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Research Article

Ordering Results of Aggregate Claim Amounts from Two Heterogeneous Portfolios

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Abstract: In this paper, we discuss the stochastic comparison of two classical surplus processes in a one-year insurance period. Under the Marshall-Olkin extended Weibull random aggregate claim amounts, we establish some sufficient conditions for the comparison of aggregate claim amounts in the sense of the usual stochastic order. Applications of our results to the Value-at-Risk and ruin probability are also given. The obtained results show that the heterogeneity of the risks in a given insurance portfolio tends to make the portfolio volatile, which in turn leads to requiring more capital. We also obtain some sufficient conditions for comparing aggregate non-random claim amounts with different occurrence frequency vectors in terms of increasing convex order.

***Keywords*:** increasing convex order, usual stochastic order, multivariate chain majorization, aggregate claim amounts, value-at-risk, ruin probability

MSC: 62P05, 60E15

1. Introduction

Consider the classical surplus process $U(t)$ given by

$$U(t) = u + ct - \sum_{i=1}^{N(t)} Z_i,$$

where $u = U(0)$, c, Z_i and $N(t)$ denote an initial surplus, constant premium, independent random claims and a given counting process, respectively. Now consider a situation when there are n policyholders in the given portfolio in a one-year insurance period. Under this assumption, the above classical surplus process $U(t)$ for $t = 1$ and $N(t) = n$ can be restated as

$$U(1) = u + c - \sum_{i=1}^{n} I_{p_i} X_{\lambda_i}, \tag{1}$$

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where the random variable X_{λ_i} denotes the total of random claims that can be made in an insurance period and I_{p_i} denotes a Bernoulli random variable associated with X_{λ_i} defined as follows: $I_{p_i} = 1$ whenever the i-th policyholder makes random claim X_{λ_i} and $I_{p_i} = 0$ whenever he/she does not make a claim. This sort of restating the conditional classical surplus process $U(1)$ as the surplus process in (1) can be made for almost all insurance contracts.

In this paper, we present the usual stochastic comparison between two classical surplus processes that can be restated as in (1). Since u and c are two constant values, to study any stochastic comparison, we just have to consider only $\sum_{i=1}^{n} I_{p_i} X_{\lambda_i}$. Indeed, the random variable

$$S_n(\boldsymbol{\lambda}, \boldsymbol{p}) = \sum_{i=1}^{n} I_{p_i} X_{\lambda_i}$$

is of interest in various fields of probability and statistics. In particular, in actuarial science, it corresponds to the aggregate claim amount in a portfolio of risks.

Some of the related quantities in actuarial sciences such as Value-at-Risk, stop-loss premium for cumulative claims, and ruin probability in finite time, include the sum of random variables. In general, finding a distribution of the sum of the random variables is difficult. So, we usually use simulation methods for the approximation of its distribution. In this paper, we find a lower bound for the survival function of this sum and its related quantity by using the concept of stochastic order. This reason can be considered as the other motivation for the main results of this paper.

The problem of comparing the numbers of claims and aggregate claim amounts with respect to some well-known stochastic orders is of interest on both theoretical and practical grounds. In this direction, [1] showed that more dispersion of the p_i's, according to the vector majorization, implies more dispersion of the total number of claims with respect to the convex order. Specifically, they proved that

$$(p_1, \cdots, p_n) \overset{m}{\succeq} (p_1^*, \cdots, p_n^*) \Rightarrow \sum_{i=1}^{n} I_{p_i} \leq_{cx} \sum_{i=1}^{n} I_{p_i^*}, \tag{2}$$

where $I_{p_1^*}, \cdots, I_{p_n^*}$ are independent Bernoulli random variables, independent of the X_{λ_i}'s, with $E(I_{p_i^*}) = p_i^*, i = 1, \cdots, n$. Ma [2] then extended the result in (2) to the case of aggregate claim amounts with the same amount of claims, and different claims, and showed that if $X_{\lambda_1}, \cdots, X_{\lambda_n}$ are non-negative exchangeable random variables (i.e., X_{λ_i} are identically distributed but not necessarily independent), then

$$(p_1, \cdots, p_n) \overset{m}{\succeq} (p_1^*, \cdots, p_n^*) \Rightarrow S_n(\boldsymbol{\lambda}, \boldsymbol{p}) \leq_{cx} S_n(\boldsymbol{\lambda}, \boldsymbol{p}^*) \tag{3}$$

$$(h(p_1), \cdots, h(p_n)) \overset{m}{\succeq} (h(p_1^*), \cdots, h(p_n^*)) \Rightarrow S_n(\boldsymbol{\lambda}, \boldsymbol{p}) \geq_{st} S_n(\boldsymbol{\lambda}, \boldsymbol{p}^*), \tag{4}$$

where $h(p) = -\log p$ or $(1-p)/p$. For the case when $(h(p_1), \cdots, h(p_n)) \in D_n^+$ and $(h(p_1^*), \cdots, h(p_n^*)) \in D_n^+$, where $D_n^+ = \{(x_1, \cdots, x_n) : x_1 \geq \cdots \geq x_n \geq 0\}$, Ma [2] also proved that (4) holds if $X_{\lambda_1}, \cdots, X_{\lambda_n}$ are independent non-negative random variables such that $X_{\lambda_1} \leq_{st} \ldots \leq_{st} X_{\lambda_n}$. This was followed up by Frostig [3] and Hu and Ruan [4] who established some sufficient conditions for comparing the aggregate claim amounts with respect to the symmetric supermodular, and multivariate usual and symmetric stochastic orders see [5], respectively. Denuit and Frostig [6] studied the effect of an increase in heterogeneity (in the sense of chain majorization) on the individual model of risk theory according to increasing convex order. Khaledi and Ahmadi [7] discussed stochastic comparison of two aggregate claims corresponding to two possible different individual risk models in the sense of usual stochastic order. Barmalzan et al. [8] presented a complete version of the results of Khaledi and Ahmadi [7] to the more general case.

For any $1 \leq i \leq j \leq n$, denote the permutation

$$\tau_{ij}(a-1, \cdots, a_i, \cdots, a_j, \cdots, a_n) = (a_1, \cdots, a_j, \cdots, a_i, \cdots, a_n).$$

A multivariate real function $g(x)$ is said to be arrangement increasing (AI) if $g(x) \geq g(t_{ij}(x))$, for any $x \in \mathbb{R}^n$ with $x_i x_j$

and $1 \le i < j \le n$. For any (i, j), such that $1 \le i < j \le n$, let $\Delta_{ij} g(x) = g(x) - g(t_{ij}(x))$, and denote

$$\mathcal{G}_{ws}^{ij} = \left\{ g(x) : \Delta_{ij} g(x) \text{ is increasing in } x_j \right\}$$

A multivariate real function $g(x)$ is said to be weakly stochastic arrangement increasing (WSAI) if $E(g(X)) \ge E(g(_{ij}(X)))$, for any $g \in \mathcal{G}_{ws}^{ij}$ and $1 \le i < j \le n$, such that the expectations exist.

In the context of claim sizes having increasing and concave survival functions with respect to parameter and sum of any two of them having Schur-concave survival function, Li and Li [9] study the usual stochastic order on aggregate claim amounts with a random occurrence frequency vector, which serves as a duality of Theorem 3.5 of Zhang and Zhao [10]. These authors also improve the sufficient condition on the usual stochastic order of aggregate claim amounts due to Theorem 4.6 of Zhang and Zhao [10] by relaxing the AI joint density of the claim sizes to the WSAI claim sizes.

In this paper, under the Marshall-Olkin extended Weibull random aggregate claim amounts, we establish some sufficient conditions for the comparison of aggregate claim amounts in the sense of the usual stochastic order. Also, we establish the increasing convex order between aggregate claim amounts of constant claim sizes when the matrix of parameters changes to another matrix in terms of chain majorization order.

The rest of this paper is organized as follows. Section 2 presents some basic concepts that will be used in the subsequent developments. Section 3 deals with stochastic orderings between order statistics arising from two sets of independent heterogeneous Marshall-Olkin extended Weibull random variables in terms of the vector majorization between the scale parameters. In Section 4, we use the Marshall-Olkin extended Weibull distribution as the claim amount distribution and establish some sufficient conditions for the comparison of aggregate claim amounts in the sense of the usual stochastic order. In Section 5, we have also obtained some sufficient conditions for comparing aggregate non-random claim amounts with different occurrence frequency vectors in terms of increasing convex order. Finally, some direct applications of these results in the context of Value-at-Risk and ruin probability are presented in Section 6.

2. Preliminaries

In this section, we present the definitions of some well-known concepts relating to stochastic orders and majorization that are most pertinent to the results established in the subsequent sections.

Definition 2.1. Suppose X and Y are two non-negative continuous random variables with distribution functions $F(t) = P(X \le t)$ and $G(t) = P(Y \le t)$, and survival functions $\bar{F}(t) = 1 - F(t)$ and $\bar{G}(t) = 1 - G(t)$, respectively.

(i) X is said to be larger than Y in the usual stochastic order (denoted by $X \ge_{st} Y$) if $\mathbb{E}(\phi(X)) \ge \mathbb{E}(\phi(Y))$ for all increasing functions $\phi : \mathbb{R} \to \mathbb{R}$ when the involved expectations exist;

(ii) X is said to be larger than Y in the increasing convex order (denoted by $X \ge_{icx} Y$) if $\mathbb{E}(\phi(X)) \ge \mathbb{E}(\phi(Y))$ for all increasing and convex functions $\phi : \mathbb{R} \to \mathbb{R}$ when the involved expectations exist.

The usual stochastic order does not always hold and therefore it is a partial order. Consequently, it is important to provide more tools to compare random variables. The stochastic order is characterized by the comparison of the expectations of increasing transformations of the random variables. If we restrict our attention to a subset of these transformations, then it is possible to provide a weaker partial criterion to compare random variables, which is the case of the increasing convex order. It is worthwhile to note that the usual stochastic orders implies the increasing convex order.

Theorem 2.1. [5] Let $g(\cdot)$ be an increasing (decreasing) real-valued function. Then, $X \ge_{st} Y$ implies $g(X) \ge_{st} (\le_{st}) g(Y)$.

Theorem 2.2. [5] Two random variables X and Y satisfy $X \ge_{st} Y$ if and only if there exist two random variables $\tilde{X}$ and $\tilde{Y}$, such that $X \overset{st}{=} \tilde{X}$ and $Y \overset{st}{=} \tilde{Y}$ and $P(\tilde{X} > \tilde{Y}) = 1$, where $\overset{st}{=}$ means the same distribution on both sides of the equality.

Let $x = (x_1, \cdots, x_n)$ and $y = (y_1, \cdots, y_n)$ be two vectors in $\mathbb{R}^n$. We denote $x \le y$ if $x_i \le y_i$, for $i = 1, \cdots, n$. Let ϕ be a a multivariate function with domain in $\mathbb{R}^n$. If $\phi(x) \le \phi(y)$ whenever $x \le y$, then we say that the function ϕ is increasing. A multivariate version of the usual stochastic order is presented in the next definition.

Definition 2.2. Suppose $\mathbf{X} = (X_1, \cdots, X_n)$ and $\mathbf{Y} = (Y_1, \cdots, Y_n)$ are two random vectors. Then, $\mathbf{X}$ is said to be larger than $\mathbf{Y}$ in the usual multivariate stochastic order (denoted by $\mathbf{X} \geq_{st} \mathbf{Y}$) if $\mathbb{E}(\phi(\mathbf{X})) \geq \mathbb{E}(\phi(\mathbf{Y}))$ for all increasing functions $\phi : \mathbb{R}^n \to \mathbb{R}$ when the involved expectations exist.

The multivariate stochastic ordering implies component-wise usual stochastic ordering. Interested readers may refer to [11] and [5] for comprehensive discussions on univariate and multivariate stochastic orders.

Definition 2.3. For two vectors $\mathbf{a} = (a_1, \cdots, a_n)$ and $\mathbf{b} = (b_1, \cdots, b_n)$, let $\{a_{1:n}, \cdots, a_{n:n}\}$ and $\{b_{1:n}, \cdots, b_{n:n}\}$ denote the increasing arrangements of their components, respectively. Then, the vector $\mathbf{a}$ is said to majorize the vector $\mathbf{b}$ (denoted by $\mathbf{a} \overset{m}{\succeq} \mathbf{b}$) if

$$\sum_{j=1}^{i} a_{j:n} \leq \sum_{j=1}^{i} b_{j:n} \text{ for } i = 1, \cdots, n-1$$

and

$$\sum_{j=1}^{n} a_{j:n} = \sum_{j=1}^{n} b_{j:n}.$$

A square matrix Π is said to be a permutation matrix if each row and column has a single unit, and all other entries to be zero. There are $n!$ such matrices of size $n \times n$, each of which is obtained by interchanging rows (or columns) of the identity matrix I. The matrix of a T-transform (T-transformation) has the form $T = w\mathrm{I} + (1 - w)\Pi$, where $0 \leq w \leq 1$ and Π is a permutation matrix (see Chapter 2 of [12]).

Definition 2.4. Suppose $A = \{a_{ij}\}$ and $B = \{b_{ij}\}$ are $m \times n$ matrices. Then, A is said to chain majorize B (denoted by $A >> B$) if there exists a finite set of $n \times n$ T_i-transform matrices, $i = 1, ..., k$, such that $B = AT_1T_2 ...T_k$. For an elaborate discussion on the theory of vector and matrix majorizations and their applications, we refer the readers to Marshall et al. (2011).

Lemma 2.1. [12] Suppose $A = \{a_{ij}\}$ and $B = \{b_{ij}\}$ are $m \times n$ matrices. A differentiable function $\phi : \mathbb{R}^{m\times n} \to \mathbb{R}$ satisfies

$$\phi(A) \geq \phi(B) \tag{5}$$

for all A and B, such that $A >> B$ if and only if

(i) $\phi(A) = \phi(A\Pi)$ for all permutation matrices Π;

(ii) $\sum_{i=1}^{m}(a_{ik} - a_{ij})[\phi_{ik}(A) - \phi_{ij}(A)]$ for all $j, k = 1, \cdots, n$, where $\phi_{ij}(A) = \partial\phi(A)/\partial a_{ij}$.

Lemma 2.2. [13] If ϕ is a positive function and $\log \phi$ is convex, then ϕ is convex.

3. Usual multivariate stochastic order between order statistics

In this section, we examine stochastic comparisons of order statistics from independent heterogeneous Marshall-Olkin extended Weibull (MOEW) random variables, with respect to the vector majorization between their vectors of scale parameters. The results obtained in this section help us to use the MOEW distribution as the claim amount distribution in the subsequent discussions.

Marshall and Olkin [14] originally proposed a new family of distributions by adding a shape parameter to a specified distribution. Specifically, suppose H is a baseline distribution function with support $\mathbb{R}^+$ and corresponding survival function $\bar{H}$. Then, they introduced the distribution

$$F(t;\alpha) = \frac{H(t)}{1 - \bar{\alpha}\bar{H}(t)}, t \in \mathbb{R}^+, 0 \leq \alpha \leq 1, \bar{\alpha} = 1 - \alpha. \tag{6}$$

Let us now use the Weibull distribution with $\bar{H}(t) = e^{-(\lambda t)^\beta}$ in (6). We then have

$$F(t;\alpha,\lambda,\beta) = \frac{1-e^{-(\lambda t)^\beta}}{1-\bar{\alpha}e^{-(\lambda t)^\beta}}, t, \lambda, \beta \in \mathbb{R}^+, 0 \le \alpha \le 1, \bar{\alpha} = 1-\alpha. \quad (7)$$

The family of distributions in (7) is called MOEW distribution with shape parameters α, β, and scale parameter λ (denoted by MOEW(α, λ, β)). For additional discussion on the MOEW distribution and its applications, we refer the readers to [15] and [16].

Theorem 3.1. [17] Suppose $X_{\lambda_1}, \cdots, X_{\lambda_n}$ are independent non-negative random variables with $X_{\lambda_i} \sim F(\lambda_i x)$, $i = 1, \cdots, n$. Assume that F is an absolutely continuous distribution function with hazard rate function r. If $r(x)$ and $xr(x)$ are decreasing and increasing in $x \in \mathbb{R}^+$, respectively, then

$$(\lambda_1, \cdots, \lambda_n) \overset{m}{\succ} (\lambda_1^*, \cdots, \lambda_n^*) \Rightarrow (X_{1:n}, \cdots, X_{n:n}) \ge_{st} (X_{1:n}^*, \cdots, X_{n:n}^*).$$

where $X_{i:n}$ and $X_{i:n}^*, i = 1, \cdots, n$ are the order statistics corresponding to X_{λ_i}'s and $X_{\lambda_i^*}$'s, respectively.

We shall now show that the MOEW distribution also satisfies the conditions of Theorem 3.1. For this purpose, we first need the following lemma.

Lemma 3.1. Suppose $X \sim MOEW(\alpha, 1, \beta)$ with hazard rate r. Then,

(i) $r(x)$ decreasing in $x \in \mathbb{R}^+$ for any $0 < \alpha \le 1$ and $0 < \beta \le 1$.

(ii) $xr(x)$ is increasing in $x \in \mathbb{R}^+$ for any $\alpha > 0$.

Proof. (i) The hazard rate function of X is $r(x) = \beta x^{\beta-1} / (1-\bar{\alpha}e^{-x^\beta})$. Taking derivative of $r(x)$ with respect to x, it readily follows that

$$(r(x))' \overset{sgn}{=} (\beta - 1)(1-\bar{\alpha}e^{-x^\beta}) - \beta\bar{\alpha}x^\beta e^{-x^\beta},$$

where $a \overset{sgn}{=} b$ means that a and b have the same sign. Now, from the assumption $0 < \alpha \le 1$ and $0 < \beta \le 1$, the desired result follows.

(ii) From (7), it is easy to observe that $xr(x) = \beta x^\beta / (1-\bar{\alpha}e^{-x^\beta})$. Then, we obtain

$$(xr(x))' \overset{sgn}{=} 1-\bar{\alpha}e^{-x^\beta} - \bar{\alpha}x^\beta e^{-x^\beta} = m(x).$$

It can be readily seen that which for $0 < \alpha \le 1$ ($\alpha \ge 1$) is non-negative (non-positive). Therefore, for $0 < \alpha \le 1$ ($\alpha \ge 1$), we have $m(x) \ge m(0) = 1-\alpha (m(x) \ge \lim_{x\to\infty} m(x) = 1)$. Thus, for any $\alpha > 0$, we see that $m(x) > 0$, which completes the proof of the lemma.

Theorem 3.2. Under the assumptions of Theorem 3.1, if $X_{\lambda_i} \sim MOEW(\alpha, \lambda_i, \beta)$, then for $0 < \alpha \le 1$ and $0 < \beta \le 1$,

$$(\lambda_1, \cdots, \lambda_n) \overset{m}{\succ} (\lambda_1^*, \cdots, \lambda_n^*) \Rightarrow (X_{1:n}, \cdots, X_{n:n}) \ge_{st} (X_{1:n}^*, \cdots, X_{n:n}^*).$$

where $X_{i:n}$ and $X_{i:n}^*, i = 1, \cdots, n$ are the order statistics corresponding to X_{λ_i}'s and $X_{\lambda_i^*}$'s, respectively.

Proof. The proof is immediately obtained from Theorem 3.1 and Lemma 3.1. It is worthwhile to note that the concept of majorization is a way of comparing two vectors of same dimension, in terms of the dispersion of their components for which the order $(\lambda_1, \cdots, \lambda_n) \overset{m}{\succ} (\lambda_1^*, \cdots, \lambda_n^*)$ results in the λ_i's being more dispersive than the λ_i^*'s, for a fixed sum. Then, based on Theorem 3.2, if λ_i's being more dispersive than the λ_i^*'s, then the vector of order statistics corresponding to λ_i's is larger than the other with respect to the usual stochastic order.

The following corollary is a direct consequence of Theorem 3.2, which shows that the survival function of the convolution of independent heterogeneous MOEW random variables is Schur-convex in the vector of scale parameters.

Corollary 3.1. Under the assumptions of Theorem 3.2, for $0 < \alpha \le 1$ and $0 < \beta \le 1$,

$$(\lambda_1,\cdots,\lambda_n)\overset{m}{\succ}(\lambda_1^*,\cdots,\lambda_n^*)\Rightarrow\sum_{i=1}^{n}X_{\lambda_i}\geq_{st}\sum_{i=1}^{n}X_{\lambda_i^*}$$

4. Usual stochastic order between aggregate claim amounts

In this section, we discuss the stochastic order of aggregate claim amounts with respect to the usual stochastic order. We also explain how we can use the MOEW distribution as the claim amount distribution. For this purpose, let us set

$$\mathcal{U}_n=\left\{(\mathbf{x},\mathbf{y})'=\begin{pmatrix}x_1,\dots,x_n\\ y_1,\dots,y_n\end{pmatrix}:x_i,y_j>0 \text{ and } (x_i-x_j)(y_i-y_j)\geq 0, i,j=1,\cdots,n\right\}.$$

Theorem 4.1. [8] Suppose $X_{\lambda_1},\cdots,X_{\lambda_n}$ are independent non-negative random variables with X_{λ_i} having survival function $\bar{F}(\cdot;\lambda_i)$, where $\lambda_i>0$ for $i=1,\cdots,n$, and that $I_{p_1},\cdots,I_{p_n}$ are independent Bernoulli random variables, independent of the X_{λ_i}'s, with $E(I_{p_i})=p_i, i=1,\cdots,n$. In addition, suppose the following two conditions hold:

(i) $\bar{F}(\cdot;\lambda_i)$, is a decreasing convex function with respect to λ_i, $i=1,\cdots,n$;

(ii) The survival function of $\sum_{i=1}^{n}X_{\lambda_i}$ is Schur-convex in λ.

Then, for $(\boldsymbol{\lambda},\boldsymbol{h}(\boldsymbol{p}))'\in\mathcal{U}_n$ and $(\boldsymbol{\lambda}^*,\boldsymbol{h}(\boldsymbol{p}^*))'\in\mathcal{U}_n$, the survival function of $S_n(\boldsymbol{\lambda},\boldsymbol{p})$ is Schur-convex in $(\boldsymbol{\lambda},\boldsymbol{h}(\boldsymbol{p}))$ with respect to multivariate chain majorization, where $h(p)=-\log p$ or $h(p)=(1-p)/p$.

Next, we prove that the MOEW distribution can be used as the claim amount distribution and then it is satisfied in the conditions of Theorem 4.1.

Theorem 4.2. Suppose $X_{\lambda_1},\cdots,X_{\lambda_n}$ are independent random variables with $X_{\lambda_i}\sim \text{MOEW}(\alpha,\lambda_i,\beta)$, $i=1,\cdots,n$, and that $I_{p_1},\cdots,I_{p_n}$ are independent Bernoulli random variables, independent of the X_{λ_i}'s, with $E(I_{p_i})=p_i$, $i=1,\cdots,n$. Then, for $0<\alpha\leq 1$, $0<\beta\leq 1$ and $(\boldsymbol{\lambda},\boldsymbol{h}(\boldsymbol{p}))'\in\mathcal{U}_n$, the survival function of $S_n(\boldsymbol{\lambda},\boldsymbol{p})$ is Schur-convex in $(\boldsymbol{\lambda},\boldsymbol{h}(\boldsymbol{p}))'$ with respect to multivariate chain majorization, where $h(p)=-\log p$ or $h(p)=(1-p)/p$.

Proof. From (7), we have

$$\bar{F}(t;\alpha,\lambda,\beta)=\frac{\alpha e^{-(\lambda t)^\beta}}{1-\bar{\alpha}e^{-(\lambda t)^\beta}},\quad t\in\mathbb{R}^+.$$

For fixed $t>0$, let us define the function $m(\lambda)=\log\bar{F}(t;\alpha,\lambda,\beta),\lambda>0.$ It is to observe that

$$m''(\lambda)=-(\beta-1)\lambda^{\beta-2}-\frac{(\beta-1)\bar{\alpha}\lambda^{\beta-2}\left(1-\bar{\alpha}e^{-(\lambda t)^\beta}\right)-\beta\bar{\alpha}t^\beta\lambda^{2\beta-1}e^{-(\lambda t)^\beta}}{\left(1-\bar{\alpha}e^{-(\lambda t)^\beta}\right)^2},$$

which is non-negative for any $0<\alpha\leq 1$, $0<\beta\leq 1$, and so $m(\lambda)$ is convex with respect to λ for any $0<\alpha\leq 1$, $0<\beta\leq 1$. Now, from Lemma 2.2, we can conclude that for any $0<\alpha\leq 1$, $0<\beta\leq 1$ and fixed $t>0,\bar{F}(t;\lambda)$ is convex with respect to λ. Moreover, we can readily see that $\bar{F}(t;\lambda)$ is decreasing with respect to λ. These observations confirm Condition (i) of Theorem 4.1. In addition, from Corollary 3.1, we readily observe that the survival function of $\sum_{i=1}^{n}X_{\lambda_i}$ is Schur-convex in λ for any $0<\alpha\leq 1$. So, Condition (ii) of Theorem 4.1 is also satisfied, which completes the proof of the theorem.

The following example provides an illustration for the result in Theorem 4.2.

Example 4.1. Suppose X_{λ_1}, X_{λ_2}, X_{λ_3} are independent random variables with $X_{\lambda_i}\sim\text{MOEW}(0.1,\lambda_i,0.5)$, $i=1,2,3$, and that I_{p_1}, I_{p_2}, I_{p_3} are independent Bernoulli random variables, independent of the X_{λ_i}'s, with $E(I_{p_i})=p_i$, i = 1, 2, 3. Also, let

$$\begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ -\log p_1 & -\log p_2 & -\log p_3 \end{pmatrix} = \begin{pmatrix} 5 & 3 & 4 \\ 6 & 1 & 3 \end{pmatrix}, \begin{pmatrix} \lambda_1^* & \lambda_2^* & \lambda_3^* \\ -\log p_1^* & -\log p_2^* & -\log p_3^* \end{pmatrix} = \begin{pmatrix} 3.88 & 4.12 & 4 \\ 3.2 & 3.8 & 3 \end{pmatrix}.$$

It is then easy to observe that both matrices are in $\mathcal{U}_3$. Consider T-transform matrices T_1 and T_2 as follows:

$$T_1 = 0.4\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 0.6\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, T_2 = 0.8\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 0.2\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Then, we have $(\boldsymbol{\lambda}^*, -\mathbf{log}\ \boldsymbol{p}^*)' = (\boldsymbol{\lambda}, -\mathbf{log}\ \boldsymbol{p})T_1 T_2$, which based on Definition 2.4, implies that

$$\begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ -\log p_1 & -\log p_2 & -\log p_3 \end{pmatrix} \gg \begin{pmatrix} \lambda_1^* & \lambda_2^* & \lambda_3^* \\ -\log p_1^* & -\log p_2^* & -\log p_3^* \end{pmatrix}.$$

Now, according to Theorem 4.2, it then follows that $S_3(\boldsymbol{\lambda}, \boldsymbol{p}) \geq_{st} S_3(\boldsymbol{\lambda}^*, \boldsymbol{p}^*)$.

Example 4.2. Suppose X_{λ_1} and X_{λ_2} are independent random variables with $X_{\lambda_i} \sim$ MOEW(1, λ_i, 1), i = 1, 2, and suppose I_{p_1} and I_{p_2} are independent Bernoulli random variables, independent of the X_{λ_i}'s, with $E(I_{p_i}) = p_i$, i = 1, 2. Set

$$\begin{pmatrix} \lambda_1 & \lambda_2 \\ -\log p_1 & -\log p_2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 6 & 0.8 \end{pmatrix} \text{ and } \begin{pmatrix} \lambda_1^* & \lambda_2^* \\ -\log p_1^* & -\log p_2^* \end{pmatrix} = \begin{pmatrix} 2.9 & 2.1 \\ 1.32 & 5.48 \end{pmatrix}.$$

Consider the T-transform matrix T as

$$T = 0.1\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 0.9\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Now, with this setting and based on Part (ii) of Definition 2.5, we observe that

$$\begin{pmatrix} \lambda_1 & \lambda_2 \\ -\log p_1 & -\log p_2 \end{pmatrix} \gg \begin{pmatrix} \lambda_1^* & \lambda_2^* \\ -\log p_1^* & -\log p_2^* \end{pmatrix},$$

and these matrices are not in $\mathcal{U}_2$. On the other hand, the survival function of $S_2(\boldsymbol{\lambda}, \boldsymbol{p})$, for $x > 0$, is obtained as

$$\begin{aligned} \bar{F}(x, \lambda, p) &= P(I_{p_1} X_{\lambda_1} + I_{p_2} X_{\lambda_2} > x) \\ &= P(I_{p_1} X_{\lambda_1} + I_{p_2} X_{\lambda_2} > x \mid I_{p_1} = 1, I_{p_2} = 0) P(I_{p_1} = 1, I_{p_2} = 0) \\ &\quad + P(I_{p_1} X_{\lambda_1} + I_{p_2} X_{\lambda_2} > x \mid I_{p_1} = 0, I_{p_2} = 1) P(I_{p_1} = 0, I_{p_2} = 1) \\ &\quad + P(I_{p_1} X_{\lambda_1} + I_{p_2} X_{\lambda_2} > x \mid I_{p_1} = 1, I_{p_2} = 1) P(I_{p_1} = 1, I_{p_2} = 1) \\ &= p_1(1 - p_2)\bar{F}(x; \lambda_1) + p_2(1 - p_1)\bar{F}(x; \lambda_2) + p_1 p_2 \bar{F}(x; \lambda_1, \lambda_2), \end{aligned} \tag{8}$$

where $\bar{F}(x; \lambda_1, \lambda_2)$ denotes the survival function of $X_{\lambda_1} + X_{\lambda_2}$. Now, by using the above matrices in (8), we observe that

$$\bar{F}(0.1, \lambda, p) \simeq 0.03026 > 0.2026 \simeq \bar{F}(0.1, \lambda^*, p^*),$$

$$\bar{F}(0.8, \lambda, p) \simeq 0.0189 < 0.0271 \simeq \bar{F}(0.8, \lambda^*, p^*).$$

Thus, these survival functions cross, which means that $S_2(\boldsymbol{\lambda}, \boldsymbol{p}) \ngeq_{st} S_2(\boldsymbol{\lambda}^*, \boldsymbol{p}^*)$.

5. Increasing convex order between aggregate non-random claim amounts

In this section, we discuss the comparison of aggregate non-random claim amounts with respect to increasing convex order. For this purpose, first, we establish the following lemma.

Lemma 5.1. Assume $\phi: \mathbb{R} \to \mathbb{R}$ is a differentiable, increasing, and convex function. Then, we have

$$\phi(a+x) - \phi(a) - \phi(x) + \phi(0) \geq 0,\ 0 \leq x \leq a.$$

Proof. Let us set $g(x) = \phi(a+x) - \phi(a) - \phi(x) + \phi(0)$. Taking the derivative of $g(x)$ with respect to x, it readily follows that $g'(x) = \phi'(a+x) - \phi'(x)$. Because ϕ is increasing and convex, g is also increasing and then $g(x) \geq g(0) = 0$. So, the desired result is immediately obtained.

Theorem 5.1. Suppose $\lambda_1, \cdots, \lambda_n$ are non-random claim size and that $I_{p_1}, \cdots, I_{p_n}$ are independent Bernoulli random variables corresponding to λ_i, with $E(I_{p_i}) = p_i$, $i = 1, \cdots, n$. Further, suppose $\lambda_1^*, \cdots, \lambda_n^*$ are non-random claim size and that $I_{p_1^*}, \cdots, I_{p_n^*}$ are independent Bernoulli random variables corresponding to λ_i, with $E(I_{p_i}) = p_i$, $i = 1, \cdots, n$. Assume that $h: [0,1] \to (0, \infty)$ is decreasing and convex and $xh'(x)$ is increasing in x. Then, for $(\boldsymbol{\lambda}, \boldsymbol{p})' \in \mathcal{U}_n$ and $(\boldsymbol{\lambda}^*, \boldsymbol{p}^*)' \in \mathcal{U}_n$, we have

$$(\boldsymbol{\lambda}, \boldsymbol{h}(\boldsymbol{p}))' \gg (\boldsymbol{\lambda}^*, \boldsymbol{h}(\boldsymbol{p}^*))' \Rightarrow \sum_{i=1}^{n} \lambda_i I_{p_i} \geq_{icx} \sum_{i=1}^{n} \lambda^* I_{p_i^*}.$$

Proof. From $(\boldsymbol{\lambda}, \boldsymbol{p})' \in \mathcal{U}_n$ and $(\boldsymbol{\lambda}^*, \boldsymbol{p}^*)' \in \mathcal{U}_n$, without loss of generality, we assume $\lambda_1 \geq \lambda_2 \cdots \geq \lambda_n, \lambda_1^* \geq \lambda_2^* \cdots \geq \lambda_n^*$, $p_1 \geq p_2 \cdots \geq p_n$ and $p_1^* \geq p_2^* \cdots \geq p_n^*$. First, we prove the result for the case when $n = 2$. Let $a_i = h(p_i)$, for $i = 1, 2$. For any increasing and convex function ϕ, we have

$$\begin{aligned}
\Gamma(\lambda, a) &:= E[\phi(\lambda_1 I_{p_1} + \lambda_2 I_{p_2})] \\
&= \phi(\lambda_1 + \lambda_2) p_1 p_2 + \phi(\lambda_1) p_1 (1 - p_2) + \phi(\lambda_2) p_2 (1 - p_1) + \phi(0)(1 - p_1)(1 - p_2) \\
&= \phi(\lambda_1 + \lambda_2) h^{-1}(a_1) h^{-1}(a_2) + \phi(\lambda_1) h^{-1}(a_1)(1 - h^{-1}(a_2)) \\
&\quad + \phi(\lambda_2) h^{-1}(a_2)(1 - h^{-1}(a_1)) + \phi(0)(1 - h^{-1}(a_1))(1 - h^{-1}(a_2)).
\end{aligned}$$

To establish the desired result, we have to check Conditions (i) and (ii) of Lemma 2.1. Clearly, $\Gamma(\boldsymbol{\lambda}, \boldsymbol{a})$ is permutation invariant on $\mathcal{U}_2$, which confirms Condition (i) of Lemma 2.1. On the other hand, for $i = j$, consider the function Λ as

$$\Lambda(\boldsymbol{\lambda}, \boldsymbol{a}) = (\lambda_1 - \lambda_2)\left(\frac{\partial \Gamma(\boldsymbol{\lambda}, \boldsymbol{a})}{\partial \lambda_1} - \frac{\partial \Gamma(\boldsymbol{\lambda}, \boldsymbol{a})}{\partial \lambda_2}\right) + (a_1 - a_2)\left(\frac{\partial \Gamma(\boldsymbol{\lambda}, \boldsymbol{a})}{\partial a_1} - \frac{\partial \Gamma(\boldsymbol{\lambda}, \boldsymbol{a})}{\partial a_1}\right).$$

The partial derivatives of $\Gamma(\boldsymbol{\lambda}, \boldsymbol{a})$ with respect to λ_1 and λ_2 are

$$\begin{aligned}
\frac{\partial \Gamma(\boldsymbol{\lambda}, \boldsymbol{a})}{\partial \lambda_1} &= \phi'(\lambda_1 + \lambda_2) h^{-1}(a_1) h^{-1}(a_2) + \phi'(\lambda_1) h^{-1}(a_1)(1 - h^{-1}(a_2)) \\
\frac{\partial \Gamma(\boldsymbol{\lambda}, \boldsymbol{a})}{\partial \lambda_2} &= \phi'(\lambda_1 + \lambda_2) h^{-1}(a_1) h^{-1}(a_2) + \phi'(\lambda_1) h^{-1}(a_2)(1 - h^{-1}(a_1)),
\end{aligned}$$

respectively. Similarly, the partial derivatives of $\Gamma(\boldsymbol{\lambda}, \boldsymbol{a})$ with respect to a_1 and a_2 are

$$\begin{aligned}
\frac{\partial \Gamma(\boldsymbol{\lambda}, \boldsymbol{a})}{\partial a_1} &= h^{-1}(a_2) \frac{\partial h^{-1}(a_1)}{\partial a_1}\left[\phi(\lambda_1 + \lambda_2) - \phi(\lambda_1) - \phi(\lambda_2) + \phi(0)\right] + \frac{\partial h^{-1}(a_1)}{\partial a_1}(\phi(\lambda_1) - \phi(0)) \\
\frac{\partial \Gamma(\boldsymbol{\lambda}, \boldsymbol{a})}{\partial a_2} &= h^{-1}(a_1) \frac{\partial h^{-1}(a_2)}{\partial a_2}\left[\phi(\lambda_1 + \lambda_2) - \phi(\lambda_1) - \phi(\lambda_2) + \phi(0)\right] + \frac{\partial h^{-1}(a_2)}{\partial a_2}(\phi(\lambda_2) - \phi(0)),
\end{aligned}$$

respectively. Then, we have

$$\begin{aligned}
\Lambda(\boldsymbol{\lambda},\boldsymbol{a}) &= (\lambda_1-\lambda_2)\left(\frac{\partial\Gamma(\boldsymbol{\lambda},\boldsymbol{a})}{\partial\lambda_1}-\frac{\partial\Gamma(\boldsymbol{\lambda},\boldsymbol{a})}{\partial\lambda_2}\right)+(a_1-a_2)\left(\frac{\partial\Gamma(\boldsymbol{\lambda},\boldsymbol{a})}{\partial a_1}-\frac{\partial\Gamma(\boldsymbol{\lambda},\boldsymbol{a})}{\partial a_1}\right)\\
&= (\lambda_1-\lambda_2)\left(\phi'(\lambda_1)h^{-1}(a_1)(1-h^{-1}(a_2))-\phi'(\lambda_2)h^{-1}(a_2)(1-h^{-1}(a_1))\right)\\
&+(a_1-a_2)\left[\phi(\lambda_1+\lambda_2)-\phi(\lambda_1)-\phi(\lambda_2)+\phi(0)\right]\left(h^{-1}(a_2)\frac{\partial h^{-1}(a_1)}{\partial a_1}-h^{-1}(a_1)\frac{\partial h^{-1}(a_2)}{\partial a_2}\right)\\
&+(a_1-a_2)\left(\frac{\partial h^{-1}(a_1)}{\partial a_1}(\phi(\lambda_1)-\phi(0))-\frac{\partial h^{-1}(a_2)}{\partial a_2}(\phi(\lambda_2)-\phi(0))\right).
\end{aligned}$$

Let us set $\lambda_1 \geq \lambda_2 > 0$. From increases and convexity properties of ϕ, we can conclude that

$$\phi(\lambda_1)-\phi(0)) \geq \phi(\lambda_2)-\phi(0)) > 0 \text{ and } \phi'(\lambda_1) \geq \phi'(\lambda_2) > 0$$

and then based on Lemma 5.1, we have

$$\phi(\lambda_1+\lambda_2)-\phi(\lambda_1)-\phi(\lambda_2)+\phi(0) > 0.$$

Since h is decreasing and $xh'(x)$ is increasing in x, for $a_1 \leq a_2$, we have

$$\begin{aligned}
A &=: h^{-1}(a_2)\frac{\partial h^{-1}(a_1)}{\partial a_1}-h^{-1}(a_1)\frac{\partial h^{-1}(a_2)}{\partial a_2}\\
&= \frac{h^{-1}(a_2)}{h'(h^{-1}(a_1))}-\frac{h^{-1}(a_1)}{h'(h^{-1}(a_2))}\\
&= \frac{1}{h'(h^{-1}(a_1))h'(h^{-1}(a_2))}\left[h^{-1}(a_2)h'(h^{-1}(a_2))-h^{-1}(a_1)h'(h^{-1}(a_1))\right]\\
&\leq 0.
\end{aligned}$$

Upon combining the obtained results, we conclude that $\Lambda(\boldsymbol{\lambda}, \boldsymbol{a})$ is positive, which confirms the Condition (ii) of Lemma 2.1. So, we have

$$(\boldsymbol{\lambda},\boldsymbol{h}(\boldsymbol{p}))' \gg (\boldsymbol{\lambda}^*,\boldsymbol{h}(\boldsymbol{p}^*))' \Rightarrow \lambda_1 I_{p_1}+\lambda_2 I_{p_2} \geq_{icx} \lambda_1^* I_{p_1^*}+\lambda_2^* I_{p_2^*}$$

Using this point, we now prove the required result for $n > 2$. For each pair (i, j) with $1 \leq i < j \leq n$ and any constant $\beta \in [0.5,1]$, let us set

$$\theta_{ij}(\beta)=h^{-1}[\beta h(p_i)+(1-\beta)h(p_j)], \theta_{ji}(\beta)=h^{-1}[(1-\beta)h(p_i)+\beta h(p_j)] \quad \text{and}$$

$$\eta_{ij}(\beta)=\beta\lambda_i+(1-\beta)\lambda_j, \eta_{ji}(\beta)=(1-\beta)\lambda_i+\beta\lambda_j.$$

It is easy to observe that $\theta_{ij}(\beta) > \theta_{ji}(\beta), \eta_{ij}(\beta) > \eta_{ji}(\beta), \lambda_i > \lambda_j, p_i > p_j$ and $(\boldsymbol{\lambda},\boldsymbol{h}(\boldsymbol{p}))' \gg (\boldsymbol{\eta}_{ij}(\boldsymbol{\beta}),\boldsymbol{\theta}_{ij}(\boldsymbol{\beta}))'$. Then, we have

$$\lambda_i I_{p_i}+\lambda_j I_{p_j} \geq_{icx} \eta_{ij}(\beta)I_{\theta_{ij}(\beta)}+\eta_{ji}(\beta)I_{\theta_{ji}(\beta)}.$$

Since the increasing convex order is closed under convolution, we have

$$\lambda_i I_{p_i}+\lambda_j I_{p_j}+\sum_{l\neq\{i,j\}}\lambda_l I_{p_l} \geq_{icx} \eta_{ij}(\beta)I_{\theta_{ij}(\beta)}+\eta_{ji}(\beta)I_{\theta_{ji}(\beta)}+\sum_{l\neq\{i,j\}}\lambda_l I_{p_l},$$

which completes the proof of the theorem.

Remark 5.1. It needs to be mentioned that the conditions "h is decreasing and convex while $xh'(x)$ is increasing in x" in Theorem 5.1 are general and hold for many arbitrary functions h. For example, we can show that in (i) $h(x) = -\log x$; (ii) $h(x) = (1-x)/x$; (iii) $h(x) = \theta^{-1}(x^{-\theta} - 1)$ for $\theta > 0$ and (iv) $h(x) = (1-\log x)^{-\theta} - 1$ for $\theta > 1$, all four h are decreasing and convex and also all four $xh'(x)$ are increasing in x.

6. Some applications of the results

In this section, we describe some applications of the results in the context of Value-at-Risk and ruin probability. These results show that the heterogeneity of the risks in a given insurance portfolio tends to make the portfolio volatile, which in turn leads to requiring more capital.

6.1 *Value-at-Risk*

The Value-at-Risk, denoted by VaR, which is defined based on quantiles of a random variable plays a critical role in risk measurement; [18] for more details. Two random risks X and Y can be compared by means of their VaRs. We may have two probability levels α_0 and α_1, such that $VaR[X; \alpha_0] \leq VaR[Y; \alpha_0]$ and $VaR[X; \alpha_1] \geq VaR[Y; \alpha 1]$. So, it is reasonable to consider a situation under which $VaR[X; \alpha] \geq VaR[Y; \alpha]$ for all probability level $\alpha \in (0, 1)$. For a risk X, the VaR at level $p \in (0,1)$ is defined as

$$VaR[X; p] = F^{-1}(p) = \inf\{u : F_X(u) \geq p\}.$$

The following corollary is a direct consequence of Theorem 4.1, which shows that the larger stochastic order aggregate claim amount lead to the desirable property of uniformly larger VaR.

Corollary 6.1. Under the assumptions of Theorem 4.2, for $0 < \alpha \leq 1$, $0 < \beta \leq 1$ and $(\boldsymbol{\lambda}, \boldsymbol{h}(\boldsymbol{p}))' \in \mathcal{U}_n$,

$$\begin{pmatrix} \lambda_1 & \dots & \lambda_n \\ h(p_1) & \dots & h(p_n) \end{pmatrix} \gg \begin{pmatrix} \lambda_1^* & \dots & \lambda_n^* \\ h(p_1^*) & \dots & h(p_n^*) \end{pmatrix} \Rightarrow VaR[S_n(\boldsymbol{\lambda}, \boldsymbol{p}); p] \geq VaR[S_n(\boldsymbol{\lambda}^*, \boldsymbol{p}^*); p],$$

where $h(p) = -\log p$ or $h(p) = (1-p)/p$.

6.2 *Ruin probability for a classical surplus process*

The ruin probability for the classical surplus process $U(t)$ has been defined as $\psi(u) = P(T < \infty | U(0) = u)$, where $T = \inf\{t : U(t) \leq 0\}$. In this subsection, we consider a situation where there are n policyholders in the given portfolio in a one-year insurance period ($t = 1$, $N(1) = n$). The following theorem shows that the larger stochastic order aggregate claim amount lead to uniformly larger ruin probability.

Theorem 6.1. Consider the assumptions of Theorem 4.1. Then,

$$S_n(\boldsymbol{\lambda}, \boldsymbol{p}) \geq_{st} S_n(\boldsymbol{\lambda}^*, \boldsymbol{p}^*) \Rightarrow \psi_{S_n(\boldsymbol{\lambda}, \boldsymbol{p})}(u) \geq \psi_{S_n(\boldsymbol{\lambda}^*, \boldsymbol{p}^*)}(u),$$

when the annual premiums are assumed to be the same.

Proof. We consider a situation where there are n policyholders in the given portfolio in a one-year insurance period ($t = 1$, $N(t) = n$). Assume that $S_n(\boldsymbol{\lambda}, \boldsymbol{p}) \geq s_t\, S_n(\boldsymbol{\lambda}^*, \boldsymbol{p}^*)$. According to Theorem 2.1, we have $-S_n(\boldsymbol{\lambda}, \boldsymbol{p}) \leq s_t - S_n(\boldsymbol{\lambda}^*, \boldsymbol{p}^*)$ and so $U(1) \leq_{st} U^\star(1)$. From Theorem 2.2 and without loss of generality, we get $P(U(t) \leq U^\star(t)) = 1$ and then $\{t : U^\star(1) < 0\} \subset \{t : U(1) < 0\}$. If $T = \inf\{t : U(1) < 0\}$ and $T^\star = \inf\{t : U^\star(1) < 0\}$, then $T \leq T^\star$. Since $\{T^\star < \infty\} \subset \{T < \infty\}$, we have $P\{T^\star < \infty | U(0) = u\} \leq P\{T < \infty | U(0) = u\}$, which completes the proof of the theorem.

The following corollaries are direct consequences of Theorem 6.1, which shows that the larger stochastic order aggregate claim amount lead to uniformly larger ruin probability.

Corollary 6.2. Under the assumptions of Theorem 3.2, for $0 < \alpha \le 1$, $0 < \beta \le 1$ and $(\boldsymbol{\lambda}, \boldsymbol{h}(\boldsymbol{p}))' \in \mathcal{U}_n$,

$$\begin{pmatrix} \lambda_1 \ldots \lambda_n \\ h(p_1) \ldots h(p_n) \end{pmatrix} \gg \begin{pmatrix} \lambda_1^* \ldots \lambda_n^* \\ h(p_1^*) \ldots h(p_n^*) \end{pmatrix} \Rightarrow \psi_{S_n(\boldsymbol{\lambda}, \boldsymbol{p})}(u) \ge \psi_{S_n(\boldsymbol{\lambda}^*, \boldsymbol{p}^*)}(u) \text{ with probability } 1,$$

when the annual premiums are assumed to be the same.

7. Concluding remarks

In this paper, we have discussed some stochastic comparisons of two MEOW random aggregate claim amounts. We have also obtained some sufficient conditions for comparing aggregate non-random claim amounts with different occurrence frequency vectors in terms of increasing convex order. Applications of our results to the VaR and ruin probability are also presented. The obtained results show that the heterogeneity of the risks in a given insurance portfolio tends to make the portfolio volatile, which in turn leads to requiring more capital.

It is of interest to note that our results do not restrict to actuarial sciences and can be used in various areas including reliability theory and survival analysis. For instance, suppose random variable X_{λ_i} presents life-length of the i-th component in a series system which may received a random shocked at binging. This random shocked may not be impact on the i-th component (set $I_{p_i} = 1$) or does (set $I_{p_i} = 0$). Thus, $Y_i = I_{p_i} X_{\lambda_i}$ admit X_{λ_i} when random shocked does not impact on the i-th component and zero when random shocked does.

As a possible generalization, it will be of interest to investigate whether the usual stochastic ordering between aggregate claim amounts can be strengthened to the hazard rate and likelihood ratio orders.

Conflict of interest

There is no conflict of interest in this study

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