Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

Research Article

Fully Regular Sets of an Imaginary Space

Rasulkhozha S. Sharafiddinov

Institute of Nuclear Physics, Uzbekistan Academy of Sciences, Ulugbek, Tashkent 100214, Uzbekistan
Email: rasulkhozha@yahoo.com

Received: 7 February 2023; **Revised:** 24 March 2023; **Accepted:** 4 April 2023

Abstract: There is no single set in an imaginary space for which an exact mathematical definition would not exist under the mathematical symmetry laws. We discuss a theory in which an imaginary number axis is strictly defined at the level of imaginary space, allowing one to formulate and prove the theorem on the basis of its internal disclosure. This new theory of a set makes it possible to introduce the notion of the full compactness of sets in an imaginary space, confirming their availability in the defined symmetry of elements of a definitely symmetrical line.

***Keywords*:** imaginary space, algebraical logic, geometrical logic, mathematically united logic, regular sets, casual sets, imaginary number axis, full compactness of a set, logic of the commutativity law, full finiteness of a set

MSC: 03A05, 03B60, 03B62, 03E99, 03F99, 03G99, 06A99, 11B99, 11U99

1. Introduction

A set of objects having a different nature must be distinguished from a set that consists only of objects of the same nature. The mathematical notion of sets may therefore be based logically on the unity of all types of mathematical symmetries and not on their absence. Such a latently internal unification, regardless of what the differences in nature of the real and imaginary points are, requires one to raise the question as to what the structural features of sets in an imaginary space are.

As was noted, however, in [1], for the first time, we can strictly define the mathematical notion of sets if and only if the very space in which they exist, as the structural subspaces separate them into two groups by the symmetry laws. The first group includes internally disclosed sets, each of which corresponds to one pair of algebraical and geometrical objects of latent unification. The second group consists of internally undisclosed sets in which there is no single object of unification. This classification, as will be seen, gives the exact mathematical definitions of fully regular and casual sets, an imaginary space, and the latent algebraical object of unification.

Thereby, one must follow the mathematical logic of the commutativity law [2] in an imaginary space at the new level, namely, at the level of the suggested theory of a set from the viewpoint of mutually crossing curved lines of images of a selected pair of elements of a set.

It is here that we must, for the first time, derive the two pairs of relations such that we can formulate eight more new definitions, two new axioms, ten new lemmas, and two new theorems, including a discussion of their proof, and everything that is connected to a latent geometrical object of unification within a set of an imaginary space. Thereby, it

DOI: https://doi.org/10.37256/cm.4420232405

will be established in a set of 12 new mathematical concepts that reflects hitherto unobserved structural properties of the imaginary objects. On this basis, it becomes possible to introduce in Appendix, recognizing that we must not confuse the names. Our purpose is to shed light on what an imaginary space is and its mathematical structure.

2. Preliminaries

One structural feature of fully regular sets of an imaginary space is, as we shall see below, the mathematical notion [3] of what unites all elements of each of them in a unified whole as an object of latent unification. At the same time, the notion of an imaginary space itself needs, in its mathematical definition, to express the ideas of the symmetry laws about how each type of positive (negative) number on an imaginary axis corresponds to a kind of higher (lower) point.

Definition 1. A nonempty space is called an imaginary one if and only if it consists of the higher and lower points of infinitely many selected systems of imaginary axes with a general center of symmetry.

This in turn requires one to present the exact mathematical definitions of fully regular and casual sets and the latent algebraical object of unification.

Definition 2. A latent algebraical object of unification of one set is the second set, such that it consists of conserving sizes of the same defined symmetry of elements of both sets.

Definition 3. The sets are called fully regular or internally disclosed ones if each of them corresponds to one pair of the algebraical and geometrical objects of latent unification of its elements.

Definition 4. The sets are called fully casual or internally undisclosed ones if none of them has neither the algebraical nor geometrical objects of latent unification of its elements.

In any nonempty space, an empty set [4] is necessarily present. The availability in it of an object of unification would imply that it is not empty.

A latent algebraical object of unification of the imaginary points, for example, for a set $\{c\}$, consisting of one element c may be a set $\{i\}$ consisting of one imaginary unity [5]. This correspondence principle, which formulates the theorem on the smallest nonempty set, may symbolically be expressed as

$$\begin{cases} \varnothing, \\ \nexists, \end{cases} \qquad \begin{cases} \{c\}, \\ \{i\}. \end{cases} \tag{1}$$

Lemma 1 (Theorem on the smallest nonempty set). No single fully regular set from the same element exists without an internally disclosed set of a higher cardinality.

We cannot therefore exclude the existence of each set from $\{c\}$ and $\{i\}$ in a kind of internally disclosed set of a higher cardinality as one of its nonempty subsets of defined symmetry. For example, in sets F and Δ, such that together they constitute a system of two sets from two elements of the same Δ symmetry

$$\begin{cases} F = \{c_{1\Delta},\ d_{1\Delta}\}, \\ \Delta = \{i,\ -i\} \end{cases} \tag{2}$$

in which either $c_{1\Delta} < d_{1\Delta}$ or $d_{1\Delta} < c_{1\Delta}$.

One element of a set F must distinguish itself from another of its elements by an individual imaginary number of a conserving size Δ of the defined Δ symmetry

$$\Delta = \begin{cases} +i & \text{for} \quad c_{1\Delta}, \\ -i & \text{for} \quad d_{1\Delta}, \\ \nexists & \text{for} \quad \text{remaining objects}. \end{cases} \tag{3}$$

The violation of this symmetry within a set F, expressed as

$$\sum \Delta \neq const \tag{4}$$

would indicate its internal undisclosure. However, according to

$$\sum \Delta = const, \tag{5}$$

it would take place if and only if in a set F, one or more elements appear from other internally disclosed classes of the defined Δ symmetry.

To reveal the algebraical [6] logic of latent unification of elements within each of internally disclosed sets of a system (2) from the viewpoint of the defined Δ symmetry, it is desirable to present (3) in the form

$$\Delta = \begin{cases} \{+i\} & \text{for} \quad \{c_{1\Delta}\}, \\ \{-i\} & \text{for} \quad \{d_{1\Delta}\}, \\ \varnothing & \text{for} \quad \text{remaining subclasses.} \end{cases} \tag{6}$$

It is also relevant to include in the discussion another fully regular set of the two elements and what unites them as a unified whole. An example of this may be a system

$$\begin{cases} O = \{c_{2\Lambda},\ d_{2\Lambda}\}, \\ \Lambda = \{i,\ -i\} \end{cases} \tag{7}$$

such that its second set comes forward in it as a conserving size

$$\Lambda = \begin{cases} +i & \text{for} \quad c_{2\Lambda}, \\ -i & \text{for} \quad d_{2\Lambda}, \\ \nexists & \text{for} \quad \text{remaining objects} \end{cases} \tag{8}$$

of the defined Λ symmetry, at which either $c_{2\Lambda} < d_{2\Lambda}$ or $d_{2\Lambda} < c_{2\Lambda}$.

Thus, an internal disclosure of a set O follows from

$$\sum \Lambda = const, \tag{9}$$

and consequently, a set Λ must be an object of unification of elements of a set O and vice versa. Therefore, to reveal the algebraical [6] logic of latent unification of elements within each of internally disclosed sets of a system (7) from the viewpoint of the defined Λ symmetry, one must present (8) in the form

$$\Lambda = \begin{cases} \{+i\} & \text{for} \quad \{c_{2\Lambda}\}, \\ \{-i\} & \text{for} \quad \{d_{2\Lambda}\}, \\ \varnothing & \text{for} \quad \text{remaining subclasses.} \end{cases} \tag{10}$$

We now remark that if

$$\sum (\Delta + \Lambda) = const \tag{11}$$

holds, then, for example, sets F and O from systems (2) and (7) constitute another class

$$\Gamma = \{F,\ O\} \tag{12}$$

such that the constancy of sum (11) itself would seem to replace it for

$$\Gamma = \{\mathcal{F},\ \mathcal{O}\} \tag{13}$$

with subclasses

$$\mathcal{F} = \{c_{1\Delta},\ c_{2\Lambda}\},\quad \mathcal{O} = \{d_{1\Delta},\ d_{2\Lambda}\}. \tag{14}$$

Of course, neither of (12) and (13) exclude a set of the two types of symmetries, namely, a set of the two types of conserving sizes

$$\mathcal{P} = \{\Delta,\ \Lambda\}. \tag{15}$$

Such a set characterizing each pair of elements or subsets of a class (12), respectively, by an individual imaginary number

$$\mathcal{P} = \begin{cases} +i & \text{for} \quad c_{1\Delta},\ c_{2\Lambda}, \\ -i & \text{for} \quad d_{1\Delta},\ d_{2\Lambda}, \\ \nexists & \text{for} \quad \text{remaining objects} \end{cases} \tag{16}$$

or according to Lemma 1, by an individual subclass

$$\mathcal{P} = \begin{cases} \{+i\} & \text{for} \quad \{c_{1\Delta}\},\ \{c_{2\Lambda}\}, \\ \{-i\} & \text{for} \quad \{d_{1\Delta}\},\ \{d_{2\Lambda}\}, \\ \varnothing & \text{for} \quad \text{remaining subclasses} \end{cases} \tag{17}$$

describes a situation where

$$\mathcal{P} = \{i,\ -i\}. \tag{18}$$

The question as to what unites all elements of each subset from (14) in a unified whole still remains open.

The preceding reasoning says that the same latent algebraical object $\mathcal{P}$ responsible for unification of elements of any subsets from F and O does not, by itself, unite the elements of each from subclasses $\mathcal{F}$ and $\mathcal{O}$, and a linearly ordered set Γ is of those classes, in any of which any pair of elements establishes one pair of relations. Its internal structure, however, encounters the condition of the commutativity law and reflects the characteristic features of the inter-ratio of the very set and the set of images of its elements. Each pair of the expected relations relates within Γ one element in any subclass from $\mathcal{F}$ and $\mathcal{O}$ to another of its elements as a latent geometrical object of their unification.

But we cannot establish them until the very imaginary space is able to formulate the theorems on the internal disclosure and undisclosure algebraical logic of sets.

3. An internal disclosure algebraical logic of sets of an imaginary space

From our earlier analysis, we find an imaginary number axis, namely, the number axis of imaginary space, allowing one to formulate and prove the theorem on an internal disclosure of an imaginary number axis.

Definition 5. A number axis is called an imaginary one if it has higher and lower imaginary points relative to its center of symmetry.

Theorem 1. To each pair of higher and lower imaginary points corresponds one pair of objects such that together they constitute a system of two sets from two elements of an imaginarily defined symmetry.

Proof of Theorem 1. An imaginary number axis

$$..., -(n+1)i,\ -ni, ..., -3i,\ -2i,\ -i,\ 0,\ i,\ 2i,\ 3i, ..., ni,\ (n+1)i, ..., \tag{19}$$

as stated in Theorem 1, has been created so that to each point corresponds one object. Thereby, it characterizes each of higher points

$$i,\ 2i,\ 3i, ..., ni,\ (n+1)i, ... \tag{20}$$

by one single object from

$$c_{1\Delta},\ c_{2\Lambda},\ c_{3\Pi}, ..., c_{n\Upsilon},\ c_{(n+1)\Phi}, ..., c_{1\Delta}, c_{2\Delta}, \tag{21}$$

where any index of a distinction from $P = \Delta, \Lambda, \Pi, ..., \Upsilon, \Phi, ...$ must be accepted as one of all types of symmetries of an imaginary space.

In a similar way, one can describe the lower points

$$-i,\ -2i,\ -3i, ..., -ni,\ -(n+1)i, ... \tag{22}$$

by their corresponding objects

$$d_{1\Delta},\ d_{2\Lambda},\ d_{3\Pi}, ..., d_{n\Upsilon},\ d_{(n+1)\Phi}, ... \tag{23}$$

such that each index P distinguishes one pair of (21) and (23) from all others.

The very availability in an imaginary number axis of the center of symmetry indicates the role of the unified principle in all systems of its objects. The first object of any pair must therefore distinguish from the second of its objects by an individual number of a conserving size of one of all types of symmetries, the unity of which constitutes the united symmetry such that its existence in an imaginary space is according to Theorem 1.

To show this, we must first group (20) and (22) and constitute the sequence

$$i,\ -i,\ 2i,\ -2i,\ 3i,\ -3i, ..., ni,\ -ni,\ (n+1)i,\ -(n+1)i, ... \tag{24}$$

The groupings of (21) and (23) suggest another sequence, such as

$$c_{1\Delta},\ d_{1\Delta},\ c_{2\Lambda},\ d_{2\Lambda},\ c_{3\Pi},\ d_{3\Pi}, ..., c_{n\Upsilon},\ d_{n\Upsilon},\ c_{(n+1)\Phi},\ d_{(n+1)\Phi}, ..., \tag{25}$$

Thus, it follows that to each set of points

$$\{i,\ -i\},\ \{2i,\ -2i\},\ \{3i,\ -3i\}, ..., \{ni,\ -ni\},\ \{(n+1)i,\ -(n+1)i\}, ... \tag{26}$$

corresponds one set of objects

$$\{c_{1\Delta},\ d_{1\Delta}\},\ \{c_{2\Lambda},\ d_{2\Lambda}\},\ \{c_{3\Pi},\ d_{3\Pi}\}, ..., \{c_{n\Upsilon},\ d_{n\Upsilon}\},\ \{c_{(n+1)\Phi},\ d_{(n+1)\Phi}\}, ... \tag{27}$$

This in turn defines the individual numbers of i and $-i$ of any conserving size $\mathcal{P} = \mathit{\Delta}, \mathit{\Lambda}, \mathit{\Pi}, ..., \mathit{\Upsilon}, \mathit{\Phi}, ...$ of all types of symmetries P having in view the existence in an imaginary space of one-to-one correspondence between the series (26) and

$$\{i,\ -i\},\ \{i,\ -i\},\ \{i,\ -i\}, ..., \{i,\ -i\},\ \{i,\ -i\}, ... \tag{28}$$

They make it possible to establish the following systems of sets:

$$\begin{cases} Q = \{c_{3\Pi},\ d_{3\Pi}\}, \\ \Pi = \{i,\ -i\}, \end{cases} \tag{29}$$

$$\begin{cases} V = \{c_{n\Upsilon},\ d_{n\Upsilon}\}, \\ \Upsilon = \{i,\ -i\}, \end{cases} \tag{30}$$

$$\begin{cases} W = \{c_{(n+1)\Phi},\ d_{(n+1)\Phi}\}, \\ \Phi = \{i,\ -i\}, \end{cases} \tag{31}$$

the first two of which were already established above.

It is not excluded, however, that

$$\sum \Pi = const, ..., \sum \Upsilon = const,\ \sum \Phi = const, ..., \tag{32}$$

and therefore, one set in each system of (29)-(31) is a fully regular object of unification of elements of the second set.

Thus, we have a general system of internally disclosed sets

$$\begin{cases} I = \{F,\ O,\ Q, ..., V,\ W, ...\}, \\ \mathcal{P} = \{\mathit{\Delta},\ \mathit{\Lambda},\ \mathit{\Pi}, ..., \mathit{\Upsilon},\ \mathit{\Phi}, ...\} \end{cases} \tag{33}$$

in which a set $\mathcal{P}$ consists of conserving sizes of all types of symmetries. Their unity establishes in an imaginary space the same defined symmetry P with the same conserving size $\mathcal{P}$, thereby confirming the validity of Theorem 1 and all consequences following from it.

So, we have learned that Theorem 1 characterizes any element or subclass of a set I of a general system (33), respectively, by an individual imaginary number

$$\mathcal{P} = \begin{cases} +i & \text{for} \quad c_{1\Delta},\ c_{2\Lambda},\ c_{3\Pi}, ..., c_{n\Upsilon},\ c_{(n+1)\Phi}, ..., \\ -i & \text{for} \quad d_{1\Delta},\ d_{2\Lambda},\ d_{3\Pi}, ..., d_{n\Upsilon},\ d_{(n+1)\Phi}, ..., \\ \nexists & \text{for} \quad \text{remaining objects} \end{cases} \tag{34}$$

or, according to Lemma 1, by an individual subset

$$\mathcal{P} = \begin{cases} \{+i\} & \text{for} \quad \{c_{1\Delta}\},\ \{c_{2\Lambda}\},\ \{c_{3\Pi}\}, ..., \{c_{n\Upsilon}\},\ \{c_{(n+1)\Phi}\}, ..., \\ \{-i\} & \text{for} \quad \{d_{1\Delta}\},\ \{d_{2\Lambda}\},\ \{d_{3\Pi}\}, ..., \{d_{n\Upsilon}\},\ \{d_{(n+1)\Phi}\}, ..., \\ \varnothing & \text{for} \quad \text{remaining subclasses} \end{cases} \tag{35}$$

such that a constancy law of the sum

$$\sum \mathcal{P} = const, \tag{36}$$

which states its internal disclosure is never violated.

To conform with these implications, the notion of an algebraical disclosure of a set in an imaginary space is logically based on the imaginarily defined symmetry of elements. But, as stated in its violation, the notion of an algebraical undisclosure of a set in an imaginary space is based logically on the imaginarily defined antisymmetry of elements. Any of the Definitions 3 and 4, together with the Definition 2, reflect the availability of a kind of theorem.

Lemma 2 (Theorem on an internal disclosure algebraical logic). There is no algebraical disclosure in a set without a strictly defined symmetry of elements.

Lemma 3 (Theorem on an internal undisclosure algebraical logic). There is no algebraical undisclosure in a set without a strictly defined antisymmetry of elements.

Finally, insofar as the definition of a real number axis [7, 8] at the new level, namely, at the level of real space, allowing one to formulate and prove the theorem on the basis of its internal disclosure, is concerned, we will start here from the requirement [1] that a real number axis including only the real points with objects must be distinguished from an imaginary number axis such that it consists of the imaginary points with objects.

A nonzero point is called a real one if and only if it has a real coordinate [2]. A nonzero point is called an imaginary one if it has an imaginary coordinate. They constitute at their unification in a unified whole one point with a complex coordinate, namely, one complex point with coordinates of the structural points.

Any complex number corresponds to a kind of complex point. Such pairs can constitute the families of the complex points as well as of the complex numbers. This gives the right to unite the real and imaginary number axes in a unified whole. Insofar as the problem of a complex number axis thus formed is concerned, the results following from its consideration call for a special presentation.

4. The commutativity images of objects of either the higher or the lower imaginary points

An algebraically disclosed set of an imaginary space can possess each of the innate properties even at any permutation of elements. This, of course, is intimately connected with the character of their spatial structure depending on the space of an imaginary number axis, namely, on the space of an imaginary base.

Returning to (33), we remark that any subclass of I is in one of its finite sets. Each of these classes corresponds to a kind of ball, within which there are points with its elements and points with images of each of them. In other words, a ball of the same internally disclosed set from I consists of the very set and the set involving those subclasses, in any of which appear images of one and only one element of the finite set. Formulating more concretely, one can symbolically present this consequence of Theorem 1 as

$$c_{NP}, d_{NP} \in I,\ c_{NP}^{(\lambda)}, d_{NP}^{(\lambda)} \in I^{(\lambda)},$$
$$f_\lambda : I \to I^{(\lambda)},\ c_{NP}^{(\lambda)} = f_\lambda(c_{NP}),\ d_{NP}^{(\lambda)} = f_\lambda(d_{NP}).$$

Here, we must keep in mind that

$$NP = 1\Delta, 2\Lambda, 3\Pi, ..., n\Upsilon, (n+1)\Phi, ...,\ \lambda \in Z_0,\ Z_0 = \{0,\ 1,\ 2, ...\}.$$

On this basis, Theorem 1 characterizes not only any element or subclass but also the corresponding images of each element or subclass of a set I of a general system (33), respectively, by an individual imaginary number

$$\mathcal{P} = \begin{cases} +i & \text{for} \quad c_{NP}^{(\lambda)}, \\ -i & \text{for} \quad d_{NP}^{(\lambda)}, \\ \nexists & \text{for} \quad \text{remaining images} \end{cases} \tag{37}$$

or, according to Lemma 1, by an individual subclass

$$\mathcal{P} = \begin{cases} \{+i\} & \text{for} \quad \{c_{NP}^{(\lambda)}\}, \\ \{-i\} & \text{for} \quad \{d_{NP}^{(\lambda)}\}, \\ \varnothing & \text{for} \quad \text{remaining subclasses} \end{cases} \tag{38}$$

such that a constancy law of the sum (36), which expresses its internal disclosure is never violated.

Definition 6. The images of each element of the same internally disclosed set constitute one of its spectra.

Lemma 4 (Theorem on the spectra of a set). There is no single spectrum in an internally disclosed set without a crossing point with another of its spectra.

Definition 6 and Lemma 4 require the revealing of the mathematical logic of the commutativity of elements of a set from the point of view of mutually crossing spectra. However, according to Definition 6, the spectrum of images of each element of any algebraically disclosed set of an imaginary space is strictly finite from above and below by images of the limited size. Therefore, in spectra

$$\begin{array}{l} f_0 : I \to I,\ c_{NP} = f_0(c_{NP}),\ d_{NP} = f_0(d_{NP}), \\ f_1 : I \to I^{(1)},\ c_{NP}^{(1)} = f_1(c_{NP}),\ d_{NP}^{(1)} = f_1(d_{NP}), \\ f_2 : I \to I^{(2)},\ c_{NP}^{(2)} = f_2(c_{NP}),\ d_{NP}^{(2)} = f_2(d_{NP}), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array}$$

there are maximally $\max\{c_{NP}^{(\lambda)}\}, \max\{d_{NP}^{(\lambda)}\}$ and minimally $\min\{c_{NP}^{(\lambda)}\}, \min\{d_{NP}^{(\lambda)}\}$ possible limits both on the images of objects (21) of higher points and on the images of objects (23) of lower points of the number axis.

Definition 7. The relation

$$c_{1\Delta}c_{2\Lambda} = c_{2\Lambda}c_{1\Delta} \tag{39}$$

between $c_{1\Delta}$ and $c_{2\Lambda}$ is a relation of commutativity only if

1. $c_{1\Delta}^{(\lambda)} \to \max\{c_{1\Delta}^{(\lambda)}\} \Rightarrow c_{2\Lambda}^{(\lambda)} \to \min\{c_{2\Lambda}^{(\lambda)}\}$,
2. $c_{2\Lambda}^{(\lambda)} \to \max\{c_{2\Lambda}^{(\lambda)}\} \Rightarrow c_{1\Delta}^{(\lambda)} \to \min\{c_{1\Delta}^{(\lambda)}\}$,
3. $c_{1\Delta}^{(\tau_1)} \ll c_{1\Delta} \to c_{2\Lambda}^{(\tau_1)} \gg c_{2\Lambda} \Rightarrow c_{1\Delta}^{(\tau_1)} = c_{2\Lambda}^{(\tau_1)} \Rightarrow [c_{1\Delta}^{(\tau_1)}]^2 = [c_{2\Lambda}^{(\tau_1)}]^2 = c_{1\Delta}^{(\tau_1)}c_{2\Lambda}^{(\tau_1)} = c_{1\Delta}^{(\lambda)}c_{2\Lambda}^{(\lambda)} = const$,
4. $c_{2\Lambda}^{(\tau_2)} \ll c_{2\Lambda} \to c_{1\Delta}^{(\tau_2)} \gg c_{1\Delta} \Rightarrow c_{2\Lambda}^{(\tau_2)} = c_{1\Delta}^{(\tau_2)} \Rightarrow [c_{2\Lambda}^{(\tau_2)}]^2 = [c_{1\Delta}^{(\tau_2)}]^2 = c_{2\Lambda}^{(\tau_2)}c_{1\Delta}^{(\tau_2)} = c_{2\Lambda}^{(\lambda)}c_{1\Delta}^{(\lambda)} = const$.

The commutativity of $c_{1\Delta}$ and $c_{2\Lambda}$ formulates the consequence of a principle that each of the two pairs of spectra of their para-images $c_{1\Delta}c_{2\Lambda}$ and $c_{2\Lambda}c_{1\Delta}$ is crossed at one of the two points τ_1 and τ_2 with some objects

$$c_{1\Delta}^{(\tau_1)} = c_{2\Lambda}^{(\tau_1)},\ c_{2\Lambda}^{(\tau_2)} = c_{1\Delta}^{(\tau_2)}, \tag{40}$$

which takes place only if

$$c_{1\Delta}^{(\tau_1)} \ll c_{1\Delta} \to c_{2\Lambda}^{(\tau_1)} \gg c_{2\Lambda},\ c_{2\Lambda}^{(\tau_2)} \ll c_{2\Lambda} \to c_{1\Delta}^{(\tau_2)} \gg c_{1\Delta}. \tag{41}$$

These connections correspond in a set $\mathcal{F}$ from (14) to those para-images of $c_{1\Delta}c_{2\Lambda}$ and $c_{2\Lambda}c_{1\Delta}$, which can be called the images of commutativity of its elements:

$$[c_{1\Delta}^{(\tau_1)}]^2 = [c_{2\Lambda}^{(\tau_1)}]^2 = c_{1\Delta}^{(\tau_1)}c_{2\Lambda}^{(\tau_1)} = c_{1\Delta}^{(\lambda)}c_{2\Lambda}^{(\lambda)} = const, \tag{42}$$

$$[c_{2\Lambda}^{(\tau_2)}]^2 = [c_{1\Delta}^{(\tau_2)}]^2 = c_{2\Lambda}^{(\tau_2)}c_{1\Delta}^{(\tau_2)} = c_{2\Lambda}^{(\lambda)}c_{1\Delta}^{(\lambda)} = const. \tag{43}$$

For the establishment of a relation of commutativity

$$d_{1\Delta}d_{2\Lambda} = d_{2\Lambda}d_{1\Delta} \tag{44}$$

between $d_{1\Delta}$ and $d_{2\Lambda}$ in a set $\mathcal{O}$ from (14), one needs use the substitutions

$$c_{1\Delta} \to d_{1\Delta},\ c_{2\Lambda} \to d_{2\Lambda} \Leftrightarrow \tau_1 \to \tau_1^*,\ \tau_2 \to \tau_2^*, \tag{45}$$

which can generalize Definition 7 to the case of objects from the lower points of the number axis. However, the question of spatial coordinates of points $\tau_1(\tau_1^*)$ and $\tau_2(\tau_2^*)$ still remains open. We can solve only the question of whether they refer to the imaginary or the real points.

Theorem 2. If a set consists only of objects of either the higher or the lower points of an imaginary axis, each of the two pairs of spectra of each pair of its elements is crossed at one of the imaginary points.

Proof of Theorem 2. A set of Theorem 2 is of sets such that within each of them one can find objects only from one sequence (21) or (23), and therefore, is a subclass of an algebraically disclosed set. A beautiful example of this may be any subclass (14), namely, each subset from $\mathcal{F}$ and $\mathcal{O}$ with established above images of the commutativity of its elements.

Turning again to (37), (39), and (42)-(45), we see that

$$\mathcal{P}_{c_{1\Delta}^{(\lambda)}} \mathcal{P}_{c_{2\Lambda}^{(\lambda)}} = -1, \quad \mathcal{P}_{c_{2\Lambda}^{(\lambda)}} \mathcal{P}_{c_{1\Delta}^{(\lambda)}} = -1, \tag{46}$$

$$\mathcal{P}_{d_{1\Delta}^{(\lambda)}} \mathcal{P}_{d_{2\Lambda}^{(\lambda)}} = -1, \quad \mathcal{P}_{d_{2\Lambda}^{(\lambda)}} \mathcal{P}_{d_{1\Delta}^{(\lambda)}} = -1, \tag{47}$$

which each of conserving sizes

$$\mathcal{P}_{c_{1\Delta}^{(\tau_1)}} = \mathcal{P}_{c_{2\Lambda}^{(\tau_1)}} = [\mathcal{P}_{c_{1\Delta}^{(\lambda)}} \mathcal{P}_{c_{2\Lambda}^{(\lambda)}}]^{1/2} = \pm i, \tag{48}$$

$$\mathcal{P}_{c_{2\Lambda}^{(\tau_2)}} = \mathcal{P}_{c_{1\Delta}^{(\tau_2)}} = [\mathcal{P}_{c_{2\Lambda}^{(\lambda)}} \mathcal{P}_{c_{1\Delta}^{(\lambda)}}]^{1/2} = \pm i, \tag{49}$$

$$\mathcal{P}_{d_{1\Delta}^{(\tau_1^*)}} = \mathcal{P}_{d_{2\Lambda}^{(\tau_1^*)}} = [\mathcal{P}_{d_{1\Delta}^{(\lambda)}} \mathcal{P}_{d_{2\Lambda}^{(\lambda)}}]^{1/2} = \pm i, \tag{50}$$

$$\mathcal{P}_{d_{2\Lambda}^{(\tau_2^*)}} = \mathcal{P}_{d_{1\Delta}^{(\tau_2^*)}} = [\mathcal{P}_{d_{2\Lambda}^{(\lambda)}} \mathcal{P}_{d_{1\Delta}^{(\lambda)}}]^{1/2} = \pm i \tag{51}$$

is compatible with Theorem 2, from which it follows the points $\tau_1(\tau_1^*)$ and $\tau_2(\tau_2^*)$ are strictly of the imaginary points, and the images of the commutativity of elements of any subclass from (14) appear only in their space.

In principle, such a circumstance requires one to consider why the internal disclosure of a set of imaginary spaces comes forward at the level of the imaginary number axis itself as a commutativity of each pair of its objects. To solve this problem, one must refer to (39) and (44), because they involve the fact that there is no single pair of objects in the same imaginary axis for which a relation of commutativity would not exist. Insofar as the fate of commutativity images of objects at both their higher and lower points is concerned, it will be illuminated in a separate work. But here we can formulate one more pair of theorems.

Lemma 5 (Theorem on a commutative pair mathematical logic). There is no mathematical disclosure in a set without commutative pairs of elements.

Lemma 6 (Theorem on an anticommutative pair mathematical logic). There is no mathematical undisclosure in a set without anticommutative pairs of elements.

Any imaginary axis in an imaginary space must be distinguished from other imaginary axes by the individual objects. Thereby, it requires one to characterize each commutative pair of each imaginary number axis both by the imaginative curved and straight lines.

Definition 8. A line is called an imaginative curved one if it unites all points with images of one and only one of the objects of each commutative pair.

Definition 9. A line is called an imaginative straight one if it unites selected points with images of each of the objects in each commutative pair.

If we choose $2i$ and $8i$ from objects of higher points of an imaginary number axis, their multiplication constitutes in whole a kind of relation $2i8i = 8i2i = (4i)^2$, in which $4i$ comes forward as the commutativity images of selected pairs of objects. Together, they satisfy the inequalities $2i < 4i$, $4i > 2i$, $4i < 8i$, $8i > 4i$ and that, consequently, the spectrum of images of each of the objects $2i$ and $8i$ has a defined directionality. An imaginative curved line of each of the imaginary objects of each imaginarily commutative pair can therefore be accepted as a curved vector [9].

5. Full finiteness of sets of an imaginary space

For completeness, we must recall that

$$c_{1\Delta}, c_{2\Lambda} \in \mathcal{F},\ c_{1\Delta}^{(\lambda)}, c_{2\Lambda}^{(\lambda)} \in \mathcal{F}^{(\lambda)},$$
$$f_{\lambda}: \mathcal{F} \to \mathcal{F}^{(\lambda)},\ c_{1\Delta}^{(\lambda)} = f_{\lambda}(c_{1\Delta}),\ c_{2\Lambda}^{(\lambda)} = f_{\lambda}(c_{2\Lambda}),$$

according to which, if one element from $\mathcal{F}$ has λ images in $\mathcal{F}^{(\lambda)}$, then either $\mathcal{F}$ or $\mathcal{F}^{(\lambda)}$ is not in a state to give a categorical answer to the question of whether or there is no one-to-one correspondence between them. The answer is still hidden in these sets. To solve this question, one must refer to the presentations

$$c_{1\Delta} = \{c_{1\Delta},\ c_{1\Delta},\ c_{1\Delta}, \ldots, c_{1\Delta},\ c_{1\Delta}, \ldots\},$$

$$c_{2\Lambda} = \{c_{2\Lambda},\ c_{2\Lambda},\ c_{2\Lambda}, \ldots, c_{2\Lambda},\ c_{2\Lambda}, \ldots\},$$

$$c_{1\Delta}^{(\lambda)} = \{c_{1\Delta},\ c_{1\Delta}^{(1)},\ c_{1\Delta}^{(2)}, \ldots, c_{1\Delta}^{(n)},\ c_{1\Delta}^{(n+1)}, \ldots\},$$

$$c_{2\Lambda}^{(\lambda)} = \{c_{2\Lambda},\ c_{2\Lambda}^{(1)},\ c_{2\Lambda}^{(2)}, \ldots, c_{2\Lambda}^{(n)},\ c_{2\Lambda}^{(n+1)}, \ldots\}$$

because they correspond in $\mathcal{F}$ to the fact that it comes forward in a set I of a general system (33) as one of its finite subclasses. This does not imply, of course, the absence for $\mathcal{F}$ of a kind of ball, in formation of which appears a role of the interratio of $\mathcal{F}$ and the set of images of its elements. Such a ball involves both λ points with each element of a set $\mathcal{F}$ and λ points with its images. Their equality expresses the full finiteness of the very finite set.

Definition 10. The finite sets of an imaginary space are called full finiteness ones if and only if within a ball of each of them a number of points with each element and a number of points with its images coincide.

The finite set $\mathcal{F}$ indicates that to each pair of points in a space, where it becomes fully finite, corresponds one and only one pair of its elements and their images. In other words, within a ball of a full finiteness set, there is no single pair of points, for which a single pair of its elements and their images would not exist. An explicit evaluation referring to the investigated set does not require, of course, the rewriting of any of its elements, and a symbolic presentation of $c_{1\Delta}$ and $c_{2\Lambda}$ in conformity with $c_{1\Delta}^{(\lambda)}$ and $c_{2\Lambda}^{(\lambda)}$ simply implies that $\mathcal{F}$ and $\mathcal{F}^{(\lambda)}$ are connected only with points not violating their one-to-one correspondence.

6. An internal disclosure geometrical logic of sets of an imaginary space

Thus, if the relations (42) and (43) relate one element within a set $\mathcal{F}$ from (14) to another from its elements, a geometrical object of their latent unification must be accepted as a definitely symmetrical line of elements.

Definition 11. A latent geometrical object of unification of one set is the second set, such that it consists of conserving points of the same defined line of elements of both sets.

A latent geometrical object of unification for a set $\{c_{1\Delta}\}$ consisting of one element $c_{1\Delta}$ may be a set $\{c_{1\Delta}^{(\lambda)}\}$ consisting of λ points with images of the same single element. A class $\{c_{1\Delta}^{(\lambda)}\}$, as noted in Definitions 6, 8, and 10, constitutes in whole a kind of imaginative curved line such that it is strictly finite from above and below by points of images of the limited size. On the other hand, Lemma 1 states that each class from $\{c_{1\Delta}\}$ and $\{c_{1\Delta}^{(\lambda)}\}$ exists in a kind of internally disclosed set of a higher cardinality as one of its nonempty subclasses. For example, in $\mathcal{F}$ and $\mathcal{F}^{(\lambda)}$, the elements constitute at their unification in a unified whole the same curvilinear triangle of both sets.

This is exactly the same as when a notion of a geometrical disclosure of a set of an imaginary space is based logically on the imaginarily symmetrical line of elements. Consequently, a notion of a geometrical undisclosure of a set in an imaginary space is logically based on the imaginarily antisymmetrical line of elements. Each of the Definitions 3 and 4, together with the Definition 11, says herewith about the existence of a kind of theorem.

Lemma 7 (Theorem on an internal disclosure geometrical logic). There is no geometrical disclosure in a set without a definitely symmetrical line of elements.

Lemma 8 (Theorem on an internal undisclosure geometrical logic). There is no geometrical undisclosure in a set without a definitely antisymmetrical line of elements.

7. An internal disclosure mathematical logic of sets of an imaginary space

Lemmas 2, 3, 7, and 8 reflect just the regularity that the Definitions 3 and 4 together with the Definitions 2 and 11 express, for each of the two sets of two elements, the idea of a unified logic within their system. It involves the notion that a mathematical disclosure of a set in an imaginary space is based imaginarily on the definitely symmetrical line of elements in their strictly defined symmetry. Conversely, a notion of a mathematical undisclosure of a set in an imaginary space is based imaginarily on the definitely antisymmetrical line of elements in their strictly defined antisymmetry. They convince us that mathematical logic [10] comes forward in a set of an internal disclosure as a united logic, allowing one to formulate one more pair of highly important theorems.

Lemma 9 (Theorem on an internal disclosure mathematical logic). There is no mathematical disclosure in a set without a definitely symmetrical line of elements of strictly defined symmetry.

Lemma 10 (Theorem on an internal undisclosure mathematical logic). There is no mathematical undisclosure in a set without a definitely antisymmetrical line of elements of strictly defined antisymmetry.

8. Concluding remarks

Definitions 2 and 3, together with Lemmas 9 and 10, confirm the fact that the sets of an imaginary space are called internally disclosed ones if each of them has a mathematical disclosure. In contrast to this, the sets in an imaginary space are called internally undisclosed ones if each of them has a mathematical undisclosure. Such regularities express, for each of the two types of sets, the idea of a kind of axiom.

Axiom 1. An internal disclosure of a set is none other than its mathematical disclosure.

Axiom 2. An internal undisclosure of a set is none other than its mathematical undisclosure.

This mathematically united logic, in turn, gives the possibility to simultaneously introduce the notion of the full compactness of sets of an imaginary space.

Definition 10. The compact sets of an imaginary space are called full compactness ones if and only if they have a mathematical disclosure.

But we cannot exclude the existence of geometrically degenerated sets such that each of them with elements of imaginarily defined symmetry has a geometrical undisclosure. Exactly the same, the very imaginary space establishes the definitely symmetrical line of elements in each of the algebraically degenerated sets, even at its algebraical undisclosure properties. Together with a geometrically degenerated set, it constitutes, in an imaginary space, the united family of mathematically degenerated sets.

From our previous analyses, we find that each of the internally disclosed subclasses of an imaginary space corresponds in set I of a general system (33) to a kind of geometrical object of unification. The building of any of them would require a detailed description, and therefore, we will include this in our future works.

Acknowledgments

I am indebted to the editors and referees for carefully reading the article and for their constructive comments and suggestions, which have helped develop the paper.

Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

Appendix

We have established a set of 12 new mathematical concepts that reflect so far unobserved latent properties of imaginary objects.

As such, an object may, according to Definition 1, be an imaginary space. A real space including only the real points must be distinguished from an imaginary space such that it consists of the imaginary points.

A nonzero point without an imaginary coordinate is called a real one. A nonzero point without a real coordinate is called an imaginary one. Together, they constitute the complex point mentioned in the third section.

An algebraical object of latent unification given by Definition 2 is called a set such that it consists of conserving sizes of *the symmetry of elements* of an algebraically disclosed set. Insofar as the very symmetry of elements is concerned, Theorem 1 characterizes each pair of objects c_{NP} of higher points and d_{NP} of lower points of an imaginary number axis by one pair of individual numbers i and $-i$ of each conserving size $\mathcal{P}$ of all types of symmetries P of an imaginary space. Their sum is compatible with (36) and, consequently, there exists a strictly defined symmetry of elements of each set having an algebraical disclosure.

A geometrical object of latent unification, as follows from Definition 11, is called a set such that it consists of conserving points *of the line of elements* of a geometrically disclosed set. Insofar as the very line of elements is concerned, it relates one element in a set to another of its elements. Simultaneously, as stated in Definitions 6 and 8, all images of each element constitute an internally disclosed set with an imaginative curved line as one of its spectra. However, according to Lemma 4, there is no single imaginative curved line in an internally disclosed set without a crossing point with another of its imaginative curved lines. This does not imply, of course, that the line of elements, the imaginative curved lines of which are crossed at one of the imaginary points, must change a number of its own values. Thereby, there appears a geometrical disclosure of each set of an imaginary space at a definitely symmetrical *line of elements*.

An algebraical disclosure of *an internally disclosed* (*fully regula*r) *set* together with its geometrical disclosure expresses the idea of Lemmas 2, 7, and 9 that it has a mathematical disclosure. Therefore, from the point of view of Lemma 5 and Axiom 1, the commutativity of each pair of objects in a mathematically disclosed set comes forward as an internal disclosure of this *fully regular set*.

However, as noted in Lemmas 3, 8, and 10, an algebraical undisclosure of an *internally undisclosed (fully casual)* set jointly with its geometrical undisclosure says that it has a mathematical undisclosure. In other words, Lemma 6 and Axiom 2 require following the logic of an anticommutativity of each pair of objects of a mathematically undisclosed set from the viewpoint of *an internal undisclosure* of this *fully casual set*.

Finally, insofar as *a definitely symmetrical line of elements of strictly defined symmetry* is concerned, Lemma 9 states that *the line of elements* is a definitely symmetrical one at a strictly defined symmetry of elements. In contrast to this, the line of elements must be a definitely antisymmetrical one if there exists a strictly defined *antisymmetry of elements*. Therefore, Lemma 10 involves a definitely *antisymmetrical line of elements of strictly defined antisymmetry.*

A comparison of Definitions 2 and 11 expresses the idea of *conserving points of the same defined symmetry of elements*. Thereby, it describes a situation when a number of points of *the line of elements*, similarly to number of points with images of each of them, is not changed at *the same defined symmetry of elements* of the very set such that it consists only of objects of either the higher or the lower points of an imaginary axis.

A set including objects at both higher and lower points of an imaginary axis in an imaginary space is called *a geometrically degenerated set*, because each of the two pairs of spectra of each pair of its elements is crossed at one of the real points. In it appears a part of the crossing of an imaginary space with a real space. But, as was mentioned in the fourth section, the implications implied by its consideration will be presented in our further works.

References

[1] Sharafiddinov RS. Internally disclosed sets of a real space. In: *Spring Western Sectional Meeting of the American Mathematical Society*. San Francisco: AMS; 2021.

[2] Korn GA, Korn TM. *Mathematical handbook for scientists and engineers: Definitions, theorems, and formulas for*

reference and review. New York: Dover Publications; 2000.
[3] Sharafiddinov RS. Unification theorems of elements of a set. In: *Spring Western Sectional Meeting of the American Mathematical Society*. Fresno: AMS; 2020.
[4] Alexandroff PS. *Introduction to the theory of sets and the general topology*. Moscow: Nauka; 1977.
[5] Sharafiddinov RS. Fully regular sets of an imaginary space. In: *Spring Western Sectional Meeting of the American Mathematical Society*. San Francisco: AMS; 2021.
[6] Halmos PR. The basic concepts of algebraic logic. *The American Mathematical Monthly*. 1956; 63(6): 363-387. Available from: https://doi.org/10.1080/00029890.1956.11988821.
[7] Wells W, Hart WW. *First year algebra*. Boston: D. C. Heath & Company; 1912.
[8] Dedekind R. *Stetigkeit und Irrationale Zahlen*. Braunschweig: Friedrich Vieweg; 1872.
[9] Sharafiddinov RS. An allgravity as a grand unification of forces. *Physics Essays Publication*. 2021; 34(3): 398-410. Available from: https://doi.org/10.4006/0836-1398-34.3.397.
[10] Henkin L. Some interconnections between modern algebra and mathematical logic. *Transactions of the American Mathematical Society*. 1953; 74: 410-427.