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Research Article

Feasibility and Stability Analysis for Basic Measles Model Using Fuzzy Parameter

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Abstract: In this article, we investigate the behaviors of the measles viral pandemic using fuzzy susceptible-infectious-recovered (SIR) model. To examine the effects of various compartment phases, we analyze disease-free equilibrium points along with basic reproduction number. The measles model is stated to be globally asymptotically stable at the disease-free equilibrium point. In order to mathematically simulate the measles, we use a first-order nonlinear differential equation. The numerical solution is computed using the Runge-Kutta method, and the model's feasibility is also covered.

***Keywords*:** SIR model, stability analysis, basic reproductive number, Runge-Kutta method, feasibility

MSC: 46N60

1. Introduction

Measles is unquestionably one of the most harmful of the various ailments that have harmed humankind throughout the years. History demonstrates that measles epidemics have happened quite often. Here, we recommend an analytical method for analyzing epidemics that is based on the established susceptible-infectious-recovered (SIR) model. We discover that this method may produce a prediction parameter that can be utilized to determine whether a local or global immunization campaign should be launched. In the SIR model, S represents the proportion of the population that is at risk of contracting the disease, I represents the proportion of the population that is already ill, and R represents the proportion of the population that has recovered.

Prior to the invention of the measles vaccine in 1963 and extensive vaccination, measles caused about 1.2 billion casualties annually. Rapid immunization has significantly decreased the number of measles deaths. From 2000 until 2017, there were only 110,000 cases of measles globally, an 80% fall from 545,000 cases.

A high body temperature appears between ten and twelve days after being exposed to the measles virus. Congestion, leaky eyes, red eyes, and a stuffy nose, as well as little white patches within the cheeks, might also appear in the early stages. After a couple of days, rashes on the face and the top of the neck will typically appear. After three days, the rashes move to the feet and hands before disappearing five to six days later. As a result, the rash will typically

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start to form.

Finding possible outbreaks in small pockets of a certain geographic area before the virus spreads to the entire country or the world is a challenge for epidemiologists. As a result, the suggested approach has been developed that enables one to decide whether an outbreak constitutes an epidemic or not based on the resolution of the SIR model's differential equations.

Most deaths associated with measles are caused by consequences. Encephalopathy, eyesight issues, irritable bowel syndrome, exhaustion, or extreme lung problems, including bronchitis, are among the most significant side effects. Undernutrition, infants, particularly those lacking vitamin A, or individuals whose physical health has been disrupted by HIV/AIDS or other disorders are more likely to get severe measles.

Certain measles consequences can be managed with intensive care that provides correct nutrition, appropriate hydration intake, and dryness therapy with an oral rehydration solution suggested by the World Health Organization. Major public health measures to lower the number of measles deaths worldwide include routine vaccination of children against the disease and bulk immunization drives in nations with high rates of cases and fatalities [1].

In order to evaluate the accuracy and sensitivities of identifying the anomalies present in the data under a minimum expense false-positive rate, an adaptive target level identification method is developed by Mohtashemi et al. [2] and Wu et al. [3] found that whenever the funding for vaccination is constrained such that perhaps the overall quantity of infected persons is as low as possible throughout the lifetime of an epidemic, an optimal design utilizing a structured susceptible-infectious (SI) model is constructed to achieve the best vaccination rates for separate categories at varying risk levels. In cases where the treatment's capability is low, a backward bifurcation was discovered by Zhou et al. [4]. The propagation of tuberculosis was discussed in this article by Side et al. [5], utilizing the Susceptible-Exposed-Infected-Recovered (SEIR) model. Utilizing data about the number of TB cases, the SEIR model for transmission of TB was examined, and calculations were run. The majority of researchers look at a model's phenomenological behavior, including steady state, fundamental reproduction numbers, local and global stability, and bifurcation analysis [6-9]. They provide a detailed check of the connections between different literary genres in order to highlight a potential possibility for future modeling of the behaviors of the disease. Vaccination and rehabilitation have been employed as functionalities for vulnerable and infected persons in an efficient control issue with an objective function. [10-17] formally describe the dominating eigenvalue of a positive linear operator as the anticipated number of subsequent implementations integrated by a particular affected person throughout its whole duration of transmissibility in a maximally vulnerable community. These studies [18-12] propose a nearly compressible model based on the characteristics of the mumps virus as described by a series of nonlinear equations. The model's numerical simulation demonstrated whether vaccination can lower the demographics' exposure and infectiousness [23-25]. The majority of research articles [26-32] furthermore performed some sensitivity analysis on the basis of numerous model parameters, where it was demonstrated that raising the immunization rate, boosting the effectiveness of vaccine administration, and educating the public about rubella all contribute to the control and subsequent eradication of the disease. Vellappandi et al. [33, 34] have derived an optimal control problem for schistosomiasis disease by using the Caputo fractional derivative. Zarin et al. [35] reformulated and analyzed a co-infection model consisting of Chagas and HIV epidemics.

In this paper, we have constructed an SIR mathematical model to study the equilibrium points, stability of measles mellitus, basic reproductive number as well as fuzzy basic reproduction number, analysis using a fuzzy system, and comparison of the analytical approach by variation iteration method and numerical approach using the Runge-Kutta method without genetic factors.

2. Preliminaries

2.1 *Fuzzy number*

Let X be a nonempty crisp set. A fuzzy subset S of X is denoted by $\widetilde{S}$ and is defined as

$$\widetilde{S} = \{(x, \mu_{s(x)}) : x \in X\}$$

where $\mu_s : X \to [0,1]$ is a membership function associated with a fuzzy set $\widetilde{S}$, which describes the degree of

belongingness of x with X.

Here, we use the membership function $\mu(x)$ to indicate the fuzzy subsets $\tilde{S}$. Also, $\mu(x)$ is called fuzzy number if X is the set of real numbers.

2.2 *Triangular fuzzy number*

A fuzzy set is called a triangular fuzzy number if the membership value can be represented by a triangular function. This function by a three parameters $F(x: a, b, c)$ [5] such as:

$$F(x: a,b,c) = \begin{cases} 0, & \text{if } x < a, \\ \dfrac{x-a}{b-a}, & \text{if } a \le x < b, \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le c, \\ 0, & \text{if } x > c, \end{cases}$$

2.3 *Fuzzy measure and fuzzy expected value*

Let Ω be a nonempty set and $P(\Omega)$ denote the set of all subsets of Ω. Then μ: $\Omega \to [0,1]$ is a fuzzy measure [5], if

(i) $\mu(\varphi) = 0$ and $\mu(\Omega) = 1$,

(ii) For $A, B \in P(\Omega)$, $\mu(A) \le \mu(B)$ if $A \subset B$.

Let μ: $\Omega \to [0,1]$ be an uncertain variable, i.e., μ is a fuzzy subset and μ a fuzzy measure on Ω.

Then, the fuzzy expected value (FEV) of μ is the real number, defined by the Sugeno measure [10].

$$\mathrm{FEV}(\mu) = \int \mu d\mu = \sup\{\min(a, k, (\alpha))\}, 0 \le \alpha \le 1$$

where $k(\alpha) = \mu\{\omega \in \Omega : \mu(\omega) \ge \alpha\}$.

2.4 *Fuzzy mathematical model*

The following SIR fuzzy mathematical model includes the transmission rate, mortality from disease rate, and recovery rate as fuzzy parameters because we assumed that there is population heterogeneity. Based on the quantity of interactions with infected (I) people and the rate of transmission of the virus, people go from S to I. When people recover from the infection, they move from the I compartment to the R compartment. Here is the virus load for the disease.

Susceptible: They are the people who are exposed to the spread of the disease out of the overall population (N).

Infected: People who exhibit infectious disease symptoms are classified as the infected population. They can transmit the disease because they are also contagious.

Recovered: Persons who have undergone treatment or taken vaccination and recovered from the infectious disease are termed the recovered population.

$$\frac{dS}{dt} = -\frac{\eta(\Omega)SI}{N} \tag{1}$$

$$\frac{dI}{dt} = \frac{\eta(\Omega)SI}{N} - \delta(\Omega)I \tag{2}$$

$$\frac{dR}{dt} = \delta(\Omega)I \tag{3}$$

$S + I + R = N$ is the total of human population.

Table 1. Parameter description

Parameters	Description of parameters
η	Transmission rate
δ	Recovery rate from infection
S	Susceptible
I	Infected
R	Recovered
Ω	Virus load

3. Stability analysis

The stability analysis of the model shows that the system is globally asymptotically stable at the disease-free equilibrium point.

3.1 *Measles-free equilibrium*

The trivial equilibrium is thought to be a measles-free equilibrium in our context. So, the measles-free equilibrium is $\lambda_0 = (0,0,0)$.

3.2 *Basic reproductive number*

The average number of secondary infections brought on by a single infected person over the course of their entire contagious lifetime is known as the basic reproduction number. The symbol for the number is R_0. The next generation matrix approach [4, 5] is used to obtain the fundamental reproduction number R_0 for the system, $R_0 = \frac{\eta S}{\delta N}$.

3.3 *Global stability analysis*

The Lyapunov function V_1 for our model is $f_1(t,S,I,R) = a_1 I'$ where $I' = \frac{dI}{dt}$ from equation (2).

We found that $\frac{df_1}{dt} = a_1[-\delta]\mathrm{I}$ by choosing a_1 as $\frac{-1}{\delta}$ it is clear that $\frac{dv_1}{dt} = 0$ if $I = 0$.

While substituting $I = 0$ in our model system of equations, we found that S approaches to 0, R approaches to 0 as t tends to infinity. Hence, by Lasalle's invariance principle, the system of equations is stable at λ_0. Hence, the system is globally asymptotically stable at λ_0.

4. Analysis of fuzzy system

We take into account the different grades of measles susceptibility and infectiousness. Because of the population's diversity, we assume in this study that the contact transmission rate and recovery rate are fuzzy variables. The probability that an infected person will spread the disease to a susceptible person after just one encounter with them. We use the membership function to express the membership function of transmission rate and recovery rate, which was first developed by Barros [30] and applied by Bhuju et al. [31] and Verma et al. [32].

Let $\eta = \eta(\Omega)$ be a representation of the virus load-dependent transmission rate demonstrated in [3].

$$\eta(\Omega)=\begin{cases}0, & \text{if } \Omega<\Omega_{\min},\\ \dfrac{\Omega-\Omega_{\min,}}{\Omega_M-\Omega_{\min,}} & \text{if } \Omega_{\min}\le\Omega\le\Omega_M,\\ 1, & \text{if } \Omega_M\le\Omega\le\Omega_{\max},\end{cases}$$

where $\Omega_{\min}$ signifies the lowest amount of virus in the society, Ω_M signifies the mid-range amount, and $\Omega_{\max}$ is the highest virus load per person in the population. Figure 1 depicts the membership function of transmission rate.

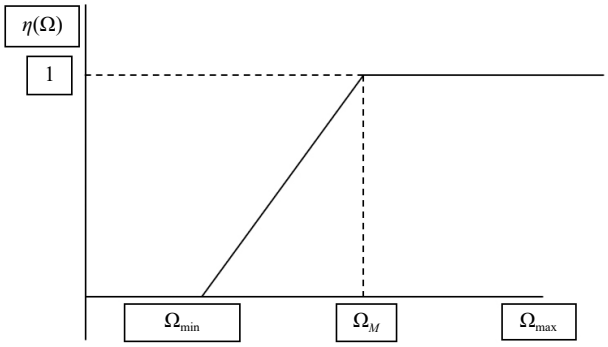

Figure 1. Membership function of transmission rate

Let's use the symbol $\delta(\Omega)$ to represent the infectious measles recovery rate that depends on viral load. As the illness is treated more slowly, the viral burden grows. Identifying the fuzzy integer that this model's recovery rate is using the following membership function

$$\delta(\Omega)\frac{(\delta_0-1)}{\Omega_{\max}}\Omega+1,\ \text{if } 0<\Omega<\Omega_{\max}$$

where Ω is the viral load, and $0<\delta_0$ is the population minimum recovery rate. Figure 2 displays the recovery membership features.

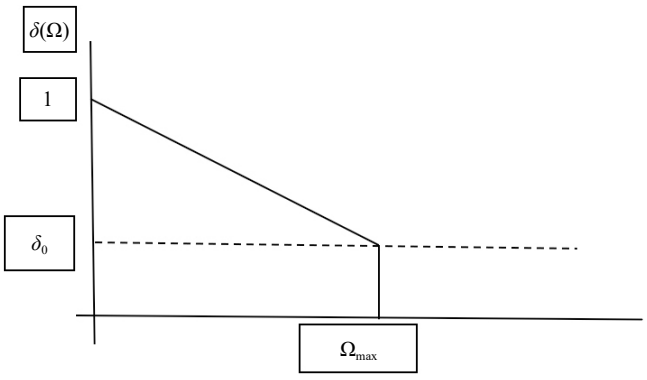

Figure 2. Membership function of recovery rate

We predict that the viral concentration may vary based on the individual. This makes it a linguistic variable that, in accordance with the expert's classification, can be mild, moderate, or heavy. Each division in the linguistic variable's

membership function is divided by [2].

$$\pi(\Omega)=\begin{cases}0, & \text{if } \Omega<\bar{\Omega}-x,\\ \dfrac{\Omega-\bar{\Omega}+x}{x}, & \text{if } \bar{\Omega}-x\le\Omega\le\bar{\Omega},\\ \dfrac{-(\Omega-\bar{\Omega}-x)}{x}, & \text{if } \bar{\Omega}<\Omega\le\bar{\Omega}+x,\\ 1, & \text{if } \Omega>\bar{\Omega}+x,\end{cases}$$

4.1 *Fuzzy basic reproduction number*

By examining the stability of the equilibrium point, the basic reproduction number R_0 can be determined. $R_0=\dfrac{\eta S}{\delta N}$ for the SIR model rises with an increase in virus load; this value cannot be a fuzzy set because it may be more than 1. We therefore introduced $\delta_0 R_0(\Omega)$ to be less than 1 as a result. Thus, $\delta_0 R_0(\Omega)\le 1$, where FEV$[\delta_0 R_0(\Omega)]$ is well-defined and is a fuzzy set. We first determine the FEV values for $R_0(\Omega)$, $\delta(\Omega)<1$, $\delta(\Omega)$, 1. Since we already know that $\delta(\Omega)<1$, we get $\dfrac{\delta_0\eta S}{\delta N}<1$. We introduce the fuzzy basic reproduction number [16] in this approach.

The fuzzy basic reproduction number is given by

$$R_{0F}=\frac{1}{\delta_0}\text{FEV}\left[\delta_0 R_0(\Omega)\right]$$

To obtain FEV, we need to define fuzzy measure of ξ where FEV$(R_0(\Omega))$ = sup $\{\min(\mathfrak{Z}, k(\mathfrak{Z}))\}$, $0\le\mathfrak{Z}\le 1$, $k(\mathfrak{Z})=\mu\{\Omega:\delta_0 R_0(\Omega)\ge\mathfrak{Z}\}=\mu(X)$, which is a fuzzy measurement. By applying the fuzzy measure, we determine FEV. The possibility measure is provided by [16] for this purpose

$$\mu(X)=\sup\pi(\Omega),\forall\Omega\in X, X\subset R$$

From FEV$(\delta_0 < R_0(\Omega))$, it is evident that $R_0(\Omega)$, where the set $X=\left[\bar{\Omega},\Omega_{\max}\right]$ is the answer to the equation given, is not diminishing one, and the answer to the following expression is $\mathfrak{Z}$,

$$\frac{\delta_0\eta S}{\delta N}=\mathfrak{Z} \tag{4}$$

Thus, $k(\mathfrak{Z})=\mu\left[\bar{\Omega},\Omega_{\max}\right]=\sup\pi(\Omega)$ with $\Omega\le\Omega\le\Omega_{\max}$, where $k(0)=1$ and $k(1)=\pi(\Omega_{\max})$.

Three categories make up the population's "amount of virus," which was thought to have linguistic importance, and each of them displays confusing behavior. There are various viral loads, from minor to extreme. This is reliant upon $\eta(\Omega)$ and $\delta(\Omega)$. In this fuzzy model, there are three different categories of people, and each has a unique rate of transmission and recovery. So, each category will be used to determine R_{0F}. We obtain fuzzy basic reproduction numbers using different amounts of virus load.

Case 1: Weak virus ($\Omega_{\min}$). The viral load in this instance is low (i.e.) when $\bar{\Omega}+x\le\Omega_{\min}$.

Here, $\eta(\Omega)=0$ and $\delta(\Omega)=\dfrac{(\delta_0-1)}{\Omega_{\max}}\Omega+1$, we have calculated FEV $[\delta_0 R_0(\Omega)]$.

$$\begin{aligned}\text{FEV}\left[\delta_0 R_0(\Omega)\right]&=\sup\left\{\min\left(\wp,(\Omega:0\ge\mathfrak{Z})\right)\right\},0\le\mathfrak{Z}\le 1\\&=\sup\left\{\min(\mathfrak{Z},\pi(\Omega))\right\},\\&=0\\R_{0F}&=\frac{1}{\delta_0}\text{FEV}\left[\delta_0 R_0(\Omega)\right]\\&=0\end{aligned}$$

In the event that $R_{0F} = 0$, it means that the disease will eventually become obsolete.

Case 2: Medium virus (Ω_M). The viral burden in this instance is medium (i.e.) when $\overline{\Omega} - x \geq \Omega_{\min}$ and $\overline{\Omega} + x \leq \Omega_M$.

Here, $\eta(\Omega) = \dfrac{\Omega - \Omega_{\min}}{\Omega_M - \Omega_{\min}}$ and $\delta(\Omega) = \dfrac{(\delta_0 - 1)}{\Omega_{\max}}\Omega + 1$. We have calculated FEV $[\delta_0 R_0(\Omega)]$.

$$\text{FEV}\left[\delta_0 R_0(\Omega)\right] = \sup\{\min(\mathfrak{Z}, k(\mathfrak{Z}))\}, 0 \leq \mathfrak{Z} \leq 1$$

when $0 < \mathfrak{Z} < 1$ with $\mathfrak{Z}$ is the solution of the following equation.

$$\frac{\delta_0 \eta S}{\delta N} = \mathfrak{Z}$$

For $0 < \mathfrak{Z} < 1$, we divide into three parts. $k(\mathfrak{Z})$ for $0 \leq \mathfrak{Z} \leq 1$

$$k(\mathfrak{Z}) = \begin{cases} 1, & \text{if } 0 < \mathfrak{Z} \leq \delta_0 R_0(\overline{\Omega}), \\ \pi(\overline{\Omega}), & \text{if } \delta_0 R_0(\overline{\Omega}) < \mathfrak{Z} \leq \delta_0 R_0(\overline{\Omega} + x), \\ 0, & \text{if } \delta_0 R_0(\overline{\Omega} + x) < \mathfrak{Z} \leq 1. \end{cases}$$

So, if $\mathfrak{Z}$, $k(\mathfrak{Z})$ is continuous, and decreasing function with $k(0) = 1$ and $k(1) = 0$. Hence, FEV$(\delta_0 R_0(\Omega))$ is the fixed point of k and R_{0F}

$$\delta_0 R_0(\overline{\Omega}) \leq \text{FEV}\left(\delta_0 R_0(\Omega)\right) \leq \delta_0 R_0(\overline{\Omega} + x),$$
$$R_0(\Omega) \leq R_{0F} \leq R_0(\Omega + x).$$

As the function $R_0(\Omega)$ is increasing and a continuous function, then by the intermediate value theorem there exists Ω with $\overline{\Omega} < \Omega < \overline{\Omega} + x$ such that $R_{0F}\, R_0(\Omega) > R_0(\overline{\Omega})$.

There is enough virus load that R_0 and $R_0(\Omega)$ are equal. Additionally, the average number of secondary cases R_{0F} is larger than the average number of secondary cases $R_0(\overline{\Omega})$ due to the medium amount of virus.

Case 3: Strong virus ($\Omega_{\max}$). In this case, the virus load is strong (i.e.) when $\overline{\Omega} + x \leq \Omega_M$ and $\overline{\Omega} + x \leq \Omega_{\max}$.

Here, $\eta(\Omega) = 1$ and $\delta(\Omega) = \dfrac{(\delta_0 - 1)}{\Omega_{\max}}\Omega + 1$, we have calculated FEV $[\delta_0 R_0(\Omega)]$.

$$\text{FEV}\left[\delta_0 R_0(\Omega)\right] = \sup\{\min(\mathfrak{Z}, k(\mathfrak{Z}))\}, 0 \leq \mathfrak{Z} \leq 1$$

when $0 < \mathfrak{Z} < 1$ with $\mathfrak{Z}$ is the solution of the following equation.

$$\frac{\delta_0 \eta S}{\delta N} = \mathfrak{Z}$$

For $0 < \mathfrak{Z} < 1$, we divide into three parts. $k(\mathfrak{Z})$ for $0 \leq \mathfrak{Z} \leq 1$

$$k(\mathfrak{Z}) = \begin{cases} 1, & \text{if } 0 < \mathfrak{Z} \leq \delta_0 R_0(\overline{\Omega}), \\ \pi(\overline{\xi}) & \delta_0 R_0(\overline{\Omega}) < \mathfrak{Z} \leq \delta_0 R_0(\overline{\Omega} + x) \\ 0, & \delta_0 R_0(\overline{\Omega} + x) < \mathfrak{Z} \leq 1. \end{cases}$$

Since the function of k is continuous and decreasing, we can directly calculate FEV $[\delta_0 R_0(\Omega)]$ and R_{0F}

$$\delta_0 R_0(\bar{\Omega}) \leq \text{FEV}(\delta_0 R_0(\Omega)) \leq \delta_0 R_0(\bar{\Omega}+x),$$
$$R_0(\bar{\Omega}) \leq \frac{1}{\delta_0} \text{FEV}(\delta_0 R_0(\Omega)) \leq R_0(\bar{\Omega}+x),$$
$$R_0(\Omega) \leq R_{0F} \leq R_0(\Omega+x).$$

Thus, $R_{0F} > 1$. We can predict that such disease will really be endemic.

If the population's transmission and recovery are not zero, we derive the fuzzy model's basic reproduction number, $R_0(\Omega) \leq R_{0F} \leq R_0(\Omega + x)$.

5. Feasibility analysis

5.1 *Feasible solution*

It is prudent to assume that the parameters employed and the variables in all classes are non-negative, that is, $t \geq 0$. We shall show evidence that, under the given non-negative initial circumstances, all model variables are non-negative.

The model's collection of possible solutions, which is positively invariant, is provided by:

$$\text{Let } \xi = \{(S, I, R) \text{ such that } N(t) = S(t) + I(t) + R(t) \to 0\}$$

The set ξ is positive invariant and attracts all the solutions in R^3

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0.$$

Hence, N is constant implies N is bounded.

All the variables in this model are positive

Let the initial data set be $(S, I, R)(0) \geq 0 \to \xi$.

The solution set of the given model is positive for all $t > 0$.

Proof. We have, from equation (1),

$$\frac{dS}{dt} = \frac{-\eta SI}{N}.$$

While integrating the above equation, $S(t) = ke^{\frac{-\beta It}{N}}, k > 0$.

When t approaches 0, $S(t)$ tends to k, which is positive.

Also, for $\frac{dI}{dt} = \frac{\eta SI}{N} - \delta I$, we get $I(t) = ke^{\left(\frac{\eta S}{N} - \delta\right)t}$ and for $\frac{dR}{dt} = \delta I$, we get $R(t) = \delta It + k$.

Hence, we prove that all the variables are positive.

5.2 *Existence of solution*

Let $f_1 = \frac{-\eta SI}{N}, f_2 = \frac{-\eta SI}{N} - \delta I$ and $f_3 = \delta I$

$$\frac{\partial f_1}{\partial S} = \frac{-\eta I}{N}, \frac{\partial f_2}{\partial S} = \frac{\eta I}{N}, \frac{\partial f_3}{\partial S} = 0, \frac{\partial f_1}{\partial I} = \frac{-\eta S}{N} - \delta, \frac{\partial f_3}{\partial I} = \delta, \frac{\partial f_1}{\partial R} = 0, = \frac{\partial f_2}{\partial R} = \frac{\partial f_3}{\partial R}$$

$\frac{\partial f_i}{\partial S}, \frac{\partial f_i}{\partial I}, \frac{\partial f_i}{\partial R}$ less than infinity where i varies from 1 to 3. It is clear that all the partial derivatives exist, continuous and

bounded. Hence, by uniqueness and existence theorem, there exist unique solution.

5.3 *Numerical approach*

The initial values for this model's numerical simulation were $S(0) = 99{,}000$, $I(0) = 1{,}000$, and $R(0) = 0$ the parameters' values are given below.

Table 2. Parameter value

Parameters	Values
η	0.2
δ	0.1
S	99,000
I	1,000
R	0

By using Runge-Kutta for fourth order, we found the following values for consecutive 100 days (Table 3) and corresponding graphs (Figures 3 to 7) are drawn.

Table 3. Susceptible, infected, recovered, basic reproduction number uing Runge-Kutta (RK) method

Days	S(RK)	I(RK)	R(RK)	$Y(R_0)$
0	99,000	1,000	0	1.98
1	98820.52	1103.075	105.1708	1.976410489
2	98622.89	1216.349	221.1821	1.97245783
3	98405.38	1340.74	349.1065	1.968107577
4	98166.13	1477.227	490.1133	1.963322516
5	97903.13	1626.85	645.4744	1.958062549
6	97614.23	1790.712	816.5717	1.952284603
7	97297.13	1969.969	1004.902	1.945942565
8	96949.36	2165.832	1212.086	1.938987258
9	96568.32	2379.555	1439.868	1.931366466
10	96151.25	2612.426	1690.128	1.923025016
11	95695.25	2865.752	1964.879	1.913904934
12	95197.28	3140.845	2266.272	1.903945695
13	94654.23	3438.997	2596.598	1.893084565
14	94062.85	3761.454	2958.28	1.881257074
15	93419.88	4109.385	3353.875	1.868397614
16	92722.01	4483.848	3786.063	1.854440199
17	91965.97	4885.742	4257.633	1.839319376
18	91148.57	5315.762	4771.47	1.822971321
19	90266.76	5774.345	5330.533	1.805335108
20	89317.71	6261.614	5937.826	1.786354154
21	88298.89	6777.316	6596.365	1.765977829
22	87208.16	7320.76	7309.141	1.744163217

Table 3. *Continued*

Days	S(RK)	I(RK)	R(RK)	$Y(R_0)$
23	86043.85	7890.755	8079.072	1.720876975
24	84804.86	8485.557	8908.949	1.696097261
25	83490.78	9102.813	9801.382	1.669815654
26	82101.95	9739.527	10758.73	1.642038994
27	80639.55	10392.04	11783.05	1.612791054
28	79105.7	11056	12875.99	1.582113937
29	77503.45	11726.44	14038.75	1.550069099
30	75836.9	12397.76	15272.03	1.516737908
31	74111.08	13063.82	16575.92	1.48222163
32	72332.04	13718.08	17949.85	1.446640799
33	70506.7	14353.69	19392.59	1.410133913
34	68642.77	14963.64	20902.18	1.372855469
35	66748.67	15540.96	22475.92	1.334973365
36	64833.29	16078.91	24110.38	1.296665756
37	62905.87	16571.13	25801.41	1.258117491
38	60975.81	17011.85	27544.21	1.219516264
39	59052.43	17396.04	29333.36	1.181048676
40	57144.82	17719.54	31162.91	1.142896376
41	55261.62	17979.19	33026.49	1.105232452
42	53410.91	18172.88	34917.38	1.068218243
43	51600.03	18299.58	36828.64	1.032000671
44	49835.51	18359.32	38753.22	0.996710195
45	48122.97	18353.17	40684.09	0.962459423
46	46467.12	18283.13	42614.3	0.929342368
47	44871.72	18152.03	44537.16	0.897434326
48	43339.62	17963.43	46446.22	0.866792305
49	41872.8	17721.46	48335.45	0.837455904
50	40472.43	17430.68	50199.23	0.809448562
51	39138.95	17095.97	52032.43	0.782779062
52	37872.16	16722.38	53830.43	0.757443197
53	36671.28	16315.01	55589.13	0.733425514
54	35535.05	15878.92	57305	0.710701055
55	34461.85	15419.04	58975	0.689237047
56	33449.72	14940.08	60596.63	0.668994477
57	32496.48	14446.51	62167.89	0.649929547
58	31599.75	13942.49	63687.24	0.631994972
59	30757.06	13431.86	65153.59	0.615141117
60	29965.85	12918.09	66566.23	0.599316979
61	29223.55	12404.33	67924.83	0.584471013
62	28527.59	11893.36	69229.4	0.570551817
63	27875.43	11387.63	70480.24	0.557508686

Table 3. *Continued*

Days	S(RK)	I(RK)	R(RK)	$Y(R_0)$
64	27264.6	10889.27	71677.89	0.545292053
65	26692.69	10400.11	72823.12	0.533853821
66	26157.38	9921.668	73916.91	0.523147616
67	25656.45	9455.222	74960.38	0.513128967
68	25187.77	9001.798	75954.79	0.503755412
69	24749.33	8562.206	76901.52	0.494986573
70	24339.21	8137.06	77802.01	0.48678417
71	23955.6	7726.799	78657.79	0.47911202
72	23596.8	7331.707	79470.43	0.471935999
73	23261.2	6951.933	80241.51	0.465223991
74	22947.29	6587.509	80972.65	0.45894582
75	22653.66	6238.366	81665.46	0.453073172
76	22378.98	5904.349	82321.56	0.447579516
77	22122	5585.23	82942.52	0.442440013
78	21881.57	5280.722	83529.93	0.437631425
79	21656.6	4990.487	84085.3	0.433132033
80	21446.08	4714.148	84610.16	0.42892154
81	21249.05	4451.297	85105.95	0.424980994
82	21064.63	4201.5	85574.1	0.421292699
83	20892.01	3964.31	86015.97	0.417840136
84	20730.39	3739.264	86432.9	0.414607891
85	20579.08	3525.896	86826.16	0.411581578
86	20437.39	3323.734	87196.98	0.408747774
87	20304.7	3132.309	87546.54	0.406093954
88	20180.42	2951.156	87875.97	0.403608428
89	20064.01	2779.817	88186.35	0.401280289
90	19954.97	2617.84	88478.7	0.399099356
91	19852.81	2464.786	88754.02	0.397056126
92	19757.09	2320.226	89013.25	0.395141727
93	19667.39	2183.745	89257.27	0.393347875
94	19583.34	2054.939	89486.93	0.391666836
95	19504.57	1933.419	89703.05	0.390091385
96	19430.74	1818.812	89906.39	0.388614773
97	19361.53	1710.757	90097.68	0.387230693
98	19296.66	1608.909	90277.6	0.385933252
99	19235.85	1512.937	90446.81	0.384716944
100	19178.83	1422.524	90605.93	0.383576619

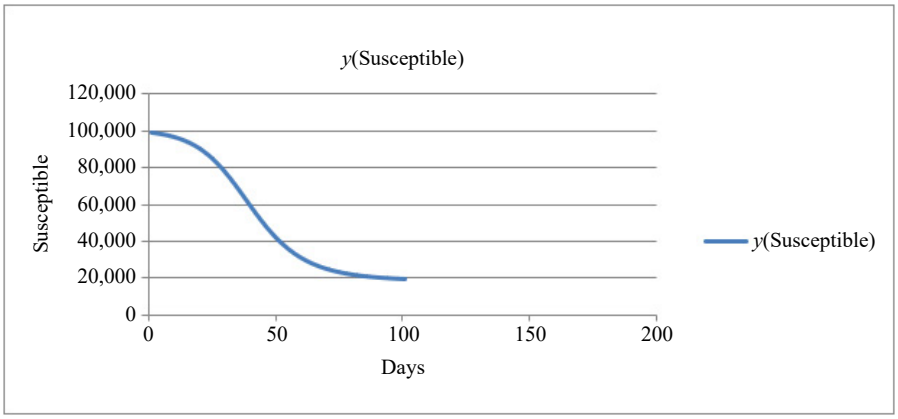

Figure 3. Susceptible population using Runge-Kutta method

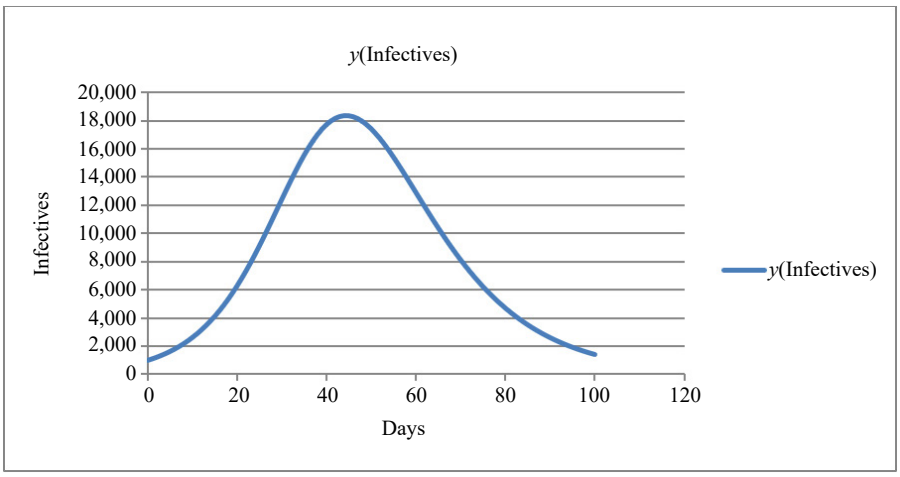

Figure 4. Infected population using Runge-Kutta method

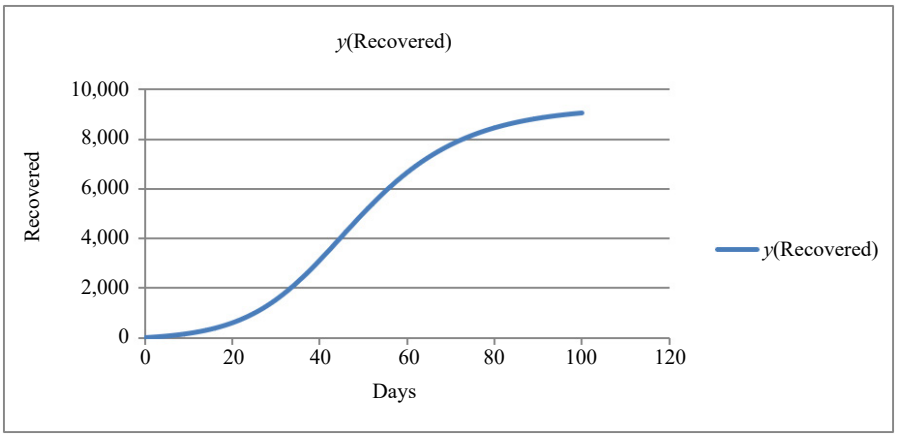

Figure 5. Recovered population using Runge-Kutta method

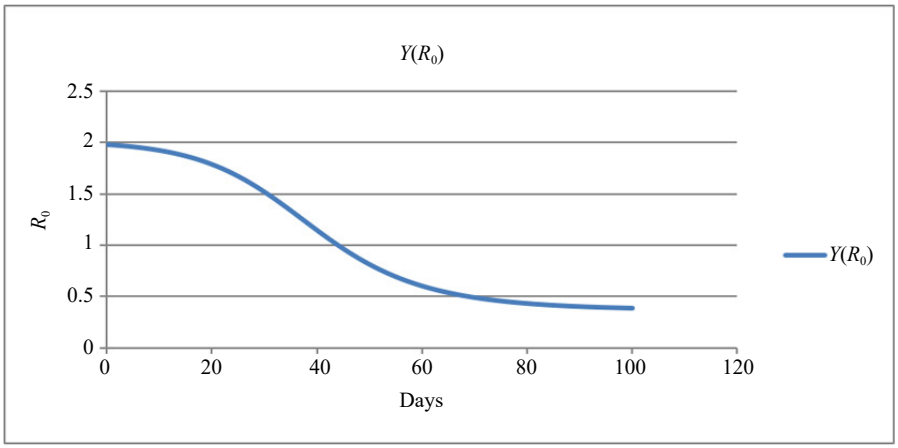

Figure 6. Graph of basic reproduction number uing Runge-Kutta method

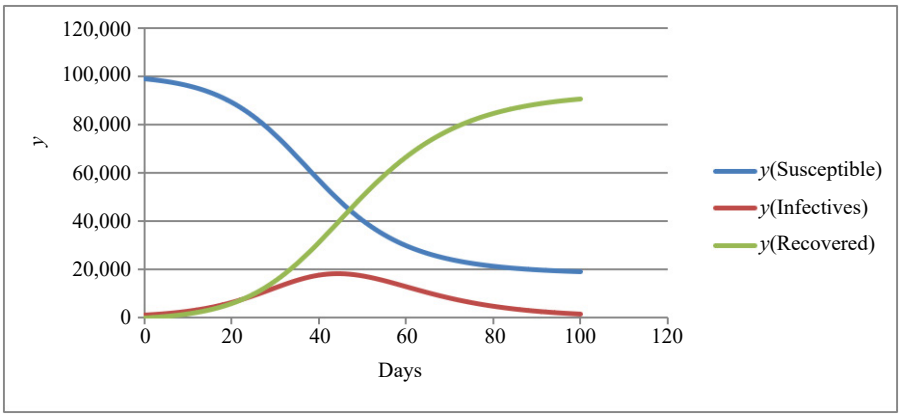

Figure 7. Comparison of susceptible, infected, and recovered population using Runge-Kutta method

6. Discussion and results

We must start implementing the following procedures in order to stop measles:

- By decreasing the transmission rate from the susceptible population to the infected compartment.
- It has been discovered that the risk of contracting measles dramatically falls if the virus is infectious, as determined by avoiding contact tracking, screening, or the illness's spread.
- Preventing the mixture of infected and recovered populations
- Using strong medical support and good treatment for the people who are infected
- Applying more effective methods to determine the confirmed cases.
- The proverb "prevention is better than cure" should be properly followed by everyone in order to ensure that the sickness doesn't spread further.
- Examining the impacts of the factors (such as tracing and screening of contacts) on the measles outbreak, an equation with piecewise constant arguments is suitable for a population with a changeable size structure.
- The Runge-Kutta method is used in this model to find numerical simulations. It is found that the equilibrium point is globally asymptotically stable under certain conditions.
- Finally, based on the data, we discovered that as infection levels rise and recovery rates rise, the susceptible population gradually declines. By comparing the R_0 graph with the infective graph, it's found that when $R_0 > 1$, infection increases, while when $R_0 < 1$, infection decreases.
- Figures 3, 4, 5, and 6 represent the susceptible, infected, recovered population and the basic reproduction

number, which are depicted using the Runge-Kutta method. Figure 7 shows the dynamical behavior of the model using the Runge-Kutta method.

7. Conclusion

In this research, we looked at a mathematical model for measles sickness in a fuzzy setting. For the models in fuzzy situations, stability analysis and fundamental reproduction numbers are explored. We used the iterative Runge-Kutta solution. While seeing the graph of R_0 and infections, it's clear that the infection increases when $R_0 > 1$ and decreases when $R_0 < 1$. In the future, we will research and analyze the applicability of the suggested methods in a different epidemic model with a fuzzy environment.

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Conflict of interest

The authors declare no conflict of interest.

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