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Research Article

A Caputo-Type Fractional-Order SQIRV Mathematical Model for Omicron Variant

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Abstract: In this study, we propose a fractional-order epidemic model of the Omicron variant. We provide analyses of the solution's positivity, boundedness, existence, and uniqueness. The steady-state solution of the proposed model is asymptotically stable and depends on the reproduction number, R_0, which is used to determine whether the disease continues to spread. Numerical simulations are investigated using various orders of the fractional derivative.

Keywords**:** Omicron, steady-states, reproduction number, stability, Caputo fractional derivative

MSC: 26A33, 34A08, 65P40

1. Introduction

Coronaviruses are monarch viruses that can infect both people and animals. Several coronaviruses have been associated with respiratory disorders in humans, including the common cold, SARS, and a newly identified disease known as COVID-19. The initial COVID-19 outbreak occurred in December 2019, and the first COVID-19 case was reported on January 30, 2020, in India. The Indian government has taken severe steps in response to a modest outbreak. Even though it was found in a wide range of countries, Omicron was still the most frequent kind everywhere on November 24, 2021. Based on a preliminary investigation, Omicron can sometimes cause a softer form of the condition. However, some individuals who contract this type of infection continue to have a chance of getting a severe illness, needing treatment, or even dying. The symptoms of earlier forms may be experienced by people who acquire the Omicron type. COVID-19 vaccination remains the best health strategy for lowering the chance of new variants arising and preventing COVID-19. The current vaccinations are intended to reduce major disease, hospitalization, and death caused by Omicron variant infection.

Various mathematical models have been developed to explain how illnesses spread in sub-populations. In light of the highly transmissible Omicron variant, mathematicians must examine the models that improved the country's awareness of COVID-19. The pandemic has affected a large population. We derive a fractional-order SQIRV model, including the quarantined compartment. Many mathematical models of COVID-19 have been developed in the absence of the vaccine and quarantine compartments [1-6]. In [7], the authors derived some novel numerical simulations on a

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model of the Omicron variant. In [8], a fractional-order model of Omicron containing heart attack effects was proposed using real data from the United Kingdom. In [9], some stability analyses were performed on a model of the Omicron variant. The fractional derivatives have been employed in the mathematical modeling of biological phenomena for a very long time. This is due to the fact that integer-order models cannot adequately explain or handle the preservation and preservation characteristics of various materials. Fractional mathematical models are useful tools for investigating infectious diseases. Many authors have formulated fractional order models for various diseases [10-13]. In [14], the authors proposed a media addiction model using the fractal-fractional derivative. Qurashi et al. [15] developed a fractal-fractional divorce epidemic model.

We examine the dynamics of an Omicron model in the form of a system of nonlinear differential equations with quarantine and vaccine compartments, which is motivated by incorporating the Caputo operator in epidemic models. To determine the condition that reduces and regulates the unique coronavirus disease spreading in the community, we create the suggested model following the disease's characteristics and formulate it in terms of the Caputo fractional differential system of equations. The existence, uniqueness, and positivity of the solution are also inferred from the papers [16-19]. This model differs from others in that it quickly eliminates the disease by incorporating quarantine and vaccinations with the proper parameters.

In Section 2, we formulated the fractional order SQIRV model for the Omicron variant. In Section 3, we discussed the stability of the proposed model. In Section 4, the existence and uniqueness of solutions to the SQIRV model are examined and derived. In Section 5, we have verified our theoretical findings with a numerical simulation of the real data of Omicron from Chennai, Tamil Nadu. In Section 6, we conclude our findings.

2. Model formulation

Here, we propose our model for analyzing the outbreaks of Omicron. The total population $N(t)$ is subdivided into state factor subpopulations of people who are susceptible individuals, quarantined individuals, infected individuals, recovered individuals, and vaccinated individuals, which are denoted as $S(t)$, $Q(t)$, $I(t)$, $R(t)$, and $V(t)$, respectively.

The following notations are used to denote the parameters in this research: Λ is the rate of human population recruitment. The natural death rate that applies to all compartments is $\mu 1$, and $\mu 2$ is the disease-induced death rate. The effective infectious contact rate between a susceptible and an infected person is $\mu 3$. $\mu 4$ is the proportion of susceptible people quarantined; $\mu 5$ is the rate of immunity loss of the recovered compartment to susceptible; $\mu 6$ is the rate at which the compartment that received the vaccination begins to lose its immunity; $\mu 7$ is the infected class's treatment rate, $\mu 8$ is the natural recovery rates as a result of quarantine; and $\mu 9$ is the proportion of vaccinated and quarantined individuals who come into contact. $\mu 10$ is the mortality rate caused by illnesses in inflicted upon afflicted persons; $\mu 11$ is the rate at which a recovered person enters a compartment that has received vaccinations; and $\mu 12$ is the rate at which an infected person moves to recovered people.

We proceed further by remembering the following definitions:

Definition 2.1. If ψ represents a continuous function on the range $[0, T]$, then the fractional derivative of Caputo is given by [20]

$$ {}^{C}D^{\delta}\psi(t) = \frac{1}{\Gamma(k-\delta)}\int_0^t (t-v)^{k-\delta-1}\frac{d^k}{dv^k}\psi(t)(v)\,dv, \tag{1}$$

where $k = [\delta] + 1$ and $[\delta]$ denotes the integer part function.

Next, we define the Riemann-Liouville integral.

Definition 2.2. The Riemann-Liouville fractional integral is given by [20]

$$ I^{\delta}\psi(t) = \frac{1}{\Gamma(\delta)}\int_0^t (t-v)^{\delta-1}\psi(t)(v)\,dv. \tag{2}$$

Therefore, the proposed fractional-order model is constructed using the integer-order model from [21, 22], given by,

$$
\begin{aligned}
&{}^{C}D^{\delta}S = \Lambda^{\delta} - \mu_{21}S - \mu_3^{\delta}SI + \mu_4^{\delta}Q + \mu_5^{\delta}R + \mu_6^{\delta}V\\
&{}^{C}D^{\delta}Q = \mu_7^{\delta}I - \mu_{22}Q - \mu_9^{\delta}QV,\\
&{}^{C}D^{\delta}I = \mu_3^{\delta}SI - \left(\mu_{23}\right)I,\\
&{}^{C}D^{\delta}R = \mu_{12}^{\delta}I + \mu_8^{\delta}Q - \mu_{24}R,\\
&{}^{C}D^{\delta}V = \mu_{11}^{\delta}R + \mu_2^{\delta}S - \mu_{25}V + \mu_9^{\delta}QV,
\end{aligned}
\tag{3}
$$

where

$$
\mu_{21} = \mu_1^{\delta} + \mu_2^{\delta}, \mu_{22} = \mu_1^{\delta} + \mu_4^{\delta} + \mu_8^{\delta}, \mu_{23} = \mu_1^{\delta} + \mu_7^{\delta} + \mu_{10}^{\delta} + \mu_{12}^{\delta}, \mu_{24} = \mu_1^{\delta} + \mu_5^{\delta} + \mu_{11}^{\delta}, \text{ and } \mu_{25} = \mu_1^{\delta} + \mu_6^{\delta}
$$

with the initial conditions: $S(0) = S_0$, $Q(0) = Q_0$, $I(0) = I_0$, $R(0) = R_0^0$, $V(0) = V_0$.

The above system (3) takes the following form:

$$
\begin{aligned}
&{}^{C}D^{\delta}X\left(t\right) = J\left(t, X\left(t\right)\right), \quad 0 < \delta \le 1,\\
&X\left(0\right) = X_0.
\end{aligned}
\tag{4}
$$

Let us define a Banach space as $B = B_1 \times B_2 \times B_3 \times B_4 \times B_5$ with $B_k = C([0; T])$; ($k = 1, 2, ..., 5$) by the norm

$$
\|X\| = \|(S, Q, I, R, V)\| = \max_{t\in[0,T]}\left[|S(t)|, |Q(t)|, |I(t)|, |R(t)|, |V(t)|\right].
$$

Consider an operator $y : B \to B$, which is defined as follows:

$$
y\left(X\right)\left(y\right) = Y_0 + \frac{1}{\Gamma(\delta)}\int_0^t \left(t - \Upsilon\right)^{\delta-1} J\left(\Upsilon, X\left(\Upsilon\right)\right)d\Upsilon.
$$

Then, the equation (4) can be solved by using Riemann-Liouvulle integral as follows: $X\left(t\right) = X_0 + \frac{1}{\Gamma(\delta)}\int_0^t \left(t - \Upsilon\right)^{\delta-1} J\left(\Upsilon, X\left(\Upsilon\right)\right)d\Upsilon$ where

$$
\begin{cases}
S\left(t\right) = S_0 + \dfrac{1}{\Gamma(\delta)}\displaystyle\int_0^t \left(t - \Upsilon\right)^{\delta-1} J\left(\Upsilon, S\right)d\Upsilon,\\
Q\left(t\right) = Q_0 + \dfrac{1}{\Gamma(\delta)}\displaystyle\int_0^t \left(t - \Upsilon\right)^{\delta-1} J\left(\Upsilon, Q\right)d\Upsilon,\\
I\left(t\right) = I_0 + \dfrac{1}{\Gamma(\delta)}\displaystyle\int_0^t \left(t - \Upsilon\right)^{\delta-1} J\left(\Upsilon, I\right)d\Upsilon,\\
R\left(t\right) = R_0 + \dfrac{1}{\Gamma(\delta)}\displaystyle\int_0^t \left(t - \Upsilon\right)^{\delta-1} J\left(\Upsilon, R\right)d\Upsilon,\\
V\left(t\right) = V_0 + \dfrac{1}{\Gamma(\delta)}\displaystyle\int_0^t \left(t - \Upsilon\right)^{\delta-1} J\left(\Upsilon, V\right)d\Upsilon,
\end{cases}
\tag{5}
$$

with

$$\begin{cases} J(\Upsilon,S) = \Lambda^\delta - \mu_{21}S - \mu_3^\delta SI + \mu_4^\delta Q + \mu_5^\delta R + \mu_d^\delta V \\ J(\Upsilon,Q) = \mu_7^\delta I - \mu_{22}Q - \mu_9^\delta QV, \\ J(\Upsilon,I) = \mu_3^\delta SI - (\mu_{23})I, \\ J(\Upsilon,R) = \mu_{12}^\delta I + \mu_8^\delta Q - \mu_{24}R, \\ J(\Upsilon,V) = \mu_{11}^\delta R + \mu_2^\delta S - \mu_{25}V + \mu_9^\delta QV. \end{cases} \tag{6}$$

2.1 *Positivity and existence of solution*

Now, the positivity of the solution should be investigated, so we define

$$R_+^5 = \{X \in R^5 \,|\, X \geq 0\} \text{ and } X(t) = (S(t), Q(t), I(t), R(t), V(t))^T.$$

The generalized mean values theorem is recalled [19].

Lemma 2.3. Let $X(\alpha) \in C[c,d]$ and ${}^C D_t^\delta XC(t) \in C(c,d]$, and $\delta \in (0,1]$, then $X(t) = X(c) + \frac{1}{\Gamma(\beta)}({}^C D_t^\delta X)(\zeta)(t-c)^\delta$ with $c \leq \zeta \leq t, \forall t \in [c,d]$.

Remark 2.4. From Lemma 2.3, it is clear that if ${}^C D_t^\delta X(t) \geq 0$, for all $t \in [c,d]$, and the function *X*(t) is increasing. If ${}^C D_t^\delta X(t) \leq 0, \forall t \in [c,d]$, then the function *U*(t) is decreasing $\forall t \in [c,d]$.

Now, we prove the positivity of the solution of the model (3)

Theorem 2.5. Assume that the initial values of the given system (3) are nonnegative and bounded in R_+^5. Then, *S*(*t*), *Q*(*t*), *I*(*t*), *R*(*t*), and *V*(*t*) are also nonnegative and bounded in $R_+^5 \forall t > 0$.

Proof. Consider the model (3) We get

$$\begin{aligned} &{}^C D_t^\delta S(t)_{S=0} = \Lambda^\delta + \mu_4^\delta Q + \mu_5^\delta R + \mu_6^\delta V \geq 0 \\ &{}^C D_t^\delta Q(t)_{Q=0} = \mu_7^\delta I \geq 0 \\ &{}^C D_t^\delta I(t)_{I=0} = 0 \\ &{}^C D_t^\delta R(t)_{R=0} = \mu_{12}^\delta I + \mu_8^\delta Q \geq 0 \\ &{}^C D_t^\delta V(t)_{V=0} = \mu_{11}^\delta R + \mu_2^\delta S \geq 0. \end{aligned}$$

The Lemma 2.3 and Remark 2.4 permit us to draw the conclusion that the solution is nonnegative and will be located within the specified realizable area.

From (3), we obtain the following:

$$\begin{aligned} &{}^C D_t^\delta N = \Lambda^\delta - \mu_1^\delta (S + Q + I + R + V) - \mu_{10}^\delta I, \\ &{}^C D_t^\delta N = \Lambda^\delta - N\mu_1^\delta. \end{aligned} \tag{7}$$

If we apply Laplace transform to equation (7), then it becomes

$$\begin{aligned} {}_s^\delta L[N(t)] - S^{\delta-1}N(0) = L[\Lambda^\delta] - \mu_1^\delta L[N(t)], \\ N(s) = \frac{\Lambda^\delta}{s(s^\delta + \mu_1^\delta)} + N(0)\frac{s^{(\delta-1)}}{s^\delta + \mu_1^\delta}. \end{aligned} \tag{8}$$

Again, by applying inverse Laplace transform to equation (8), it gives

$$N(t)=\frac{\Lambda^{\delta}}{\mu_1^{\delta}}L^{-1}\left[\frac{1}{s}\right]-\left(\frac{\Lambda^{\delta}}{\mu_1^{\delta}}-N(0)\right)L^{-1}\frac{s^{\delta-1}}{s^{\delta}+\mu_1^{\delta}},$$
$$\lim_{t\to\infty}N(t)\leq\frac{\Lambda^{\delta}}{\mu_1^{\delta}}. \tag{9}$$

So, we get the nonnegativity and boundedness of solutions for $t > 0$.

2.2 *Steady-state solutions*

We examine the brief performance of (3), which heavily depends on R_0 and the existence of its steady-state solutions. There are two steady-state solutions exist for the problem.

For $I = 0$, i.e., when the infection is absent, the steady-state solutions of (3) is found to be

$$\begin{aligned}E^0&=(S,Q,I,R,V)\\&=\left(\frac{\Lambda^{\delta}\left(\mu_1^{\delta}+\mu_6^{\delta}\right)}{\mu_1^{\delta}\left(\mu_1^{\delta}+\mu_2^{\delta}+\mu_6^{\delta}\right)},0,0,0,\frac{\Lambda^{\delta}\mu_2^{\delta}}{\mu_1^{\delta}\left(\mu_1^{\delta}+\mu_2^{\delta}+\mu_6^{\delta}\right)}\right).\end{aligned} \tag{10}$$

Also, for $I \neq 0$, i.e., when the infection is persistent, the steady-state solutions of (3) is found to be

$$\begin{aligned}E^*&=\left(S^*,Q^*,I^*,R^*,V^*\right)\\&=\left(\frac{\mu_{33}}{\mu_3^{\delta}},\frac{\mu_7^{\delta}I^*}{\mu_{22}+\mu_9^{\delta}V^*},\frac{\mu_3^{\delta}V^{\delta}-\mu_{21}\mu_{33}+\mu_6^{\delta}C}{\mu_{33}-\mu_4^{\delta}A-\mu_5^{\delta}B-\mu_6^{\delta}C},\frac{\mu_8^{\delta}Q^*+\mu_{12}^{\delta}I^*}{\mu_1^{\delta}+\mu_5^{\delta}+\mu_{11}^{\delta}},\frac{\mu_2^{\delta}\mu_{11}^{\delta}R^*+\mu_2^{\delta}\mu_{23}}{\mu_2^{\delta}\left(\mu_1^{\delta}+\mu_6^{\delta}-\mu_9^{\delta}Q^*\right)}\right)\end{aligned} \tag{11}$$

where $\mu_{33}=\mu_1^{\delta}+\mu_7^{\delta}+\mu_{10}^{\delta}+\mu_{12}^{\delta}$, $A=\frac{\mu_7^{\delta}}{\mu_{22}+\mu_9^{\delta}V^*}$, $B=\frac{\mu_{12}^{\delta}}{\mu_1^{\delta}+\mu_5^{\delta}+\mu_{11}^{\delta}}+\frac{\mu_7^{\delta}\mu_8^{\delta}}{\left(\mu_{22}+\mu_9^{\delta}V^*\right)\left(\mu_1^{\delta}+\mu_5^{\delta}+\mu_{11}^{\delta}\right)}$, $C=\frac{\mu_{11}^{\delta}B}{\mu_1^{\delta}+\mu_6^{\delta}-\mu_9^{\delta}Q^*}$.

The fundamental reproduction number R_0 is calculated by referring to [24] and [25].

Theorem 2.6. Let $Z=\{y=0|y_k,k=1,2,3....\}$ and $F_k(x)$ represent the frequency of new clinical symptoms of illness in compartment k. Also, let V_k^{+} represent the rate at which people enter compartment k through other means, and V_k^{-} represent the rate at which people leave compartment k. Then, $\dot{y}_k=F_k(y)-V_k(y),k=1,2,3,....$ and $V_k(y)=V_k^{-}-V_k^{t}$, such that the matrices V is nonsingular and F is nonnegative.

By using this theorem, the reproduction number can be calculated as

$$F=\begin{pmatrix}0&0&0&0&0\\0&0&\mu_7^{\delta}&0&0\\0&0&\mu_3^{\delta}S&0&0\\0&0&\mu_{12}^{\delta}&0&0\\0&0&0&0&0\end{pmatrix},$$

$$V=\begin{pmatrix}\mu_1^{\delta}+\mu_2^{\delta}&\mu_4^{\delta}&0&\mu_5^{\delta}&\mu_6^{\delta}\\0&\mu_1^{\delta}+\mu_4^{\delta}+\mu_7^{\delta}+\mu_8^{\delta}&0&0&0\\0&0&\mu_1^{\delta}+\mu_{10}^{\delta}+\mu_{12}^{\delta}+\mu_7^{\delta}&0&0\\0&\mu_8^{\delta}&0&\mu_1^{\delta}+\mu_5^{\delta}+\mu_{11}^{\delta}&0\\\mu_2^{\delta}&0&0&0&\mu_1^{\delta}+\mu_6^{\delta}\end{pmatrix}.$$

Then, R_0 is defined by,

$$R_0 = \rho\left(FV^{-1}\right) = \mu_3^\delta\left(\frac{\Lambda^\delta\left(\mu_1^\delta\right)+\mu_6^\delta}{\mu_1^\delta\left(\mu_1^\delta+\mu_2^\delta+\mu_6^\delta\right)}\right)\left(\frac{1}{\mu_1^\delta+\mu_7^\delta+\mu_{10}^\delta+\mu_{12}^\delta}\right). \tag{12}$$

3. Stability analysis

This section analyzes and determines the stability of steady-state solutions for the given system (3).

Theorem 3.1. The infection-free equilibrium point E_0 (11) of the given system (3) is stable if $R_0 < 1$ and unstable otherwise.

Proof. It is necessary to look at the linearization of the proposed system to discuss the stabilization of the steady-state solution of (3) at any equilibrium point (S^*, Q^*, I^*, R^*, V^*).

$$\begin{aligned}
&{}^C D^\delta S = \Lambda^\delta - \mu_{21}S - \mu_3^\delta I^* S - \mu_3^\delta S^* I + \mu_4^\delta Q + \mu_5^\delta R + \mu_6^\delta V\\
&{}^C D^\delta Q = \mu_7^\delta I - \mu_{22}Q - \mu_9^\delta V^* Q - \mu_9^\delta Q^* V,\\
&{}^C D^\delta I = \mu_3^\delta I^* S + \mu_3^\delta S^* I - \left(\mu_{23}\right)I,\\
&{}^C D^\delta R = \mu_{12}^\delta I + \mu_8^\delta Q - \mu_{24}R,\\
&{}^C D^\delta V = \mu_{11}^\delta R + \mu_2^\delta S - \mu_{25}V + \mu_9^\delta V^* Q + \mu_9^\delta Q^* V.
\end{aligned} \tag{13}$$

By taking the Laplace transform for (13), we get

$$\begin{aligned}
&s^\delta L\left[S(s)\right] - s^{\delta-1}S(0) = \Lambda^\delta - \mu_{21}L\left[S(s)\right] - \mu_3^\delta I^* L\left[S(s)\right] - \mu_3^\delta S^* L\left[I(s)\right]\\
&\qquad + \mu_4^\delta L\left[Q(s)\right] + \mu_5^\delta L\left[R(s)\right] + \mu_6^\delta L\left[V(s)\right],\\
&s^\delta L\left[Q(s)\right] - s^{\delta-1}Q(0) = \mu_7^\delta L\left[I(s)\right] - \mu_{22}L\left[Q(s)\right] - \mu_9^\delta V^* L\left[Q(s)\right] - \mu_9^\delta Q^* L\left[V(s)\right],\\
&s^\delta L\left[I(s)\right] - s^{\delta-1}I(0) = \mu_3^\delta I^* L\left[S(s)\right] - \mu_3^\delta S^* L\left[I(s)\right] - \left(\mu_{23}\right)L\left[I(s)\right],\\
&s^\delta L\left[R(s)\right] - s^{\delta-1}R(0) = \mu_{12}^\delta L\left[I(s)\right] + \mu_8^\delta L\left[Q(s)\right] - \mu_{24}L\left[R(s)\right],\\
&s^\delta L\left[V(s)\right] - s^{\delta-1}V(0) = \mu_{11}^\delta L\left[R(s)\right] + \mu_2^\delta L\left[S(s)\right] - \mu_{25}L\left[V(s)\right] + \mu_9^\delta V^* L\left[Q(s)\right]\\
&\qquad + \mu_9^\delta Q^* L\left[V(s)\right].
\end{aligned} \tag{14}$$

Thus, the system (14) becomes

$$\Delta(s).\left[L\left[S(s)\right]L\left[Q(s)\right]L\left[I(s)\right]L\left[R(s)\right]L\left[V(s)\right]\right] = \left[v_1(s)v_2(s)v_3(s)v_4(s)v_5(s)\right]$$

where

$$\begin{cases}
v_1(s) = s^{\delta-1}S(0)\\
v_2(s) = s^{\delta-1}Q(0)\\
v_3(s) = s^{\delta-1}I(0)\\
v_4(s) = s^{\delta-1}R(0)\\
v_5(s) = s^{\delta-1}V(0)
\end{cases}.$$

Hence,

$$\Delta(s)=$$
$$\begin{bmatrix} s^{\delta}+\mu_{21}+\mu_3^{\delta}I^* & -\mu_4^{\delta} & +\mu_8^{\delta}S^* & -\mu_5^{\delta} & -\mu_6^{\delta} \\ 0 & s^{\delta}+\mu_{22}+\mu_9^{\delta}V^* & -\mu_7^{\delta} & 0 & -\mu_9^{\delta}Q^* \\ -\mu_3^{\delta}I^* & 0 & s^{d}+\mu_{23}-\mu_3^{\delta}S^* & 0 & 0 \\ 0 & -\mu_8^{\delta} & \mu_{12}^{\delta} & s^{\delta}+\mu_{24} & 0 \\ \mu_2^{\delta} & -\mu_9^{\delta}V^* & 0 & -\mu_{11}^{\delta} & s^{\delta+}\mu_{25}-\mu_9^{\delta}Q^* \end{bmatrix},$$

which is a system of (14)'s characteristic matrix. Now, the proposed system's characteristic matrix at disease-free equilibrium (10) is provided by

$$\Delta(s)=$$
$$\begin{bmatrix} s^{\delta}+\mu_{21} & -\mu_4^{\delta} & \mu_3^{\delta}S^* & -\mu_5^{\delta} & -\mu_6^{\delta} \\ 0 & s^{\delta}+\mu_{22}+\mu_9^{\delta}V & -\mu_7^{\delta} & 0 & \\ 0 & 0 & s^{d}+\mu_{23}-\mu_3^{\delta}S & 0 & 0 \\ 0 & -\mu_8^{\delta} & -\mu_{12}^{\delta} & s^{\delta}+\mu_{24} & 0 \\ -\mu_2^{\delta} & -\mu_9^{\delta}V & 0 & -\mu_{11}^{\delta} & s^{\delta}+\mu_{25} \end{bmatrix},$$

From the Jacobian matrix, the eigenvalues are $-\left(\mu_1^{\delta}+\mu_4^{\delta}+\mu_7^{\delta}+\mu_8^{\delta}\right)$, $-\left(\mu_1^{\delta}+\mu_4^{\delta}+\mu_{11}^{\delta}\right)$, $\mu_3^{\delta}S-\left(\mu_1^{\delta}+\mu_7^{\delta}+\mu_{10}^{\delta}+\mu_{12}^{\delta}\right)$, and $\frac{1}{2}\left(-\left(\mu_1^{\delta}+\mu_2^{\delta}\right)-\left(\mu_1^{\delta}+\mu_6^{\delta}\right)\pm\sqrt{\left(\mu_1^{\delta}+\mu_2^{\delta}\right)^2+4\mu_2^{\delta}\mu_6^{\delta}-\mu_1^{\delta}\mu_2^{\delta}\left(\mu_1^{\delta}+\mu_6^{\delta}\right)+\left(\mu_1^{\delta}+\mu_6^{\delta}\right)^2}\right)$.

Now, the system (3) is stable if $\mu_3^{\delta}S-\left(\mu_1^{\delta}+\mu_7^{\delta}+\mu_8^{\delta}+\mu_{10}^{\delta}\right)<0$, and $\mu_1^{\delta}\left(\mu_1^{\delta}+\mu_2^{\delta}+\mu_6^{\delta}\right)>1$.

$$\Leftrightarrow \frac{\mu_3^{\delta}S}{\mu_1^{\delta}+\mu_7^{\delta}+\mu_{10}^{\delta}+\mu_{12}^{\delta}}<1, \text{ and } \mu_1^{\delta}\left(\mu_1^{\delta}+\mu_2^{\delta}+\mu_6^{\delta}\right)>1.$$

$$\Leftrightarrow \mu_3^{\delta}\left(\frac{\Lambda^{\delta}\left(\mu_1^{\delta}+\mu_6^{\delta}\right)}{\mu_1^{\delta}\left(\mu_1^{\delta}+\mu_2^{\delta}+\mu_6^{\delta}\right)}\right)\left(\frac{1}{\mu_1^{\delta}+\mu_7^{\delta}+\mu_{10}^{\delta}+\mu_{12}^{\delta}}\right)<1.$$

Thus, the steady-state solution (10) of (3) is locally asymptotically stable if $R_0 < 1$ when $I = 0$.

4. Existence and uniqueness of solution of the model

This section covers the existence and uniqueness of solutions. The following theorems provide evidence that a solution exists:

Theorem 4.1. Assume that $S(t)$, $Q(t)$, $I(t)$, $R(t)$, and $V(t)$ are nonnegative and bounded functions. Then, the system (6) satisfies Lipschitz condition.

Proof. Let $S(t)$, $Q(t)$, $I(t)$, $R(t)$, and $V(t)$ be nonnegative and bounded functions.

Then, there exists some positive constants ξ_1, ξ_2, ξ_3, ξ_4, ξ_5, such that

$$\|S(t)\|\le\xi_1, \|Q(t)\|\le\xi_2, \|I(t)\|\le\xi_3, \|R(t)\|\le\xi_4, \|V(t)\|\le\xi_5.$$

Consider the function $J(\gamma, S)$, for any S and S_1, we can get

$$
\begin{aligned}
\left\| J(\gamma, S) - J(\gamma, S_1) \right\| &= \left\| \mu_{21}(S - S_1) + \mu_2^{\delta} I (S - S_1) \right\| \\
&\le \left\| \mu_{21}(S - S_1) \right\| + \left\| \mu_2^{\delta} I (S - S_1) \right\| \\
&\le \left(\mu_{21} + \mu_2^{\delta} \left\| I(t) \right\| \right) \left\| S - S_1 \right\| \\
&\le \left(\mu_{21} + \mu_3^{\delta} \xi_3 \right) \left\| S - S_1 \right\| \\
&\le G_{J_1} \left\| S - S_1 \right\|,
\end{aligned} \tag{15}
$$

where $G_{J_1} = \mu_{21} + \mu_3^{\delta} \xi_3$. Hence, $J(\gamma, S)$ satisfies the Lipschitz condition. Similarly, we can find G_{J_i}, for $i = 2, 3, 4, 5$ so that $J(\gamma, S)$, $J(\gamma, Q)$, $J(\gamma, I)$, $J(\gamma, R)$, and $J(\gamma, V)$ satisfy the Lipschitz's conditions.

Consider the equation (5), it can be formulated as

$$
X_k(t) = \begin{cases}
S_k(t) = S_0 + \dfrac{1}{\Gamma(\delta)} \int_0^t (t - \gamma)^{\delta - 1} J(\gamma, S_{k-1}) d\gamma, \\
Q_k(t) = E_0 + \dfrac{1}{\Gamma(\delta)} \int_0^t (t - \gamma)^{\delta - 1} J(\gamma, Q_{k-1}) d\gamma, \\
I_k(t) = I_0 + \dfrac{1}{\Gamma(\delta)} \int_0^t (t - \gamma)^{\delta - 1} J(\gamma, I_{k-1}) d\gamma, \\
R_k(t) = R_0 + \dfrac{1}{\Gamma(\delta)} \int_0^t (t - \gamma)^{\delta - 1} J(\gamma, R_{k-1}) d\gamma, \\
V_k(t) = V_0 + \dfrac{1}{\Gamma(\delta)} \int_0^t (t - \gamma)^{\delta - 1} J(\gamma, V_{k-1}) d\gamma
\end{cases}
$$

The initial conditions listed in the following equations set up their first parts. When two terms are contrasted, we get the following expression

$$
\begin{cases}
\Psi_{1_k}(t) = S_k(t) - S_{k-1}(t) = \dfrac{1}{\Gamma(\delta)} \int_0^t \left[J(\gamma, S_{k-1}) - J(\gamma, S_{k-2}) \right] d\gamma, \\
\Psi_{2_k}(t) = Q_k(t) - Q_{k-1}(t) = \dfrac{1}{\Gamma(\delta)} \int_0^t \left[J(\gamma, Q_{k-1}) - J(\gamma, Q_{k-2}) \right] d\gamma, \\
\Psi_{3_k}(t) = I_k(t) - I_{k-1}(t) = \dfrac{1}{\Gamma(\delta)} \int_0^t \left[J(\gamma, S_{k-1}) - J(\gamma, S_{k-2}) \right] d\gamma, \\
\Psi_{4_k}(t) = R_k(t) - R_{k-1}(t) = \dfrac{1}{\Gamma(\delta)} \int_0^t \left[J(\gamma, R_{k-1}) - J(\gamma, R_{k-2}) \right] d\gamma, \\
\Psi_{5_k}(t) = V_k(t) - V_{k-1}(t) = \dfrac{1}{\Gamma(\delta)} \int_0^t \left[J(\gamma, V_{k-1}) - J(\gamma, V_{k-2}) \right] d\gamma,
\end{cases}
$$

where

$$
X_k(t) \begin{cases}
S_k(t) = \sum_{i=0}^{k} \Psi_{1_i}(t), \\
Q_k(t) = \sum_{i=0}^{k} \Psi_{2_i}(t), \\
I_k(t) = \sum_{i=0}^{k} \Psi_{3_i}(t), \\
R_k(t) = \sum_{i=0}^{k} \Psi_{4_i}(t), \\
V_k(t) = \sum_{i=0}^{k} \Psi_{5_i}(t),
\end{cases} \tag{16}
$$

Consider

$$\begin{aligned}\left\|\Psi_{1_k}(t)\right\| &= \left\|S_k(t)-S_{k-1}(t)\right\| = \frac{1}{\Gamma(\delta)}\int_0^t \left[J(\gamma,S_{k-1})-J(\gamma,S_{k-1})\right]d\gamma \\ &= \frac{\xi_1}{\Gamma(\delta)}\int_0^t \left\|S_{k-1}-S_{k-2}\right\| d\gamma = \frac{\xi_1}{\Gamma(\delta)}\int_0^t \left\|\Psi_{1_k-1}(t)\right\| d\gamma.\end{aligned}$$

Hence, we can get

$$\left\|\Psi_{i_k}(t)\right\| = \frac{\xi_i}{\Gamma(\delta)}\int_0^t \left\|\Psi_{i_{k-1}}(t)\right\| d\gamma \text{ for } i = 1,2,\ldots 5. \tag{17}$$

Now, the functions defined in (16) exist and smooth. We have that the functions $S(t)$, $Q(t)$, $I(t)$, $R(t)$, and $V(t)$ are bounded. Also, $J(t, S)$, $J(t, Q)$, $J(t, I)$, $J(t, R)$, and $J(t, V)$, fulfill Lipschitz's conditions, thus, we obtain the following relations:

$$\begin{cases}\left\|\Psi_{1_k}(t)\right\| \le \left\|S(0)\right\| \left\|\frac{\xi_1}{\Gamma(\delta)}t\right\|^k, \\ \left\|\Psi_{2_k}(t)\right\| \le \left\|Q(0)\right\| \left\|\frac{\xi_2}{\Gamma(\delta)}t\right\|^k, \\ \left\|\Psi_{3_k}(t)\right\| \le \left\|I(0)\right\| \left\|\frac{\xi_3}{\Gamma(\delta)}t\right\|^k, \\ \left\|\Psi_{4_k}(t)\right\| \le \left\|R(0)\right\| \left\|\frac{\xi_4}{\Gamma(\delta)}t\right\|^k, \\ \left\|\Psi_{5_k}(t)\right\| \le \left\|V(0)\right\| \left\|\frac{\xi_5}{\Gamma(\delta)}t\right\|^k.\end{cases} \tag{18}$$

As a result, the system in (18) demonstrates the presence and smoothness of the function in (17).

Theorem 4.2. Assume that $Y : B \to B$ is completely continuous and $J : [0, T] \times \mathrm{B} \to \mathbb{R}$ is continuous. Also, if there exists a constant $G_J > 0$ for $X, X_1 \in B$, along with

$$\left|J(t,X)-J(t,X_1)\right| \le G_J \left|X-X_1\right|,$$

then the system (3) has at least one solution.

Proof. Consider the sequence $\{X_k\}$ and it converges to $X \in B$. Define the remaining terms as follows after n iterations: $E_{1_k}(t), E_{2_k}(t), E_{3_k}(t), E_{4_k}(t), E_{5_k}(t)$, such that

$$\begin{aligned}&S(t)-S(0) = S_k(t)-E_{1_k}(t), \\ &Q(t)-Q(0) = Q_k(t)-E_{2_k}(t), \\ &I(t)-I(0) = I_k(t)-E_{3_k}(t), \\ &R(t)-R(0) = R_k(t)-E_{4_k}(t), \\ &V(t)-V(0) = V_k(t)-E_{5_k}(t).\end{aligned}$$

By using Lipschitz condition of $J(t, \mathrm{S})$ along with the triangle inequality, we obtain

$$\left\|E_{1_k}(t)\right\| = \frac{1}{\Gamma(\delta)}\int_0^t \left[J(\gamma,S)-J(\gamma,S_{k-1})\right]d\gamma \le \frac{\xi_1}{\Gamma(\delta)}\left\|S-S_{k-1}\right\| t.$$

Applying the above process recursively, we get

$$\left\| E_{1_k}(t) \right\| \le \left\| \frac{C_1}{\Gamma(\delta)} t \right\|^{k+1} \xi_1.$$

Then, at t_0

$$\left\| E_k(t) \right\| \le \left\| \frac{C_1}{\Gamma(\delta)} t_0 \right\|^{k+1} \xi_1.$$

Hence,

$$\lim_{k\to\infty} \left\| E_{1_k}(t) \right\| \le \lim_{k\to\infty} \left\| \frac{C_1}{\Gamma(\delta)} t_0 \right\|^{k+1} \xi_1. \tag{19}$$

For $\frac{C_1}{\Gamma(\delta)} t_0 < 1,$ equation (19) becomes $\lim_{k\to\infty} \left\| E_{1_k}(t) \right\| = 0.$

Again, by the similar way, $\left\| E_{i_k}(t) \right\| \to 0$ as $k \to \infty.$

Hence, for $t \in [0, T]$, we have $S_k(t) \to S(t)$ as $k \to \infty$

$$\begin{aligned} \left\| y(S_k) - y(S) \right\| &\le \frac{1}{\Gamma(\delta)} \max_{t\in[0,T]} \int_0^t (t-\gamma)^{\delta-1} - \left| J(\gamma, S_n(\gamma)) J(\gamma, S(\gamma)) \right| d\gamma, \\ &\le \frac{G_J}{\Gamma(\delta)} \left\| S_k - S \right\|_{t\in[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \le \frac{T^\delta G_J}{\Gamma(\delta+1)} \left\| S_k - S \right\|. \end{aligned}$$

Since $S_k \to S,$ so $\left\| y(S_k) - y(S) \right\| \to 0$ as $k \to \infty$ and $\left\| y(X_k) - y(X) \right\| \to 0$ as $k \to \infty$. Thus, the continuity of Y follows.

Let $M \subset B$ be a bounded set. Hence, by using the definition of $B, \left| J(t, X(t)) \right| \le L_J, L_J > 0, \forall X \in M$.

Thus, we obtain the following for each $X \in M$

$$\begin{aligned} \left\| y(X) \right\| &\le \frac{1}{\Gamma(\delta)} \max_{t\in[0,T]} \int_0^t (t-\gamma)^{\delta-1} - \left| J(\gamma, Y(\gamma)) \right| d\gamma \\ &\le \frac{L_J}{\Gamma(\delta)} \max_{t\in[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \le \frac{T^\delta L_J}{\Gamma(\delta+1)}. \end{aligned}$$

Hence, it is proved that Y is equicontinuous. Additionally, because of the continuity and boundedness of Y, clearly Y is compact. Thus, it is completely continuous. Let $\Psi = \{X \in B : X = \rho\, Y(X), \rho \in [0, 1]$, now it is must to prove that Ψ is bounded. If $S \in \Psi$, then for any $t \in [0,T]$, we have

$$\begin{aligned} \left\| S \right\| &\le \max_{t\to[0,T]} \left\{ \frac{\rho}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} - \left| J(\gamma, S(\gamma)) \right| d\gamma \right\} \\ &\le \frac{L_J}{\Gamma(\delta)} \max_{T\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \le \frac{T^\delta L_J}{\Gamma(\delta+1)}. \end{aligned}$$

Hence, Ψ is a bounded set. As a result, there is at least one fixed point [26], for Y. Thus, at least one solution exists for the given system (3).

Theorem 4.3. If $\frac{\xi_j}{\Gamma(\delta)} t < 1$ for $j = 1, 2, ..., 5$, then (3) has a unique solution.

Proof. Let $\{S_v(t), Q_v(t), I_v(t), R_v(t), V_v(t)\}$ be a set of solutions of system (3), then

$$\left\|S(t)-S_v(t)\right\|=\frac{1}{\Gamma(\delta)}\int_0^t\left[J(\gamma,S)-J(\gamma,S_v)\right]d\gamma=\frac{\xi_1}{\Gamma(\delta)}t\left\|S(t)-S_v(t)\right\|.$$

Thus,

$$\left(1-\frac{\xi_1}{\Gamma(\delta)}t\right)\left\|S(t)-S_1(t)\right\|\leq 0. \tag{20}$$

We have $\frac{\xi_1}{\Gamma(\delta)}t<1$ for j = 1, then from (20) we can get $\left\|S(t)-S_v(t)\right\|=0$

Thus, $S(t) = S_v(t)$. In the similar way, for j = 2, 3, 4, and 5, we have $Q(t) = Q_v(t)$, $I(t) = I_v(t)$, $R(t) = R_v(t)$, and $V(t) = V_v(t)$

Hence, the system has unique (3) solution.

5. Numerical simulations

For numerical simulations, we used data from Chennai, Tamil Nadu. The theoretical findings validate this Omicron variant pandemic data [27]. The SARS-CoV-2 strain, which is extremely transmissible and spreads quickly, was discovered in Chennai on December 15, 2021. On March 31, 2022, Chennai achieved zero-risk status for propagating Omicron viruses. In Chennai, there were 499 active cases, 42 confirmed cases, 86 cases that had been recovered, and 3,373 cases that had been vaccinated. The numerical values are simulated using Matlab. The table below lists the values of the variables and parameters.

Table 1. Values of the variables

Parameters	Values	Parameters	Values	Parameters	Values
$S(0)$	3,233	μ_1^δ	0.0870^δ	μ_7^δ	0.6707δ
$Q(0)$	744	μ_2^δ	0.0104^δ	μ_8^δ	0.0580δ
$I(0)$	499	μ_3^δ	0.1543^δ	μ_9^δ	0.2206δ
$R(0)$	86	μ_4^δ	0.2301^δ	μ_{10}^δ	0.0002δ
$V(0)$	3,373	μ_5^δ	0.0266^δ	μ_{11}^δ	0.0255δ
Λ^δ	5^δ	μ_6^δ	0.0096^δ	μ_{12}^δ	0.1723δ

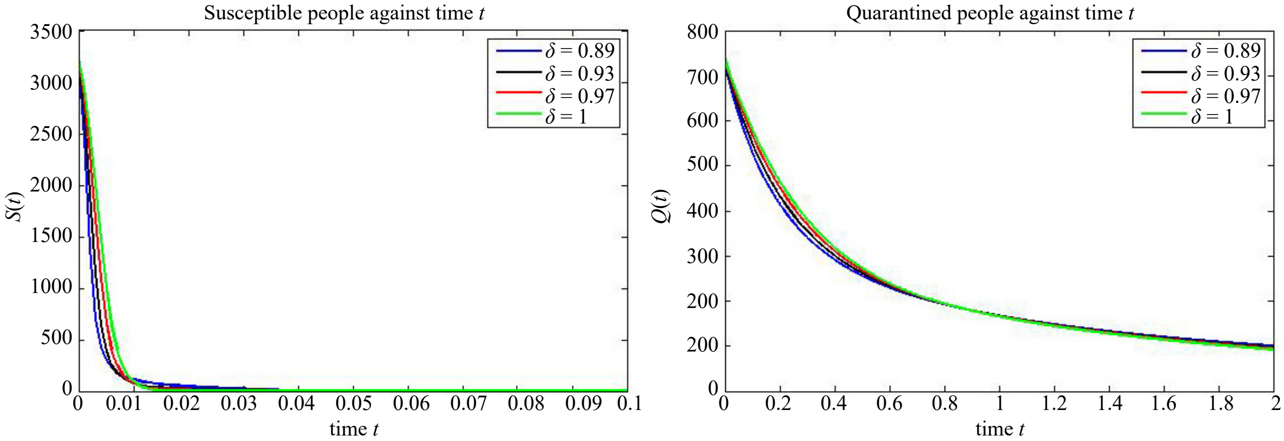

Figure 1. $S(t)$ and $Q(t)$ against time t

Figure 1 depicts the impact of the Omicron variation on susceptible and quarantined individuals in Chennai over time t. People are decreasing their tendency to get an infection due to more signicant vaccination and quarantine.

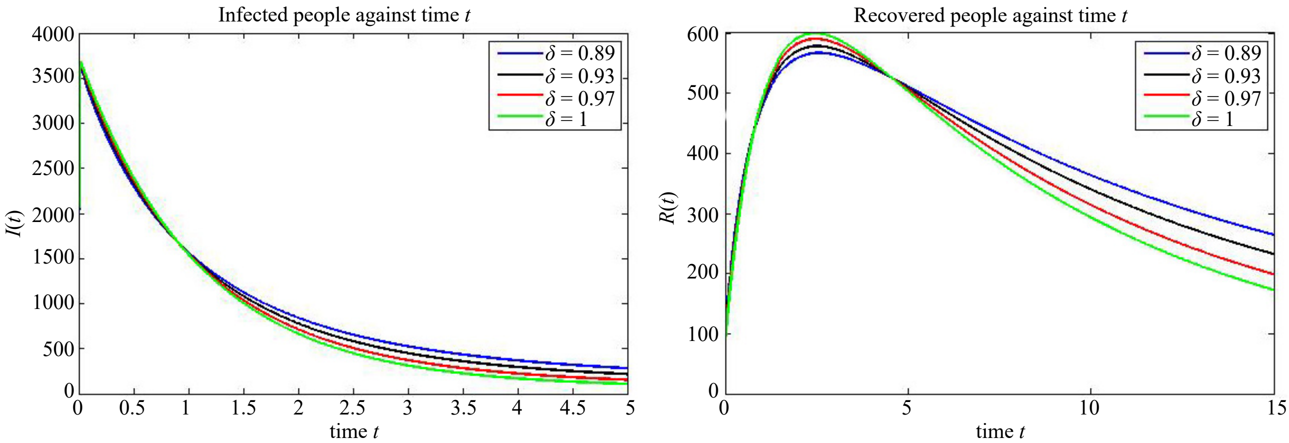

Figure 2. $I(t)$ and $R(t)$ against time t

Figure 2 displays the infected and recovered individuals in the host population with respect to time t in the Chennai district. The individuals infected tested positive for the Omicron variant, and recovered by March 31, 2022, are depicted in Figures 1 and 2. According to RT-PCR sample tests, the number of patients infected in Chennai has reduced to a low level, and no deaths have occurred.

Figure 3 depicts the correlation among the proportions of infected, isolated, and vaccinated patients in Chennai during the Omicron infection period.

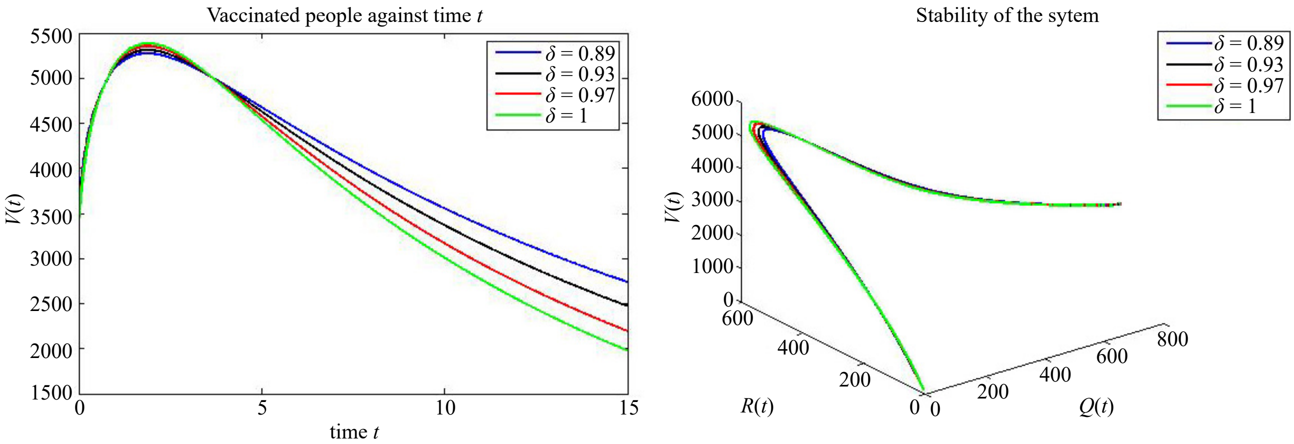

Figure 3. Vaccinated people against time t and stability between quarantined, recovered, and vaccinated people

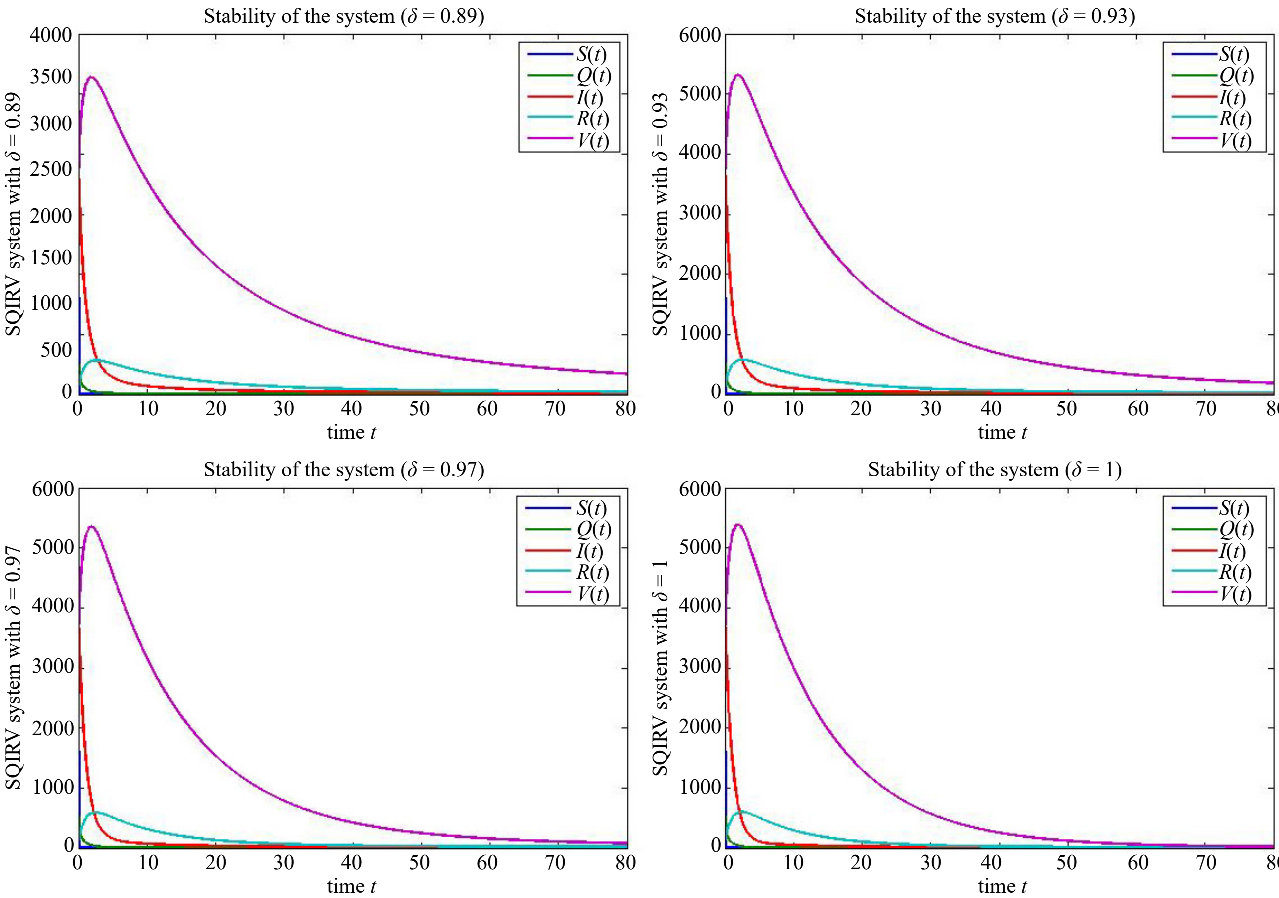

Figure 4. Omicron system's stability against time t with respect of four different δ's

The stability of the proposed mathematical model is described in Figure 4 for Omicron in Chennai. During the Omicron period, which begins on December 25 and ends on March 11, 2022, the people of Chennai suffer from an extreme rate of disease. When people received vaccinations on government recommendations, the spread of illness gradually dropped to an acceptable level. Figure 5 depicts the Chennai host population's stability graph for the developed model with various orders of δ.

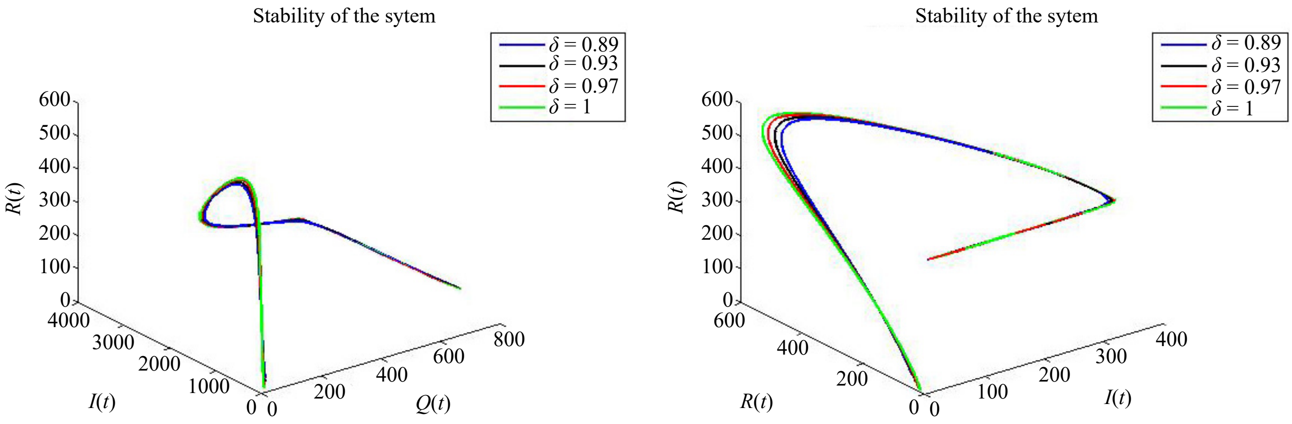

Figure 5. Stability between quarantined, recovered, and vaccinated people

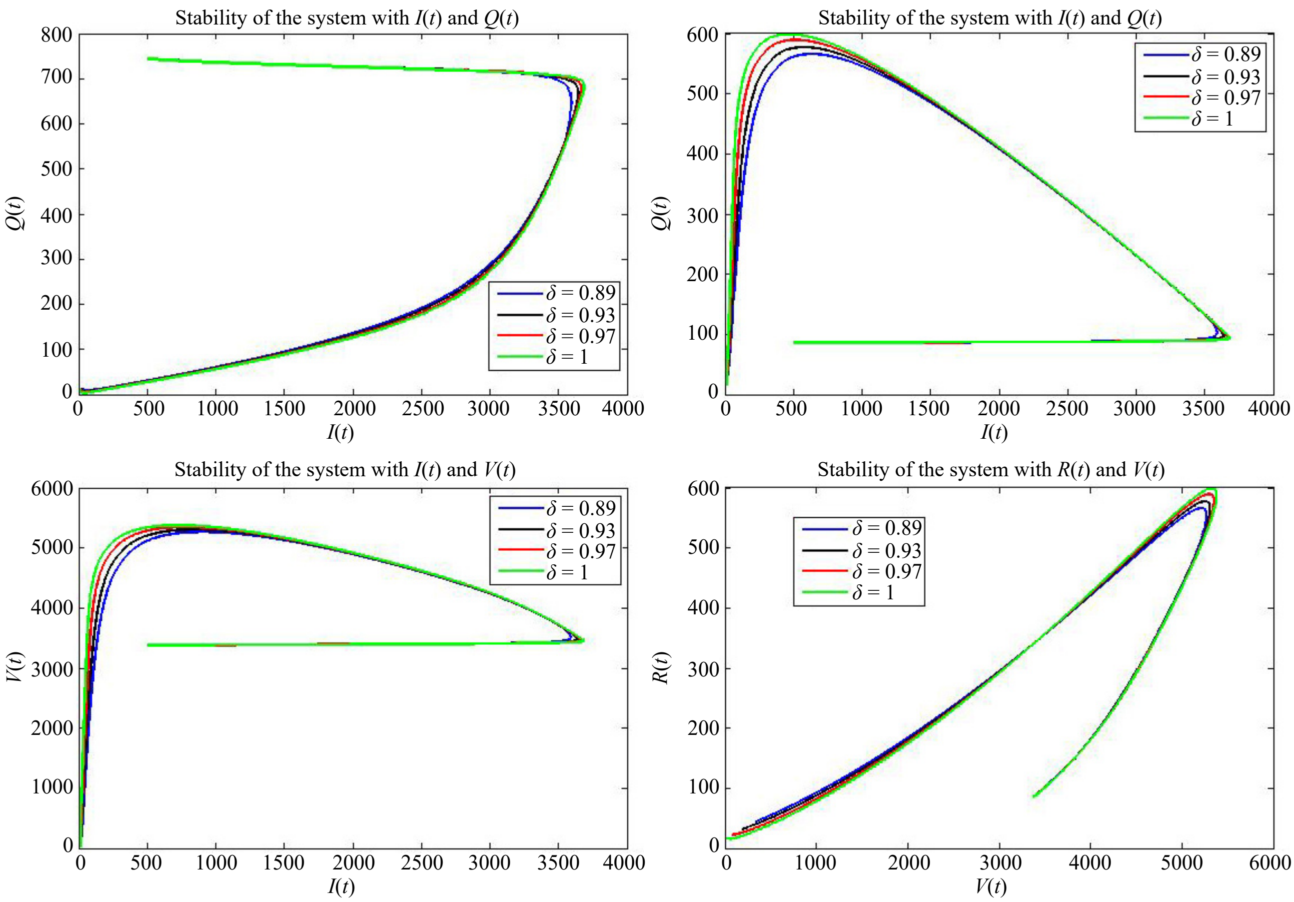

Figure 6. Stability of the system with the relation between infected individual and other individuals

Figure 6 displays the link between those who have been quarantined, recovered, and inoculated and those who have not. When the Omicron variety was originally discovered, it immediately spread. However, the transmission of the variation was stopped when the government increased the rate of vaccination and quarantine.

Figures clearly show that the disease transmission has been totally controlled after 30 days in Chennai. The fractional-order simulations show that, compared to integer-order data, the infection may take longer to leave the population depending on the different fractional-order scenarios. The biological cause of this delay in disease control may be not conforming to the quarantine, avoiding vaccination, or utilizing simple preventative measures. Derivatives with an integer order cannot capture these effects.

The infection falls after a brief rapid spread with the reproduction number $R_0 < 1$, as seen from all the figures. This has led to stability in the system throughout Tamil Nadu, including the Chennai region.

6. Conclusion

In this article, we have proposed and analyzed a Caputo-type fractional-order epidemic model for the Omicron variant. This model enhances other mathematical models by properly introducing the necessary parameters and considering the nonlinear forces of isolation and immunization. It is demonstrated that the steady-state solution is locally asymptotically stable if $R_0 < 1$. Controlling the infection rate stabilizes the stationary solution and increases the vaccinated class when $R_0 < 1$. The existence of a unique solution has been proven. We found out that the Omicron strain spreads less when the appropriate responses are taken. In the future, disease outbreaks could be predicted using the proposed model and other real data. The same model can be more broadly generalized by using many different fractional derivatives.

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Conflict of interest

The authors declare that they have no competing interests.

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