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Research Article

On Some New Approach of Paranormed Spaces

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Abstract: The object of this paper is to bring out the space $r^{\mathfrak{q}}_{\mathfrak{B}}(g, p)$ of non-absolute patterns. Also, we will structure its completeness property. Also, the Köthe duals will be determined. Moreover, the Schauder basis for it will be constructed.

***Keywords*:** infinite matrices, completeness property, basis

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1. Introduction

By $\Omega = \mathbf{C}^{\mathbf{N}}$, we indicate the set of every sequence for **C** to symbolize the complex field and $\mathbf{N} = \{0, 1, 2, \cdots\}$. We say each linear subspace of **Ω** is called a sequence space. We symbolize the bounded sequences by l_∞ and by $l(p)$ as p-absolutely convergent series.

Consider Y to be any linear space, define $\mathfrak{G}: Y \to \mathbb{R}$, and call it a paranorm for Y, holding the following axioms:

(i) $\mathfrak{G}(\Theta) = 0$,

(ii) $\mathfrak{G}(-\mho) = \mathfrak{G}(\mho)$,

(iii) $\mathfrak{G}(\mho + \zeta) \leq \mathfrak{G}(\mho) + \mathfrak{G}(\zeta)$, and

(iv) for $|c_n - c| \to 0$ and $\mathfrak{G}(\mho_n - \mho) \to 0$, imply $\mathfrak{G}(c_n \mho_n - c\mho) \to 0$ for each c's in $\mathbb{R}$ and $\mho$'s in Y, where zero vector Θ belongs to Y. Choose (p_k) as a bounded and positive number sequence having $\sup_k p_k = \mathcal{H}$ and $\mathcal{M} = \max\{1, \mathcal{H}\}$. So, as in [1] the space $l(p)$ is given by:

$$l(p) = \{\varsigma = (\varsigma_k) : \sum_k |\varsigma_k|^{pk} < \infty\}$$

and is completely paranormed with

$$\Psi_1(\mho) = \left[\sum_k |\mho_k|^{pk}\right]^{1/M},$$

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for $\frac{1}{p_k}+\frac{1}{\{p'_k\}}=1$ if $1<\inf p_k\leq\mathcal{H}<\infty$.

Consider infinite matrix $(\mathcal{W}=(w_{i,j}))$ and sequence $(\nu=(\nu_k)\in\Omega)$, then for every $(i\in\mathbb{N})$, the $\mathcal{W}$-transform $(\mathcal{W}\nu=\{(\mathcal{W}\nu)_i\})$ exists with

$$((\mathcal{W}\nu)_i=\sum_{j=0}^{\infty}w_{i,j}\nu_j).$$

As in [2], the matrix domain of $\mathcal{W}$ in $\mathcal{G}$ is

$$\mathcal{G}_{\mathcal{W}}=\left\{\nu=(\nu_j)\in\Omega:\mathcal{W}\nu\in\mathcal{G}\right\}. \tag{1}$$

For $t\in\mathbb{N}$, choose sequence of positive numbers $(\mathfrak{Q}_j)$ with $\mathcal{Q}_t=\sum_{j=0}^{t}\mathfrak{Q}_j$. Then, the matrix $R^{\mathfrak{Q}}=(r_{tj}^{\mathfrak{Q}})$ is given by

$$r_{tj}^{\mathfrak{Q}}=\begin{cases}\frac{\mathfrak{Q}_j}{\mathcal{Q}_t}, & \text{if } 0\leq j\leq t,\\ 0, & \text{if } j>t.\end{cases}$$

In [3], the author has given new techniques and introduced the spaces $U(\vartriangle)$ as follows:

$$U(\Delta)=\{\mho=(\mho_j)\in\Omega:(\vartriangle\mho_j)\in U\}$$

for $U\in\{l_\infty,c,c_0\}$ and $\vartriangle\mho_j=\mho_j-\mho_{j-1}$.

It was further analysed in [4-7] and many others as cited. The authors in [8] introduced the space $r^{\mathfrak{Q}}(\vartriangle_g^p)$ as follows:

$$r^{\mathfrak{Q}}(\Delta_g^p)=\left\{\mho=(\mho_k)\in\Omega:\sum_k\left|\frac{1}{\mathcal{Q}_k}\sum_{j=0}^{k}g_j\mathfrak{Q}_j\vartriangle\mho_j\right|^{p_k}<\infty\right\},$$

where $(0<p_k\leq H<\infty)$ and $g=(g_j)$ is a sequence, such that $g_j\neq 0$ for all $j\in\mathbb{N}$.

For each $i,j\in\mathbb{N}$, the author in [9] defined matrix $\mathfrak{B}=(b_{mk})$ as:

$$b_{mj}=\begin{cases}r, & \text{for } j=m-i\\ s, & \text{for } j=m-1\\ 0, & \text{for } 0\leq j<m-1 \text{ or } j>m,\end{cases}$$

with $r,s\in\mathbb{R}-\{0\}$. By putting $r=1$, $2=-1$, matrix $\mathfrak{B}$ reaches to matrix $\vartriangle$.

To notion of getting a new way of introducing generalized spaces with some limiting approach were studied in [4, 9, 10] and many others.

2. The space $r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p)$

The approach of this portion is to define $r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p)$, and compute its various topological structures. By $R_g^q\mathfrak{B}$-transform of a sequence $\mho=(\mho_k)$, we imply that sequence $\eta=(\eta_k)$ is related as

$$\eta_k(q) = \frac{1}{\mathfrak{Q}_k}\left\{\sum_{j=0}^{k-1}(g_j\mathfrak{Q}_j.r + g_{j+1}\mathfrak{Q}_{j+1}.s)\mho_j + g_k\mathfrak{Q}_k.r.\mho_k\right\}, (k \in \mathbb{N}). \tag{2}$$

Following various authors as in [9-20], we introduce $r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p)$ as follows:

$$r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p) = \left\{\mho = (\mho_k) \in \Omega : \eta_k(q) \in l(p)\right\}.$$

In case $r = 1$ and $s = -1$, then the set $r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p)$ gets merged to $r^q(\Delta_g^p)$ [8] and for taking $g_n = g_k$ for all $n \in \mathbb{N}$ (k fixed), yielding the results as in [6]. Also, if $(g_k) = e = (1, 1, ...)$, $s = -1$ and $r = 1$, then $r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p)$ merges to $r^q(\Delta_g^p)$ studied by Başarir [9].

Utilizing notion of (1), we redefine it as

$$r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p) = \{l(p)\}_{R_g^q\mathfrak{B}}.$$

Now, we shall now begin the following theorem without proof, which is important in the text.

Theorem 2.1. For $\mathcal{M} = \max\{1, \mathcal{H}\}$ and $0 < p_k \le \mathcal{H} < \infty$, the set $r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p)$ is complete and is paranormed by $\mathfrak{G}_{\mathfrak{B}}$, where

$$\mathfrak{G}_{\mathfrak{B}}(\mho) = \left[\sum_k \left|\frac{1}{\mathcal{Q}_k}\left(\sum_{j=0}^{k-1}(g_j\mathfrak{Q}_j.r + g_{j+1}\mathfrak{Q}_{j+1}.s)\mho_j + \mathfrak{Q}_k g_k.r\mho_k\right)\right|^{p_k}\right]^{\frac{1}{\mathcal{M}}}.$$

Theorem 2.2. For $0 < p_k \le \mathcal{H} < \infty$, the set $r_B^{\mathfrak{Q}}(g,p)$ and $l(p)$ are linearly isomorphic.

Proof. Using the notion of (2), choose the map $T : r_B^{\mathfrak{Q}}(g,p) \to l(p)$ as $\mho \to \eta = T\mho$.

Nothing to prove about the linearity of T as is obvious. Also, $\mho = \Theta$ for $T\mho = \Theta$ showing T is injective. Suppose $\eta \in l(p)$, and choose $\mho = (\mho_k)$ as

$$\mho_k = \sum_{n=0}^{k-1}(-1)^{k-n}\left(\frac{s^{k-n-1}}{r^{k-n}g_{n+1}\mathfrak{Q}_{n+1}} + \frac{s^{k-n}}{r^{k-n+1}g_n\mathfrak{Q}_n}\right)\mathfrak{Q}_n\eta_n + \frac{\mathfrak{Q}_k\eta_k}{rg_k\mathfrak{Q}_k}.$$

Then,

$$\begin{aligned}\mathfrak{G}_B(\mho) &= \left[\sum_k \left|\frac{1}{\mathfrak{Q}_k}\sum_{j=0}^{k-1}(g_j\mathfrak{Q}_j r + g_{j+1}\mathfrak{Q}_{j+1}s)\mho_j + \frac{g_k\mathfrak{Q}_k r\mho_k}{\mathfrak{Q}_k}\right|^{p_k}\right]^{\frac{1}{\mathcal{M}}} \\ &= \left[\sum_k \left|\sum_{j=0}^{k}\delta_{kj}\eta_j\right|^{p_k}\right]^{\frac{1}{\mathcal{M}}} \\ &= \left[\sum_k |\eta_k|^{p_k}\right]^{\frac{1}{\mathcal{M}}} = \Psi_1(\eta) < \infty,\end{aligned}$$

for

$$\delta_{kj} = \begin{cases} 1, & \text{when } k \ne j, \\ 0, & \text{when } k = j. \end{cases}$$

Consequently, $\mho \in r_B^{\mathfrak{Q}}(g,p)$. Hence, T is surjective as well as preserves paranorm, yielding T as linear bijection, and the result follows.

3. Duals and basis of $r_{\mathfrak{B}}^{\mathfrak{Q}}(g,p)$

Here, the determination of basis and duals of $r_B^{\mathfrak{Q}}(g,p)$ will be given.

Definition 3.1. Köthe duals: For the spaces $\mathcal{K}$ and $\mathcal{L}$ define $(\Lambda(\mathcal{K},\mathcal{L}))$ as follows:

$$\Lambda(\mathcal{K},\mathcal{L}) = \left\{\upsilon = (\upsilon_j) \in \Omega : \upsilon x = (\upsilon_j x_j) \in \mathcal{L} \, \forall \, x = (x_j) \in \mathcal{K}\right\}.$$

Therefore, as in [1], the Köthe duals by above representation are defined as

$$K^{\alpha} = \Lambda(K,\ell_1), K^{\beta} = \Lambda(K,cs) \text{ and } K^{\gamma} = \Lambda(K,bs).$$

Theorem 3.1 (i): For each $k \in \mathbb{N}$ with $1 < p_k \le \mathcal{H} < \infty$, construct the sets $D_1(g,p)$ and $D_2(g,p)$ as:

$$D_1(g,p) = \bigcup_{B>1} \{a = (a_k) \in \Omega : \\ \sup_{K \in F} \sum_k \left| \sum_{n \in K} \left[\nabla_g(k,n) a_n \mathfrak{Q}_k + \frac{a_n}{g_n \mathfrak{Q}_n} \mathfrak{Q}_n \right] B^{-1} \right|^{p'_k} < \infty\}$$

and

$$D_2(g,p) = \bigcup_{B>1} \{a = (a_k) \in \Omega : \\ \sum_k \left| \left[\left(\frac{a_k}{r g_k \mathfrak{Q}_k} + \nabla_g(k,n) \sum_{i=k+1}^{n} a_i \right) \mathfrak{Q}_k \right] B^{-1} \right|^{p'_k} < \infty\},$$

where

$$\nabla_g(k,n) = (-1)^{n-k} \left(\frac{s^{n-k-1}}{r^{n-k} g_{k+1} \mathfrak{Q}_{k+1}} + \frac{s^{n-k}}{r^{n-k+1} g_k \mathfrak{Q}_k} \right).$$

Then,

$$\left[r_B^{\mathfrak{Q}}(g,p)\right]^{\alpha} = D_1(g,p), \left[r_B^{\mathfrak{Q}}(g,p)\right]^{\beta} = D_2(g,p) \cap cs = \left[r_B^{\mathfrak{Q}}(g,p)\right]^{\gamma}.$$

(ii): Let $0 < p_k \le 1$, for each $k \in \mathbb{N}$. Consider the sets $D_3(g,p)$ and $D_4(g,p)$ as given below:

$$D_3(g,p) = \{a = (a_k) \in \Omega : \\ \sup_{K \in F} \sup_k \left| \sum_{n \in K} \left[\nabla_g(k,n) a_n \mathfrak{Q}_k + \frac{a_n}{r g_n \mathfrak{Q}_n} \mathfrak{Q}_n \right] B^{-1} \right|^{p_k} < \infty\}$$

and

$$D_4(g,p) = \{a = (a_k) \in \Omega : \\ \sup_k \left\| \left[\left(\frac{a_k}{r g_k \mathfrak{Q}_k} + \nabla_g(n,k) \sum_{i=k+1}^{n} a_i \right) \mathfrak{Q}_k \right] \right\|^{p_k} < \infty\}.$$

Then, we have

$$\left[r_B^{\mathfrak{Q}}(g,p)\right]^{\alpha} = D_3(g,p), \left[r_B^{\mathfrak{Q}}(g,p)\right]^{\beta} = D_4(g,p) \cap cs = \left[r_B^{\mathfrak{Q}}(g,p)\right]^{\gamma}.$$

To establish Theorem 3.1, the following lemmas are needed.

Lemma 3.1 (see, [17]) **(i):** For an integer $B > 1$, if $1 < p_k \le H < \infty$, then $C \in (l(p):l_1)$ if

$$\sup_{K \in F} \sum_{k \in \mathbb{N}} \left| \sum_{n \in K} c_{nk} B^{-1} \right|^{p_k'} < \infty.$$

(ii): Let $0 < p_k \le 1$. Then, $C \in (l(p):l_\infty)$ if

$$\sup_{K \in F} \sup_{k \in \mathbb{N}} \left| \sum_{n \in K} c_{nk} \right|^{p_k} < \infty.$$

Lemma 3.2 (see, [21]) **(i):** For an integer $B > 1$, if $1 < p_k \le \mathcal{H} < \infty$. Then, $C \in (l(p):l_\infty)$ if

$$\sup_n \sum_k \left| c_{nk} B^{-1} \right|^{p_k'} < \infty. \tag{3}$$

(ii): For $0 < p_k \le 1$ with $k \in \mathbb{N}$, then $C \in (l(p):l_\infty)$ if

$$\sup_{n,k \in \mathbb{N}} | c_{nk} |^{p_k} < \infty. \tag{4}$$

Lemma 3.3 (see, [21]). For $0 < p_k \le H < \infty$ with each $k \in \mathbb{N}$, we have $C \in (l(p):c)$ if (3) and (4) hold along with

$$\lim_n c_{nk} = \beta_k. \tag{5}$$

Proof of Theorem 3.1. First choose $1 < pk \le H < \infty$ and define $a = (a_n) \in \Omega$, then (2) yields

$$\begin{aligned} a_n \mho_n &= \sum_{k=0}^{n-1} \nabla_g(k,n) a_n \mathcal{Q}_k \eta_k + \frac{a_n \mathcal{Q}_n \eta_n}{r.\mathfrak{Q}_n} g_k^{-1} \\ &= \sum_{k=0}^{n} c_{nk} \eta_k = (C\eta)_n, \end{aligned} \tag{6}$$

where $C = (c_{nk})$ is defined by

$$c_{nk} = \begin{cases} \nabla_g(k,n) a_n \mathcal{Q}_k, & \text{if } 0 \le k \le n-1, \\ \dfrac{a_n \mathcal{Q}_n}{r g_n \mathfrak{Q}_n}, & \text{if } k = n, \\ 0, & \text{if } k > n, \end{cases}$$

Clearly from (6) with Lemma 3.1, we deduce that $a\mho = (a_n \mho_n) \in l_1$ whenever $\mho = (\mho_n) \in r_B^{\mathfrak{Q}}(g,p)$ if $C\eta \in l_1$ whenever $\eta \in l(p)$, yielding $\left[r_B^{\mathfrak{Q}}(g,p)\right]^{\alpha} = D_1(g,p)$.

Now for $n \in \mathbb{N}$, consider

$$\sum_{k=0}^{n} a_n \mho_n = \sum_{k=0}^{n}\left(\frac{a_k}{r.\mathfrak{Q}_k} g_k^{-1} + \nabla_g(k,n)\sum_{i=k+1}^{n} a_i\right)\mathcal{Q}_k \eta_k = (D\eta)_n, \tag{7}$$

with $D = (d_{nk}),$ given by

$$d_{nk} = \begin{cases} \left(\dfrac{a_k}{rg_k \mathfrak{Q}_k} + \nabla_g(k,n)\displaystyle\sum_{i=k+1}^{n} a_i\right)\mathcal{Q}_k, & \text{if } 0 \le k \le n, \\ 0, & \text{if } k > n, \end{cases}$$

Now, from (7) and Lemma 3.2 yields that $a\mho = (a_n \mho_n) \in cs$ whenever $\mho = (\mho_n) \in r_B^{\mathfrak{Q}}(g,p)$ if $D\eta \in c$ for $\eta \in l(p)$. Consequently, from (7), we have

$$\sum_k \left|\left[\left(\frac{a_k}{rg_k \mathfrak{Q}_k} + \nabla_g(n,k)\sum_{i=k+1}^{n} a_i\right)\mathcal{Q}_k\right] B^{-1}\right|^{p_k'} < \infty, \tag{8}$$

and $\lim_n d_{nk}$ is finite, thereby yielding $\left[r_B^{\mathfrak{Q}}(g,p)\right]^{\beta} = D_2(g,p) \cap cs.$

As established above, with Lemma 3.3 along (8) yields $a\mho = (a_k \mho_k) \in bs$ whenever $\mho = (\mho_n) \in r_B^{\mathfrak{Q}}(g,p),$ if and only if $D\eta \in l_\infty$ whenever $\eta = (\eta_k) \in l(p).$ Consequently, by applying the same condition, we deduce that $\left[r_B^{\mathfrak{Q}}(g,p)\right]^{\gamma} = D_2(g,p) \cap cs.$

Definition 3.2. Basis: If space $\mathfrak{G}$ is paranormed by $\mathfrak{B}$ contains a sequence $(\wp_n)$, and every $\varsigma \in \mathfrak{G}$, we can find one and only one (α_n), such that

$$\lim_n \mathfrak{B}\left(\varsigma - \sum_{i=0}^{n} \alpha_i \wp_i\right) = 0,$$

where (α_n) represents sequence of scalars, then $(\wp_n)$ is a Schauder basis for $\mathfrak{G}$. The series $\sum \alpha_i \wp_i$ having the sum ς is then said to be as expansion of ς w.r.t. $(\wp_n)$ and is expressed as $\varsigma = \sum \alpha_i \wp_i$.

Theorem 3.2. Let $b^{(m)}(\mathfrak{Q}) = \{b_n^{(m)}(\mathfrak{Q})\}$ be defined as elements of $r_B^{\mathfrak{Q}}(g,p)$ as

$$b_n^{(m)}(\mathfrak{Q}) = \begin{cases} \dfrac{\mathcal{Q}_m}{rg_m \mathfrak{Q}_m} + \nabla_g(n,m)\mathcal{Q}_m, & \text{if } 0 \le n \le m, \\ 0, & \text{if } n > m, \end{cases}$$

for each fixed $m \in \mathbb{N}$. Then, $\{b^{(m)}(\mathfrak{Q})\}$ is a basis for $r_B^{\mathfrak{Q}}(g,p)$ having a unique representation of the form

$$\mho = \sum_m \lambda_m(\mathfrak{Q}) b^{(m)}(\mathfrak{Q}) \tag{9}$$

for any $x \in r_B^{\mathfrak{Q}}(g,p)$ with $\lambda_m(\mathfrak{Q}) = (R_g^{\mathfrak{Q}} B\mho)_m \; \forall m \in \mathbb{N}$ and $0 < p_m \le H < \infty$.

Proof. For $0 < p_j \le \mathcal{H} < \infty$, trivially, $\{b^{(j)}(\mathfrak{Q})\} \subset r_B^{\mathfrak{Q}}(g,p)$ as

$$R_g^{\mathfrak{Q}} B b^{(j)}(\mathfrak{Q}) = e^{(j)} \in l(p) \text{ for } j \in \mathbb{N},$$

where the sequence $e^{(j)}$ having only non-zero term as 1 at *j*th place.

Let $\mho \in r_B^q(g,p)$ be given. For every non-negative integer κ, we put

$$\mho^{[\kappa]} = \sum_{m=0}^{\kappa} \lambda_m(\mathfrak{Q}) b^{(m)}(\mathfrak{Q}). \tag{10}$$

Now, taking $R_g^q B$ to (10), and with the help of (9), we see that

$$R_u^q B \mho^{[\kappa]} = \sum_{m=0}^{\kappa} \lambda_m(\mathfrak{Q}) R_g^q B b^{(m)}(\mathfrak{Q}) = \sum_{m=0}^{\kappa} (R_g^q B \mho)_m e^{(m)}$$

and

$$\left(R_g^q B\left(\mho - \mho^{[\kappa]}\right)\right)_i = \begin{cases} 0, & \text{if } \; 0 \le i \le \kappa \\ \left(R_g^q B \mho\right)_i, & \text{if } \; i > \kappa \end{cases}$$

with $i, \kappa \in \mathbb{N}$. Also, for $\varepsilon > 0$, we can find an integer κ_0, such that

$$\left(\sum_{i=\kappa}^{\infty} |(R_g^q B \mho)_i|^{p_k} \right)^{\frac{1}{\mathcal{M}}} < \frac{\varepsilon}{2}$$

for all $\kappa \ge \kappa_0$. Hence,

$$\begin{aligned} \mathfrak{G}_B\left(\mho - \mho^{[\kappa]}\right) &= \left(\sum_{i=\kappa}^{\infty} |(R_g^q B \mho)_i|^{p_i} \right)^{\frac{1}{\mathcal{M}}} \\ &\le \left(\sum_{i=\kappa_0}^{\infty} |(R_g^q B \mho)_i|^{p_k} \right)^{\frac{1}{\mathcal{M}}} \\ &< \frac{\varepsilon}{2} < \varepsilon \end{aligned}$$

for each $\kappa \ge \kappa_0$, which proves that $\mho \in r_B^q(g, p)$ is represented as (9).

To prove this representation for $\mho \in r_B^q(g, p)$ given by (9) is unique. We assume on the contrary that there do exists another representation given by $\mho = \sum_j \mu_j(\mathfrak{Q}) b^j(\mathfrak{Q})$. But, as in Theorem 3.1, the $T : r_B^q(g, p) \to l(p)$ is continuous, so we have

$$\begin{aligned} (R_g^q B \mho)_n &= \sum_j \mu_j(\mathfrak{Q}) \left(R_g^q B b^j(\mathfrak{Q})\right)_n \\ &= \sum_j \mu_j(\mathfrak{Q}) e_n^{(j)} = \mu_n(\mathfrak{Q}) \end{aligned}$$

for each $n \in \mathbb{N}$, contradicting $(R_g^q B)_n = \lambda_n(\mathfrak{Q})$. Therefore, it follows that representation (9) is unique.

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Availability of data and material

Data sharing is not applicable to this article, as no data sets were generated or analyzed during the current study.

Conflict of interest

It is declared that the author has no conflict of interest.

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