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Research Article

Optimization of a Two-Warehouse EOQ Model for Non-Instantaneous Deteriorating Items with Polynomial Demand, Advance Payment, and Shortages

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Abstract: In this paper, a two-warehouse inventory model with polynomial time-dependent demand for non-instantaneous deteriorating items is taken into consideration. As advertisement plays a vital role in business, more advertisements are to be made by the retailer to increase demand in the competitive market. In order to meet the high demand, it is necessary to maintain two warehouses to store the goods in inventory: one owned warehouse with limited capacity and one rented warehouse with unlimited capacity. In order to increase sales, the retailer anticipates a discount from the manufacturer, and in turn, the retailer is equipped to pay a portion of the purchase price in advance. The model aims to optimize the overall profit by determining the appropriate order quantity and cycle length. Furthermore, numerical examples are provided for the validation of the proposed model.

***Keywords*:** advance payment, deterioration, intensity of advertisement, capacity constraint

MSC: 90B05, 90B06

Nomenclature

A	Frequency of advertisement in each cycle
α	Deterioration rate in own warehouse ($0 < \alpha < 1$)
B	Backorder quantity
β	Deterioration rate in rental warehouse ($0 < \beta < 1$)
C_A	Advertisement cost per advertisement
Ch_1	Holding cost per unit in rental warehouse
Ch_2	Holding cost per unit in own warehouse
C_o	Ordering cost in each order
C_P	Purchase cost per unit
C_S	Shortage cost per unit
C_T	Transportation cost/unit
d	Discount rate availed by the retailer for the advance payment

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μ_1	Lifetime of items in own warehouse
μ_2	Lifetime of items in rental warehouse ($\mu_1 < \mu_2$)
Q	Order quantity in each cycle
s	Selling price
σ	Portion of advance payment paid by the retailer ($0 < \sigma < 1$)
T	Cycle length
t_0	Time at which the inventory in the rental warehouse becomes zero
t_1	Time at which the inventory in its warehouse becomes zero
W	Maximum capacity in own warehouse

1. Introduction

Mathematical concepts have been applied to many real-life problems, mostly inventory control. In the business sector, the manufacturer has to maintain the inventory level of the products they have produced. In this regard, they have to make many decisions, such as how much to order and when to order to promote sales, avoid shortages. Buying and storing inventory above normal levels effectively reduces the risk of such market fluctuations when supply and demand are infrequent and seasonal. A warehouse is generally used for central storage or the systematic accumulation of goods. Throughout the year, a warehouse's inventory efficiently meets customer demand.

Businesses use two types of warehouses: owned warehouses (OW) and rented warehouses (RW). Major companies, manufacturers, and retailers all have their OW. Due to an unexpected rise in demand for certain products, businesses may need to use rental warehousing to store additional items. In modern cultural inventory control, this two-warehouse system is prevalent. Because rental storage facilities have high inventory costs, suppliers and retailers prioritize meeting customer demand from RW. Many different inventory models have been developed as a result of this two-warehouse system.

Deterioration is defined as a decline in the utility or marginal value of a product, as well as decline, failure, wastage, vapor pressure, unsustainability, and fraud. Inventory models typically consider non-deteriorating goods that never deteriorate and spontaneously deteriorating goods that begin to deteriorate as soon as they are added to inventory. However, most physical products gradually deteriorate over time. As a result, biological materials in stock degradation are a significant concern, and dealing with such semi-perishable materials is essential.

Nowadays, marketing strategy, advertising, and sales promotion play a vital role in developing business opportunities. Such actions may occur in mass/electronic/traditional media and through sales agents who promote the service in unique ways that can have a long-lasting impact on society. Hence it is assumed that the intensity and frequency of relevant advertisements and the selling price of the applicable product significantly impact consumer demand for the appropriate product. Advertising is one of the most important factors influencing the popularity of a product among consumers. This will effectively increase the demand for the product as a result. Advertising is essential to raise consumer awareness about introducing a new product or redesigning an existing one.

In a business operation, it is assumed that retailers will pay suppliers after receiving the goods from them; however, this might not always be the case. Globalization has made today's marketing environments highly competitive. As a result, suppliers make a variety of offers to retailers in an attempt to win their business. One of the most notable and standard policies to attract more customers is the permitted delay in payment. Compared to competitors, this policy has a positive effect on marketing. In this regard, wholesalers and suppliers offer various incentives to their retailers to promote sales. Also, to attract and make them regular customers, the manufacturer should use different marketing strategies such as price discounts, quantity discounts, trade discounts, seasonal discounts, etc, to the retailers. When a retailer places a purchase order for a product, the supplier may provide a reduction for the advance payment.

It is evident from the existing body of literature that researchers have looked at both linear and exponential demand rates when dealing with time-varying demand. A linearly time-varying demand implies a uniform change in the product's demand rate per unit of time, which is uncommon in the real market. Because sales for any product cannot change rapidly, massively time-varying consumption denotes a very rapid change in demand, which is also unusual. As a result, the more feasible non-linear demand must be regarded.

Many researchers have studied the two-warehouse inventory model with scarcity and fixed demand rate, and some of them have investigated how advertising plays a vital role in their business. Trade credit is necessary to make a business viable. The following section discusses relevant commodity modeling literature that addresses prepayment, scarcity, advertising, and deterioration.

2. Literature review

In an economic setting, two well-recognized frameworks for building the objective function of inventory or production-inventory models are the net present value approach and the average cost/profit approach discussed by Ghiami [1]. The challenge is the need for insight into the probability distributions of the selling duration and future market prices, which also results in uncertainty in the random demands. He et al. [2] provided a model to maximize the expected total revenue. Van Donselaar et al. [3] examined a multi-item periodic review inventory system with exogenous lot sizes and back ordering to mitigate inventory holding costs while adhering to the requirement that the total fill rate correspond to a target level.

Dash et al. [4] described the topic of an integrated supply chain model with Weibull distribution and promotional cum market demand for deteriorating items. Sarkar et al. [5] developed a model where demand and time depend on partial regression with shortages in all cycles, and time-squared inventory replenishment is based on the principle of deterioration over time. Bhunia and Shaikh [6] analyzed a deterministic inventory model with linearly increasing demand, shortages, and different levels of item deterioration in both warehouses. Mohan [7] discussed time-dependent holding costs with non-linear order, time-dependent breakdown, and scrap value. Bhunia et al. [8] examined a twowarehouse inventory model for flawed items with varying time, partial backlog, and variable demand based on marketing strategy and time. Bera et al. [9] investigated an appropriate partial posterior two-storage inventory model for perishable goods with uncertainty.

Bose et al. [10] built a quantity discount model for perishable goods with time-dependent linear lead time and scarcity under wage growth and period cash discounting. Thangam [11] analyzed price discounting and lot-sizing policies for perishable items in a supply chain using an advanced payment scheme and optimal two-tier trade credits. Zhang et al. [12] investigated the optimal order conditions for the retailer, established the existence and integrity of the optimal prepayment solution, and examined the supplier's pricing policy for the retailer's prepayment. Rahman et al. [13] proposed an appropriate approach to inventory organization for perishable goods with hybrid demand based on stock and selling price under part balances with a certain fixed ratio. Gupta et al. [14] discussed the deteriorating inventory model with allowable payment delays and price-sensitive investment demand. Khanra et al. [15] developed an EOQ model for a declining item with a time-dependent quadratic order and a billing delay. Min and Zhou [16] formulated capacity control and the need for sustained dependence with stock-based sales rates and a partial backlog. Khan et al. [17] analyzed is performed when the business manager considers it more cost-effective to deal with a no-end situation involving salvage values than a zero-end situation involving a conventional selling price, especially with the increase in online transactions for mixed cash and prepayment contracts. Taleizadeh [18] created an EOQ model for an evaporating item with partial back ordering and progress payments, a known price increase, and a partial, delayed payment. Sivalingam et al. [19] proposed algorithm uses the amount of information gathered from averages, subtraction, standard deviation, and hybrid population members to explore the search space to reach a near-optimal or quasi-optimal solution. Priya and Senbagam [20] highlighted the topic of non-linear time-varying demand variable deterioration and EOQ share dependence for time-dependent deteriorating products with partial time delays. Dehghani et al. [21] proposed that Average and Subtraction-Based Optimizer (ASBO) is mathematically modeled with the ability to solve optimization problems. Twenty-three test functions, including uni-modal and multi-modal, have been employed to evaluate ASBO's performance in effectively solving optimization problems. Dey et al. [22] pointed out the model for goods under inflation and the time value of money, two storage inventory problems with dynamic demand and frequency lead-time over a finite time horizon.

San-José et al. [23] hypothesized that the effects of a power function dependent on the frequency of advertisement and a process dependent on both the selling price and time combine multiplicatively to influence an item's demand rate. Jaggi et al. [24] covered managing demand under inflationary conditions using the two-warehouse limited backlogging explicit formula for perishable products with a linear demand trend. Dehghani and Trojovsky's [25] proposed the idea of Hybrid Leader Based Optimization (HLBO) to lead the algorithmic population under the direction of a hybrid leader. The stages of HLBO are mathematically modeled in two phases: exploration and exploitation. Khan et al. [26] examined two distinct inventory models for perishable goods developed under linearly time-dependently increasing holding costs. In contrast, the demand for the product is influenced by its selling price and the frequency of its advertising. Meena et al. [27] developed an inventory model for non-instantaneous deteriorating items with price and advertisement-dependent demand under a suitable payment delay. By enacting a cap and tax policy, investing in green technology, and promoting products, Kar et al. [28] created a sustainable, flexible production model with a single type of interchangeable product production. He also discussed that the demand for each manufacturer depends on the price of the products sold in online, offline, and print advertisements.

It is evident that the majority of the inventory models for non-instantaneously deteriorating items discussed above were not developed in scenarios where demand that is dependent on advertisement and quadratic time, in conjunction with advance payment and prepayment discounts, are investigated. This comprehensive review of the literature reveals that no authors have considered the combination of prepayment, quadratic time-dependent and advertising-dependent demand, and fully backlogged non-perishables. Since this model has a novel managerial strategy that aids a retailer in minimizing the total inventory cost, we have made an effort to investigate these issues concurrently and derive a comprehensive model to determine the optimal total profit. To corresponding optimization problem is formulated and solved by using MATLAB software.

The next phase of the paper will be clarified as follows: The assumptions used in this model are described in Section 3 of this paper. In Section 4, the formulation of the proposed model is developed. In Section 5, we have presented numerical examples to validate the proposed model. Based on the numerical examples, an in-depth analysis is made in Section 6. In Section 7, a fruitful conclusion has been reached, according to the analysis done.

3. Assumptions

To formulate the proposed model, the following assumptions are made:

1. A single item is considered over the infinite planning horizon.

2. Shortages are allowed and fully back-ordered.

3. The supplier offers a discount to the retailer for advance payment to promote sales. Here, the supplier expects some portion of the purchase cost in advance while ordering the items, and the rest can be paid at delivery.

4. Deterioration takes place after the lifetime of the items (i.e., the deterioration is non-instantaneous).

5. The demand rate D is a deterministic polynomial function of time and frequency of advertisement and is given by $D = A^{\gamma}(v_1 + v_2 t + v_3 t^2)$, where A is a positive integer, and v_1, v_2, v_3 and γ are positive constants.

6. There are two warehouses: the OW has a limited capacity of W units, and the RW has an unlimited capacity. The items of the RW were consumed first, followed by the units of the OW for economical reasons. Khare and Sharma [29] and Ghosh and Chaudhuri [30] described the implications of a time-minimizing ratio inventory management system with a contribution margin.

4. Model formulation

The inventory system is established as follows: Each cycle begins with Q units of goods entering the inventory system. Priority is given to removing the backlog, after which the W units are placed in their warehouse, while the remaining inventory is stored in a rented warehouse. Only after the goods in the rented warehouse are exhausted are the goods in the own warehouse consumed.

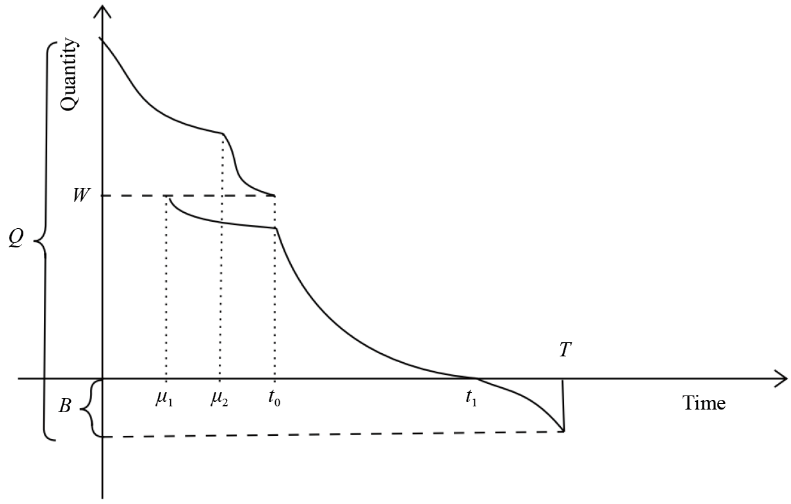

Figure 1. Inventory level

4.1 *Inventory level in RW*

The inventory level in the warehouse is changed during the interval, as in Figure 1. For $0 \le t \le \mu_2$, the inventory level decreases due to demand only. In $\mu_2 \le t \le t_0$, the inventory level reduces due to the demand and deterioration. This is given by the differential equation

$$\frac{dI_1(t)}{dt} = -D = -A^{\gamma}(v_1 + v_2 t + v_3 t^2) \text{ for } 0 \le t \le \mu_2. \tag{1}$$

From the differential equation, we can able to find the value of $I_1(t)$

$$I_1(t) = Q - B - W - A^{\gamma}\left(v_1 t + \frac{v_2 t^2}{2} + \frac{v_3 t^3}{3}\right) \text{ for } 0 \le t \le \mu_2 \tag{2}$$

with initial condition $I_1(0) = Q - B - W$ for $0 \le t \le \mu_2$.

During the interval, the inventory level is given by the differential equation

$$\frac{dI_2(t)}{dt} + \beta I_2(t) = -A^{\gamma}(v_1 + v_2 t + v_3 t^2) \text{ for } \mu_2 \le t \le t_0. \tag{3}$$

We can find the value of $I_2(t)$ from the differential equation.

$$I_2(t) = \frac{-A^{\gamma}}{\beta^3}\left[e^{\beta(t_0 - t)} A_1 - e^{\beta(\mu_2 - t)} A_2\right] \tag{4}$$

where

$$A_1 = \beta^2(v_1 + v_2t_0 + v_3t_0^2) - \beta(v_2 + 2v_3t_0) + 2v_3,$$

$$A_2 = \beta^2(v_1 + v_2\mu_2 + v_3\mu_2^2) - \beta(v_2 + 2v_3\mu_2) + 2v_3.$$

4.2 *Inventory level in OW*

In Figure 1, during the interval $0 \le t \le \mu_1$, the inventory level in its warehouse remains constant and equals W. In $\mu_1 \le t \le t_0$, inventory level decreases due to deterioration only at the rate of α. The inventory level drops due to deterioration and demand in $t_0 \le t \le t_1$. Thus, we have

$$J_1(t) = W \;\text{ for }\; 0 \le t \le \mu_1. \tag{5}$$

Because during the interval $[\mu_1,\, t_0]$, the OW inventory is reduced by deterioration at a rate of α and is given by the differential equation

$$\frac{dJ_2(t)}{dt} + \alpha J_2(t) = 0 \;\text{ for }\; \mu_1 \le t \le t_O. \tag{6}$$

Thus, we have

$$J_2(t) = e^{-\alpha t} + W - e^{-\alpha\mu_1} \;\text{ with }\; J_2(\mu_1) = W. \tag{7}$$

Hence, $W = e^{-\alpha\mu_1}$.

During the interval $[t_0,\, t_1]$, the OW inventory is reduced by demand and deterioration at a rate of α and is given by the differential equation,

$$\frac{dJ_3(t)}{dt} + \alpha J_3(t) = -A^{\gamma}(v_1 + v_2t + v_3t^2) \;\text{ for }\; t_0 \le t \le t_1 \tag{8}$$

from the above equation, we can find $J_3(t)$ as

$$J_3(t) = -\frac{-A^{\gamma}}{\alpha^3}\left[e^{\alpha(t_1-t)}A_3 - e^{\alpha(t_0-t)}A_4\right] \tag{9}$$

with $J_3(t_0) = J_2(t_0)$, where

$$A_3 = \alpha^2(v_1 + v_2t_1 + v_3t_1^2) - \alpha(v_2 + 2v_3t_1) + 2v_3,$$

$$A_4 = \alpha^2(v_1 + v_2t_0 + v_3t_0^2) - \alpha(v_2 + 2v_3t_0) + 2v_3.$$

Hence, we get

$$W = e^{-\alpha\mu_1} - e^{\alpha t_0} - \frac{-A^\gamma}{\alpha^3}\left[e^{\alpha(t_1-t_0)}A_3 - A_4\right].$$

During $t_1 \le t \le T$, there is a shortage is in inventory

$$\frac{dJ_4(t)}{dt} = -D \text{ for } t_1 \le t \le T. \tag{10}$$

Proceeding with the equation, we get

$$J_4(t) = -\left(v_1 t + \frac{v_2 t^2}{2} + \frac{v_3 t^3}{3}\right) \tag{11}$$

with $J_4(T) = -B$.

Hence, we get

$$B = \left(v_1 T + \frac{v_2 T^2}{2} + \frac{v_3 T^3}{3}\right). \tag{12}$$

4.3 *Model's associated cost per cycle*

Ordering cost C_0.

Advertisement cost AC_A.

Transportation cost $C_T Q$.

Holding cost in the RW

$$Ch_1\left[(Q-B-W)\left(\mu_2 - A^\gamma\left(\frac{v_1\mu_2^2}{2} + \frac{v_2\mu_2^3}{6} + \frac{v_3\mu_2^4}{12}\right)\right) - \frac{A^\gamma}{\beta^4}\left[-(A_1+A_2) + A_2 e^{\beta(\mu_2-t_0)} + A_1 e^{\beta(t_0-\mu_2)}\right]\right].$$

Holding cost in the OW

$$Ch_2\left[W\mu_1 + \left(\frac{e^{-\alpha t_0}}{-\alpha} - \frac{e^{-\alpha\mu_1}}{-\alpha}\right) + (w - e^{-\alpha\mu_1})(t_0-\mu_1) + \frac{A^\gamma}{\alpha^3}\left[\frac{-A_4}{\alpha}(e^{\alpha(t_0-t_1)} - 1) + \frac{A_3}{\alpha}(1 - e^{\alpha(t_1-t_0)})\right]\right]. \tag{13}$$

Shortage cost

$$C_s A^\gamma\left(\frac{v_1(T^2-t_1^2)}{2} + \frac{v_2(T^3-t_1^3)}{6} + \frac{v_3(T^4-t_1^4)}{12}\right). \tag{14}$$

Purchase cost $C_p Q$.

Discount availed $d(C_p Q)\sigma$.

Sales revenue sQ.
Actual purchase cost $(1-d\sigma)C_pQ$.
Total cost per cycle

$$\begin{aligned} TC &= C_0+AC_A+C_TQ \\ &+Ch_1\left[(Q-B-W)\left(\mu_2-A^{\gamma}\left(\frac{v_1\mu_2^2}{2}+\frac{v_2\mu_2^3}{6}+\frac{v_3\mu_2^4}{12}\right)\right)-\frac{A^{\gamma}}{\beta^4}\left[-(A_1+A_2)+A_2e^{\beta(\mu_2-t_0)}+A_1e^{\beta(t_0-\mu_2)}\right]\right] \\ &+Ch_2\left[W\mu_1+\left(\frac{e^{-\alpha t_0}}{-\alpha}-\frac{e^{-\alpha\mu_1}}{-\alpha}\right)+(w-e^{-\alpha\mu_1})(t_0-\mu_1)+\frac{A^{\gamma}}{\alpha^3}\left[\frac{-A_4}{\alpha}(e^{\alpha(t_0-t_1)}-1)+\frac{A_3}{\alpha}(1-e^{\alpha(t_1-t_0)})\right]\right] \\ &+C_sA^{\gamma}\left(\frac{v_1(T^2-t_1^2)}{2}+\frac{v_2(T^3-t_1^3)}{6}+\frac{v_3(T^4-t_1^4)}{12}\right)+(1-d\sigma)C_pQ. \end{aligned} \tag{15}$$

Moreover, the total profit for this model is

$$TP = sQ-\left[\begin{array}{l} C_0+AC_A+C_TQ \\ +Ch_1\left[\begin{array}{l}(Q-B-W)\left(\mu_2-A^{\gamma}\left(\frac{v_1\mu_2^2}{2}+\frac{v_2\mu_2^3}{6}+\frac{v_3\mu_2^4}{12}\right)\right) \\ -\frac{A^{\gamma}}{\beta^4}\left[-(A_1+A_2)+A_2e^{\beta(\mu_2-t_0)}+A_1e^{\beta(t_0-\mu_2)}\right]\end{array}\right] \\ +Ch_2\left[\begin{array}{l}W\mu_1+\left(\frac{e^{-\alpha t_0}}{-\alpha}-\frac{e^{-\alpha\mu_1}}{-\alpha}\right)+(w-e^{-\alpha\mu_1})(t_0-\mu_1) \\ +\frac{A^{\gamma}}{\alpha^3}\left[\frac{-A_4}{\alpha}(e^{\alpha(t_0-t_1)}-1)+\frac{A_3}{\alpha}(1-e^{\alpha(t_1-t_0)})\right]\end{array}\right] \\ +C_sA^{\gamma}\left(\frac{v_1(T^2-t_1^2)}{2}+\frac{v_2(T^3-t_1^3)}{6}+\frac{v_3(T^4-t_1^4)}{12}\right)+(1-d\sigma)C_pQ \end{array}\right]. \tag{16}$$

5. Numerical example

The values of the parameters mentioned above are reasonable, but they were all chosen at random. With the assistance of the documented algorithm, we were able to resolve the proposed problem.

Example 1 Let us consider the values ordering cost $C_o = 500$ per order, advertisement cost $C_A = 10$, selling price $s = 110$, purchasing cost $C_p = 25$ per unit, shortage cost $C_s = 1$ per unit, holding cost for rental warehouse $Ch_1 = 2$ per unit, own warehouse holding cost is $Ch_2 = 1.5$ per unit, a portion of advance payment at the time of ordering $\sigma = 0.7$, discount rate availed by the retailer = 20%, deterioration rate in OW $\alpha = 0.2$, deterioration rate in RW $\beta = 0.15$, $\gamma = 0.3$, $v_1 = 59.75$, $v_2 = 80.5$, $v_3 = 105$, a lifetime of an item in OW $\mu_1 = 0.49$ year, lifetime of the item in rental warehouse $\mu_2 = 0.4$ year, values adapted from Khare and Sharma [18] for $t_o = 0.6$, $t_1 = 0.78$, $T = 1.005$, then we get total profit € 82,321.08. The optimum frequency level is 5, optimum total cost € 27,684.72.

Example 2 Suppose high frequency of advertisement is $A = 20$, ordering cost $C_o = 500$ per order, advertisement cost $C_A = 25$, selling price $s = 110$; purchasing cost $C_p = 25$ per unit, shortage cost $C_s = 1$ per unit, holding cost for RW

$Ch_1 = 2$ per unit, OW holding cost is $Ch_2 = 1.5$ per unit, a portion of advance payment at the time of ordering $\sigma = 0.7$, discount rate availed by the retailer = 20%, deterioration rate in OW $\alpha = 0.2$, deterioration rate in RW $\beta = 0.15$, $\gamma = 0.3$, $v_1 = 59.75$, $v_2 = 80.5$, $v_3 = 105$, a lifetime of an item in OW $\mu_1 = 0.49$ year, a lifetime of the item in RW $\mu_2 = 0.4$ year, $t_o = 0.6$, $t_1 = 0.78$, $T = 1.005$, then we get total profit € 63,468.59, optimum total cost € 35,564.36.

6. Discussion

The following significant perspectives can be noticed from the above numerical examples:

• From Table 1, we get the optimum advertisement frequency and total profit.

Table 1. Comparison of frequency of advertisement and total profit

A	Total cost (TC)	Total profit (TP)
1	27,708.57	82,297.23
2	27,694.79	82,311.01
3	27,688.41	82,317.39
4	27,685.52	82,320.28
5	27,684.72	82,321.08
6	27,685.35	82,320.45
7	27,687.01	82,318.78
8	27,689.48	82,316.32
9	27,692.57	82,313.23
10	27,696.19	82,309.61
11	27,700.23	82,305.57
12	27,704.65	82,301.15
13	27,709.38	82,296.42
14	27,714.38	82,291.41
15	27,719.64	82,286.16
16	27,725.1	82,280.69
17	27,730.77	82,275.03
18	27,736.6	82,269.19
19	27,742.6	82,263.2
20	27,748.74	82,257.06

• Figure 2 shows how advertisements increase cycle length and create demand to improve sales.

• Figure 3 demonstrates how using MATLAB will ensure maximum profit and minimal total cost.

• Selling price makes liberal impacts of total profit and γ, v_1, v_2, v_3 are positive values that greatly impact total cost beyond the ordering cost.

• Owning a warehouse diminishes holding costs, but renting a warehouse raises holding costs, raising the total cost of the inventory.

• Here, the high cycle length and frequent advertising cause the back-order quantity to rise. Sales and retailer goodwill are impacted.

• By examining the two examples, we can see that the total expenditure will rise significantly if the cost and frequency of advertisements rise.

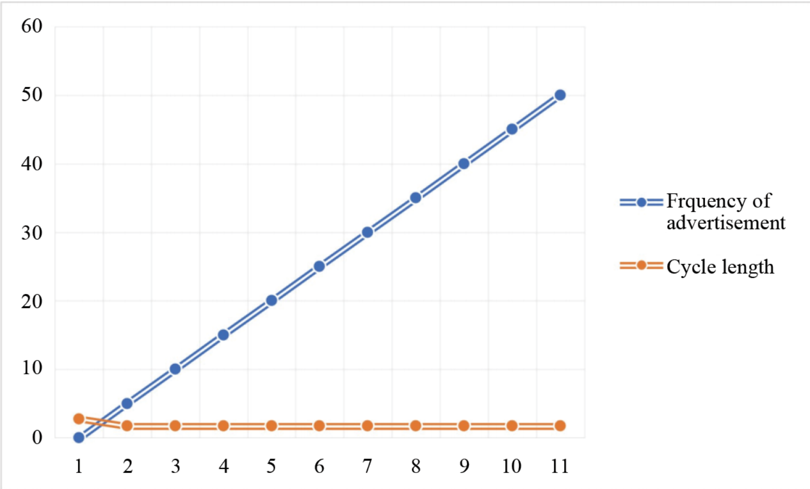

Figure 2. Comparison of advertisement and cycle length

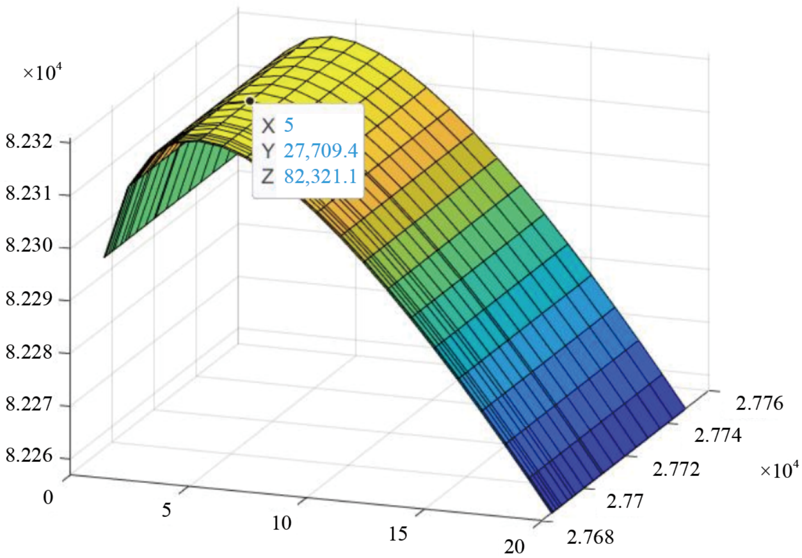

Figure 3. Optimal profit

7. Conclusion

Due to the intense competition in the marketing environment, suppliers make various offers to their retailers, including advance payments and discounts on the product's price. In this case, we combined these concepts and created a two-warehouse scheduling problem. We solved the proposed model using a numerical example and determined which possibility is more economically advantageous. In addition, we customized a two-warehouse inventory model with a depreciated product affected by time, selling price, and amount of media advertising. An advertisement induces consumers to buy a product. Therefore, it has a significant impact on attracting customers, and this impact directly affects the demand for the product. We have looked at this particular allocation problem using a similar demand function in this paper.

Conflict of interest

There is no conflict of interest in this study.

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