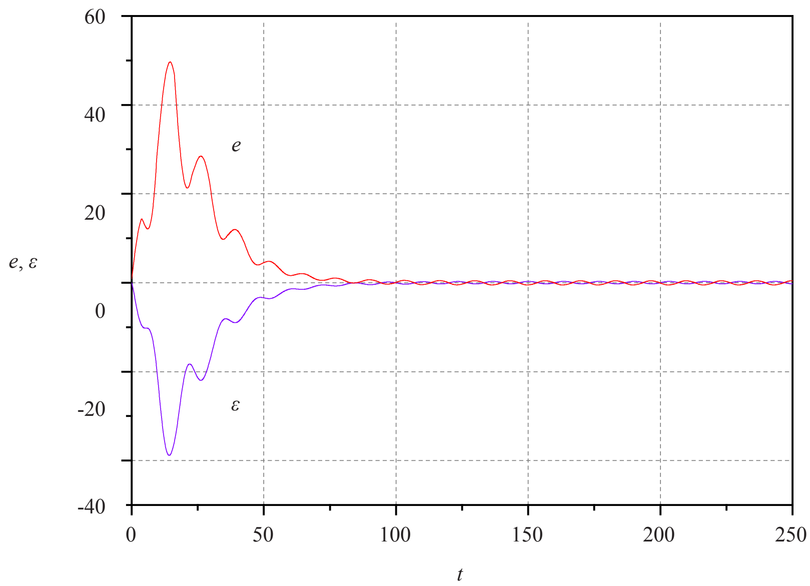

Figure 10. Outputs modification of systems AS_X, AS_Z

Figure 10 shows the change in errors e, ε. The accuracy of obtained parameter estimates is shown in Figure 11, where $\mathcal{N}_{\beta,\mu,n}(t) = \|(\Delta\beta(t))^2 + (\Delta\mu(t))^2 + (\Delta n(t))^2\|$, $\|.\|$ is the Euclidean norm.

Figure 12 demonstrates the adaptive system work with $\mathcal{M}_{\mu\beta n}$ in $(\varepsilon, \mathcal{N}_{\beta,\mu,n})$ and $(\varepsilon, \hat{\beta})$ spaces. We see that the tuning process is nonlinear. It depends on the main circuit AS_X work of the adaptive system and the uncertainty estimation.

So, simulation results confirm the exponential dissipativity of the designed system. The obtained results are applicable to the analysis of robotic and macaronis systems.

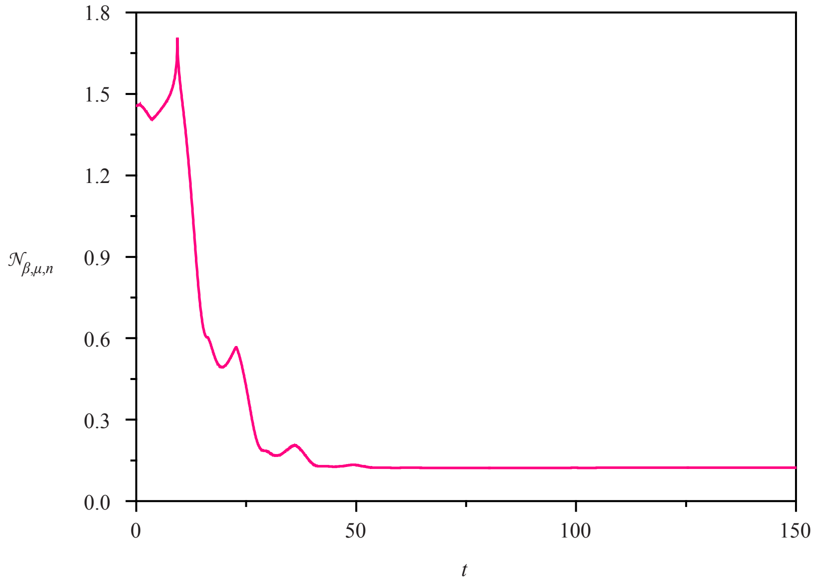

Figure 11. Changing $\mathcal{N}_{\beta,\mu,n}(t)$ for adaptive system with $\mathcal{M}_{\mu\beta n}$

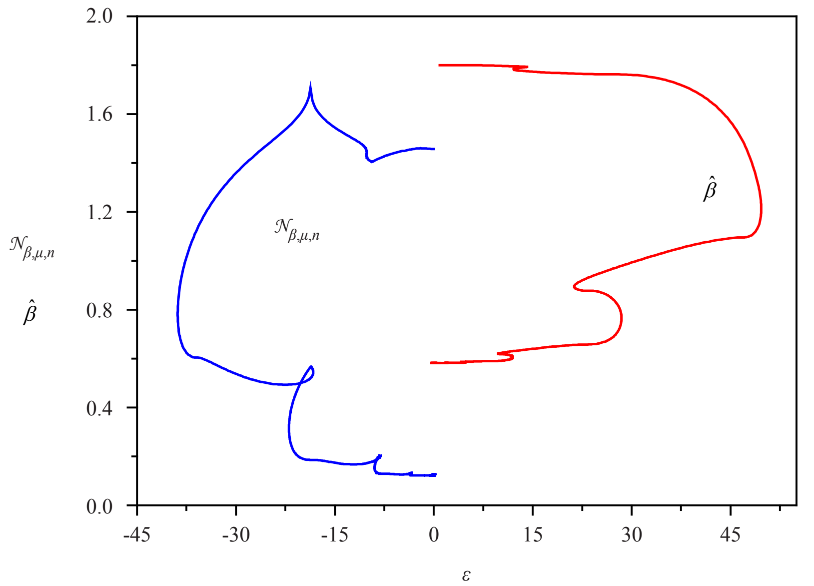

Figure 12. Adaptive system tuning with $\mathcal{M}_{\mu\beta n}$ in (ε, $\mathcal{N}_{\beta,\mu,n}$) and ($\varepsilon$, $\hat{\beta}$) spaces

8. Conclusion

We propose the adaptive identification method of system parameters with the Bouc-Wen hysteresis. We relate the fundamental problem of the BWH identification to ensuring the stability of the assessment system. The proposed identification method is based on the use of adaptive observers. Algorithms for the adaptive observer are designed and the trajectories limitation in the adaptive system is shown. An approach is proposed to estimate the uncertainty about the hysteresis state. This estimation is used to adjust the parameters of the hysteresis model. We consider BWH modifications and propose adaptive algorithms for estimating their parameters. The Lyapunov vector function method are used to evaluate the identification system quality in coordinate and parametric spaces. We prove processes exponential dissipativity of in an adaptive system. It shows that the exponential dissipation domain of the system determines by the level of derivative output. We study the influence of input on BWH parameters identification.

Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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Appendix A

A.1 *Proof of Theorem 1*

Consider the Lyapunov function $V(t)$ (36). Then $\dot{V}(t)$

$$\dot{V} = -k_x e^2 + \dot{V}_K - \dot{V}_K \le -2k_x V_e. \tag{A.1}$$

Apply the condition (i) theorem 1. As $\dot{V}(t) < 0$, the AS_X-system is stable. Integrate $\dot{V}(t)$ on the time and obtain

$$V(t_0) - 2k_x \int_{t_0}^{t} V_e(\tau) d\tau \ge V(t). \tag{A.2}$$

Get from (A.2) to all trajectories of the system AS_X belong to the area $G_t = \{(e, \Delta K) : V(t) \le V(t_0)\}$. We get an estimate for the AS_X-system

$$\int_{t_0}^{t} 2k_x V_e(\tau) d\tau \le V(t_0) - V(t). \tag{A.3}$$

□

Appendix B

B.1 *Proof of Theorem 3*

Determine $\dot{V}_{\varepsilon,\beta,\gamma}$

$$\dot{V}_{\varepsilon\beta\gamma} = -k_z \varepsilon^2 + \varepsilon\left(\beta\eta_\beta + \gamma\eta_\gamma + \Delta\dot{x}\right) + \dot{V}_{\beta,\gamma} - \dot{V}_{\beta,\gamma}$$

$$= -k_z \varepsilon^2 + \varepsilon\left(\beta\eta_\beta + \gamma\eta_\gamma + \Delta\dot{x}\right). \tag{B.1}$$

Since, $\tilde{\dot{x}}$ is function x, $\tilde{\dot{x}} = \sigma\dot{x}$, where $\sigma \approx 1$. We have showed that ε_z is the estimation z. Therefore, present η_β as

$$\eta_\beta = |\dot{x}||z|^n \, sign(z) - |\tilde{\dot{x}}||\hat{z}|^n \, sign(\hat{z}) \cong |\dot{x}||\varepsilon|^n, \tag{B.2}$$

for $\forall t > t_\varepsilon$. Similarly

$$\eta_\beta = |\dot{x}||z|^n \, sign(z) - |\tilde{\dot{x}}||\hat{z}|^n \, sign(\hat{z}) \cong |\dot{x}||\varepsilon|^n, \tag{B.3}$$

Considering the assumption 1 and the boundedness of trajectories AS_X-system, we obtain $|\dot{x}| \le \upsilon$ for $\forall t > t_0$, where $\upsilon > 0$. Then

$$\dot{V}_{\varepsilon\beta\gamma} \leq -k_z\varepsilon^2 + \beta|\varepsilon||\eta_\beta| + \gamma|\varepsilon||\eta_\gamma| + |\varepsilon||\Delta\dot{x}|$$

$$\leq -k_z\varepsilon^2 + \beta\upsilon|\varepsilon|^{n+1} + \gamma\upsilon|\varepsilon|^{n+1} + |\varepsilon||\Delta\dot{x}|$$

$$\leq -k_z\varepsilon^2 + \upsilon(\beta+\gamma)|\varepsilon|^{n+1} + \delta_\Delta|\varepsilon| \tag{B.4}$$

where $|\Delta\dot{x}| \leq \delta_\Delta$, $\delta_\Delta \geq 0$.

Let

$$\tilde{g}_1(t) = \sup_{\varepsilon\in\Omega}\frac{|\varepsilon|^{n+1}(t)}{V_\varepsilon(t,\varepsilon)},\ g_1 = \max_t \tilde{g}_1(t). \tag{B.5}$$

Then $|\varepsilon|^{n+1}(t) \leq g_1\varepsilon^2(t)$ and transform (B.4) to the form

$$\dot{V}_{\varepsilon\beta\gamma} \leq -k_z\varepsilon^2 + \upsilon(\beta+\gamma)g_1\varepsilon^2 + \delta_\Delta|\varepsilon|$$

$$= -(k_z - \upsilon(\beta+\gamma)g_1)\varepsilon^2 + \delta_\Delta|\varepsilon|, \tag{B.6}$$

where $k_z - \upsilon(\beta+\gamma)g_1 > 0$.

Apply the inequality

$$-aq^2 + bq \leq -\frac{a}{2}q^2 + \frac{b^2}{2a}. \tag{B.7}$$

Then (B.6)

$$\dot{V}_{\varepsilon\beta\gamma} \leq -(k_z - \upsilon(\beta+\gamma)g_1)\varepsilon^2 + \delta_\Delta|\varepsilon|$$

$$\leq -\frac{k_z - \upsilon(\beta+\gamma)g_1}{2}\varepsilon^2 + \frac{1}{2(k_z - \upsilon(\beta+\gamma)g_1)}(\delta_\Delta)^2$$

$$\leq -(k_z - \upsilon(\beta+\gamma)g_1)V_\varepsilon + \frac{1}{2(k_z - \upsilon(\beta+\gamma)g_1)}(\delta_\Delta)^2. \tag{B.8}$$

Integrate (B.8) and obtain the estimation

$$\int_{t_o}^{t}(k_z - \upsilon(\beta+\gamma)g_1)V_\varepsilon(\tau) + \frac{1}{2(k_z - \upsilon(\beta+\gamma)g_1)(t-t_0)}(\delta_\Delta)^2 \leq V_{\varepsilon\beta\gamma}(t_0) - V_{\varepsilon\beta\gamma}(t). \tag{B.9}$$

The left part (B.9) is nonnegative and $V_\varepsilon(t)$ satisfies conditions of theorem 3. Therefore, all trajectories AS_Z-system is limited. □

B.2 *Proof of Theorem 4*

Consider $\dot{V}_\varepsilon$

$$\dot{V}_\varepsilon = -k_z\varepsilon^2 + \varepsilon\left(\beta\eta_\beta + \gamma\eta_\gamma + \Delta\dot{x}\right) + \varepsilon\left(\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right) + \Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^n\right). \tag{C.1}$$

Evaluate the second and third summands on the right side (C.1).

Lemma C1. The estimation

$$\left|\eta_\beta\right| \le \upsilon\left|\varepsilon\right|^n$$

is fair for $\eta_\beta = \left|\dot{x}\right|\left|z\right|^n sign(z) - \left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z})$.

Lemma C1 proof.

As $|\dot{x}| \le \upsilon$, then

$$\left|\eta_\beta\right| \cong \upsilon\left|\left|z\right|^n sign\left(z\right) - \sigma\left|\hat{z}\right|^n sign\left(\hat{z}\right)\right|$$

$$\le \upsilon\left(\left|z-\hat{z}\right|^n + \left|\hat{z}\right|^n - \left|\hat{z}\right|^n\right) \le \upsilon\left|z-\hat{z}\right|^n = \upsilon\left|\varepsilon\right|^n. \qquad \square$$

Similarly, the estimation has the form $\left|\eta_\gamma\right| \le \upsilon\left|\varepsilon\right|^n$ for $\eta_\gamma = \dot{x}\left|z\right|^n - \tilde{\dot{x}}\left|\hat{z}\right|^n$. It is based on the proof of the lemma C1. Then

$$\left|\varepsilon\left(\beta\eta_\beta + \gamma\eta_\gamma + \Delta\dot{x}\right)\right| \le \tilde{\upsilon}\left(\beta+\gamma\right)\left|\varepsilon\right|^{n+1} + \delta_\Delta\left|\varepsilon\right|, \tag{C.2}$$

where $\tilde{\upsilon} = \upsilon\left(\beta+\gamma\right)$.

Consider the last item in the right member (C.1).

Lemma C2. The estimation

$$\left|\varepsilon\left(\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right) + \Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^n\right)\right| \le \omega\upsilon\left|\varepsilon\right|^{n+1}\left\{\left|\Delta\beta\right| + \left|\Delta\gamma\right|\right\}, \tag{C.3}$$

is fair for $\varepsilon(\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z}) + \Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^n)$, where $\omega > 0$ is such that $|\varepsilon - \varepsilon_z| \le \omega|\varepsilon|$.

Lemma C2 proof. Transform (C.3) to the form

$$\left(\varepsilon\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right) + \varepsilon\Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^n\right)\Big|_{\hat{z}=\varepsilon-\varepsilon_z} = \varepsilon\left|\tilde{\dot{x}}\right|\left|\varepsilon-\varepsilon_z\right|^n\left(\Delta\beta sign\left(\varepsilon-\varepsilon_z\right) + \Delta\gamma sign\left(\tilde{\dot{x}}\right)\right). \tag{C.4}$$

Let $\omega > 0$ exist such that $|\varepsilon - \varepsilon_z| \le \omega|\varepsilon|$. Then (C.4)

$$|\varepsilon||\tilde{\dot{x}}||\varepsilon-\varepsilon_z|^n\left|\Delta\beta sign(\varepsilon-\varepsilon_z)+\Delta\gamma sign(\tilde{\dot{x}})\right|$$

$$\leq \omega\upsilon|\varepsilon|^{n+1}\left|\Delta\beta sign(\varepsilon-\varepsilon_z)+\Delta\gamma sign(\tilde{\dot{x}})\right|$$

$$\leq \omega\upsilon|\varepsilon|^{n+1}\left\{|\Delta\beta|+|\Delta\gamma|\right\}. \tag{C.5}$$

(C.3) is result (C.5). □

Consider (C.5) and transform $\omega\upsilon|\varepsilon|^{n+1}\{|\Delta\beta|+|\Delta\gamma|\}$ into the form

$$\omega\upsilon|\varepsilon|^{n+1}\left\{|\Delta\beta|+|\Delta\gamma|\right\}\leq 0.5\varepsilon^{2(n+1)}+0.5\lambda\left(\Delta\beta^2+\Delta\gamma^2\right), \tag{C.6}$$

where $\lambda > 1$.

As

$$(\Delta\beta)^2+(\Delta\gamma)^2=2\cdot 0.5\left[\chi_\beta^{-1}\chi_\beta(\Delta\beta)^2+\chi_\gamma^{-1}\chi_\gamma(\Delta\gamma)^2\right]\leq 2\chi V_{\beta,\gamma},$$

where $\chi = \max(\chi_\beta, \chi_\gamma)$, then (C.6) receives the form

$$\omega\upsilon|\varepsilon|^{n+1}\left\{|\Delta\beta|+|\Delta\gamma|\right\}\leq 0.5\omega\upsilon\varepsilon^{2(n+1)}+\lambda\chi\omega\upsilon V_{\beta,\gamma}. \tag{C.7}$$

As

$$-k_z\varepsilon^2+\delta_\Delta|\varepsilon|\leq -\frac{1}{2}k_z\varepsilon^2+\frac{1}{2k_z}(\delta_\Delta)^2, \tag{C.8}$$

then (C.1), considering lemmas C1 and C2, and inequalities (C.7) and (C.8), we present as

$$\dot{V}_\varepsilon \leq -k_z\varepsilon^2+\tilde{\upsilon}|\varepsilon|^{n+1}+\delta_\Delta|\varepsilon|+0.5\omega\upsilon\varepsilon^{2(n+1)}+\lambda\chi\omega\upsilon V_{\beta,\gamma}$$

$$\leq -\frac{1}{2}k_z\varepsilon^2+\tilde{\upsilon}|\varepsilon|^{n+1}+0.5\omega\upsilon\varepsilon^{2(n+1)}+\lambda\chi\omega\upsilon V_{\beta,\gamma}+\frac{1}{2k_z}(\delta_\Delta)^2. \tag{C.9}$$

Consider functions $\tilde{g}_1(t)$ (B.5) and

$$g_2(t)=\sup_{\varepsilon\in\Omega}\frac{|\varepsilon|^{2(n+1)}(t)}{V_\varepsilon(t,\varepsilon)},\ g_2=\max_t \tilde{g}_2(t).$$

Then (C.9)

$$\dot{V}_\varepsilon \leq -\frac{1}{2}\left(k_z-2\tilde{\upsilon}g_1-\omega\upsilon g_2\right)V_\varepsilon+\lambda\chi\omega\upsilon V_{\beta,\gamma}+\frac{1}{2k_z}(\delta_\Delta)^2. \tag{C.10}$$

Obtain the estimation for the derivative $V_{\beta,\gamma}(t) = 0.5\chi_\beta^{-1}(\Delta\beta)^2 + 0.5\chi_\gamma^{-1}(\Delta\gamma)^2$:

$$\dot{V}_{\beta,\gamma} = -\varepsilon\Delta\beta\left|\tilde{x}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right) - \varepsilon\Delta\gamma\tilde{x}\left|\hat{z}\right|^n . \tag{C.11}$$

Let $c_1 > 0$, $c_2 > 0$ exist such that

$$\varepsilon\Delta\gamma\tilde{x}\left|\hat{z}\right|^n = c_2\left[\left(\Delta\gamma\right)^2\left(\tilde{x}\left|\hat{z}\right|^n\right)^2 + \varepsilon^2\right], \tag{C.12}$$

$$\varepsilon\Delta\beta\left|\tilde{x}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right) = c_1\left[\left(\Delta\beta\right)^2\left(\left|\tilde{x}\right|\left|\hat{z}\right|^n\right)^2 + \varepsilon^2\right]. \tag{C.13}$$

Then (C.11)

$$\dot{V}_{\beta,\gamma} = -c\varepsilon^2 - \left(\tilde{x}\left|\hat{z}\right|^n\right)^2\left(c_1\left(\Delta\beta\right)^2 + c_2\left(\Delta\gamma\right)^2\right), \tag{C.14}$$

where $c = c_1 + c_2$.

Let $\overline{c} = \min(c_1, c_2)$. Then

$$c_1\left(\Delta\beta\right)^2 + c_2\left(\Delta\gamma\right)^2 \geq \overline{c}\left(\left(\Delta\beta\right)^2 + \left(\Delta\gamma\right)^2\right) \geq 2\overline{c}\,\overline{\chi}V_{\beta,\gamma}. \tag{C.15}$$

where $\overline{\chi} = \min(\chi_\beta, \chi_\gamma)$.

Apply (C.14) and (C.15) write as

$$\dot{V}_{\beta,\gamma} \leq -c\varepsilon^2 - 2\overline{c}\,\overline{\chi}\left(\tilde{x}\left|\hat{z}\right|^n\right)^2 V_{\beta,\gamma}. \tag{C.16}$$

$\tilde{x}$ bounded $(\tilde{x})^2 \geq \pi_x \geq 0$, therefore,

$$\dot{V}_{\beta,\gamma} \leq -c\varepsilon^2 - 2\overline{c}\chi\left(\tilde{x}\left|\hat{z}\right|^n\right)^2 V_{\beta,\gamma} \leq -c\varepsilon^2 - 2\overline{c}\chi\pi_x\left|\hat{z}\right|^{2n} V_{\beta,\gamma},$$

$$\dot{V}_{\beta,\gamma} \leq -c\varepsilon^2 - 2\chi\pi_x\overline{c}\left(\varepsilon - \varepsilon_z\right)^{2n} V_{\beta,\gamma}. \tag{C.17}$$

The variable ε boundedness follows from theorem 3, and the boundedness ε_z is the boundedness result ε. Therefore, $\left(\varepsilon - \varepsilon_z\right)^{2n} \geq c_z$, where $c_z \geq 0$. So

$$\dot{V}_{\beta,\gamma} \leq -c\varepsilon^2 - 2\chi\pi_x\overline{c}c_z V_{\beta,\gamma}. \tag{C.18}$$

Let $d_s \triangleq \chi\pi_x\overline{c}c_z$. Then (C.14) write as

$$\dot{V}_{\beta,\gamma} \leq -d_s V_{\beta,\gamma} + 2\sqrt{cd_s}\,\varepsilon\sqrt{V_{\beta,\gamma}}. \tag{C.19}$$

Use the inequality (B.7) and obtain the estimation for $\dot{V}_{\beta,\gamma}$

$$\dot{V}_{\beta,\gamma} \leq -\frac{d_s}{2} V_{\beta,\gamma} + cV_{\varepsilon}. \tag{C.20}$$

So, the following system of inequalities is fair for the AS_Z-system

$$\begin{bmatrix} \dot{V}_{\varepsilon} \\ \dot{V}_{\beta,\gamma} \end{bmatrix} \leq \underbrace{\begin{bmatrix} -\left(k_z - 2\tilde{\upsilon} g_1 - \omega \upsilon g_2\right) & \lambda \chi \omega \upsilon \\ c & -\dfrac{d_s}{2} \end{bmatrix}}_{A_{\varepsilon}} \begin{bmatrix} V_{\varepsilon} \\ V_{\beta,\gamma} \end{bmatrix} + \underbrace{\begin{bmatrix} \dfrac{1}{2k_z} \\ 0 \end{bmatrix}}_{B_{\varepsilon}} \left(\delta_{\Delta}\right)^2. \tag{C.21}$$

Let $V_{\tilde{\rho}}(t) \leq \tilde{s}_{\tilde{\rho}}(t),\ \forall (t \geq t_0) \,\&\, (V_{\tilde{\rho}}(t_0) \leq \tilde{s}_{\tilde{\rho}}(t_0)),\ \tilde{\rho} = \varepsilon(\beta, \gamma)$, and $\tilde{S} = [\tilde{s}_{\varepsilon} \tilde{s}_{\beta,\gamma}]^T$. The comparison system for (C.17) has the form

$$\dot{\tilde{S}} = A_{\varepsilon} \tilde{S} + B_{\varepsilon} \left(\delta_{\Delta}\right)^2. \tag{C.22}$$

We have the estimation for the system AS_Z from (C.22)

$$V_{\varepsilon,\beta,\gamma}(t) \leq e^{A_{\varepsilon}\left(t - t_0\right)} \tilde{S}(t_0) + \left(\delta_{\Delta}\right)^2 \int_{t_0}^{T} e^{A_{\varepsilon}\left(t - \tau\right)} B_{\varepsilon} d\tau, \tag{C.23}$$

where $V_{\varepsilon,\beta,\gamma} = [V_{\varepsilon} V_{\beta,\gamma}]^T$ if

$$k_z > 2\tilde{\upsilon} g_1 - \omega \upsilon g_2,\ \left(k_z - 2\tilde{\upsilon} g_1 - \omega \upsilon g_2\right) d_s > 2c\lambda \chi \omega \upsilon,\ d_s > 0. \qquad \square$$