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Research Article

# Analysis of a Single Server Queueing System Controlled by a Random Switch

**Akshaya Ramesh, S. Udayabaskaran***

Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Avadi, Chennai, Tamilnadu 600062, India
E-mail: s_udayabaskaran@hotmail.com

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**Abstract:** In this paper, a single server queueing system operating in a doubly stochastic environment is analysed. The random environment makes transitions among $N$ levels controlled by a random switch which performs the task of assigning a job to the server. The queueing system resides in level $k$ of the environment till the completion of the service of the last customer in the system and immediately reports to the random switch to get a new service job. The random switch generates a set up time and waits till a customer arrives and assigns a job to the server in any one of the $N$ levels with a positive probability governed by a binomial distribution. In each level $r$ of the environment, the queueing system behaves like $M(\lambda_r)/M(\mu_r)/1$ queue subject to the condition that the server reports to the random switch immediately after performing exhaustive service for getting a new assignment. For this model, time-dependent state probabilities are explicitly found and the corresponding steady-state probabilities are deduced. Some key performance measures are also obtained. A numerical study is also made.

***Keywords***: single server queue, doubly stochastic environment, exhaustive service, random switch/set up time

**MSC:** 60K25, 90B22

## 1. Introduction

Several authors have studied single server queueing systems subject to randomly occurring disasters (see, Sengupta [1], Yechiali [2], Chakravarthy [3], Krishna Kumar et al. [4], Sudhesh [5], Paz and Yechiali [6], Udayabaskaran and Dora Pravina [7], Kim and Kim [8], Ammar et al. [9]). In particular, Paz and Yechiali [6] have analyzed the steady-state behaviour of an $M/M/1$ queue operating in random environment subject to disasters where the underlying environment is described by a $n$-level continuous-time Markov chain. They have not obtained transient solution for the queueing model. Udayabaskaran and Dora Pravina [7] have obtained time-dependent probabilities for the queueing model of Paz and Yechiali [6]. Recently, Ammar et al. [9] have extended the queueing model of Paz and Yechiali [6] by incorporating customer impatience during repair time, and obtained transient solution for the state probabilities of the system. In the above previous research works, all customers were washed out at the epoch of every disaster. When catastrophes and flushing out of customers are allowed, the queueing system starts from scratch after every catastrophe. This renders


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the analysis of the queueing system in a different set up. However, if the system works in a disaster-free environment, the analysis requires a quite careful approach. The merit of our prosed model is that it does not include disasters and loss of customers. The demerit of the model is that it cannot be implemented in a catastrophic environment. There is no loss of customers in our model. The disaster models are quite different from our model. They are concerned about loss of customers only. Further, immediately after the occurrence of every disaster, the server goes for repair and during the repair time, customers are allowed to join the system. Immediately after the repair, the server is ready for offering service with a probability even if no customer had joined the system during repair time. In the present model, we consider an assignment process wherein the server is put into service immediately after the first customer had arrived only. This prevents the server to remain idle in the queueing system.

Queueing systems with set up times for servers under repair have been studied recently by Phung-Duc [10] and Karunakaran and Maragatha Sundari [11]. In the present paper, we propose a single server queueing model which avoids catastrophes and flushing out of customers, but includes exhaustive service and set up time for server controlled by a random switch. The paper is organized as follows: Section 2 describes the model. Section 3 derives the integral equations for the time-dependent probabilities of the system. In section 4, explicit expressions for the transient probabilities are obtained. Section 5 deduces the steady-state probabilities. In section 6, we obtain performance measures. Section 7 provides a numerical illustration for the performance measures.

## 2. Model description

Consider a single server queueing system operating in a random environment. The random environment makes transitions among $N$ levels controlled by a random switch. The random switch puts the queueing system in any one of the $N$ levels with a positive probability. In each level $r$ of the environment, the queueing system behaves like an $M(\lambda_r)/M(\mu_r)/1$ queue with the server reporting to the random switch immediately after performing exhaustive service. The queueing system resides in level $k$ till the completion of the service of the last customer in the system and immediately reports to the random switch to get a new service job. The random switch waits till a customer arrives with arrival rate $\lambda_0$ and immediately assigns the server to perform service in any one of the $N$ levels, say $r$ with a positive probability $q_r$ such that $q_r = \binom{N-1}{r-1} p^{N-r}(1-p)^{r-1},\ 0 < p < 1;\ r = 1, 2, \cdots, N.$

### 2.1 *Notations*

$Pr(A)$: Probability of event $A$

$\int_0^t f(u)g(t-u)du$ : $f(t)©g(t)$ Convolution of $f(t)$ and $g(t)$

$\int_0^\infty e^{-st} f(t)dt = f^*(s)$ : $L[f(t)]$ Laplace transform of $f(t)$

$L^{-1}[F(s)]$: Inverse Laplace transform of $F(s)$

$f^{(r)}(t)$: $r$-fold convolution of $f(t)$

## 3. Governing equations

Let $X(t)$ be the number of customers in the system and $L(t)$ be the level of the random environment at time $t$. We designate level 0 when the server is at the random switch. The joint process $\{(X(t), L(t)) | t \geq 0\}$ is Markov. The state space of the two dimensional stochastic process is given by

$$\Omega = \{(0,0)\} \cup \{(j,k) \mid j = 1, 2, \cdots; k = 1, 2, \cdots, N\}.$$

The state transition diagram is given below (Figure 1):

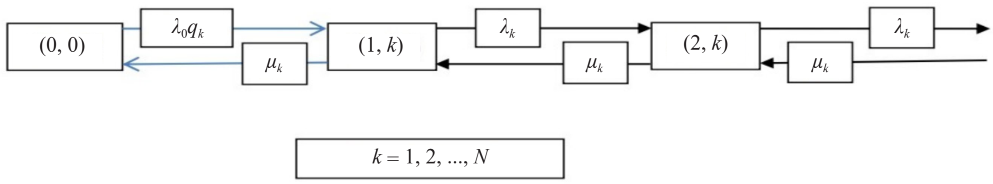


**Figure 1.** State transition-rate diagram

We define the state probabilities as follows:

$$p(j,k,t) = Pr[X(t) = j, L(t) = k],\ j = 0,1,\cdots;\ k = 0,1,\cdots,N.$$

We assume $X(0) = 0$, and $L(0) = 0$. Denoting the convolution $\int_0^t f(u)g(t-u)du$ as $f(t)©g(t)$, and using the imbedding technique of Bellman et al. [12], we obtain the following governing equations:

$$p(0,0,t) = e^{-\lambda_0 t} + \sum_{k=1}^{N} \mu_k p(1,k,t)©e^{-\lambda_0 t}; \tag{1}$$

$$p(1,k,t) = [p(0,0,t)\lambda_0 q_k + p(2,k,t)\mu_k]©e^{-(\lambda_k+\mu)t},\ k = 1,2,\cdots,N; \tag{2}$$

$$p(j,k,t) = [p(j-1,k,t)\lambda_k + p(j+1,k,t)\mu_k]©e^{-(\lambda_k+\mu_k)t},\ j = 2,3,\cdots;\ k = 1,2,\cdots,N. \tag{3}$$

## 4. Transient solution

Taking Laplace transform on both sides of (1)-(3), we get

$$(s+\lambda_0)p^*(0,0,s) = 1 + \sum_{k=1}^{N} p^*(1,k,s)\mu_k; \tag{4}$$

$$(s+\lambda_k+\mu_k)p^*(1,k,s) = p^*(0,0,s)\lambda_0 q_k + p^*(2,k,s)\mu_k,\ k = 1,2,\cdots,N; \tag{5}$$

$$(s+\lambda_k+\mu_k)p^*(j,k,s) = p^*(j-1,k,s)\lambda_k + p^*(j+1,k,s)\mu_k,\ j = 2,3,\cdots;\ k = 1,2,\cdots,N. \tag{6}$$

To solve (4)-(6), we consider partial generating functions

$$G_k^*(u,s) = \sum_{j=1}^{\infty} p^*(j,k,s)u^j,\ k = 1,2,\cdots,N.$$

By using (5) and (6), we get

$$G_k^*(u,s) = \frac{p^*(0,0,s)\lambda_0 q_k u^2 - \mu_k p^*(1,k,s)u}{(s+\lambda_k+\mu_k)u - \lambda_k u^2 - \mu_k},\ k = 1,2,\cdots,N. \tag{7}$$

The zeros of the denominator of right hand side of (9) are given by

$$\xi_{k,1}(s)=\frac{(s+\lambda_k+\mu_k)-\sqrt{(s+\lambda_k+\mu_k)^2-4\lambda_k\mu_k}}{2\lambda_k}, \tag{8}$$

$$\xi_{k,2}(s)=\frac{(s+\lambda_k+\mu_k)+\sqrt{(s+\lambda_k+\mu_k)^2-4\lambda_k\mu_k}}{2\lambda_k}. \tag{9}$$

The properties of $\xi_{k,1}(s)$ and $\xi_{k,2}(s)$ are

$$\xi_{k,1}(s)+\xi_{k,2}(s)=\frac{(s+\lambda_k+\mu_k)}{\lambda_k}, \tag{10}$$

$$\xi_{k,1}(s)\xi_{k,2}(s)=\frac{\mu_k}{\lambda_k}, \tag{11}$$

$$|\xi_{k,1}(s)|<1,\quad |\xi_{k,2}(s)|>1, \tag{12}$$

$$(s+\lambda_k+\mu_k)u-\lambda_k u^2-\mu_k\equiv\lambda_k(u-\xi_{k,1}(s))(\xi_{k,2}(s)-u). \tag{13}$$

Since $G_k^*(u, s)$ is analytic in $|u|\leq 1$, the numerator of right hand side of (9) should vanish at $\xi_{k,1}(s)$. Consequently, we obtain

$$p^*(1,k,s)=\frac{\lambda_0 q_k\xi_{k,1}}{\mu_k}p^*(0,0,s). \tag{14}$$

Substituting (13) and (14) in (7) and simplifying, we get

$$G_k^*(u,s)=\frac{p^*(0,0,s)q_k\lambda_0\xi_{k,1}(s)u}{(\mu_k-\lambda_k\xi_{k,1}u)},\ k=1,2,\cdots,N. \tag{15}$$

Equation (15) yields the transient solution of the state probabilities of the queueing system. These probabilities are stated and proved in the following theorem:

**Theorem 1** Under the condition $\lambda_k<\mu_k$, $k=1, 2, \cdots, N$, the time-dependent state probabilities of the queueing system are given by

$$p(0,0,t)=\sum_{r=0}^{\infty}(-1)^r\int_0^t f^{(r)}(u)du, \tag{16}$$

$$p(j,k,t)=\frac{\lambda_0 qk}{\lambda_k}\left(\frac{\lambda_k}{\mu_k}\right)^{(j/2)}p(0,0,t)©e^{-(\lambda_k+\mu_k)t}\frac{jI_j(2\sqrt{\lambda_k\mu_k}t)}{t},\ t>0;\ j=1,2,\cdots;\ k=1,2,\cdots,N, \tag{17}$$

where $I_n(t)$ is the modified Bessel function of $n$-th order and $f^{(r)}(t)$ is the $r$-fold convolution of

$$f(t)=\lambda_0\sum_{k=1}^{N}\frac{q_k}{\lambda_k}e^{-(\lambda_k+\mu_k)t}\sum_{n=1}^{\infty}\left(\frac{\lambda_k}{\mu_k}\right)^{(n/2)}\frac{nI_n(2\sqrt{\lambda_k\mu_k}t)}{t},\ t>0.$$

**Proof.** Assuming $\lambda_k<\mu_k$, $k=1, 2, \cdots, N$, the expression on right hand side of (15) can be expanded as a power series in $u$ inside the domain $|u|<1$ and thus, we obtain

$$G_k^*(u,s)=p^*(0,0,s)\lambda_0 q_k\sum_{j=1}^{\infty}\frac{\lambda_k^{j-1}\xi_{k,1}^j u^j}{\mu_k^j}. \tag{18}$$

By the definition of $G_k^*(u, s)$, equating the coefficients of $u^j$ in (18), we obtain

$$p^*(j,k,s)=p^*(0,0,s)\lambda_0 q_k\frac{\lambda_k^{j-1}\xi_{k,1}^j u^j}{\mu_k^j},\ j=1,2,\cdots;\ k=1,2,\cdots,N. \tag{19}$$

By the total probability law, we have

$$p(0,0,t)+\sum_{k=1}^{N}p(j,k,t)=1. \tag{20}$$

By taking Laplace transform on both sides of (20), we get

$$p^*(0,0,s)+\sum_{k=1}^{N}\sum_{j=1}^{\infty}p^*(j,k,s)=\frac{1}{s}. \tag{21}$$

Substituting (19) in (21) and solving for $p^*(0, 0, s)$, we get

$$p^*(0,0,s)=\frac{1}{s[1+F(s)]}, \tag{22}$$

where

$$F(s)=\lambda_0\sum_{k=1}^{N}\frac{q_k\xi_{k,1}(s)}{\mu_k-\lambda_k\xi_{k,1}(s)}. \tag{23}$$

The function $F(s)$ has the following properties: $\lim_{s\to 0}sF(\mathrm{s})=0$, $\lim_{s\to 0}sF(s)=\lambda_0$. Denoting the inverse Laplace transform of $F(\mathrm{s})$ by $f(t), f(t)$ has the properties: $f(0)=\lambda_0$, $f(\infty)=0$ and (23) yields

$$f(t)=\lambda_0\sum_{k=1}^{N}\frac{q_k}{\lambda_k}e^{-(\lambda_k+\mu_k)t}\sum_{n=1}^{\infty}\left(\frac{\lambda_k}{\mu_k}\right)^{(n/2)}\frac{nI_n(2\sqrt{\lambda_k\mu_k}t)}{t},\ t>0, \tag{24}$$

where we have used the result (see Abramowitz and Stegun [13], page 1024:29.3.53)

$$L^{-1}\left(\frac{\left(s-\sqrt{s^2-a^2}\right)^n}{a^n}\right)=\frac{n}{t}I_n(at),\ n>0. \tag{25}$$

By taking inverse laplace transform on both sides of (22), we get explicitly the result (16). By taking inverse Laplace transform on both sides of (19), we get explicitly (17). Thus the proof is complete.

Now, we proceed to deduce the steady-state probabilities.

## 5. Steady-state probabilities

By definition and the final value theorem of Laplace transform, we have

$$\pi(0,0)=\lim_{t\to\infty}p(0,0,t)=\lim_{s\to 0}sp^*(0,0,s),$$

$$\pi(j,k)=\lim_{t\to\infty}p(j,k,t)=\lim_{s\to 0}sp^*(j,k,s),\ j=1,2,\cdots;\ k=1,2,\cdots,N.$$

By equation (22) and using the fact that $\lim_{s\to 0}\xi_{k,1}(s)=1$, we get

$$\pi(0,0)=\frac{1}{1+\lambda_0\sum_{k=1}^{N}\frac{q_k}{\mu_k-\lambda_k}}. \tag{26}$$

By using (19), we get

$$\pi(j,k)=\frac{\lambda_0 q_k\lambda_k^{j-1}}{\mu_k^j\left[1+\lambda_0\sum_{k=1}^{N}\frac{q_k}{\mu_k-\lambda_k}\right]},\ j=1,2,\cdots;\ k=1,2,\cdots,N. \tag{27}$$

Thus, we have proved the following theorem:

**Theorem 2** Under the condition $\lambda_k<\mu_k$, $k=1,2,\cdots,N$, the steady-state probabilities are given by

$$\pi(0,0)=\frac{1}{1+\lambda_0\sum_{k=1}^{N}\frac{q_k}{\mu_k-\lambda_k}},$$

$$\pi(j,k)=\frac{\lambda_0 q_k\lambda_k^{j-1}}{\mu_k^j\left[1+\lambda_0\sum_{k=1}^{N}\frac{q_k}{\mu_k-\lambda_k}\right]},\ j=1,2,\cdots;\ k=1,2,\cdots,N.$$

Next, we proceed to explore some new performance measures of the present queueing model.

## 6. Performance measures

In this section, we derive a few important measures of system performance such as mean rate of assignments, assignment completions and visits to a particular level of the environment.

### 6.1 *Stationary mean rate of occurrence of assignments*

Let $A(t)$ be the number of times the server gets assignments up to time $t$. Since the assignment is done from the state (0, 0, 0) upon the arrival of a customer to the system, and the assignment can be done to any one of the $N$ levels of the environment, we get

$$E[A(t)] = \sum_{k=1}^{N} \int_0^t P(0, 0, u) \lambda_0 q_k du. \tag{28}$$

Consequently, the stationary mean rate a of occurrences of assignments is given by

$$a = \lim_{t \to \infty} \frac{E[A(t)]}{t} = \sum_{k=1}^{N} \pi(0, 0) \lambda_0 q_k = \pi(0, 0) \lambda_0. \tag{29}$$

### 6.2 *Stationary mean rate of occurrence of assignment completions*

Let $B(t)$ be the number of times assignment completions occur up to time $t$. Since an assignment completion occurs only when service completion takes place at the state $(1, k)$, $k = 1, 2, \cdots, N$. Therefore, the expected number of assignment completions up to time $t$ is given by

$$E[B(t)] = \sum_{k=1}^{N} \int_0^t P(1, k, u) \mu_k du. \tag{30}$$

Consequently, the stationary mean rate $b$ of occurrences of assignment completions is given by

$$b = \lim_{t \to \infty} \frac{E[B(t)]}{t} = \sum_{k=1}^{N} \pi(1, k) \mu_k. \tag{31}$$

Using the equation (5), we obtain

$$b = \lambda_0 \pi(0, 0) = \sum_{k=1} \pi(1, k) \mu_k = a. \tag{32}$$

### 6.3 *Stationary mean rate of occurrence of visits to k- the level of the environment*

Let $C_k(t)$ be the number of times the server is assigned to the $k$-th level of the environment that occur up to time $t$. Since an assignment to the $k$-level is done from the state (0, 0), the expected number of assignments to the $k$-th level of the environment up to time $t$ is given by

$$E[C_k(t)] = \lambda_0 q_k \int_0^t P(0, 0, u) du. \tag{33}$$

Consequently, the stationary mean rate $c_k$ of occurrences of assignments to the $k$-th level of the environment is given by

$$c_k = \lim_{t\to\infty} \frac{E[C_k(t)]}{t} = \lambda_0 q_k \pi(0,0). \tag{34}$$

## 6.4 *Probability that the server is idle*

Let $\kappa(t)$ be the probability that the server is idle at time $t$. Since the server is idle when the system is in the state (0, 0), we obtain

$$\kappa(t) = P(0,0,t). \tag{35}$$

consequently, the steady-state probability $\kappa$ that the server is idle is given by

$$\kappa = \lim_{t\to\infty} P(0,0,t) = \pi(0,0). \tag{36}$$

We now proceed to illustrate the performance of the present queueing model by a numerical study.

# 7. A numerical example

Let $N = 5$ and assume the following values for the other parameters of the system:

$$\lambda_0 = 2.0;\ \lambda_1 = 3.0;\ \mu_1 = 5.0;\ \lambda_2 = 4.0;\ \mu_2 = 8.0;\ \lambda_3 = 5.0;\ \mu_3 = 11.0;$$

$$\lambda_4 = 6.0;\ \mu_4 = 14.0;\ \lambda_5 = 7.0;\ \mu_5 = 17.0;\ p = 0.6.$$

The steady-state probabilities are given by the following Table 1:

**Table 1.** Steady-state probabilities for $p = 0.6$ $\pi(0, 0) = 0.7519249$

| $j$ | $\pi(j, 1)$ | $\pi(j, 2)$ | $\pi(j, 3)$ | $\pi(j, 4)$ | $\pi(j, 5)$ |
|---|---|---|---|---|---|
| 1 | 0.0077 | 0.02887 | 0.04725 | 0.03712 | 0.01146 |
| 2 | 0.00462 | 0.01444 | 0.02148 | 0.01591 | 0.00472 |
| 3 | 0.00277 | 0.00722 | 0.00976 | 0.00682 | 0.00194 |
| 4 | 0.00166 | 0.00361 | 0.00444 | 0.00292 | 0.0008 |
| 5 | 0.001 | 0.0018 | 0.00202 | 0.00125 | 0.00033 |
| 6 | 0.0006 | 0.0009 | 0.00092 | 0.00054 | 0.00014 |
| 7 | 0.00036 | 0.00045 | 0.00042 | 0.00023 | 0.00006 |
| 8 | 0.00022 | 0.00023 | 0.00019 | 0.0001 | 0.00002 |
| 9 | 0.00013 | 0.00011 | 0.00009 | 0.00004 | 0.00001 |
| 10 | 0.00008 | 0.00006 | 0.00004 | 0.00002 | 0 |

Now, by varying $p$, we study the behaviour of the performance measures $a$, $b$, $c_k$ and $\kappa$. Table 2 provides the variation of $a$, the stationary mean rate of occurrences of assignments as a function of $p$. Figure 2 exhibits that the stationary mean rate of occurrences of assignments increases as $p$ increases. Since $a = b$, the stationary mean rate $b$ of occurrences of completion of assignments as a function of $p$ also increases as $p$ increases.

**Table 2.** $a$ as a function $p$

| $p$ | $a$ | $p$ | $a$ | $p$ | $a$ | $p$ | $a$ |
|---|---|---|---|---|---|---|---|
| 0.05 | 1.0499370 | 0.30 | 1.2864880 | 0.55 | 1.4739195 | 0.80 | 1.6001024 |
| 0.10 | 1.0994931 | 0.35 | 1.3287922 | 0.60 | 1.5038499 | 0.85 | 1.6190710 |
| 0.15 | 1.1482861 | 0.40 | 1.3688130 | 0.65 | 1.5313042 | 0.90 | 1.6363666 |
| 0.20 | 1.1959434 | 0.45 | 1.4063943 | 0.70 | 1.5563960 | 0.95 | 1.6521740 |
| 0.25 | 1.2421155 | 0.50 | 1.4414414 | 0.75 | 1.5792721 | 1 | 1.6666667 |

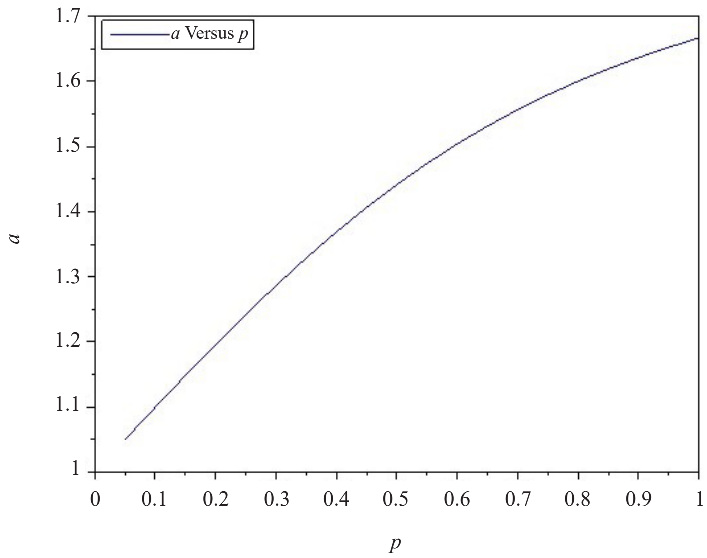


**Figure 2.** $a$ as a function $p$

**Table 3.** $c_5$ as a function $p$

| $p$ | $c_5$ | $p$ | $c_5$ | $p$ | $c_5$ | $p$ | $c_5$ |
|---|---|---|---|---|---|---|---|
| 0.05 | 0.0000066 | 0.30 | 0.0104206 | 0.55 | 0.1348728 | 0.80 | 0.6554019 |
| 0.10 | 0.0001099 | 0.35 | 0.0199402 | 0.60 | 0.1948989 | 0.85 | 0.8451652 |
| 0.15 | 0.0005813 | 0.40 | 0.0350416 | 0.65 | 0.2733474 | 0.90 | 1.0736201 |
| 0.20 | 0.0019135 | 0.45 | 0.0576710 | 0.70 | 0.3736907 | 0.95 | 1.3457061 |
| 0.25 | 0.0048520 | 0.50 | 0.0900901 | 0.75 | 0.4996915 | 1 | 1.6666667 |

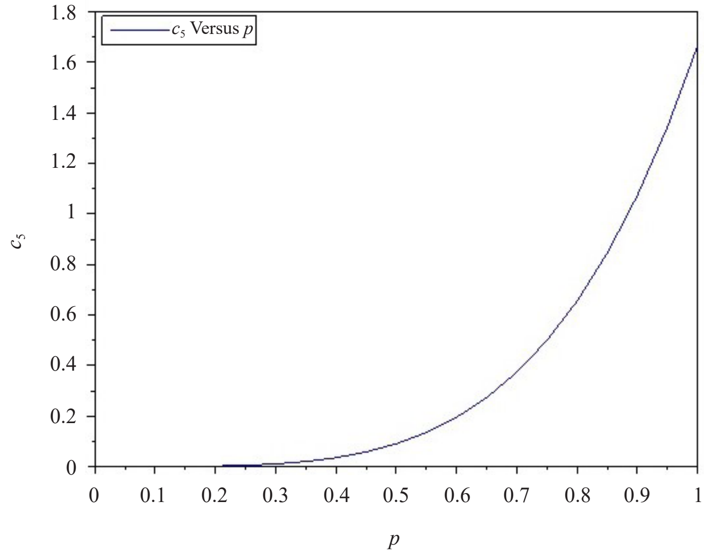


**Figure 3.** $c_5$ as a function $p$

**Table 4.** $\kappa$ as a function $p$

| $p$ | $\kappa$ | $p$ | $\kappa$ | $p$ | $\kappa$ | $p$ | $\kappa$ |
|---|---|---|---|---|---|---|---|
| 0.05 | 0.5249685 | 0.30 | 0.6432440 | 0.55 | 0.7369597 | 0.80 | 0.8000512 |
| 0.10 | 0.5497466 | 0.35 | 0.6643961 | 0.60 | 0.7519249 | 0.85 | 0.8095355 |
| 0.15 | 0.5741430 | 0.40 | 0.6844065 | 0.65 | 0.7656521 | 0.90 | 0.8181833 |
| 0.20 | 0.5979717 | 0.45 | 0.7031972 | 0.70 | 0.7781980 | 0.95 | 0.8260870 |
| 0.25 | 0.6210577 | 0.50 | 0.7207207 | 0.75 | 0.7896360 | 1 | 0.8333333 |

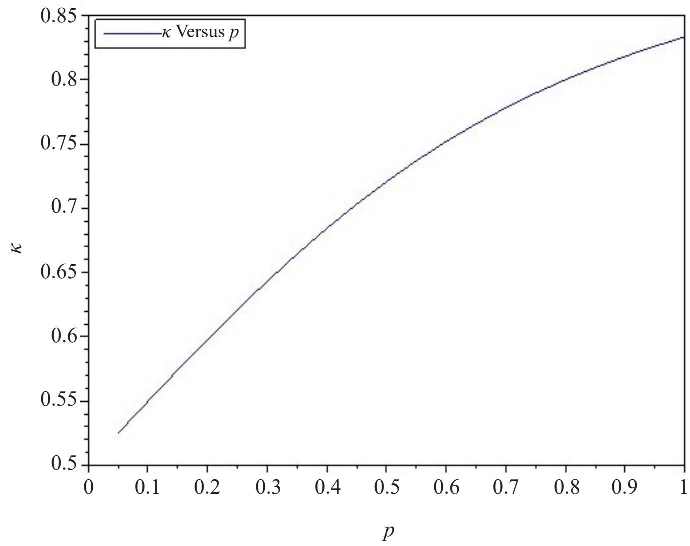


**Figure 4.** $\kappa$ as a function $p$

Table 3 provides the variation of $c_5$, the mean stationary rate of occurrences of assignments to the 5-th level of the environment as a function of $p$. Figure 3 portrays that $c_5$ increases as $p$ increases.

In Table 4, we present the stationary probability $\kappa = \pi(0, 0)$ that the server is idle by varying $p$ from 0.05 to 1. Figure 4 shows that $\kappa$ increases as $p$ increases.

Using the time-dependent solution for $P(0, 0, t)$ in (16), we computed the transient probability $\kappa(t) = P(0, 0, t)$ that the server is idle at time $t$ by fixing $p = 0.6$ and varying $t$ from 0.01 to 1.0. The computed values are tabulated in Table 5 and plot them in Figure 5 which shows that $\kappa(t)$ decreases as $t$ increases.

**Table 5.** $\kappa(t)$ as a function $t$

| $t$ | $P(0, 0, t)$ | $t$ | $P(0, 0, t)$ | $t$ | $P(0, 0, t)$ | $t$ | $P(0, 0, t)$ |
|---|---|---|---|---|---|---|---|
| 0.01 | 0.989983 | 0.11 | 0.9876164 | 0.21 | 0.9867125 | 0.31 | 0.9862467 |
| 0.02 | 0.9895874 | 0.12 | 0.9874909 | 0.22 | 0.9866524 | 0.32 | 0.9862129 |
| 0.03 | 0.9892465 | 0.13 | 0.9873758 | 0.23 | 0.986596 | 0.33 | 0.9861809 |
| 0.04 | 0.98895 | 0.14 | 0.9872697 | 0.24 | 0.986543 | 0.34 | 0.9861505 |
| 0.05 | 0.9886897 | 0.15 | 0.9871717 | 0.25 | 0.9864931 | 0.35 | 0.9861215 |
| 0.06 | 0.9884594 | 0.16 | 0.987081 | 0.26 | 0.986446 | 0.36 | 0.986094 |
| 0.07 | 0.9882542 | 0.17 | 0.9869966 | 0.27 | 0.9864016 | 0.37 | 0.9860679 |
| 0.08 | 0.9880701 | 0.18 | 0.9869182 | 0.28 | 0.9863596 | 0.38 | 0.986043 |
| 0.09 | 0.9879041 | 0.19 | 0.9868449 | 0.29 | 0.9863199 | 0.39 | 0.9860192 |
| 0.10 | 0.9877535 | 0.20 | 0.9867765 | 0.30 | 0.9862823 | 0.40 | 0.9859966 |

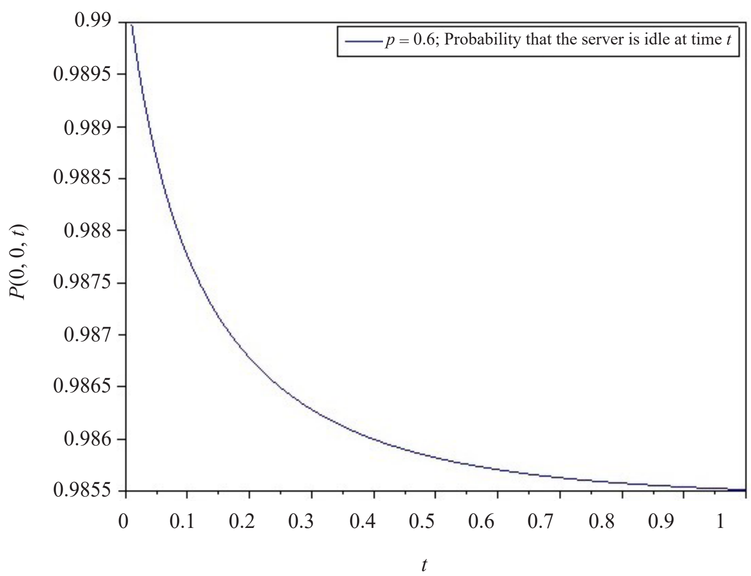


**Figure 5.** $\kappa(t)$ as $t$ varies

## 8. Conclusion

In the present paper, we formulated a single server queueing system operating in a doubly stochastic environment. The random environment has $N$ levels and the server was assigned to serve in any one of the levels of the environment controlled by a random switch. In each level $r$ of the environment, the queueing system was modelled as a Markovian queue where the server reported to the random switch immediately after performing exhaustive service for getting a new assignment with a positive probability governed by a binomial distribution. We derived time-dependent state probabilities and also the corresponding steady-state probabilities. Some key performance measures were also obtained. A numerical study was also made to highlight the transient and steady-state results.


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## Conflict of interest

The authors have no conflict of interest either wholly or partially in the content of the article.


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