Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

UNIVERSAL WISER PUBLISHER

Research Article

A Novel Study on the Maize Streak Virus Epidemic Model Using Caputo-Fabrizio Fractional Derivative

M. Chandra Malar[1], **M. Gayathri**[1], **A. Manickam**[2*]

[1]Department of Mathematics, Marudupandiyar College (Arts & Science), (Affiliated to Bharathidasan University, Tiruchirappalli-620 024), Trichy Main Road, Vallam Post, Thanjavur-613 403, Tamil Nadu, India
[2]School of Advanced Sciences & Languages, Department of Mathematics, VIT Bhopal University, Bhopal-Indore Highway, Kothrikalan, Sehore, Madhya Pradesh 466114, India
Email: manickammaths2011@gmail.com

Received: 27 January 2023; **Revised:** 14 February 2023; **Accepted:** 15 February 2023

Abstract: In this paper, we simulate a maize streak virus (MSV) epidemic model using the Caputo-Fabrizio fractional derivative. We solve a nonlinear fractional-order model of the MSV using a recently proposed efficient numerical method. We perform several graphical simulations to explore the proposed model dynamics. The proposed investigations justify the usefulness of the recently proposed scheme in epidemiology. The Caputo-Fabrizio type fractional-order generalization of the model and implementation of the method are the key features of this study.

Keywords: maize streak virus, fractional-order model, Caputo-Fabrizio derivative, numerical scheme, graphical simulations

2010 MSC: 26A33, 65D30, 92D25, 93A30

1. Introduction

Over 80 wild and domesticated grasses are infected by the maize streak virus (MSV), which also affects maize, its main host, and causes maize streak disease (MSD). It is a Mastrevirus-related maize disease that is spread by insects and is widespread in sub-Saharan Africa and close-by Indian Ocean island nations, including Madagascar, Mauritius, and La Reunion. The A-strain of MSV causes infrequent outbreaks of maize stripe disease in Africa's maize-growing areas. MSV was first reported by South African entomologist Claude Fuller in a 1901 publication under the name "mealie variegation" [1, 2].

The researchers have proposed some studies concerning mathematical modeling of MSV. In [3, 4], the authors derived a model of optimal control problems for MSV. In [5], Alemneh et al. proposed an eco-epidemic model of the transmission of MSV in maize plants. In [6], the authors investigated a model considering pest invasion for MSV pathogen interaction. In [7], the authors proposed a fractional-order model of MSV.

Fractional derivatives are widely used to describe various real-world phenomena incorporating memory effects in systems [8-11]. Recently, fractional derivatives have been used in several areas, such as mechanics [12], ecology [13],

https://doi.org/10.37256/cm.4320232383

neural networking [14], optics [15], chemical reactor theory [16], image processing [17], and concrete research [18]. Several deadly epidemics have been modeled by using fractional-order operators. In [19], the researchers modeled the spread of Chlamydia in the United States using a Caputo derivative. A novel fractional-order model of the COVID-19 epidemic was proposed in [20]. In [21], a fractional-order model of ecological disease was considered. In [22], the authors modeled the dynamics of tuberculosis. In [23], fractal-fractional derivatives were incorporated to deal with a malaria model. In [24], a model of Lassa hemorrhagic fever was proposed. In [25], the authors proposed a fractional yellow fever virus model with an efficient numerical approach. In [26], the authors used a modified predictor-corrector scheme for the numerical solution of a fractional-order SIR (Susceptible, Infectious, Recovered) model with COVID-19 outbreaks. In [27], the analysis of the spread of infectious diseases and the effects of consciousness programs by media using three fractional operators was proposed. In [28], the authors used Caputo derivatives to propose fractional-order models of rabies and the canine distemper virus. In [29], huanglongbing disease in citrus tree populations was modeled using Caputo derivatives. In [30], the authors used Atangana-Baleanu and Caputo derivatives to model mosaic disease. In order to consider the applications of fractional derivatives in other fields, the authors in [31] used the generalized Caputo derivative to investigate a psychological model. The authors in [32, 33] investigated fractional-order chaotic systems. In [34], a novel fractional-order Lagrangian was derived. In [35], a fractional-order model of the Stuxnet virus was proposed. In [36], a fractional-order model describing wind-influenced projectile motion was proposed.

The models with integer-order operators do not include memory effects in the system because of their local nature. Therefore, we consider a fractional-order model in terms of the Caputo-Fabrizio fractional derivative. The paper is organized as follows: In Section 2, we recall some preliminaries. The fractional-order model is proposed in Section 3. In Section 4, we derive the numerical solution of the model using a recently proposed numerical scheme [37], where we provide proof of stability and error estimation. To explore the dynamics of the proposed model, several graphs are plotted in Section 5. In Section 6, we conclude our findings.

2. Preliminaries

Here we recall the following definitions:

Definition 1. *Consider* $q \in [1, \infty)$ *and* Ω *be open subset of* $\mathbb{R}$. *The Sobolev space* $H^q(\Omega)$ *is defined by*

$$H^q(\Omega) = \left\{ f \in L^2(\Omega) : D^\alpha f \in L^2(\Omega), \forall |\alpha| \le 2 \right\}.$$

Definition 2. [38] *For the function* $g \in H^1(a, b)$; $b > a : \beta \in [0, 1]$, *the Caputo-Fabrizio (C-F) fractional derivative of order* β *is defined by*

$${}_{a}^{CF}D_t^{\beta}\left(g(t)\right) = \frac{M(\beta)}{1-\beta}\int_a^t g'(s)\exp\left[-\frac{\beta(t-s)}{1-\beta}\right]ds, \tag{1}$$

where $M(\beta)$ *is a normalized function with* $M(0) = M(1) = 1$.

The relative C-F fractional integral is defined by

$${}^{CF}I^{\beta}g(t) = (1-\beta)g(t) + \beta\int_a^t g(s)ds. \tag{2}$$

3. Model dynamics

In the articles [3, 5], the authors derived an integer-order nonlinear model to describe the transmission dynamics of MSV. Later, the authors in [7] proposed a Caputo-type fractional-order version of that model. Here we propose the same model but consider the Caputo-Fabrizio fractional derivative with an exponential decay-type kernel. The model dynamics are given as follows:

$$
\begin{cases}
{}_{0}^{CF}D_t^{\beta} S(t) = g^{\beta} S(t)\left(1 - \dfrac{S(t)+I(t)}{k}\right) - \dfrac{\Omega_1^{\beta} S(t) Y(t)}{a+S(t)}, \\
{}_{0}^{CF}D_t^{\beta} I(t) = \dfrac{\Omega_1^{\beta} S(t) Y(t)}{a+S(t)} - d_1^{\beta} I(t), \\
{}_{0}^{CF}D_t^{\beta} H(t) = \alpha^{\beta} - \dfrac{\Omega_2^{\beta} I(t) H(t)}{c+I(t)} - d_2^{\beta} H(t), \\
{}_{0}^{CF}D_t^{\beta} Y(t) = \dfrac{b^{\beta}\Omega_2^{\beta} I(t) H(t)}{c+I(t)} - d_3^{\beta} Y(t),
\end{cases} \tag{3}
$$

where ${}_{0}^{CF}D_t^{\beta}$ is the Caputo-Fabrizio fractional derivative with order β. The model contains four different classes, which are the susceptible maize population class $S(t)$, infected maize $I(t)$, susceptible leafhopper vector $H(t)$, and infected leafhopper vector $Y(t)$. The power β is used on the parameters for making equal time dimensions. The reason for incorporating the Caputo-Fabrizio derivative is that its kernel is non-singular, while the previous Caputo generalization [7] had a singular kernel. The model parameters are described in Table 1 with numerical values.

Table 1. Description of model parameters [3, 5, 7]

Parameters	Description	Values
g	Intrinsic growth rate of maize	0.0005
k	Carrying capacity	10,000
Ω_1	Infection and predation rate of infected leafhopper on susceptible maize plant	0.45
a	Half saturation rate of susceptible maize with infected plant	0.4
d_1	Death rate of infected maize	0.008
α	Recruitment rate of susceptible leafhopper	0.02
Ω_2	Infection and predation rate of susceptible leafhopper on infected plants	0.04
c	Half saturation rate of susceptible leafhopper with infected plants	0.6
d_2	Mortality rate of susceptible leafhopper	0.0303
b	Conversion rate of infected leafhopper	0.45
d_3	Mortality rate of infected leafhopper	0.0303

Using the inequalities

$$
\begin{cases}
X_1(t,S,I,H,Y) = g^{\beta} S(t)\left(1 - \dfrac{S(t)+I(t)}{k}\right) - \dfrac{\Omega_1^{\beta} S(t) Y(t)}{a+S(t)}, \\
X_2(t,S,I,H,Y) = \dfrac{\Omega_1^{\beta} S(t) Y(t)}{a+S(t)} - d_1^{\beta} I(t), \\
X_3(t,S,I,H,Y) = \alpha^{\beta} - \dfrac{\Omega_2^{\beta} I(t) H(t)}{c+I(t)} - d_2^{\beta} H(t), \\
X_4(t,S,I,H,Y) = \dfrac{b^{\beta}\Omega_2^{\beta} I(t) H(t)}{c+I(t)} - d_3^{\beta} Y(t)
\end{cases} \tag{4}
$$

and

$$K(t)=\begin{cases}S(t)\\ I(t)\\ H(t)\\ Y(t)\end{cases},\quad K_0(t)=\begin{cases}S_0(t)\\ I_0(t)\\ H_0(t)\\ Y_0(t)\end{cases},\quad X(t,K(t))=\begin{cases}X_1(t,S,I,H,Y)\\ X_2(t,S,I,H,Y)\\ X_3(t,S,I,H,Y)\\ X_4(t,S,I,H,Y)\end{cases}, \tag{5}$$

the proposed Caputo-Fabrizio model (3) can be expressed in the following form of initial value problem (IVP)

$$\begin{aligned}{}_0^{CF}D_t^{\beta}K(t)&=X(t,K(t)),t\in[0,T],\\ K(0)&=K_0.\end{aligned} \tag{6}$$

4. Numerical simulations

Here we establish the numerical solution of the given model (3) using the recently proposed scheme given in [37]. In this regard, let us consider the above-given IVP (6)

$${}_0^{CF}D_t^{\beta}K(t)=X(t,K(t)),t\in[0,T], \tag{7}$$

$$K(0)=K_0, \tag{8}$$

where $0<\beta<1$, ${}_0^{CF}D_t^{\beta}$ denote the C-F fractional derivative of order β.

From the definition, the equation (7) can be written as

$$\frac{M(\beta)}{1-\beta}\int_0^t K'(s)\exp\left[-\frac{\beta(t-s)}{1-\beta}\right]ds=X(t,K(t)). \tag{9}$$

Using the fundamental theorem, above expression is expressed by

$$K(t)-K(0)=\frac{1-\beta}{M(\beta)}X(t,K(t))+\frac{\beta}{M(\beta)}\int_0^t X(s,K(s))ds. \tag{10}$$

We split the interval [0, T] into n-equal sections with step size $0<h<1$. Consider $n\in\mathbb{N}$ nodes t_0; t_1; … ; t_n and $h=\frac{t_n}{n}$, $t_n=nh$. We assume that after the discretization, the solution $K(t)$ is known up to t_n and we need to calculate it at $t=t_{n+1}$. Replacing t by t_{n+1} in equation (10) gives

$$K(t_{n+1})-K(0)=\frac{1-\beta}{M(\beta)}X(t_{n+1},K(t_{n+1}))+\frac{\beta}{M(\beta)}\int_0^{t_{n+1}} X(t,K(t))dt. \tag{11}$$

Replacing t by t_n in equation (10) gives

$$K(t_n)-K(0)=\frac{1-\beta}{M(\beta)}X(t_n,K(t_n))+\frac{\beta}{M(\beta)}\int_0^{t_n} X(t,K(t))dt. \tag{12}$$

Subtracting equation (12) from equation (11) gives

$$K\left(t_{n+1}\right)-K\left(t_n\right)=\frac{1-\beta}{M(\beta)}\left[X\left(t_{n+1},K\left(t_{n+1}\right)\right)-X\left(t_n,K\left(t_n\right)\right)\right]+\frac{\beta}{M(\beta)}\int_{t_n}^{t_{n+1}}X\left(t,K(t)\right)dt. \tag{13}$$

By applying trapezoidal implicit rule to equation (13), we get

$$\begin{aligned}K\left(t_{n+1}\right)-K\left(t_n\right)&=\frac{1-\beta}{M(\beta)}\left[X\left(t_{n+1},K\left(t_{n+1}\right)\right)-X\left(t_n,K\left(t_n\right)\right)\right]\\&\quad+\frac{\beta}{M(\beta)}\left[X\left(t_{n+1},K\left(t_{n+1}\right)\right)+X\left(t_n,K\left(t_n\right)\right)\right]\frac{\left(t_{n+1}-t_n\right)}{2}\\&=K\left(t_n\right)+\frac{1-\beta}{M(\beta)}\left[X\left(t_{n+1},K\left(t_{n+1}\right)\right)-X\left(t_n,K\left(t_n\right)\right)\right]\\&\quad+\frac{h\beta}{2M(\beta)}\left[X\left(t_{n+1},K\left(t_{n+1}\right)\right)+X\left(t_n,K\left(t_n\right)\right)\right]\\&=K\left(t_n\right)+\left(\frac{h\beta}{2M(\beta)}-\frac{1-\beta}{M(\beta)}\right)X\left(t_n,K\left(t_n\right)\right)+\left(\frac{h\beta}{2M(\beta)}+\frac{1-\beta}{M(\beta)}\right)X\left(t_{n+1},K\left(t_{n+1}\right)\right).\end{aligned} \tag{14}$$

Then, equation (14) is in the form of $K(t_{n+1}) = U(t) + CX(t_{n+1}, K(t_{n+1}))$, where

$$U=K\left(t_n\right)+\left(\frac{h\beta}{2M(\beta)}-\frac{1-\beta}{M(\beta)}\right)X\left(t_n,K\left(t_n\right)\right)$$

is a known part, $C=\left(\frac{h\beta}{2M(\beta)}-\frac{1-\beta}{M(\beta)}\right)$ is a constant, $N = CX$ and $K(t_{n+1})$ are unknown parts.

Therefore, from equation (14), we define the predictor formulae for the model (3) by

$$S_{n+1}^{P_1}=U=S_n+\left(\frac{h\beta}{2M(\beta)}-\frac{1-\beta}{M(\beta)}\right)\left(g^{\beta}S_n\left(t_n\right)\left(1-\frac{S_n\left(t_n\right)+I_n\left(t_n\right)}{k}\right)-\frac{\Omega_1^{\beta}S_n\left(t_n\right)Y_n\left(t_n\right)}{a+S_n\left(t_n\right)}\right), \tag{15}$$

$$S_{n+1}^{P_1}=\left(\frac{h\beta}{2M(\beta)}+\frac{1-\beta}{M(\beta)}\right)\left(g^{\beta}S_{n+1}\left(t_{n+1}\right)\left(1-\frac{S_{n+1}\left(t_{n+1}\right)+I_{n+1}\left(t_{n+1}\right)}{k}\right)-\frac{\Omega_1^{\beta}S_{n+1}\left(t_{n+1}\right)Y_{n+1}\left(t_{n+1}\right)}{a+S_n+1\left(t_{n+1}\right)}\right), \tag{16}$$

$$I_{n+1}^{P_1}=U=I_n\left(\frac{h\beta}{2M(\beta)}+\frac{1-\beta}{M(\beta)}\right)\left(\frac{\Omega_1^{\beta}S_n\left(t_n\right)Y_n\left(t_n\right)}{a+S_{n+1}\left(t_{n+1}\right)}-d_1^{B}I_n\left(t_n\right)\right), \tag{17}$$

$$I_{n+1}^{P_2}=\left(\frac{h\beta}{2M(\beta)}+\frac{1-\beta}{M(\beta)}\right)\left(\frac{\Omega_1^{\beta}S_{n+1}\left(t_{n+1}\right)Y_{n+1}\left(t_{n+1}\right)}{a+S_{n+1}\left(t_{n+1}\right)}-d_1^{B}I_{n+1}\left(t_{n+1}\right)\right), \tag{18}$$

$$H_{n+1}^{P_1}=U=H_n+\left(\frac{h\beta}{2M(\beta)}-\frac{1-\beta}{M(\beta)}\right)\left(\alpha^{\beta}-\frac{\Omega_2^{\beta}I_n\left(t_n\right)H_n\left(t_n\right)}{c+I_n(t)}-d_1^{B}H_n(t)\right), \tag{19}$$

$$H_{n+1}^{P_2} = \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left(\alpha^\beta - \frac{\Omega_2^\beta I_{n+1}(t_{n+1}) H_{n+1}(t_{n+1})}{c + I_{n+1}(t_{n+1})} - d_2^\beta H_{n+1}(t_{n+1})\right), \tag{20}$$

$$Y_{n+1}^{P_1} = U = Y_n + \left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)}\right)\left(\frac{b^\beta \Omega_2^\beta I_n(t_n) H_n(t_n)}{c + I_n(t_n)} - d_3^\beta Y_n(t_n)\right), \tag{21}$$

$$Y_{n+1}^{P_2} = \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left(\frac{b^\beta \Omega_2^\beta I_{n+1}(t_{n+1}) H_{n+1}(t_{n+1})}{c + I_{n+1}(t_{n+1})} - d_3^\beta Y_{n+1}(t_{n+1})\right). \tag{22}$$

Using predictor, we define the corrector by

$$\begin{aligned}
S_{n+1}^{C} &= S_{n+1}^{P_1} + \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left(g^\beta \left(S_{n+1}^{P_1}(t_{n+1}) + S_{n+1}^{P_2}(t_{n+1})\right)\right) \\
&\times \left(1 - \frac{\left(S_{n+1}^{P_1}(t_{n+1}) + S_{n+1}^{P_2}(t_{n+1})\right) + \left(I_{n+1}^{P_1}(t_{n+1}) + I_{n+1}^{P_2}(t_{n+1})\right)}{k}\right) \\
&- \left(\frac{\Omega_1^\beta \left(S_{n+1}^{P_1}(t_{n+1}) + S_{n+1}^{P_2}(t_{n+1})\right)\left(Y_{n+1}^{P_1}(t_{n+1}) + Y_{n+1}^{P_2}(t_{n+1})\right)}{a + \left(S_{n+1}^{P_1}(t_{n+1}) S_{n+1}^{P_2}(t_{n+1})\right)}\right), \\
I_{n+1}^{C} &= I_{n+1}^{P_1} + \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right) \\
&\times \left(\frac{\Omega_1^\beta \left(S_{n+1}^{P_1}(t_{n+1}) + S_{n+1}^{P_2}(t_{n+1})\right)\left(Y_{n+1}^{P_1}(t_{n+1}) + Y_{n+1}^{P_2}(t_{n+1})\right)}{a + \left(S_{n+1}^{P_1}(t_{n+1}) + S_{n+1}^{P_2}(t_{n+1})\right)} - d_1^\beta \left(I_{n+1}^{P_1}(t_{n+1}) + I_{n+1}^{P_2}(t_{n+1})\right)\right), \\
H_{n+1}^{C} &= H_{n+1}^{P_1} + \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right) \\
&\times \left(\alpha^\beta - \frac{\Omega_2^\beta \left(I_{n+1}^{P_2}(t_{n+1}) + I_{n+1}^{P_2}(t_{n+1})\right)\left(H_{n+1}^{P_1}(t_{n+1}) + H_{n+1}^{P_2}(t_{n+1})\right)}{c + \left(I_{n+1}^{P_1}(t_{n+1}) I_{n+1}^{P_2}(t_{n+1})\right)} - d_2^\beta \left(H_{n+1}^{P_1}(t_{n+1}) + H_{n+1}^{P_2}(t_{n+1})\right)\right), \\
Y_{n+1}^{C} &= Y_{n+1}^{P_1} + \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right) \\
&\times \left(\frac{b^\beta \Omega_2^\beta \left(I_{n+1}^{P_1}(t_{n+1}) + I_{n+1}^{P_2}(t_{n+1})\right)\left(H_{n+1}^{P_1}(t_{n+1}) + H_{n+1}^{P_2}(t_{n+1})\right)}{c + \left(I_{n+1}^{P_1}(t_{n+1}) + I_{n+1}^{P_2}(t_{n+1})\right)} - d_3^\beta \left(Y_{n+1}^{P_1}(t_{n+1}) + Y_{n+1}^{P_2}(t_{n+1})\right)\right).
\end{aligned} \tag{23}$$

4.1 *Error analysis*

From [37], the truncation error (T_n) is defined by

$$\begin{aligned}
T_n = \frac{K(t_{n+1}) - K(t_n)}{h} &- \left(\frac{\beta}{2M(\beta)} - \frac{1-\beta}{hM(\beta)}\right) X\left(t_n, K(t_n)\right) \\
&- \left(\frac{\beta}{2M(\beta)} + \frac{1-\beta}{hM(\beta)}\right) X\left(t_{n+1}, K^{P_1}(t_n + 1) + K^{P_2}(t_{n+1})\right).
\end{aligned} \tag{24}$$

From corrector equation,

$$0 = \frac{K_{n+1} - K_n}{h} - \left(\frac{\beta}{2M(\beta)} - \frac{1-\beta}{hM(\beta)} \right) X(t_n, K_n)$$
$$- \left(\frac{\beta}{2M(\beta)} + \frac{1-\beta}{hM(\beta)} \right) X\left(t_{n+1}, K_{n+1}^{P_1} + K_{n+1}^{P_2}\right). \tag{25}$$

From the global error's definition: $e_n = K(t_n) - K_n$ and subtracting equation (25) from equation (24), we have

$$e_{n+1} = e_n + \left(\frac{h\beta}{2M(\beta)} \right) \left[X\left(t_n, K(t_{n+1})\right) - X(t_n, K_n) \right] + \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)} \right)$$
$$\left[X\left(t_{n+1}, K^{P_1}(t_{n+1}) K^{P_2}(t_{n+1})\right) - X\left(t_{n+1}, K_{n+1}^{P_2}\right) \right] + hT_n. \tag{26}$$

Theorem 1. *Consider that X in equation* (7) *satisfies the Lipschitz constraint i.e., there exists a Lipschitz constant $L > 0$ follows $\left|X(t,x) - X(t,y)\right| \le L|x-y|$ The term K_{n+1}^C is the corrector of IVP* (7) (8)*. If $K(t_{n+1})$ and K_{n+1} denote the exact and numerical solutions at t_{n+1}, respectively, then the global error e_{n+1} is defined by*

$$\left|e_{n+1}\right| \le \frac{e^{T\beta L} - 1}{L} \tilde{T}. \tag{27}$$

Proof: From equation (26)*, we have*

$$\left|e_{n+1}\right| \le \left|e_n\right| + \left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)} \right) \left| X\left(t_n, K(t_n)\right) - X(t_n, K_n) \right| + \left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)} \right)$$
$$\left| X\left(t_{n+1}, K^{P_1}(t_{n+1}) + K^{P_2}(t_{n+1})\right) - X\left(t_{n+1}, K_{n+1}^{P_1} + K_{n+1}^{P_2}\right) \right| + h\left|T_n\right|$$
$$\le \left|e_n\right| + L\left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)} \right)\left|e_n\right| + L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)} \right)\left| K^{P_1}(t_{n+1}) - K_{n+1}^{P_1} \right|$$
$$+ L\left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)} \right)\left| K^{P_2}(t_{n+1}) - K_{n+1}^{P_2} \right| + h\left|T_n\right|. \tag{28}$$

Using (21) *and* (22)*, we get*

$$\left| K^{P_1}(t_{n+1}) - K_{n+1}^{P_1} \right| \le \left|e_n\right| + L\left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)} \right)\left|e_n\right|,$$

and

$$\left| K^{P_2}(t_{n+1}) - K_{n+1}^{P_2} \right| \le L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)} \right)\left|e_n\right| + L^2\left(\frac{h^2\beta^2}{4\left(M(\beta)\right)^2} - \frac{(1-\beta)^2}{\left(M(\beta)\right)^2} \right)\left|e_n\right|.$$

Using above inequalities in (28) *and resolving it, we arrive at*

$$
\begin{aligned}
&|e_{n+1}| \\
&\le |e_n| + L\left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)}\right)|e_n| + L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left\{|e_n| + L\left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)}\right)|e_n|\right\} \\
&+L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left\{L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)|e_n| + L^2\left(\frac{h^2\beta^2}{4(M(\beta))^2} - \frac{(1-\beta)^2}{(M(\beta))^2}\right)|e_n|\right\} + h|T_n| \\
&\le \left\{1 + \frac{Lh\beta}{M(\beta)} + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2(M(\beta))^2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8(M(\beta))^3}\right)\right\}|e_n| + h|T_n| \\
&\le \left\{1 + \frac{Lh\beta}{M(\beta)} + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2(M(\beta))^2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8(M(\beta))^3}\right)\right\}|e_n| + h\tilde{T}, \qquad (29)
\end{aligned}
$$

where $r = 0, \cdots, n$ *and* $\tilde{T} = \max_{0<r<n}|T_r|$. *Here we see that global error* e_{n+1} *is bounded by the truncation error* $\tilde{T}$ *for* $M(\beta) = 1$.

$$
\begin{aligned}
|e_{n+1}| &\le \left\{1 + Lh\beta + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8}\right)\right\}|e_n| + h\tilde{T} \\
&\le \left\{1 + Lh\beta + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8}\right)\right\}^{n+1}|e_0| \\
&\quad + \left(\frac{\left\{1 + Lh\beta + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8}\right)\right\}^n - 1}{\left\{1 + Lh\beta + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8}\right)\right\} - 1}\right)h\tilde{T}. \qquad (30)
\end{aligned}
$$

Taking $|e_0| = 0$, *from* (30), *we have*

$$
\begin{aligned}
|e_{n+1}| &\le \frac{\left\{1 + Lh\beta + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8}\right)\right\}^n - 1}{\left\{1 + Lh\beta + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8}\right)\right\} - 1}h\tilde{T} \\
&\le \frac{e^{nh\beta L} - 1}{L}\tilde{T}. \qquad (31)
\end{aligned}
$$

By taking $nh = T$ *in* (31), *we get the proof.*

4.2 *Stability analysis*

Consider the IVP with perturbed initial condition

$$
\begin{aligned}
&{}_0^{CF}D_t^{\beta}\left(\tilde{K}(t)\right) = X\left(t, \tilde{K}(t)\right), t \in [0, T], \\
&\tilde{K}(0) = \tilde{K}_0,
\end{aligned}
$$

where $0 < \beta < 1$.

The perturbed solution achieved from the scheme is given by

$$\tilde{K}_n^{P_1} = U + \tilde{K}_{n-1} + \left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)}\right) X\left(t_{n-1}, \tilde{K}_{n-1}\right),$$
$$\tilde{K}_n^{P_2} = \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right) X\left(t_n, \tilde{K}_n^{P_1}\right)$$
$$\tilde{K}_n^{C} = \tilde{K}_n^{P_1} + \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right) X\left(t_n, \tilde{K}_n^{P_1} + \tilde{K}_n^{P_2}\right).$$

Theorem 2. *Consider the solution K_n derived from the algorithm* (15)-(23) *at the stage t_n with initial condition $K(0) = K_0$. Let $\tilde{K}_n$ be the solution investigated from the same algorithm* (15)-(23) *with changed initial condition $\tilde{K}_0 = K_0 + \delta$. If X satisfy the Lipschitz condition with Lipschitz constant $L > 0$, then there exist two positive constants h' and k such that*

$$\left|K_n - \tilde{K}_n\right| \le \delta k \forall hn < T, h \in (0, h'), whenever \left|\delta_0\right| \le \left|\delta\right|.$$

Proof: At the n^{th} step

$$\begin{aligned}
&\left|K_n - \tilde{K}_n\right| \\
&= \left|K_n^{P_1} + \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right) X\left(t_n, K_n^{P_1} + K_n^{P_2}\right) - \tilde{K}_n^{P_1} - \left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right) X\left(t_n, \tilde{K}_n^{P_1} + \tilde{K}_n^{P_2}\right)\right| \\
&\le \left|K_n^{P_1} - \tilde{K}_n^{P_1}\right| + L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left|K_n^{P_1} - \tilde{K}_n^{P_1}\right| + L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left|K_n^{P_2} - \tilde{K}_n^{P_2}\right| \\
&= \left[1 + L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\right]\left|K_n^{P_1} - K_n^{P_1}\right| + L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left|K_n^{P_2} - \tilde{K}_n^{P_2}\right| \\
&\le \left[1 + L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\right]\left[\left|K_{n-1} - \tilde{K}_{n-1}\right| + L\left(\frac{h\beta}{2M(\beta)} - \frac{1-\beta}{M(\beta)}\right)\left|K_{n-1} - \tilde{K}_{n-1}\right|\right] \\
&\qquad + L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left[L\left(\frac{h\beta}{2M(\beta)} + \frac{1-\beta}{M(\beta)}\right)\left|K_{n-1} - \tilde{K}_{n-1}\right|\right. \\
&\qquad\qquad \left. + L^2\left(\frac{h^2\beta^2}{4\left(M(\beta)\right)^2} - \frac{(1-\beta)^2}{\left(M(\beta)\right)^2}\right)\left|K_{n-1} - \tilde{K}_{n-1}\right|\right] \\
&\le \left\{1 + \frac{Lh\beta}{M(\beta)} + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2\left(M(\beta)\right)^2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8\left(M(\beta)\right)^3}\right)\right\}\left|K_{n-1} - \tilde{K}_{n-1}\right|.
\end{aligned}$$

Taking $M(\beta) = 1$ in above inequality, we get

$$\left|K_n - \tilde{K}_n\right| \le \left\{1 + Lh\beta + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8}\right)\right\}\left|K_{n-1} - \tilde{K}_{n-1}\right|.$$

Using above given inequality, we have

$$\begin{aligned}
\left|K_n - \tilde{K}_n\right| &\le \left\{1 + Lh\beta + L^2\left(\frac{h^2\beta^2 + 2h\beta}{2}\right) + L^3\left(\frac{h^3\beta^3 + 2h^2\beta^2}{8}\right)\right\}^n \left|K_0 - \tilde{K}_0\right| \\
&\le e^{Lnh\beta}\delta_0 = e^{LT\beta}\delta_0 = k\delta_0,
\end{aligned}$$

which proves the stability of the method.

5. Graphical observations

Here we plot the numerical solution of the proposed model derived from the algorithm (15)-(23). The simulation is performed in *Mathematica*. The initial values are taken as $S(0) = 1{,}000$, $I(0) = 20$, $H(0) = 100$, and $Y(0) = 0$. In Figure 1, the model classes are plotted at fractional orders $\beta = 1, 0.95, 0.90, 0.80$. Here we notice from Figure 1b that the population of infected maize decreases when the fractional order decreases. Some two-dimensional (2D) plots between the model compartments are given in Figure 2.

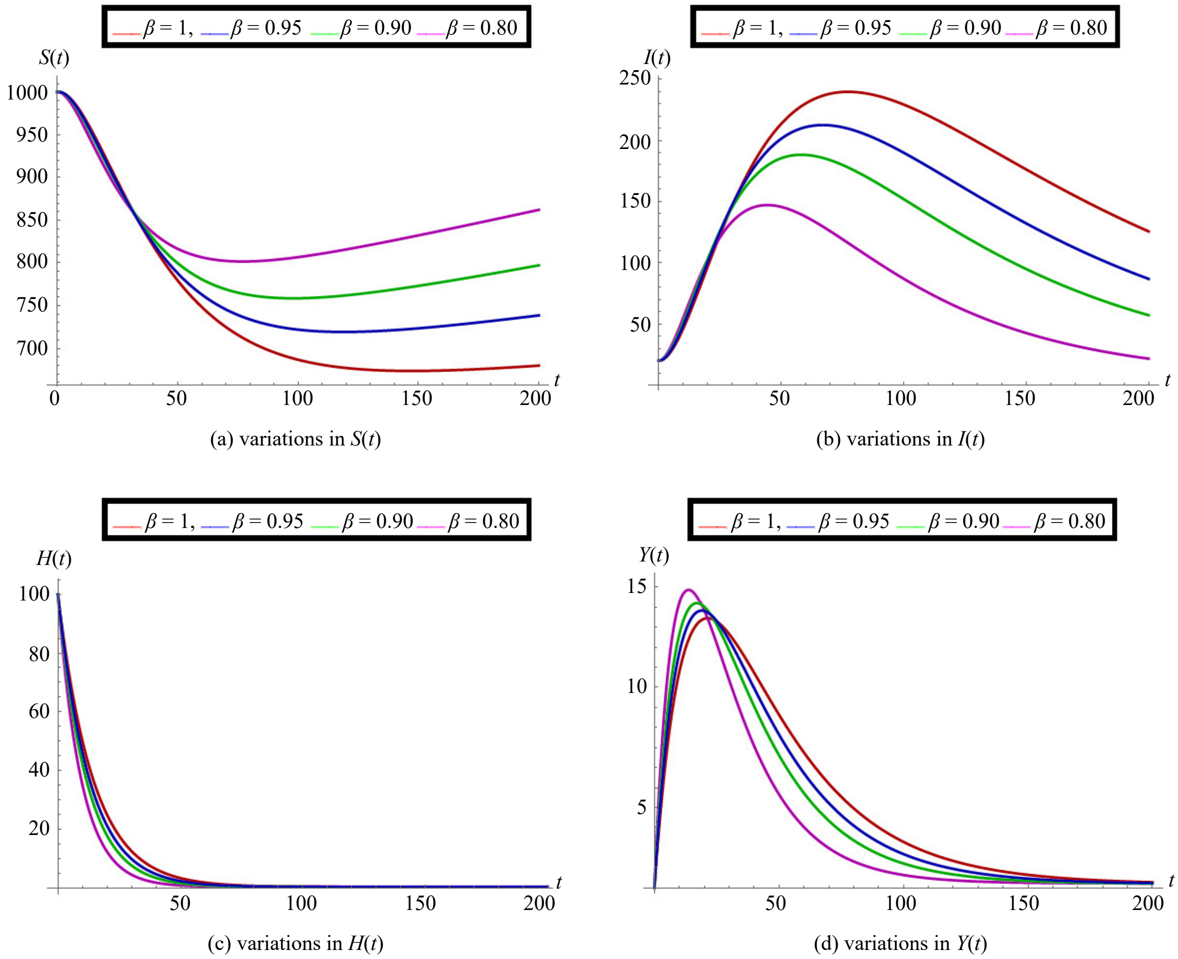
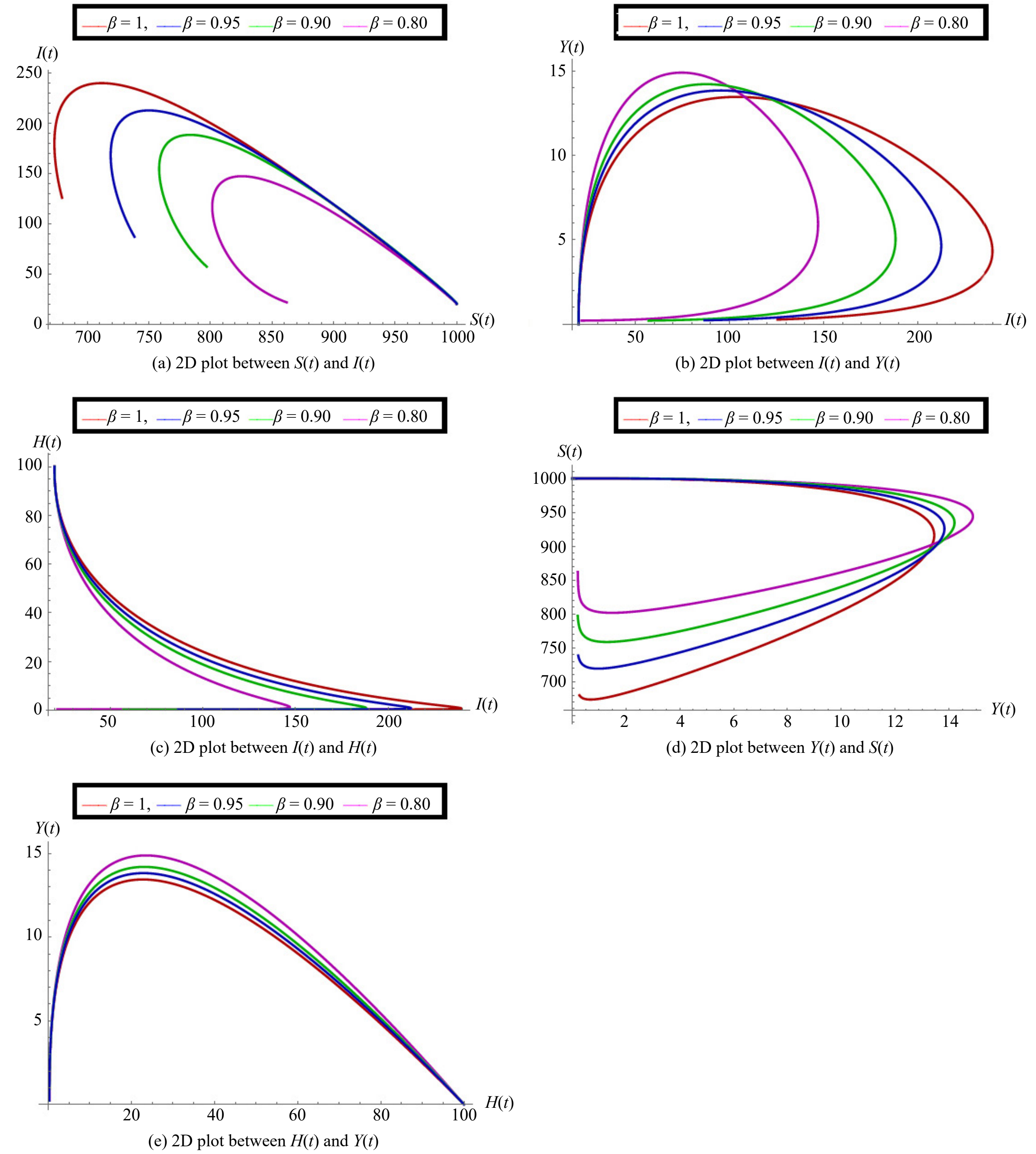


Figure 1. Plots of the compartments at $\beta = 1, 0.95, 0.90, 0.80$



(a) 2D plot between $S(t)$ and $I(t)$

(b) 2D plot between $I(t)$ and $Y(t)$

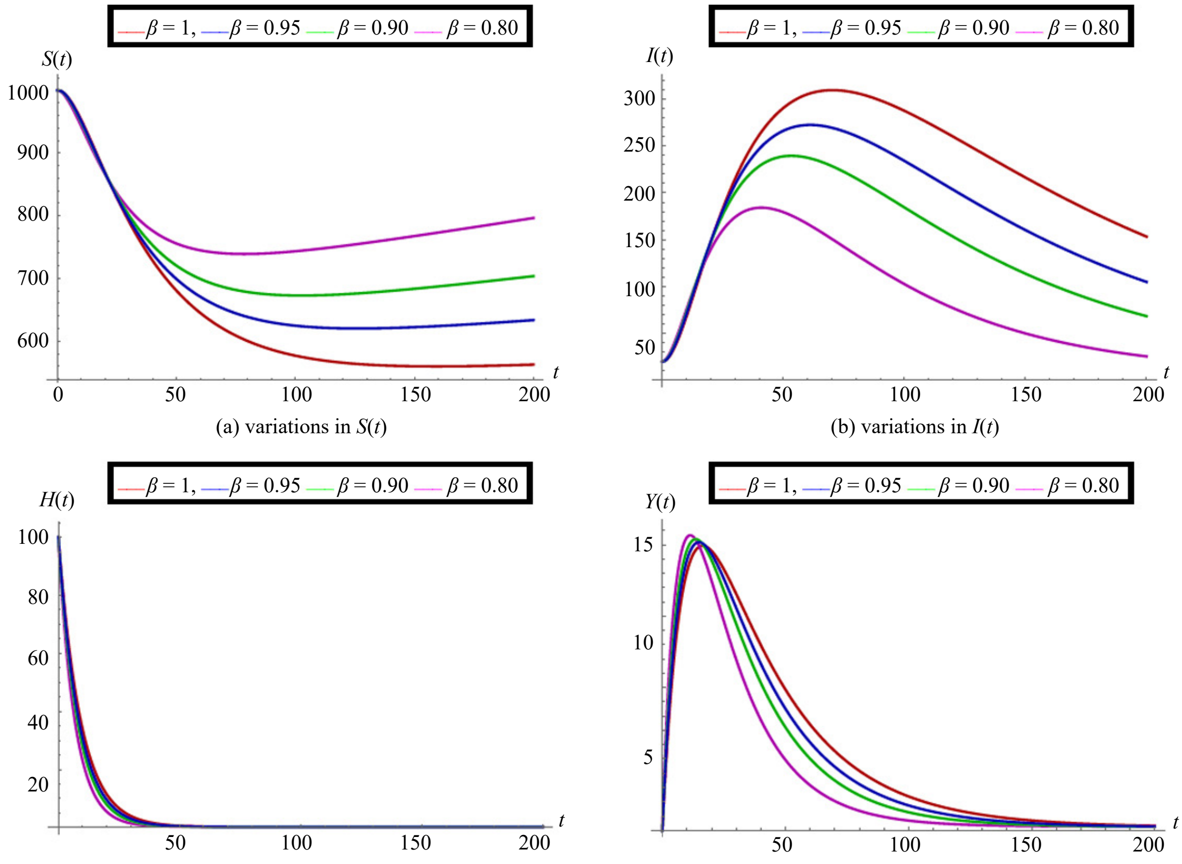
(c) 2D plot between $I(t)$ and $H(t)$

(d) 2D plot between $Y(t)$ and $S(t)$

(e) 2D plot between $H(t)$ and $Y(t)$

Figure 2. Comparative changes in the compartments at $\beta = 1, 0.95, 0.90, 0.80$

Figure 3 is plotted by taking $\Omega_1 = 0.85$ to explore the influence of the rate of infection and predation of infected leafhopper. The variations in the class $I(t)$ can be seen in Figure 3b compared to the previous case (Figure 1b).



(a) variations in $S(t)$ (b) variations in $I(t)$

(c) variations in $H(t)$ (d) variations in $Y(t)$

Figure 3. Plots of the compartments at β = 1, 0.95, 0.90, 0.80 for Ω_1 = 0.85

Figure 4 is plotted by taking Ω_2 = 0.08 to explore the influence of the rate of infection and predation of susceptible leafhopper.

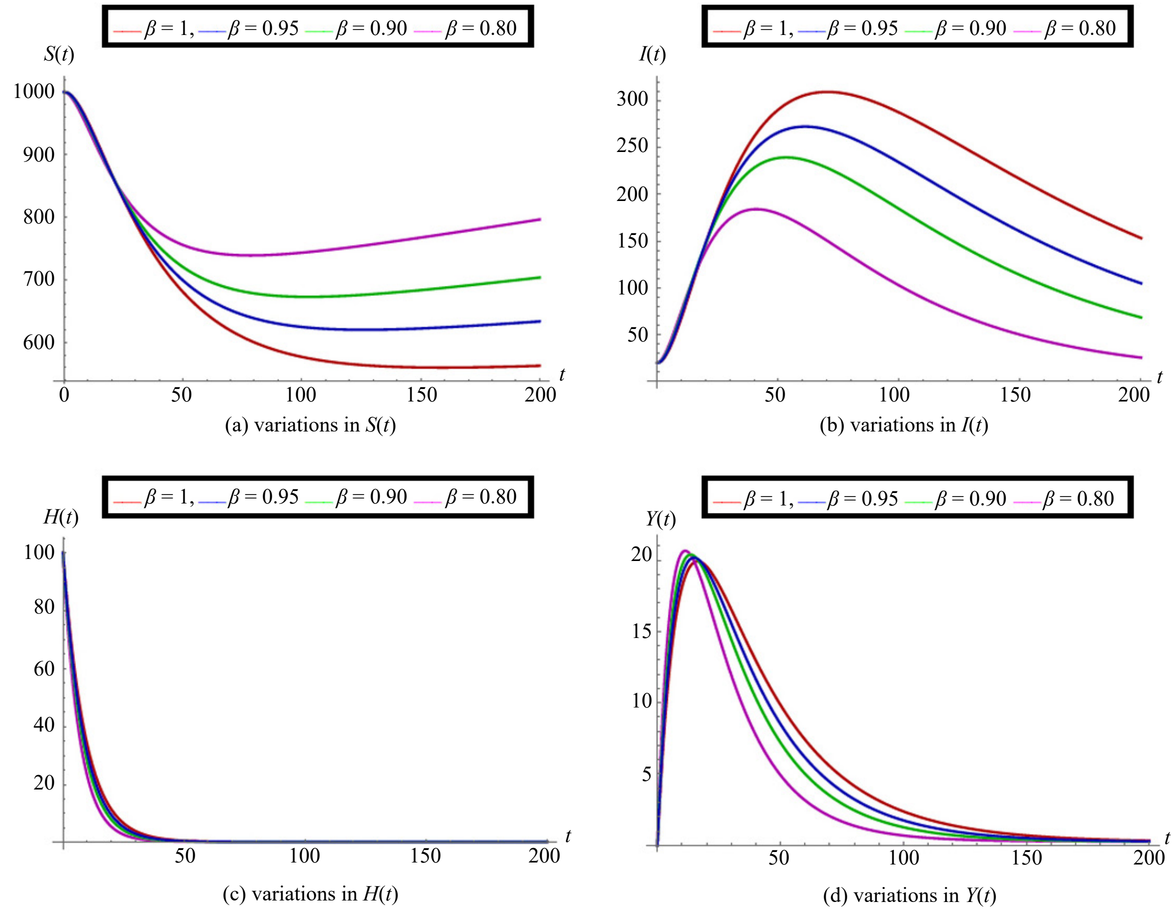
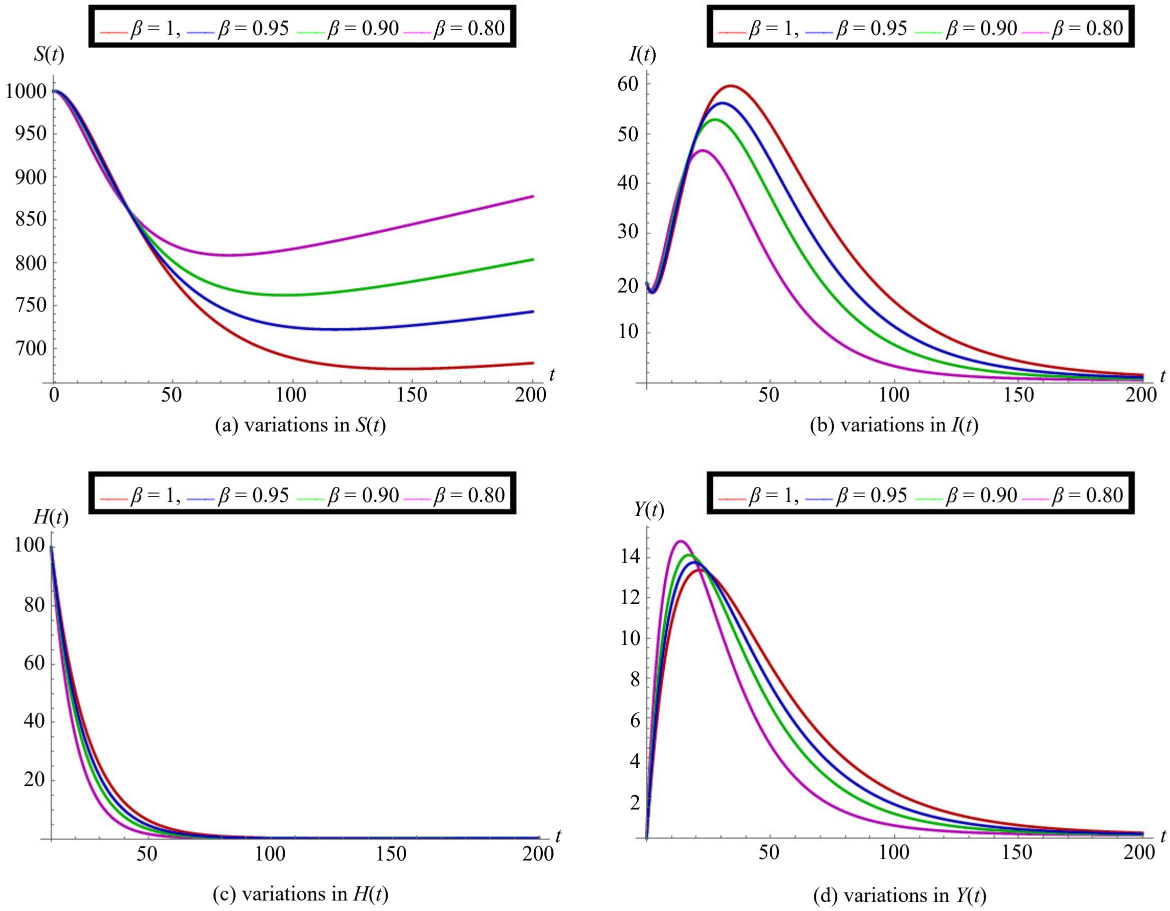


Figure 4. Plots of the compartments at $\beta = 1, 0.95, 0.90, 0.80$ for $\Omega_2 = 0.08$

Figure 5 is plotted by taking $d_1 = 0.09$ to verify the influence of mortality rate of infectious maize population. In Figure 6, all classes are plotted together at particular fractional order values.



(a) variations in $S(t)$ (b) variations in $I(t)$

(c) variations in $H(t)$ (d) variations in $Y(t)$

Figure 5. Plots of the compartments at $\beta = 1, 0.95, 0.90, 0.80$ for $d_1 = 0.09$

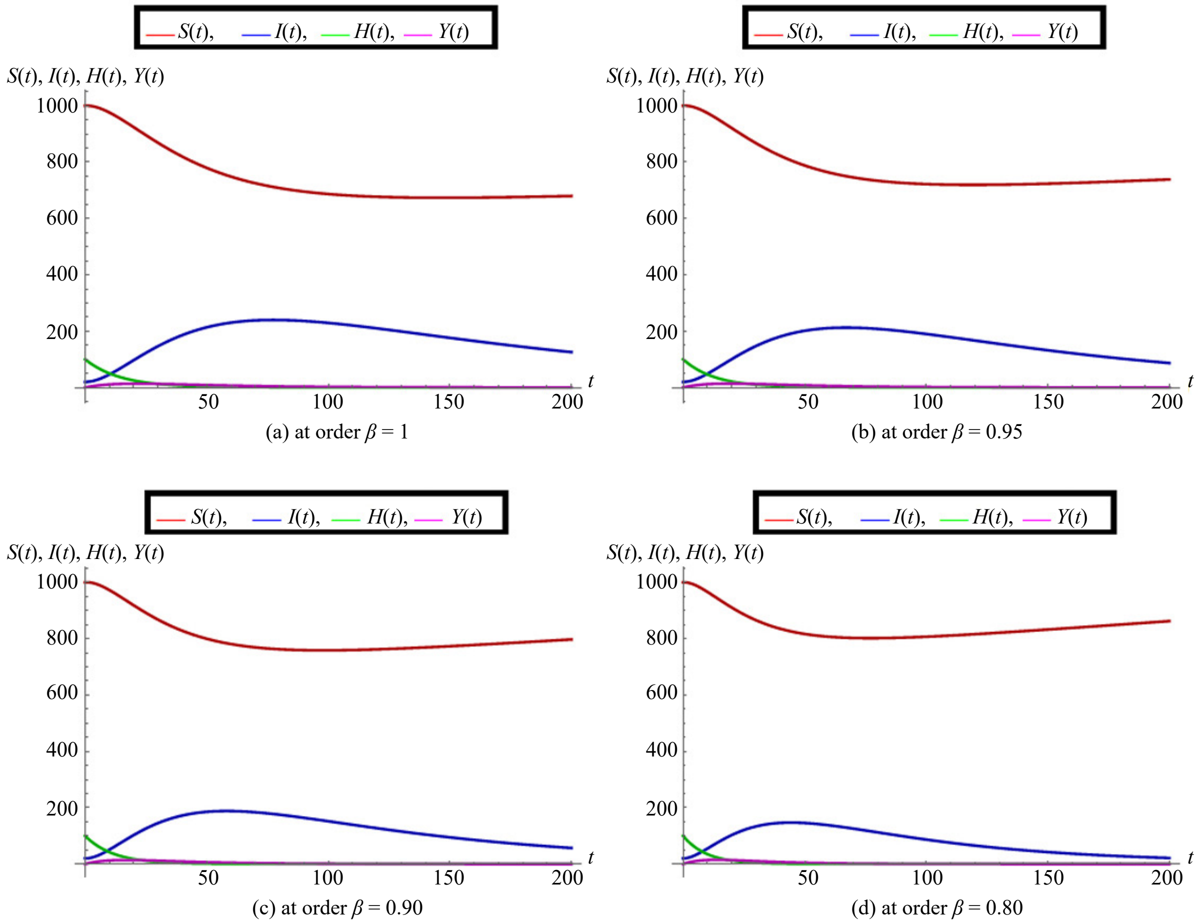


Figure 6. Plots of the model classes at random values of β

From the numerical and graphical simulations, we justify that the Caputo-Fabrizio fractional derivative worked well to analyze the given disease dynamics. Also, the given numerical scheme is fully suitable to solve such types of nonlinear models.

6. Conclusion

In this article, we have considered a nonlinear mathematical model of MSV disease using the Caputo-Fabrizio fractional derivative. The numerical solution of the proposed model has been derived using a recently published numerical method. Several graphs have been plotted to justify the solution obtained. In this study, we have justified the applicability of the given scheme for solving epidemic models. In the future, the given scheme can be utilized to simulate other types of problems. Moreover, the given MSV model can be proposed in the sense of any other derivative.

Availability of data and materials

All the data is included in the manuscript.

Competing interests

The authors declare that they have no competing interests.

References

[1] Mesfin T, Den Hollander J, Markham PG. Cicadulina species and maize streak virus in Ethiopia. *International Journal of Pest Management*. 1991; 37(3): 240-244. Available from: https://doi.org/10.1080/09670879109371592.

[2] Bosque-Prez NA. Eight decades of maize streak virus research. *Virus Research*. 2000; 71(1-2): 107-121. Available from: https://doi.org/10.1016/S0168-1702(00)00192-1.

[3] Alemneh HT, Kassa AS, Godana AA. An optimal control model with cost effectiveness analysis of Maize streak virus disease in maize plant. *Infectious Disease Modelling*. 2021; 6: 169-182. Available from: https://doi.org/10.1016/j.idm.2020.12.001.

[4] Alemneh HT, Makinde OD, Theuri DM. Optimal control model and cost effectiveness analysis of maize streak virus pathogen interaction with pest invasion in maize plant. *Egyptian Journal of Basic and Applied Sciences.* 2020; 7(1): 180-193. Available from: https://doi.org/10.1080/2314808X.2020.1769303.

[5] Alemneh HT, Makinde OD, Theuri DM. Ecoepidemiological model and analysis of MSV disease transmission dynamics in maize plant. *International Journal of Mathematics and Mathematical Sciences*. 2019; 7965232. Available from: https://doi.org/10.1155/2019/7965232.

[6] Alemneh, HT, Makinde OD, Theuri DM. Mathematical modelling of MSV pathogen interaction with pest invasion on maize plant. *Global Journal of Pure and Applied Mathematics*. 2019; 15(1): 55-79. Available from: https://www.ripublication.com/gjpam19/gjpamv15n1_06.pdf.

[7] Kumar P, Erturk VS, Vellappandi M, Trinh H, Govindaraj V. A study on the maize streak virus epidemic model by using optimized linearization-based predictor-corrector method in Caputo sense. *Chaos, Solitons & Fractals*. 2022; 158: 112067. Available from: https://doi.org/10.1016/j.chaos.2022.112067.

[8] Kilbas AA, Srivastava HM, Trujillo JJ. *Theory and applications of fractional differential equations*. Elsevier Science; 2006.

[9] Angstmann CN, Jacobs BA, Henry BI, Xu Z. Intrinsic discontinuities in solutions of evolution equations involving fractional Caputo-Fabrizio and Atangana-Baleanu operators. *Mathematics.* 2020; 8(11): 2023. Available from: https://doi.org/10.3390/math8112023.

[10] Diethelm K. *The analysis of fractional differential equations: An application-oriented exposition using differential operators of Caputo typ*e. Heidelberg: Springer; 2010. Available from: https://doi.org/10.1007/978-3-642-14574-2.

[11] Podlubny I. *Fractional differential equations: An introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications*. San Diego: Academic Press; 1998.

[12] Erturk VS, Godwe E, Baleanu D, Kumar P, Asad J, Jajarmi A. Novel fractional-order Lagrangian to describe motion of beam on nanowire. *Acta Physica Polonica A*. 2021; 140(3): 265-272. Available from: https://doi.org/10.12693/APhysPolA.140.265.

[13] Din A, Khan FM, Khan ZU, Yusuf A, Munir T. The mathematical study of climate change model under nonlocal fractional derivative. *Partial Differential Equations in Applied Mathematics*. 2022; 5: 100204. Available from: https://doi.org/10.1016/j.padiff.2021.100204.

[14] Viera-Martin E, Gómez-Aguilar JF, Solis-Pérez JE, Hernández-Pérez JA, Escobar-Jiménez RF. Artificial neural networks: A practical review of applications involving fractional calculus. *The European Physical Journal Special Topics.* 2022; 231(10): 2059-2095. Available from: https://doi.org/10.1140/epjs/s11734-022-00455-3.

[15] Erturk VS, Ahmadkhanlu A, Kumar P, Govindaraj V. Some novel mathematical analysis on a corneal shape model by using Caputo fractional derivative. *Optik*. 2022; 261: 169086. Available from: https://doi.org/10.1016/j.ijleo.2022.169086.

[16] Erturk VS, Alomari AK, Kumar P, Murillo-Arcila M. Analytic solution for the strongly nonlinear multi-order fractional version of a BVP occurring in chemical reactor theory. *Discrete Dynamics in Nature and Society*. 2022; 8655340. Available from: https://doi.org/10.1155/2022/8655340.

[17] Yang Q, Chen D, Zhao T, Chen Y. Fractional calculus in image processing: A review. *Fractional Calculus and Applied Analysis*. 2016; 19(5): 1222-1249. Available from: https://doi.org/10.1515/fca-2016-0063.

[18] Kumar P, Govindaraj V, Erturk VS, Abdellattif MH. A study on the dynamics of alkali-silica chemical reaction by using Caputo fractional derivative. *Pramana.* 2022; 96(3): 128. Available from: https://doi.org/10.1007/s12043-

022-02359-2.
[19] Vellappandi M, Kumar P, Govindaraj V. Role of fractional derivatives in the mathematical modeling of the transmission of Chlamydia in the United States from 1989 to 2019. *Nonlinear Dynamics*. 2022; 111: 4915-4929. Available from: https://doi.org/10.1007/s11071-022-08073-3.
[20] Naik PA, Yavuz M, Qureshi S, Zu J, Townley S. Modeling and analysis of COVID-19 epidemics with treatment in fractional derivatives using real data from Pakistan. *The European Physical Journal Plus*. 2020; 135(10): 795. Available from: https://doi.org/10.1140/epjp/s13360-020-00819-5.
[21] Kumar P, Erturk VS. Environmental persistence infuences infection dynamics for a butterfly pathogen via new generalised Caputo type fractional derivative. *Chaos, Solitons & Fractals*. 2021; 144: 110672. Available from: https://doi.org/10.1016/j.chaos.2021.110672.
[22] Khan MA, Ullah S, Farooq M. A new fractional model for tuberculosis with relapse via Atangana-Baleanu derivative. *Chaos, Solitons & Fractals.* 2018; 116: 227-238. Available from: https://doi.org/10.1016/j.chaos.2018.09.039.
[23] Gómez-Aguilar JF, Córdova-Fraga T, Abdeljawad T, Khan A, Khan H. Analysis of fractal-fractional malaria transmission model. *Fractals.* 2020; 28(8): 2040041. Available from: https://doi.org/10.1142/S0218348X20400411.
[24] Atangana A. A novel model for the lassa hemorrhagic fever: Deathly disease for pregnant women. *Neural Computing and Applications*. 2015; 26(8): 1895-1903. Available from: https://doi.org/10.1007/s00521-015-1860-9.
[25] Baishya C, Achar SJ, Veeresha P, Kumar D. Dynamical analysis of fractional yellow fever virus model with efficient numerical approach. *Journal of Computational Analysis & Applications*. 2023; 31(1): 140-157.
[26] Gao W, Veeresha P, Cattani C, Baishya C, Baskonus HM. Modified predictor-corrector method for the numerical solution of a fractional-order SIR model with 2019-nCoV. *Fractal and Fractional*. 2022; 6(2): 92. Available from: https://doi.org/10.3390/fractalfract6020092.
[27] Veeresha P. Analysis of the spread of infectious diseases with the effects of consciousness programs by media using three fractional operators. In: Singh H, Srivastava HM, Baleanu D. (eds.) *Methods of mathematical modelling.* Academic Press; 2022. p.113-135. Available from: https://doi.org/10.1016/B978-0-323-99888-8.00007-3.
[28] Kumar P, Erturk VS, Yusuf A, Nisar KS, Abdelwahab SF. A study on canine distemper virus (CDV) and rabies epidemics in the red fox population via fractional derivatives. *Results in Physics*. 2021; 25: 104281. Available from: https://doi.org/10.1016/j.rinp.2021.104281.
[29] Kumar P, Erturk VS, Nisar KS. Fractional dynamics of huanglongbing transmission within a citrus tree. *Mathematical Methods in the Applied Sciences.* 2021; 44(14): 11404-11424. Available from: https://doi.org/10.1002/mma.7499.
[30] Kumar P, Erturk VS, Almusawa H. Mathematical structure of mosaic disease using microbial biostimulants via Caputo and Atangana-Baleanu derivatives. *Results in Physics.* 2021; 24: 104186. Available from: https://doi.org/10.1016/j.rinp.2021.104186.
[31] Kumar P, Erturk VS, Murillo-Arcila M. A complex fractional mathematical modeling for the love story of Layla and Majnun. *Chaos, Solitons & Fractals.* 2021; 150: 111091. Available from: https://doi.org/10.1016/j.chaos.2021.111091.
[32] Baleanu D, Sajjadi SS, Jajarmi A, Defterli Ö. On a nonlinear dynamical system with both chaotic and nonchaotic behaviors: a new fractional analysis and control. *Advances in Difference Equations*. 2021; 2021(1): 234. Available from: https://doi.org/10.1186/s13662-021-03393-x.
[33] Baleanu D, Zibaei S, Namjoo M, Jajarmi A. A nonstandard finite difference scheme for the modeling and nonidentical synchronization of a novel fractional chaotic system. *Advances in Difference Equations*. 2021; 2021(1): 308. Available from: https://doi.org/10.1186/s13662-021-03454-1.
[34] Jajarmi A, Baleanu D, Vahid KZ, Pirouz HM, Asad JH. A new and general fractional Lagrangian approach: A capacitor microphone case study. *Results in Physics*. 2021; 31: 104950. Available from: https://doi.org/10.1016/j.rinp.2021.104950.
[35] Kumar P, Govindaraj V, Erturk VS, Nisar KS, Inc M. Fractional mathematical modeling of the Stuxnet virus along with an optimal control problem. *Ain Shams Engineering Journal.* 2022; 14(7): 102004. Available from: https://doi.org/10.1016/j.asej.2022.102004
[36] Veeresha P, Ilhan E, Baskonus HM. Fractional approach for analysis of the model describing wind-influenced projectile motion. *Physica Scripta.* 2021; 96(7): 075209. Available from: https://doi.org/10.1088/1402-4896/abf868.
[37] Mahatekar Y, Scindia PS, Kumar P. A new numerical method to solve fractional differential equations in terms of Caputo-Fabrizio derivatives. *Physica Scripta*. 2023; 98(2): 024001. Available from: https://doi.org/10.1088/1402-4896/acaf1a.

[38] Caputo M, Fabrizio M. A new definition of fractional derivative without singular kernel. *Progress in Fractional Differentiation and Applications*. 2015; 1(2): 73-85.