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Research Article

An EOQ Model Under the Condition of Permissible Delay in Payments with Allowed Stock-Out Cost and Lead Time

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Abstract: From this present study, derive the two consecutive demands between the time intervals: economic order quantity and total annual variable cost. The solution for this inventory model is optimizing the total annual variable cost. Here, given an arithmetical example and sensitivity analysis for the provision of the inventory model, assume the planning horizon and replenishment rate are infinite. To the best of our knowledge, this is the first study to find out the total annual variable cost using various costs under the condition of a permissible delay in payments with an allowed stock-out cost and lead time.

***Keywords*:** carrying cost, economic order quantity, interest payable, interest earned, lead time, planning horizon, replenishment rate, stock-out cost

MSC: 46N10

Nomenclature

A	Permissible delay in settling accounts
C	Carrying cost per item per year excluding interest charges
$C(L)$	Lead time crashing cost
$C(T)$	Total annual variable cost
d	Annual demand
I_c	Interest charges per Rs. investment in stock per year
I_e	Interest which can be earned per Rs. in a year
O	Cost of placing one order
p	Unit purchase price
$q(T)$	Economic order quantity (EOQ)
r	Stock-out cost per unit
s	Unit selling price
T	Time interval between two successive orders

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1. Introduction

By streamlining inventory processes, traders can verify product availability while cutting inventory costs and minimizing superfluous stock. Because of this, the inventory optimization verifies the proper inventory supply to reach the target level while using the least amount of capital. The majority of retailers use deterministic continuous review in accordance with inventory theory. The main goal is to retain inventory items on hand and place further orders as needed. The best model for forecasting client orders and shipping times is this one.

A long time ago, inventories were thought to be regarded as a country's or a person's wealth of measurements and influence. Due to significant advancements in production and fluctuations in product life, inventories are now seen as a far greater risk than a nation's riches of measures and power. The application of essential procedures in inventory management is now required by the idea of inventories, which is referred to as inventory control. If not, inventory control ensures that the right amount and quality of the product is made accessible when needed, taking into account the need to be economical with regard to carrying costs, ordering costs, setup costs, shortfall costs, manufacturing costs, purchase costs, and working capital. The time gap between placing the orders and receiving the orders is called delivery lag or lead time. The time horizon, otherwise called the planning period for which the inventory is to be meticulous, The planning period is divided into two types: one is finite and the other is infinite.

Aggarwal and Jaggi [1] carried out the ordering policies of deteriorating items under permissible delay in payments. Sepehri et al. [2] established the joint pricing and inventory model for deteriorating items with maximum lifetime and controllable carbon emissions under a permissible delay in payments. Taleizadeh et al. [3] recognized an EOQ model with partial delayed payment and partial backordering. Sarkar [4] introduced an EOQ model with delays in payments and stock-dependent demand in the presence of imperfect production. Sarkar [5] obtained an EOQ model with delays in payments and a time-varying deterioration rate. Jaggi et al. [6] framed an EOQ model with a permissible delay in payments in a fuzzy environment. Huang [7] coined an integrated inventory model under conditions of order processing cost reduction and permissible delay in payments. Huang et al. [8] formulated an optimal integrated vendor-buyer inventory policy under conditions of order processing time reduction and permissible delay in payments. Yao and Lee [9] invented fuzzy inventory with a backorder for fuzzy order quantity. Kaushik [10] created an inventory model with a permissible delay in payment and different interest rate charges. Cárdenas-Barrón et al. [11] approached an EOQ inventory model with nonlinear stock-dependent holding costs, nonlinear stock-dependent demand, and trade credit. Meena et al. [12] concluded inventory management in an inflationary environment under delay in payments with shortages. Poswal et al. [13] innovated an investigation and analysis of a fuzzy EOQ model for price-sensitive and stock-dependent demand under shortages. Chen et al. [14] implemented a retailer's economic order quantity when the supplier offers a conditionally permissible delay in payments linked to order quantity. Tripathi [15] computed an innovative approach to the EOQ structure for decaying items with time-sensitive demand, cash discounts, shortages, and a permissible delay in payments. Ghiami et al. [16] organized a two-echelon inventory model for a deteriorating item with stock-dependent demand, partial backlogging, and capacity constraints. Zadeh [17] first introduced the fuzzy set.

The key objective is to formulate the economic order quantity model under the condition of a permissible delay in payments with an allowed stock-out cost and a lead time for finding the optimized total cost.

2. Methodology

2.1 *Assumptions*

The following assumptions are made in this developed inventory model:

- Infinite planning horizon.
- Infinite replenishment rate.
- Stock-out cost is allowed.
- Lead time or delivery lag is allowed.
- Delivery lag crashing cost is taken by the form $C(L) = aL^{-b}$, $a > 0$, $0 < b \leq 0.5$.
- During that period, A account is not settled, and the merchant can earn interest. And they have to settle the account at A; for that, they want to pirate money at some specific rate of interest in order to get the balance stocks financed for the period A to T.

3. Model formulation

3.1 *Proposed inventory model*

Annual variable cost = Cost of placing orders + Cost of stock carrying + Cost of stock-out + Lead time crashing cost + Interest payable during the time $(T - A)$ – Interest earned (1)

$$\text{Cost of placing orders} = \frac{O}{T} \tag{2}$$

$$\text{Cost of stack carrying} = \frac{dTC}{2} \tag{3}$$

$$\text{Cost of stock-out} = — \tag{4}$$

$$\text{Lead time crashing cost} = C(L) = a\ L^{-b}, a > 0,\ 0 < b < 0.5 \tag{5}$$

For the calculated interest payable and interest earned, there are two possible cases, viz., $T \geq A$ and $T \leq A$.

Case 1: If $T \geq A$:

The cost of interest charges for the products retained in stock. The products that are being sold before the replenishment account is established are used to earn interest. The replenishment account is established, the situation is upturned, and efficiently, the products still in stock have to be financed at the interest rate I_c. The stock level at the time of settling the replenishment account is $d(T - A)$ and interest owed during the time is $(T - A)$.

$$\begin{aligned}\text{Interest payable per unit time} &= \frac{dpI_c\,(T-A)^2}{2T} \\ &= \frac{dpI_cT}{2} + \frac{dpI_cA^2}{2T} - dpI_cA\end{aligned} \tag{6}$$

The extreme accumulated amount is d. As if $T \geq A$ and earned interest during the payment period A.

Then,

$$\text{interest earn per unit time} = \frac{dsA^2I_e}{2T}. \tag{7}$$

The total annual variable cost per unit time is

$$C(T) = \frac{O}{T} + \frac{dTC}{2} + \frac{r}{T} + a\,L^{-b} + \frac{dpI_cT}{2} + \frac{dpI_cA^2}{2T} - dpI_cA - \frac{d\,sA^2\,I_e}{2T}. \tag{8}$$

The necessary and sufficient conditions for $C(T)$ to be minimum are

$$\frac{dC(T)}{dT} = 0 \text{ and } \frac{d^2C(T)}{dT^2} \geq 0$$

$$\frac{dC(T)}{dT} = -\frac{O}{T^2} + \frac{dC}{2} - \frac{r}{T^2} + \frac{dpI_c}{2} - \frac{dpI_c\,A^2}{2T^2} + \frac{dsA^2I_e}{2T^2} \tag{9}$$

$$\frac{d^2C(T)}{dT^2}=\frac{2O}{T^3}+\frac{2r}{T^3}+\frac{dpI_cA^2}{T^3}-\frac{dsA^2I_e}{T^3}\geq 0$$

$$\frac{dC(T)}{dT}=0 \tag{10}$$

$$T=\left[\frac{2O+2r+(pI_c-sI_e)dA^2}{d\,(C+pI_c)}\right]^{1/2}. \tag{11}$$

EOQ:

$$q(T)=dT=\left[\frac{d[2O+2r+(pI_c-sI_e)dA^2]}{(C+pI_c)}\right]^{1/2}. \tag{12}$$

Case 2: If $T \leq A$:

In this condition, the merchant can earn interest on income created from the sales up to T. And take interest paid will be zero. The extreme accumulated total is dTs, if $T \leq A$ and interest earned during the payment period A. Then,

$$\text{interest earned per unit}=\left[\frac{dsT^2}{2}+dTs\,(A-T)\right]\frac{I_e}{T}$$
$$=ds\left(A-T/2\right)I_e. \tag{13}$$

The total annual variable cost per unit time is

$$C(T)=\frac{O}{T}+\frac{dTC}{2}+\frac{r}{T}+a\,L^{-b}-dsAI_e+\frac{dsTI_e}{2}. \tag{14}$$

The necessary and sufficient conditions for $C(T)$ to be minimum are

$$\frac{dC(T)}{dT}=0 \text{ and } \frac{d^2C(T)}{dT^2}\geq 0$$

$$\frac{dC(T)}{dT}=-\frac{O}{T^2}+\frac{dC}{2}-\frac{r}{T^2}+\frac{dsI_e}{2} \tag{15}$$

$$\frac{d^2C(T)}{dT^2}=\frac{2O}{T^3}+\frac{2r}{T^3}\geq 0$$

$$\frac{dC(T)}{dT}=0. \tag{16}$$

Time interval:

$$T=\left[\frac{O+r}{d\,(C+sI_e)}\right]^{1/2} \tag{17}$$

EOQ:

$$q(T) = dT = \left[\frac{d[O + r]}{(C + sI_e)}\right]^{1/2} \tag{18}$$

4. Numerical example

The proposed model has been illustrated by the example.

Let d =1,250 units/year, C = Rs. 25 per unit per year, O = Rs. 20,000 per order, p = Rs. 1,800 per unit, s = Rs. 200 per unit, r = Rs. 5 per unit, A = 0.05 year, I_c = Rs. 10 per investment in a stock per year, I_e = Rs. 8 per year, a = 2, b = 0.4, L = 40.

Solution

Case 1:

T = 0.0637

EOQ = $q(T)$ = 61.95

Total annual variable cost = Rs. 308,947.29

Case 2:

T = 0.0992

EOQ = $q(T)$ = 124.05

Total annual variable cost = Rs. 102,413.76

5. Sensitivity analysis

Case 1:

Table 1. Sensitivity analysis for varying demand value (d) in Case 1

s No.	*d*	*T*	q^*	*TC* (Rs.)
1	1,200	0.0642	77.04	309,105.39
2	1,250	0.0637	79.59	308,947.29
3	1,300	0.0631	82.03	308,677.77
4	1,350	0.0626	84.51	308,308.18
5	1,400	0.0621	86.94	307,846.73

Case 2:

Table 2. Sensitivity analysis for varying demand value (d) in Case 2

s No.	*d*	*T*	q^*	*TC* (Rs.)
1	1,200	0.1013	121.56	104,250.69
2	1,250	0.0992	124.05	102,413.76
3	1,300	0.0973	126.49	100,374.83
4	1,350	0.0955	128.93	98,228.46
5	1,400	0.0938	131.32	95,970.88

6. Concluding remarks

It is imperative to note that this is the first study demonstrating that the EOQ inventory model is a study about the circumstances of a permissible delay in payments with an allowed stock-out cost and lead time. From this prescribed inventory model, obtain the values using arithmetical analysis and sensitivity analysis to find the solutions for time interval, EOQ, and the total annual variable cost. In the sensitivity analysis, we change the demand values; we get different values in time, economic order quantity, and total cost. From the sensitivity analysis, we observed that as demand increased, the time interval and total cost decreased. For future studies, this inventory model can extend to stock-dependent demand and deteriorating items in a fuzzy sense.

Data availability

No data was used for the research described in the article.

Conflict of interest

The authors have declared that they have no known competition, financial interest, or personal relationship that could have appeared to influence the work reported in this research article.

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