Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

Research Article

Impact of Inspection Error of an Economic Production Quantity Model with Acceptance Sampling, Defective Items, and Rework

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Received: 7 February 2023; **Revised:** 19 April 2023; **Accepted:** 21 July 2023

Abstract: In this model, the manufacturer needs to inspect and rework defective items due to the flawed manufacturing process. When checking for defective products, two types of errors, namely Type 1 error and Type 2 error, may occur at the inspection stage. If a lot of defective materials are rejected during the inspection, a non-destructive screening process separates the material into reworkable and non-defective categories. This study includes inspection rate insufficiency (IRI) and inspection rate sufficiency (IRS) based on the relationship between inspection and production rates. Rework Priority Policy (RPP) is followed in every scenario. This paper aims to minimize the inspection error and optimize the total profit. Numerical examples help demonstrate the value of this well-established model. A sensitivity analysis of this study was carried out to check its accuracy.

Keywords: rework, imperfect inspection, imperfect production quantity, acceptance sampling

MSC: 90B05, 90B06

Nomenclature

a	Selling price per unit
α	Manufacturer inspection rate
c_{h_1}	Holding cost for perfect items/unit /unit time
c_{h_2}	Holding cost for defective items/unit/unit time
c_i	Number of shipment cycle production time
c_{is}	Cost of inspection per unit
c_p	Manufacturer's production cost/unit
c_{rw}	The manufacturer's rework rate $R_w > D$
c_t	Transportation cost per unit
D	Demand
L_i	The period during which items are inspected by the manufacturer

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L_p	Total production quantity per production run
L_r	The time for defective items to be reworked by the manufacturer
m_1	Length of the cycle in sufficient inspection
m_2	Length of the cycle in insufficient inspection
P_r	Production rate, where ($P_r > D$)
P_q	Production quantity with perfect quantity while manufacturing for delivery for the first lot q_1 to the retailer
μ	Defective rate
Q	Total production quantity per production run
R_w	The manufacturer rework rate
T_{L_i}	The time for manufacturer's production run (for $i = 1$ IRI scenario, $i = 2$ IRS scenario)

1. Introduction

Various standard variables in the dielectric material, fabrication, fabrics, and petrochemical industries, such as raw material quality variation, internal material flaws, equipment failures, and working conditions, always impact the manufacturer's production process. As a consequence, Sobhani et al. [1] and Guha and Bose [2] discussed the manufacturer's flawed manufacturing process may result in the production of goods of inferior quality. To ensure that the market's needs can be met by flawless commodities, such defective things must be screened out. Regarding the inspection procedure, the producer should check all completed products during the manufacturing cycle. This requires a specific amount of resources, such as time, equipment, and a workforce (Ullah & Kang [3], Sarkar & Saren [4]). However, the inspection and rework processes are time-consuming, which can delay the manufacturer's on-time delivery and make meeting the retailer's order deadlines even more challenging. The Rosenblatt and Lee [5] study is one of the first to examine how flawed production processes affect economic production quantity (EPQ) and economic order quantity (EOQ). A factory's quality control rate may be lower than producibility because of a lack of manual labor or sophisticated test equipment, creating the IRI situation discussed by Zhou et al. [6]. An EPQ model for a batch production facility and manufacturer capable of producing products of varying quality. The manufacturer has lots of goods, and systems are expected to be used to evaluate the quality of each batch of products. The manufacturer's inspection rate may play a variety of functions depending on the inspection conditions for the timely delivery of flawless goods to the store. This work aims to determine the implications of the observation rate under the IRI and IRS scenarios on the effective logistics strategy, which is thus the first goal of this work. Because the acceptance sampling procedure is imperfect, two different errors may occur. Type 1 errors occur when a lot is deemed unacceptable when it is acceptable. Lot acceptance errors of Type 2 occur when a lot is accepted when it should not have been. In particular, the notable contributions of this paper can be explained as follows: As a result of the relationship between the vendor's checking and fabrication rate, the probability of error occurs during inspection time when this study examines the entire beneficial production policy using two alternative scenarios, which include inspection rate insufficiency (IRI) and inspection rate sufficiency (IRS).

To maximize production and profit, we consider the manufacturer in this task. Items can be produced with imperfections. In the interim, time misclassification may also happen. Here, we choose the Rework Productivity Policy (RPP) for asynchronous rework production time. Using appropriate numerical examples and sensitivity analysis, we demonstrated the effect caused by changes in various parameters. To the best of our knowledge, no one has taken the impact of inspection error and rework for defective goods into consideration up to this point. As a result, our approach offers a fresh managerial perspective that enables a provider to maximize the system's total profit. We will discuss the problem description, the numerical analysis, and the conclusions in the following sections.

2. Literature review

More plausible scenarios are included in the traditional economic production scale model. Numerous scholars have already thought about more effective operational strategies that consider defective products based on EPQ, where the manufacturer is personally liable for quality assurance and whose inspection rate exceeds the demand rate. Al-Salamah [7] developed an EPQ model to examine the scenario involving destructive and accurate acceptance sampling methods and defective manufacturing and inspection. Cunha et al. [8] implemented an EPQ model that considers faulty items with a portion of back-orders, giving a discount for packages of substandard quality. Marchi et al. [9] used the EPQ model to solve the energy efficiency problem; the EPQ model incorporates manufacturing, accuracy, and dependability training with defective materials. Numerous academics have also underlined how important it is for the producers to be in charge of the checking accuracy procedures in the supply system. Su [10] A cashback rewards policy created a holistic inventory model that accounts for defective items and assumes that the manufacturer undertakes 100% scrutiny processes with minimal checking durations. Al Hanbali et al. [11] created a maintenance model including parts quality, lead-time, and inspection errors, and they increased the optimal cost consistency rate as average deterioration per hour. Chen [12] tried to improve the cost of the joint economic lot-size problem (JELP) model and operational strategies for damaged and perishable goods under RPP. In other words, the manufacturer decides whether to repair defective products, and the rework process must be fully finished before the shipment arrives (this situation can be considered a notable case of RPP). The above investigations were carried out in the IRS situation, where the supplier's performance rate sets the maximum limit of readily available flawless quality materials due to the inspector's adequate inspection rate — the IRI scenario. Given the situation, it is essential to determine how IRI affects capacity utilization and develop relevant integrated operational solutions for defective items. In addition, the manufacturer's production system may provide repair or rework facilities, allowing the company to directly apply rework policies for faulty items. Taleizadeh et al. [13] The manufacturer spends a lot of time on quality. The review and rework process will begin after the inspection process is inspected by Zhou et al. [6]. Bazan et al. [14] invigilated that the manufacturer's inspection rate is not less than its manufacturing rate. The processing interval of a vendor can be viewed as equivalent to its analysis process, and the repair process can be carried out after the production process is complete, as finished goods can be inspected until they are manufactured. Bräuer and Buscher [15] discussed the implications of both the relaunch policies on current production policies and the preferences of inclusive warehouse management towards these policies, which are the second topic of this paper and will be addressed in further research on texts. Hsu and Hsu [16] and Yoo et al. [17] designed an inventory model where items are screened, with the potential for Type 1 and Type 2 errors to result in the misclassification of goods. Jaber and Khan [18] considered an economic order quantity model in which items are misclassified at fixed rates based on quality and imperfect screening imperfections. Zheng et al. [19] discussed that after describing the average deterioration rate, the likelihood ratio sequence, and the monotonicity of the average failure time in the average deterioration rate, the original acceptance test is used as an acceptance index to simplify the problem successfully. Cheng et al. [20] developed a new approach that allows consumers to examine the product's operational reliability indicators without needlessly rejecting unreliable samples, which provides a higher chance of product acceptance than traditional acceptance sampling schemes. Salameh and Jaber [21] developed the EPQ model, which takes into account goods with flaws that can be discovered through screening and combined into a single batch that will be sold after the production cycle. Wang et al. [22] explored the optimization of deterioration control limits and periodic inspection intervals and the effect of inspection error.

Malik et al. [23] discussed the essentials of setting up a good coordination framework to resolve SC disagreements. coordination in a two-member SC with a flexible production system under stochastic demand and buyer service level constraints. Kalantari and Taleizadeh [24] proposed an EPQ model for estimating the number of shipments, the size of replenishment lots, and the selling price with rework and multi-shipments into a single model. Chiu et al. [25] proposed that the EPQ model includes several deliveries and a known percentage of defective products. After the production process, the defective goods are either scrapped or repaired. Zhang et al. [26] stated that the impact of preliminary studies with two types of defects is considered in the composite design problem. The quality loss depends on equipment wear and tear and the state of the manufacturing process. Karthick and Uthayakumar [27] detail that defective products are exposed due to equipment malfunctions during production, and the buyer undergoes a quality inspection process to identify faulty products. Cárdenas-Barrón et al. [28] alleviate the existing studies with integer values for quantity of product and freight variety. In this approach, the producer will keep inventory to avoid shortages: one for finished goods produced during the previous cycle and another for goods still in production during the current process. To summarize, the research omits both analyzing the integrated system's preference for various rework policies and incorporating IRI and IRS situations to determine the effects of the quality control rate on integrated and coordinated approaches. Because of this, no studies have examined the impact of verification and revamping procedures on throughput to ensure the viability of derived solutions while considering defective items.

Table 1. Comparison of our model with existing literature

Literature	Model	Reworking policy	Inspection scenario		Error	
		RPP	IRI	IRS	Type 1	Type 2
Su [10]	EPQ			✓		
Lin et al. [29]	EPQ	✓	✓	✓		
Hsu and Hsu [16]	EOQ		✓			
Al-Salamah [7]	EPQ		✓			
Sarkar and Saren [4]	EPQ			✓		
Chen [12]	EPQ		✓			
Marchi et al. [9]	EPQ		✓			
Khan et al. [30]	EOQ		✓			
Kalantari and Taleizadeh [24]	EOQ		✓			
Salameh and Jaber [21]	EOQ	✓				
Rosenblatt and Lee [5]	EOQ/EPQ	✓				
Hsu and Hsu [31]	EPQ				✓	✓
Cárdenas-Barrón et al. [28]	EOQ	✓				
Taleizadeh et al. [13]	EPQ	✓				
Ullah and Kang [3]	EOQ					
Zhou et al. [6]	EOQ	✓				
This paper	EPQ	✓	✓	✓	✓	✓

3. Assumptions

1. A single manufacturer and retailer comprise the incorporated inventory control system.
2. The demand rate is constant and known over the planning horizon.
3. The asynchronous rework policy and the rework procedure are at work here.
4. As per Jaber and Khan [18], the suggested equal-sized distribution policy, the retailer's purchase, and the manufacturing policies of the manufacturer are as follows: given the proportion of faulty items, the manufacturer continuously produces Q units at P_r.
5. The processing and supervision processes are carried out concurrently. Also, two views can be derived from the relationship between the manufacturer's safety check rate and production rate:

The retailer orders q_i (i.e., DT_i) Q units during the order cycle with perfect quality. Then, new units of the highest possible quality are checked and shipped to the merchant once each cycle until the manufacturer's inventory level is zero.

6. A lot is accepted only if the sample contains no defective items and no missorted items due to a Type 1 error, denoted by E_1, or if the model includes more than one defective item and misclassification due to a Type 2 error, marked by E_2.
7. The producer verification rate is lower than its consumption rate, according to Bazan et al. [14].

4. Model formulation

The retailer's and the manufacturer's order policies can be expressed as follows, using the approach to equal-sized supplies that Jaber and Khan [18] suggested. The retailer orders q_i (i.e., DT_i) units of flawless quality during the order cycle T_i according to market requirements D. Given the number of defective goods, the producer consistently turns out units at a rate of R_p. Then, until the manufacturer's inventory is depleted, new units with the highest possible quality are examined and sent to the retailer once every cycle.

4.1 *Classification of acceptance probability P_a*

The basic definition of sampling is the probability that the lot will be accepted and, therefore, shipped to the retailer is

$$P_a = P(X \geq 1)E_2 + P(X = 0)(1 - E_1).$$

The quantity of flawed items in the sample of n is the random variable X in this formulation. Therefore, a lot is only accepted if it contains no defective items and no misclassification due to a Type 1 error, denoted by E_1, or if it contains more than one defective item and there is a misclassification due to a Type 2 error, denoted by E_2. We consider E_1 and E_2 constants and aspects of the sampling procedure. The likelihood that the lot won't be approved and sent for screening is $1 - P_a$. The random variable X is the number of defective items in an n-item sample. If X is approximately a binomial random variable with a probability distribution function, if the lot size is assumed to be large

$$f(x) = \binom{n}{x} \mu^x (1 - \mu)^{n-x}. \tag{1}$$

If the acceptance number X is set to zero, the likelihood that the sample contains no defective items is

$$P(X = 0) = (1 - \mu)^n. \tag{2}$$

Therefore, the likelihood the lot will be accepted as described in

$$P_a = (1 - (1 - \mu)^n)E_2 + (1 - \mu)^n (1 - E_1) = (1 - \mu)^n (1 - E_1 - E_2) + E_2. \tag{3}$$

4.2 *Rework priority policy under IRI scenario*

Rework-priority policies require the start of the reworking procedure by the manufacturer for damaged goods before sending finished goods to the retailer. In Figure 1, we consider a firm with a flawed production system. Manufactured goods are divided into two categories: defective and non-defective. As products are produced in batches or lots, the manufacturer will still have a substantial inventory of manufactured goods. It is presumed that the acceptance number is zero. Although lots can be misclassified with frequent Type 1 and Type 2 errors, the inspection procedure is not flawless. The anticipated lot size will be sent to the perfect goods warehouse if P_a is the likelihood that a lot is accepted. The manufacturing rate of the enterprise, in this case, is more significant than its checking rate ($\alpha < P_r$), which causes the production cycle of the enterprise to be shorter than its inspection period, i.e., $L_r < T_{L_1}$ and consider $Q = m_i DT_i$ (i = 1 IRI scenario, i = 2 IRS scenario). After the entire inspection procedure is finished, defective products are reworked in these cases. Since damaged goods can be repaired and distributed as usual.

1. The producer creates Q total bunches of units (i.e., $m_1 q_1$) with a production rate of P_r during the production process L_p (note the dotted line in Figure 1).
2. As part of the inspection procedure L_i, the manufacturer's perfect products are gathered at a rate of $\alpha(1 - \mu)$, which includes $q_1(1 - \mu)$ units of perfect items.
3. For the remainder of the first cycle ($L_p - L_i$), the producer begins to fix partially flawed products into flawless ones at a rapid rate of R_w, guaranteeing the manufacturer's inventory for perfect items equals the retailer's first

lot q_1. Every T_1 cycle, the manufacturer completes the reworking process and sends q_1 units to the store in the highest possible quality.

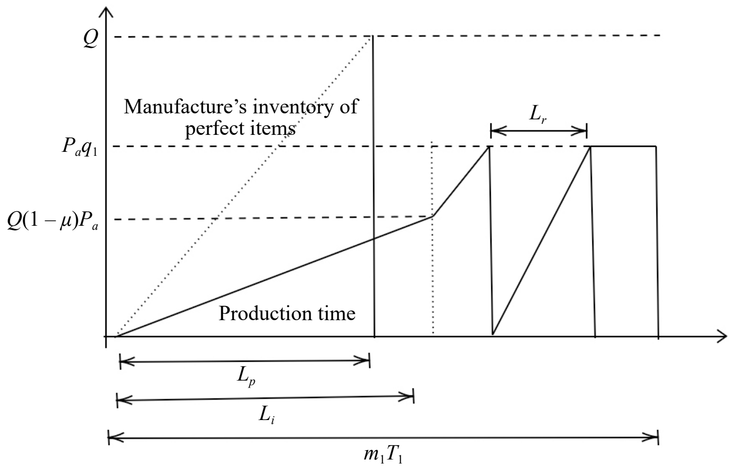

Figure 1. IRI inventory level

Additionally, in Figure 2, lines AC and CD represent the manufacturer's quality control rate for perfect items ($\alpha(1-\mu)$) and the manufacturer's repair rate (R_w) for defective items, respectively. Line AB represents the manufacturer's production rate (P_r), including inadequate and perfect items. Under RPP, $Q = T_{L_1} = m_1 D T_1$, then the total production is

$$\frac{Q}{P_r}. \tag{4}$$

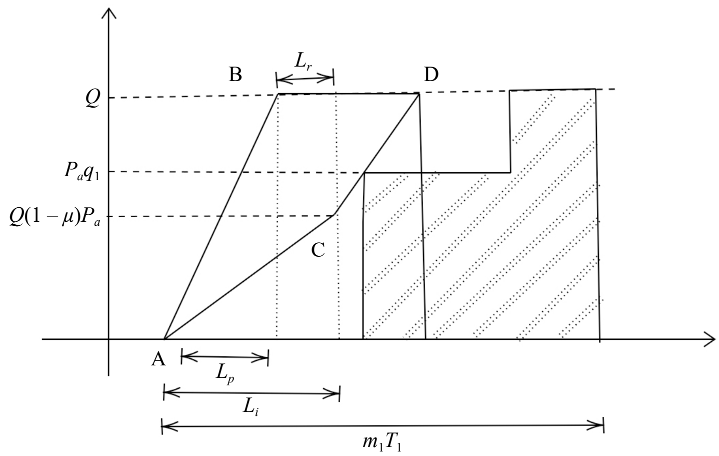

Figure 2. IRI supply chain inventory model

During the inspection, the perfect quantity is

$$\alpha(1-\mu), \tag{5}$$

which is including $Q(1-\mu)$.

Meanwhile, an asynchronous rework policy is applied to defective items. The manufacturer will charge for returned items. Depending on their quality, returned items are sent to the rework stage or to salvage. The lot is sent to the screening stage, where the products are separated into non-defective and reworkable if the inspector deems the lot unacceptable. A percentage of lots of products can be fixed, and these products can only be sold in the market. Reworkable items can be fixed at R_w per time unit using the rework procedure. The manufacturer thus maintains three different types of inventory: work-in-process, inventory of non-defective goods sent to the main market, and inventory of reworkable goods.

A manufacturer who produces large lot size needs to send their product to the primary market by acceptance in cycle T_1.

The retailer's first order quantity is

$$q_1 P_a. \tag{6}$$

The length of the production is

$$L_p = \frac{Q}{P_r} = \frac{m_1 D T_1}{P_r}. \tag{7}$$

The length of the reworked item, which includes misclassification goods, is

$$L_r = \frac{Q(1-P_a)\mu}{R_w} = \frac{m_1 T D \mu (1-P_a)}{R_w}. \tag{8}$$

The length of the inspection time is

$$L_i = \frac{Q P_a (1-\mu)}{\alpha(1-\mu)} = \frac{m_1 D T_1 P_a}{\alpha}. \tag{9}$$

Since total production amount for the perfect item during T_{L_1} is equal to retailer order quantity,

$$T_{L_1} = q_1(1-\mu)P_a + (Q - q_1[(1-\mu)(1-P_a) + \mu R_w]). \tag{10}$$

Reworked for the defective items

$$L_r = \frac{Q\mu}{R_w} = \frac{m_1 D T_1 \mu}{R_w}. \tag{11}$$

A flawless production process leads to flawless production quantity is

$$\begin{aligned} q_1 &= Q(1-\mu)P_a + R_w(T_{L_i} - L_i)(1-P_a) \\ T_{L_1} &= \frac{D T_1}{(1-P_a)R_w}\left(1 - m_1(1-\mu)P_a - \frac{R_w(1-P_a)}{\alpha}\right). \end{aligned} \tag{12}$$

Under the rework priority policy, to fulfill the retailer's initial order lot, it is also necessary to rework partially damaged items and produce and inspect q_1 units. The following statement can be summarized by noting that the manufacturer's number of shipments should not be fewer than 1, $\frac{1}{1-\mu} \geq m_1 \geq 1$.

Based on the Figure 1, ABCD manufacturer's inventory with defective items is

$$\frac{Q}{2}(2L_i - L_p) - \frac{1}{2}(L_i(1-\mu)Q) + \frac{1}{2}L_r\mu Q = \frac{(m_1DT_1)^2}{2}\left[\frac{2P_a}{\alpha} - \frac{1}{P_r} - \frac{1}{2\alpha}(1-\mu)qP_a^2 + \frac{1}{2}\frac{\mu(1-P_a)}{R_w}\right]. \tag{13}$$

The manufacturer's accumulated inventory of perfect items is

$$m_1DT_1\left[2(m_1-1) + \frac{2Dt_1}{(1-P_a)R_w}(1-m_1(1-\mu)P_a) - \frac{R_w(1-P_a)}{\alpha} - \frac{m_1DT_1}{P_r}\right]$$
$$+\frac{(m_1DT_1)^2}{2}\left(\frac{1-\mu}{\alpha} - \frac{1}{P_r} + \frac{\mu^2}{R_w}\right) - \frac{m(m-1)}{2}DT_1^2. \tag{14}$$

Setup cost per production

$$\frac{c_m}{m_1T_1}. \tag{15}$$

Production cost per unit is

$$\frac{Qc_p}{m_1T_1}. \tag{16}$$

Inspection cost per production is

$$Qc_{is}. \tag{17}$$

Rework cost per production is

$$C_{rw}\mu(1-P_a). \tag{18}$$

The manufacturer's total production of goods for flawless items during the manufacturing period

$$\frac{c_{h_1}D^2T_1^2}{2(1-P_a)R_w}\left(1-m_1(1-\mu)P_a - \frac{R_w(1-P_a)}{\alpha}\right). \tag{19}$$

The holding cost for defective items incurred by the producer is

$$\frac{c_{h_2}m_1^2D^3T_1^3}{4}\left[\frac{2P_a}{\alpha} - \frac{1}{P_r} - \frac{1}{2\alpha}(1-\mu)qP_a^2 + \frac{1}{2}\frac{\mu(1-P_a)}{R_w}\right]. \tag{20}$$

Expected sales revenue is

$$a\left(Q(1-\mu)P_a + Q(1-\mu)(1-P_a) + \mu R_w\right). \tag{21}$$

The total cost included the manufacturer's setup cost, production cost, inventory holding cost for defective items and perfect items, and transportation cost.

The total cost per unit of time in RPP is

$$TC_1 = \frac{c_m}{m_1 T_1} + \frac{c_p Q}{m_1 T_1} + Qc_{is} + c_t Q + c_{rw}\mu(1-P_a) + \frac{c_{h_1} D^2 T_1^2}{2(1-P_a)R_w}\left(1 - m_1(1-\mu)P_a - \frac{R_w(1-P_a)}{\alpha}\right)$$
$$+\frac{c_{h_2} m_1^2 D^3 T_1^3}{2}\left[\frac{2P_a}{\alpha} - \frac{1}{P_r} - \frac{1}{2\alpha}(1-\mu)q_1 P_a^2 + \frac{1}{2}\frac{\mu(1-P_a)}{R_w}\right]. \tag{22}$$

The expected net total profit is = Sales revenue – Total cost

$$TP_1 = a\left(Q(1-\mu)P_a + Q(1-\mu)(1-P_a) + \mu R_w\right)$$
$$-\left[\begin{array}{l}\frac{c_m}{m_1 T_1} + \frac{c_p Q}{m_1 T_1} + Qc_{is} + c_t Q + c_{rw}\mu(1-P_a) + \frac{c_{h_1} D^2 T_1^2}{2(1-P_a)R_w}\left(1 - m_1(1-\mu)P_a - \frac{R_w(1-P_a)}{\alpha}\right) \\ +\frac{c_{h_2} m_1^2 D^3 T_1^3}{2}\left[\frac{2P_a}{\alpha} - \frac{1}{P_r} - \frac{1}{2\alpha}(1-\mu)q_1 P_a^2 + \frac{1}{2}\frac{\mu(1-P_a)}{R_w}\right]\end{array}\right] \tag{23}$$

where $P_a = (1-E_1-E_2)(1-\mu)^n + E_2$.

4.3 *Discussion of the IRS scenario*

In this part, the manufacturer's inspection rate exceeds its production rate, i.e., ($\alpha \geq P_r$). Under these conditions, the inspection process can be concluded when the entire production run is finished, indicating that $L_i = L_P = \frac{m_2 D T_2}{P_r}$. Later, the remanufacturing process can start at the end of the entire manufacturing process. Moreover, the variation of inventory level under the RPP is shown in Figure 3.

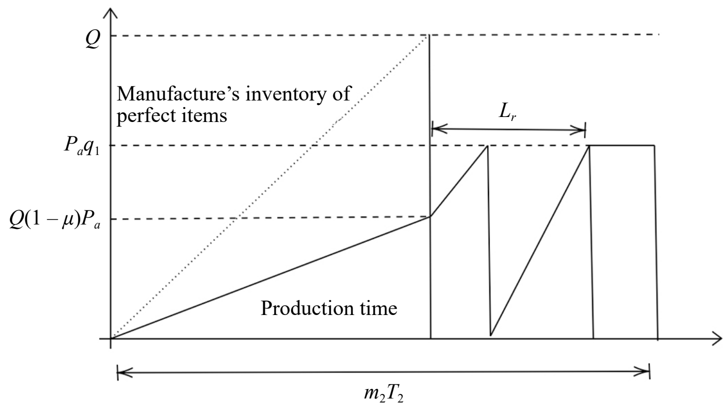

Figure 3. IRS inventory level

4.3.1 *Rework priority policy under IRS scenario*

Under the IRS of RPP, the manufacturer's production amount includes manufacturing and safety checks, and $T_{L_2} - L_i$ is a partial reworking process. Because for perfect items, the retailer's first order lot equals the manufacturer's production volume amount during T_{L_2}, we obtain

$$q_1 = (1-\mu)T_2 P_r + R_w(T_{L_2} - L_p),$$

which can be rewritten as

$$T_{L_2} = \frac{DT_2}{R_w}\left[1 - \frac{(1-\mu)P_r}{D} - \frac{m_2 R_w}{P_r}\right]. \tag{24}$$

Under this circumstance, $L_i \leq T_{L_2}$ can transform $\frac{1}{1-\mu} \geq m_2 \geq 1.$ As a result, in both the IRI and IRS scenarios, the probability of failure always limits the manufacturer's number of RPP consignments. Furthermore, it is clear that

$$L_r = \frac{m_2 DT_2 \mu}{R_w}. \tag{25}$$

In addition, under the IRS method, Figure 4 shows that the manufacturer production rate for perfect items is $Q(1-\mu)$. According to the graph, ABDC includes ABC and BCD, indicating the manufacturer's level of inventory was compiled with defective items per manufacturing

$$= \left((1-P_a)L_r\mu + L_p\mu\right)\frac{m_2 DT_2}{2} = \frac{(m_2 DT_2)^2 \mu}{2}\left(\frac{(1-P_a)\mu}{R_w} + \frac{1}{P_r}\right). \tag{26}$$

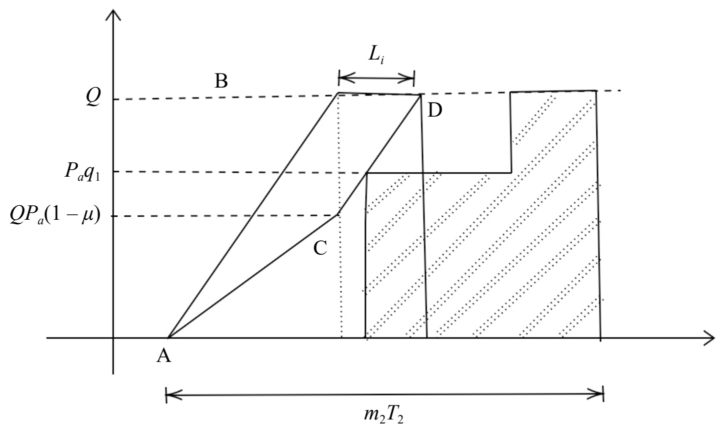

Figure 4. IRS supply chain

Furthermore, perfect products from the manufacturer's stock level is

$$= \frac{m_2 DT_2}{2}[2(m_2-1)T_2 + 2T_{L_2} - L_p] - \frac{(m_2 DT_2)^2 \mu}{2}\left(\frac{(1-P_a)\mu}{R_w} + \frac{1}{P_r} - \frac{m_2(m_2-1)}{2}DT_2\right)$$

$$= \frac{DT_2}{2}\left[2m_2(m_2-1)T_2 + \frac{2DT}{R_w}\left[1 - \frac{(1-\mu)P_r}{D} - \frac{m_2 R_w}{P_r} - \frac{m_D T_2}{P_r}\right] - \mu\frac{m^2 DT_2}{2}\left(\frac{(1-P_a)\mu}{R_w}\frac{1}{P_r}\right) - \frac{m_2(m_2-1)T_2}{2}\right]. \tag{27}$$

The total cost function per unit time under RPP can be formulated as follows: Expected sales revenue is

$$a\left(Q(1-\mu)P_a + Q(1-\mu)(1-P_a) + \mu R_w\right) \tag{28}$$

$$TC_2 = \frac{c_m}{m_2T_2} + \frac{c_pQ}{m_2T_2} + Qc_{is} + c_tQ + c_{rw}\mu(1-P_a) + \frac{c_{h_1}D^2T_2^2}{2}$$
$$\left[2m_2(m_2-1)T_2 + \frac{2DT}{R_w}\left[1 - \frac{(1-\mu)P_r}{D} - \frac{m_2R_w}{P_r} - \frac{m_DT_2}{P_r}\right] - \mu\frac{m^2DT_2}{2}\left(\frac{(1-P_a)\mu}{R_w} + \frac{1}{P_r}\right) - \frac{m_2(m_2-1)T_2}{2}\right]$$
$$+\frac{c_{h_2}(m_2^2(DT_2)^3\mu}{2}\left(\frac{(1-P_a)\mu}{R_w} + \frac{1}{P_r}\right) \tag{29}$$

Expected total profit = Sales revenue – Total cost

$$TP_2 = a\left(Q(1-\mu)P_a + Q(1-\mu)(1-P_a) + \mu R_w\right)$$
$$-\left(\begin{array}{l}\dfrac{c_m}{m_2T_2} + \dfrac{c_pQ}{m_2T_2} + Qc_{is} + c_tQ + C_{rw}\mu(1-P_a) + \dfrac{Ch_1D^2T_2^2}{2} \\ \left[\begin{array}{l} 2m_2(m_2-1)T_2 + \dfrac{2DT}{R_w}\left[1 - \dfrac{(1-\mu)P_r}{D} - \dfrac{m_2R_w}{P_r} - \dfrac{m_2DT_2}{P_r}\right] \\ -\mu\dfrac{m_2DT_2}{2}\left(\dfrac{(1-P_a)\mu}{R_w} + \dfrac{1}{P_r}\right) - \dfrac{m_2(m_2-1)T_2}{2}\end{array}\right] + \dfrac{ch_2(m_2^2(DT_2)^3\mu}{2}\left(\dfrac{(1-P_a)\mu}{R_w} + \dfrac{1}{P_r}\right)\end{array}\right), \tag{30}$$

where $P_a = (1-E_1-E_2)(1-\mu)^n + E_2$.

5. Solution procedure

Our aim is to find the maximum profit.

Theorem 1. The total profit function in equation TP_2 (22) is concave from an unknown variable Q.

Proof. It is necessary to verify the profitability of the IRI scenario and to check its concavity in terms of the differential equation TP_1 (22) concerning Q

$$P_1Q = a\left((1-\mu)P_a + (1-\mu)(1-P_a) + \mu R_w\right)$$
$$-\left[\frac{c_p}{m_1T_1} + c_{is} + c_t + c_{rw}\mu(1-P_a) + c_{h_2}DT_1Q\left[\frac{2P_a}{\alpha} - \frac{1}{P_r} - \frac{1}{2\alpha}(1-\mu)q_1P_a^2 + \frac{1}{2}\frac{\mu(1-P_a)}{R_w}\right]\right] \tag{31}$$

where $P_a = (1-E_1-E_2)(1-\mu)^n + E_2$.

$$\frac{\partial^2 TP_1}{\partial Q^2} = \left[c_{h_2}DT_1\left[\frac{2P_a}{\alpha} - \frac{1}{P_r} - \frac{1}{2\alpha}(1-\mu)q_1P_a^2 + \frac{1}{2}\frac{\mu(1-P_a)}{R_w}\right]\right] \tag{32}$$

where $P_a = (1-E_1-E_2)(1-\mu)^n + E_2$.

$$\text{i.e.,} \frac{\partial^2 TP_1}{\partial Q^2} > 0.$$

Therefore, it satisfied the IRI scenario.

Result 1.1. From the equation (31), $\dfrac{\partial TP_1}{\partial Q} = 0$. We can find the Q value as

$$Q_1 = \frac{a\left((1-\mu)P_a + (1-\mu)(1-P_a) + \mu R_w\right) - (\frac{c_p}{m_1 T_1} + c_{is} + c_t + c_{rw}\mu(1-P_a)}{c_{h_2} D T_1 \left[\frac{2P_a}{\alpha} - \frac{1}{P_r} - \frac{1}{2\alpha}(1-\mu) q_1 P_a^2 + \frac{1}{2}\frac{\mu(1-P_a)}{R_w}\right]}. \tag{33}$$

Theorem 2. The total profit function in equation TP_2 (29) is concave from an unknown variable Q.

Proof. Using a differential equation TP_2 (29) to test the IRS scenario to see if it can result in greater profitability concerning Q

$$TP_2 Q = a\left((1-\mu)P_a + (1-\mu)(1-P_a) + \mu R_w\right) - \left(\frac{c_p}{m_2 T_2} + c_{is} + c_t + c_{rw}\mu(1-P_a) + c_{h_2}\left(Q(DT_2)\mu\left(\frac{(1-P_a)\mu}{R_w} + \frac{1}{P_r}\right)\right)\right. \tag{34}$$

where $P_a = (1-E_1-E_2)(1-\mu)^n + E_2$.

Similarly, calculate the second-order derivative to check convexity,

$$\frac{\partial^2 TP_2}{\partial Q^2} = c_{h_2} D T_2 \mu \left(\frac{(1-P_a)\mu}{R_w} + \frac{1}{P_r}\right) \tag{35}$$

where $P_a = (1-E_1-E_2)(1-\mu)^n + E_2$.

$$\text{i.e.,} \frac{\partial^2 TP_2}{\partial Q^2} > 0.$$

So, it satisfied the IRS scenario.

Result 2.1. From the equation (34), $\frac{dTP_1}{dQ} = 0$.

We can find the Q value as

$$Q_2 = \frac{a\left((1-\mu)P_a + (1-\mu)(1-P_a) + \mu R_w\right) - \left(\frac{c_p}{m_2 T_2} + c_{is} + c_t + c_{rw}\mu(1-P_a)\right)}{c_{h_2} D T_2 \mu \left(\frac{(1-P_a)\mu}{R_w} + \frac{1}{P_r}\right)}. \tag{36}$$

6. Numerical examples

Take the example of a manufacturer who uses an imperfect manufacturing process. Before a lot can be dispatched to a store or market, it must undergo an approval sampling stage. Two methods are available to the manufacturer for assessing quality characteristics: destructive and non-destructive. A lot must not contain damaged goods to be sent to a retailer. In rejecting lots, they will proceed to a more costly screening stage, separating them into non-defective, reworkable categories. For two alternative rework policies, numerical examples are run under the IRI or IRS scenario to determine the best-integrated production inventory solution that considers defective goods. To demonstrate the feasibility of the operational solutions derived from the integrated system, the performance of the integrated system is compared.

6.1 *Base case*

The curves have been created for two values of E_1 and E_2 and three values of the probability of a defective item, μ

= 0.1,0.2, and 0.3. Figure 5 shows the case when E_1 = 0.05 and E_2 = 0.04, whereas the sold lines are for that scenario. The sample sizes were chosen to range from 1 to 15. Table 2 shows that when the length of the cycle increases, the level of probability of acceptance will reduce.

After that defective rate, μ = 0.7 has the probability of acceptance being stable and maintained at 0.05. The same observation holds for sample sizes extending beyond the plot's right side and the plot's right side. The sampling scheme shows its maximum ability to detect the probability of error for sample sizes in the center of the range. From Table 2, μ = 0.1 and 0.2, the difference between the two defective rates at cycle length m = 5 is 0.2391, a significant difference. So, we conclude that P_a is less responsive to μ for small and large n and more sensitive to μ for a medium sample size of n.

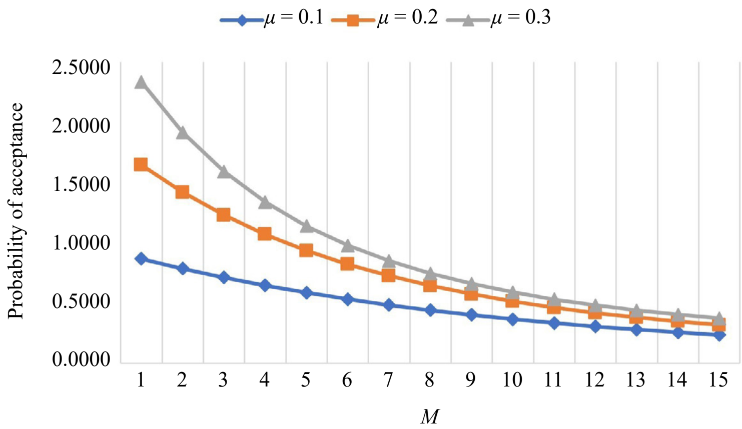

Figure 5. Defective rate versus probability of acceptance

Table 2. Defective rate and probability of acceptance

m	μ = 0.1	μ = 0.2	μ = 0.3
1	0.8690	0.7780	0.6870
2	0.7871	0.6324	0.4959
3	0.7134	0.5159	0.3621
4	0.6471	0.4227	0.2685
5	0.5873	0.3482	0.2029
6	0.5336	0.2886	0.1571
7	0.4853	0.2408	0.1249
8	0.4417	0.2027	0.1025
9	0.4026	0.1721	0.0867
10	0.3673	0.1477	0.0757
11	0.3356	0.1282	0.0680
12	0.3070	0.1125	0.0626
13	0.2813	0.1000	0.0588
14	0.2582	0.0900	0.0562
15	0.2374	0.0820	0.0543

Example 1. Let us assume the values in the IRI scenario. Take cycle values is one-quarter period: a = 280 per unit,

c_s = 1.2 per unit, c_{is} = 0.6 per unit, c_m = 0.5 per unit, R_w = 1.85 per unit, c_p = 0.75 per unit, c_t = 0.8 per unit, c_{rw} = 2.82 per unit, c_{h_1} = 0.4 per unit, c_{h_2} = 0.7 per unit, μ = 0.1, m_1 = 15, p_a = 0.2374, T_1 = 1, D = 150 per cycle, α = 320 per cycle, P_r = 350. Put the values in equation (33) and get the Q value for the IRI equation (31); and the total profit is 73784230.54. Since $\alpha < P_r$, an increased upper limit of m_1 and an increase in μ eventually cause the integrated system to perform better under RPP. In particular, if μ is above a certain threshold, the probability of acceptance is 0.05.

Example 2. In the IRS scenario, i.e., $\alpha > P_r$, let us take values from the previous example except α = 350 per cycle and P_r = 320, applying values in the equation (36) and we get Q = 1186.37, values together in the equation (34) that's the total profit is 196926073.36. In comparison of Examples 1 and 2, the IRS scenario gets 7.8% more profit than the IRI scenario at $m_1 = m_2 = 15$, i.e., same length of time. The defective increase is 0.5, and the cycle length is 15; the IRS scenario profit is higher than the IRI scenario profit. There is a 52.74% difference in profit between the IRS and other scenarios. As a result, IRI scenario drops its profit dramatically if defective rates increase in each scenario.

7. Sensitivity analysis

Through sensitivity analysis, acceptance sampling has significantly affected lot sizes, estimated net profit, and expected cycle length. Ideal lot sizes are positively affected by the probability of defective material but negatively predict net profit.

1. Selling price, manufacturing costs, inspection costs, manufacturing costs, and transportation costs do not change from the order of magnitude of the ideal. Still, there are differences in defective goods and holding costs for demand.
2. Table 3 shows that in the IRI scenario, if demand decreases by 50%, profit will decrease by 23.74%, demand will decrease by 33.33%, and if demand increases by 50%, profit will increase by 57.82% and increase by 37.83%.
3. Table 3 indicates the IRS scenario: if demand had decreased by 50%, profits would have decreased by 58.71%, whereas if demand had increased by 50%, profits would have increased by 92.97%.
4. The transportation, inspection, production, and rework costs for both the IRI and IRS scenarios have no bearing on the overall profit or ideal order quantity.
5. Rework is critical in all scenarios, with the IRI scenario decreasing profit by up to 67% and raising it by up to 65.62%. The IRS scenario improved profit by up to 93.62% and dropped it by up to 63.57%.
6. Figure 6 demonstrates the variation in demand between the IRI and IRS situations.

Table 3. Parameter analysis

Parameters	Changes (%)	IRI		IRS	
		Q	TP_1	Q	TP_2
a	-50.00	1320.2586	1329532402	545.5709	150468609.18
	-25.00	1320.2586	1329615591	824.3376	150550737.12
	25.00	1320.2586	1329781970	1381.8711	150814629.57
	50.00	1320.2586	1329865159	1660.6379	150996394.08
c_p	-50.00	1320.2586	1329698781	1103.1044	150666077.2
	-25.00	1320.2586	1329698781	1103.1044	150666077.2
	25.00	1320.2586	1329698781	1103.1044	150666077.2
	50.00	1320.2586	1329698781	1103.1044	150666077.2
c_{is}	-50.00	1320.2586	1329699177	1103.8385	150666534.4
	-25.00	1320.2586	1329698979	1103.4714	150666305.8
	25.00	1320.2586	1329698583	1102.7373	150665848.7
	50.00	1320.2586	1329698385	1102.3703	150665620.1
c_m	-50.00	1320.2586	1329698781	1103.1044	150666077.3

Table 3. *Continued*

Parameters	Changes (%)	IRI		IRS	
		Q	TP_1	Q	TP_2
	-25.00	1320.2586	1329698781	1103.1044	150666077.3
	25.00	1320.2586	1329698781	1103.1044	150666077.2
	50.00	1320.2586	1329698781	1103.1044	150666077.2
R_w	-50.00	1317.8919	1326991616	503.9463	538068205.6
	-25.00	1319.0441	1328310356	793.9562	252307837.4
	25.00	1321.4108	1331014484	1434.1825	102028692.6
	50.00	1322.563	1332328702	1787.0031	74761984.29
c_p	-50.00	1320.2586	1329698830	1103.2420	150666149.4
	-25.00	1320.2586	1329698855	1103.1732	150666113.3
	25.00	1320.2586	1329698756	1103.0356	150666041.1
	50.00	1320.2586	1329698731	1102.9667	150666005.1
	25.00	1320.2586	1329698807	1103.1044	188267292.1
	50.00	1320.2586	1329698833	1103.1044	225868507.1
c_{h_2}	-50.00	659.62632	165956862.2	2181.1905	150922953.2
	-25.00	990.82261	562144371.2	1465.0207	150752346.2
	25.00	1649.6945	2593868852	884.4458	150613887.3
	50.00	1980.3878	4487112843	738.2254	150578922.1
D	-50.00	990.19392	561075241	2181.1905	18875988.84
	-25.00	1485.2909	1893163590	1465.2039	63109456.69
	25.00	2475.4848	8763543304	884.5126	296952614.8
	50.00	2970.5818	15143075272	738.2254	517921193.2

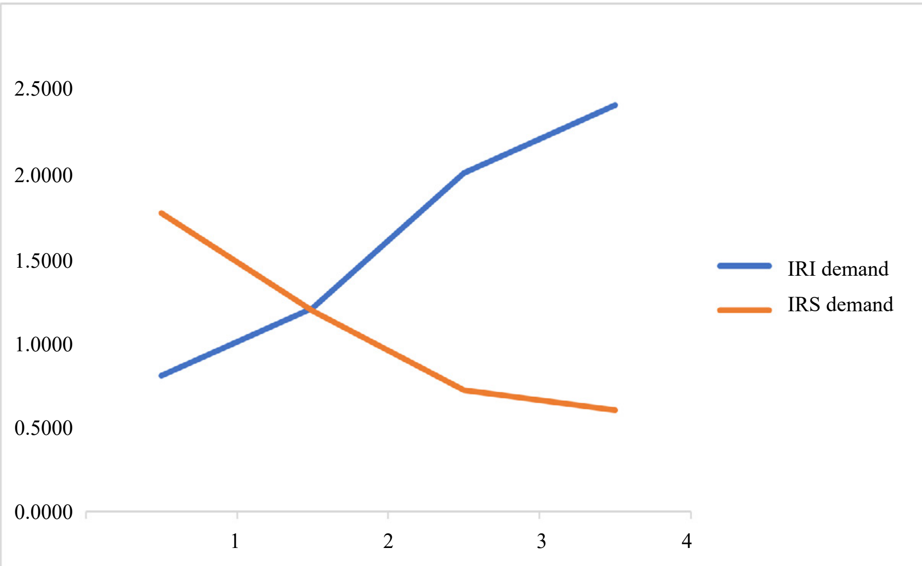

Figure 6. IRI scenario demand versus IRS scenario demand

7. In Figure 7, defective item holding costs are also an important factor in determining the quantity to order.

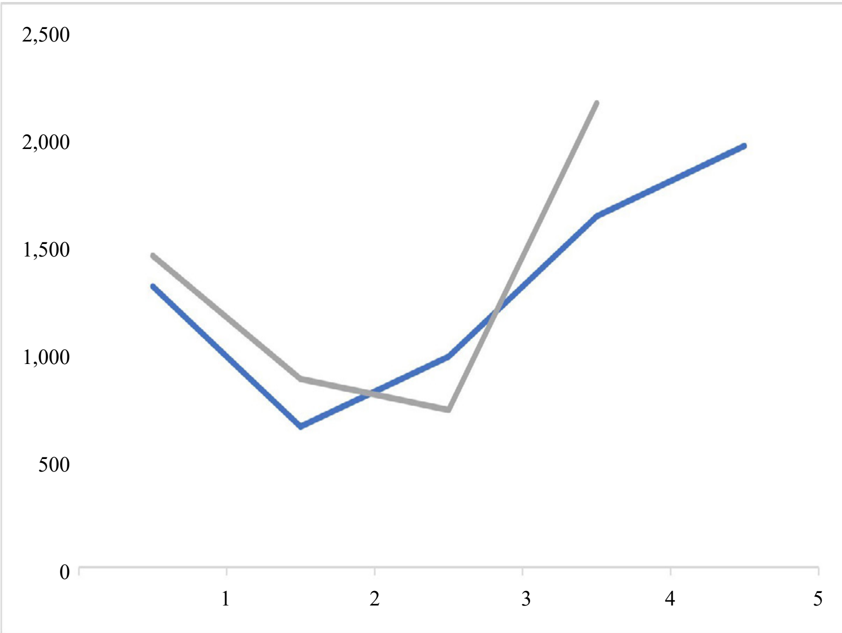

Figure 7. Quantity variation about holding cost

8. Figures 8 and 9 illustrate that profitability and quantity go down dramatically as the defective rate increases.

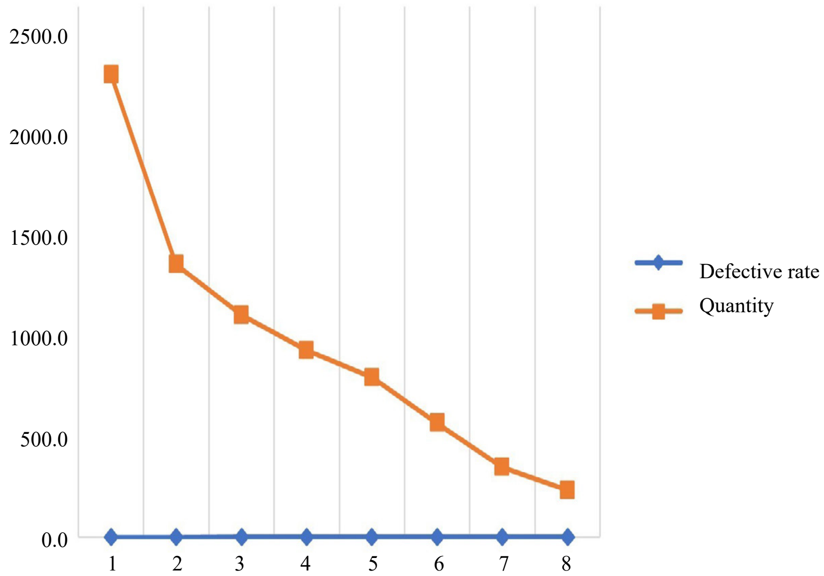

Figure 8. Quantity rate versus defective rate

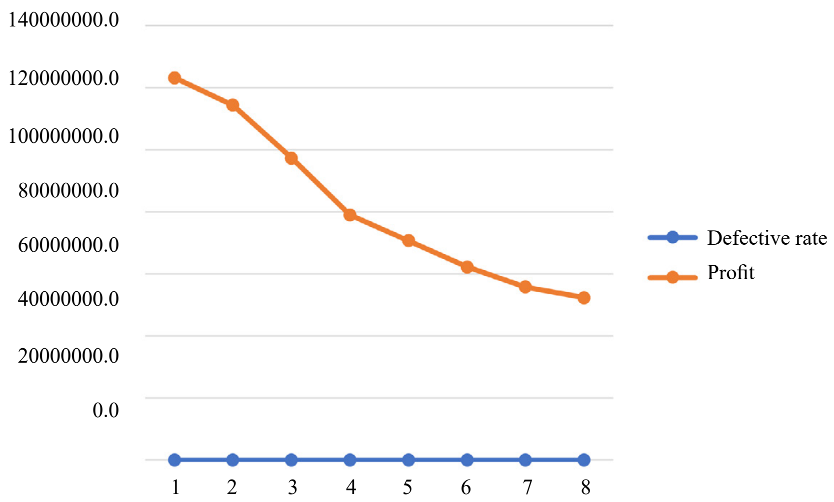

Figure 9. Profit rate versus defective rate

9. The analysis of the impact of Type 1 error probability concluded that it is more cost-effective to raise the lot size for higher values of mistake probability.
10. It is discovered that a smaller lot size and a longer cycle time are required as the probability increases. Additionally, the effects of the other model parameters have been explored.

8. Conclusion

This article presents the EPQ model for imperfect quality, the single acceptance model and misclassification errors, defective goods with an equal export policy, and manufacturer control over production, inspection, and rework processes. The IRI and IRS scenarios are the foundations upon which this study bases its analysis of the interaction between the manufacturer's total production and the inspection rate. Additionally, two separate rework policies are applied to the number of shipments in each situation. Optimal lot sizes that maximize objective functions are identified and demonstrated, and annualized net profit functions are also developed. The manufacturer's inspection and reworking processes may need to be fixed, which could significantly lower the impeccable production volume of the manufacturer's goods per unit of time and make it more challenging for the manufacturer to complete the product. The analysis of the impact of Type 1 error probability concluded that it is more expensive to raise the lot size for higher values of mistake probability. A study of the EPQ model is conducted to determine how it reacts to changes in the likelihood of Type 1 and Type 2 errors. It is discovered that, as the probability increases, a smaller lot size and a longer cycle time are required. The impacts of the other model parameters have also been studied and analyzed.

8.1 *Further research*

In additional research, the suggested paradigm does not permit the backorder. Trade credits and cost savings can help with this. Using the Joint Economic Lot-size Problem (JELP) model as an extension, this solution resolves the issues with traditional optimization strategies. It may be possible to divide the primary and secondary markets by reworking the priority problem and rewriting the delivery priority problem for multiple retailers.

Acknowledgments

We are grateful for the referees' suggestions that improved the content and presentation of our article.

Conflict of interest

The authors declare that they do not have any competing financial interests or personal relationships that may appear to have influenced the work reported in this paper.

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