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Research Article

Profit Analysis to an Industrial System Possessing Active Redundancy form using Geometric Distribution

Pankaj[1], **Jasdev Bhatti**[2], **Mohit Kumar Kakkar**[3]
Chitkara University Institute of Engineering and Technology, Chitkara University, Punjab, India.
E-mail: jasdev.bhatti@chitkara.edu.in

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Abstract: This paper elaborates on the approach towards reliability analysis for the active redundant system with a dual nature of repair strategy according to the kind of failure. The reliability characteristics using regenerative techniques are analyzed stochastically for the system, which includes two parallel units acting in an active redundancy form. A transition model has been designed for the working mechanism discussed in the paper and evaluated with the help of a geometric distribution possessing distinct repair. Regenerative techniques and geometric distribution were used to derive the numerical equations for reliability parameters such as availability, mean time to system failure, and repair mechanisms following down period of the system. A graphical analysis of the repair/failure rate with respect to the profit function has also been presented.

***Keywords*:** active redundancy, geometric distribution, redundancy techniques, stochastic modelling, steady-state probability distribution

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1. Introduction

Reliability engineering is a branch of engineering that applies scientific knowledge to a component, product, or plant as a technique to make sure that it will operate as per the requirements for a defined period of time in a particular environment without malfunctioning. A product's lifetime is necessary to maintain, and in order to achieve this, the maximum focus is laid upon dependability, which is the ability of a system or component to function as per the requirement for an established period of time under well-defined conditions. To make the system's performance better, researchers have proposed a variety of reliability-enhancing strategies, that includes redundancy, preventative maintenance, and prioritization. It requires a lot of work and preparation over many years to analyze the effectiveness of industrial models. There is a continuous advancement of new technologies for production and maintenance methods, due to which the working processes in the industrial sector change annually. Any industry's success depends on two crucial factors: when new designs are incorporated, and good or cutting-edge maintenance techniques are offered to its clients as necessary variables.

A considerable effort has been put in for many years in order to make a visible change in the dependability assessment of industrial models. In 2016, Hua et al. [14,15] pioneered the unit degradation paths approach in reliability analysis. They raised an important research issue in his analysis of systems with connected unit degradation mechanisms. They developed a method for listing all successful occurrences, which are arranged sequences of failures

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represented by system state transition routes, and obtained closed-form equations for determining the odds of successful events. Fang et al. [12] investigated the dependability of balanced engine series systems with 'm' sectors, each of which has a cold standby system with 'n' engines. Two models for the new balanced system were developed, and some performance indicators, such as formulas for system dependability, lifespan distributions, and means and variances of system lifetimes, are provided. Dong et al. [10,11-13] examined the wiener processes-based reliability with degradation systems. In order to describe the dynamic behaviors of a generic k-out-of-n:G warm standby system while taking into consideration the inspection features and maintenance impacts on the behaviors of the system, in 2020, Wu et al. [21] devised an analytical technique. The nonlinear program is optimized using the Markov process to reduce the estimated total cost. Chillar et al. [7,8,16] used the principles of deterioration and maintenance due to random shocks to examine the system's dependability. To perform maintenance and repairs on the unit, a single server instantly visits the system. If the unit is damaged by shocks, maintenance is required. The semi-Markov process [18] and the regenerating point approach [2] are used to get a variety of reliability characteristics, such as mean time to system failure (MTSF), availability [19], and profit function [7]. J. Bhatti et al. [4-6] proposed a dependability model for the car repair industry. Moreover, the inspection approach was created as a tool to assess failure and choose a repair plan that is focused on efficiency in terms of both time and money. The adaptive surrogate-based network reliability study was given by Dehghani et al. [9] using the merging of Bayesian additive regression trees and Monte Carlo simulation. Kumar et al. [17] examined a single-unit system model by including Weibull distribution for failure, preventative maintenance, and various other parameters. A two-stage degradation model is put forth, in which the system degradation levels in the first and second stages are represented by correlated bivariate Wiener processes and univariate Wiener processes [20]. Adlakha et al. [1] explored a two-unit cold standby communication system in which both units are originally packed and re-joined when needed. The system continues to function until a need arises or it fails after some time has passed after the operational unit fails. In 2019, Bhardwaj et al. [3] developed a mathematical model of how long a diesel locomotive can run with regard to a decreasing trend using neural networks. For the time series data of the parameter used as the train data for the neural network, interpolation is performed.

The current paper focuses on analyzing the food packaging system, which consists of two similar kinds of units that are well structured in active redundancy mode. This transition model (Figure 1) can be fairly applied to the food packaging industry, where the two machines described through A_0 are both in full operation in stage S_0 and are used to pack various types of foods or vegetables. As we know, during the operative mode of any machines there is always a fair chance of failure. Hence, in the current discussed machines, the same possibility of distinct nature of failures, i.e., minor and major failure, is very well introduced. The distinct repairman mechanism with different costs and times of repair has been introduced to resolve the above failures. S_1 to S_4 in the transition model clearly signifies the situations when one machine is in repair mode after facing either a minor or major type of failure and the other unit is satisfying the need to package the food items by operating with extra working time. The dual nature of a repairman with a repair rate r_1 is involved in inspecting the nature of failures. The benefit of including the dual nature of repairman 1 is that it reduces the cost of different inspections and minor repairs. In case the repairman finds that the machines needs repair for regular service or minor repairs like incoming power issues, main disconnect off, web brakes due to damaged transfer roll surface, poor web tracking, poor seals, or poor package cutoff. But in case the first repairman finds that the issue is bigger and needs a more efficient repairman and time he passes out the failed unit to the second repairman with repair rate r_2. The major issues like end and fin seal problem, back stand problems, unwind problems, etc. As we know, there is always the possibility of a system shutdown due to a complete failure of all units. The possibilities of these situations have also been discussed in the current paper through stages S_5 to S_8. In these situation, the working hours of the repairman increased with the increased cost of getting the system into operative mode by repairing at least one failed machine.

Table 1. Nomenclature

Symbol	Description
A_0	Units that fall under category A are operating.
A_{r1}	Upon failure, unit A will need inspection and minor repairs.
A_{r2}	Repair the unit A if it is not fixed at the initial inspection.
r_2	Repair rate of major failure.
A_{r1w}	Unit A is now awaiting a turn to enter the r1 stage of inspection and repair.
p_1 ,p_2	Probability that unit's A and B will fail.
r_1/s_1	Successful/unsuccessful probability rate for minor failure inspection and its repair.
r_2/s_2	Successful/Un-successful probability rate for major repair.
a/b	Probability of inspecting and repairing the minor/major failure.
©	Convolution of the two functions of non-negative variable.
P_{ij}	Steady state transition probability from state Si to Sj
q_{ij}/Q_{ij}	Probability and cumulative density function of the first transition from regenerating state "i" to "j".

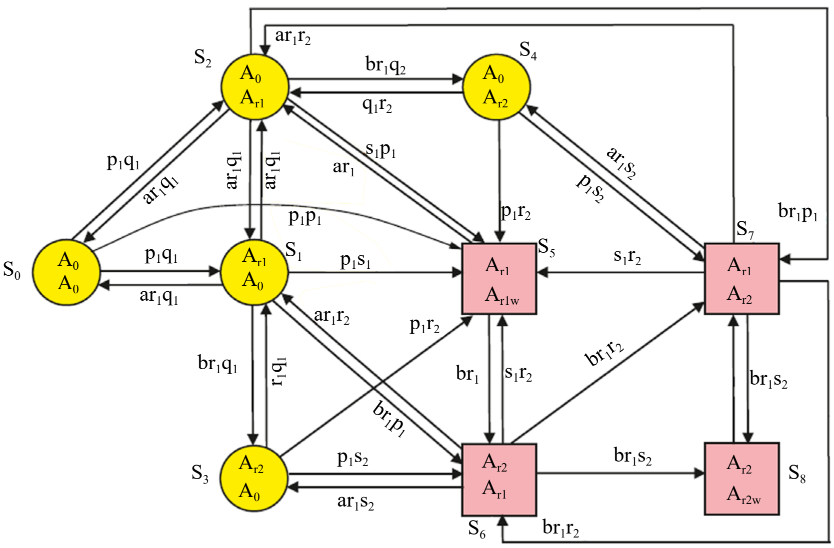

Figure 1. Transition Model

Operative States:

$S_0 = (A_0, A_0)$, $S_1 = (A_{r1}, A_0)$, $S_2 = (A_0, A_{r1})$, $S_3 = (A_{r2}, A_0)$, $S_4 = (A_0, A_{r2})$

Failed States:

$S_5 = (A_{r1}, A_{r1w})$, $S_6 = (A_{r2}, A_{r1})$, $S_7 = (A_{r1}, A_{r2})$, $S_8 = (A_{r2}, A_{r2w})$

2. Transition probabilities and Sojourn times

2.1. Transition Probabilities

As the system experiences discrete failures like less numbers of failures in small scale industries, the distribution of failures before a success is geometric. According to the following definition, the cumulative density function Q_{ij} of the initial passage time from regeneration state "i" to "j" is:

$$Q_{01}(t)=\frac{p_1q_1(1-(q_1q_1)^{t+1})}{1-q_1q_1},\quad Q_{02}(t)=\frac{q_1p_1(1-(q_1q_1)^{t+1})}{1-q_1q_1},\quad Q_{05}(t)=\frac{p_1p_1(1-(q_1q_1)^{t+1})}{1-q_1q_1},$$

$$Q_{10}(t)=\frac{ar_1q_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},\quad Q_{12}(t)=\frac{ar_1p_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},\quad Q_{13}(t)=\frac{br_1q_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},$$

$$Q_{15}(t)=\frac{s_1p_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},\quad Q_{16}(t)=\frac{br_1p_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},\quad Q_{20}(t)=\frac{ar_1q_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},$$

$$Q_{21}(t)=\frac{ar_1p_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},\quad Q_{24}(t)=\frac{br_1q_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},\quad Q_{25}(t)=\frac{s_1p_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},$$

$$Q_{27}(t)=\frac{br_1p_1(1-(s_1q_1)^{t+1})}{1-s_1q_1},\quad Q_{31}(t)=\frac{r_2q_1(1-(q_1s_2)^{t+1})}{1-s_2q_1},\quad Q_{35}(t)=\frac{p_1r_2(1-(q_1s_2)^{t+1})}{1-s_2q_1},$$

$$Q_{36}(t)=\frac{p_1s_2(1-(q_1s_2)^{t+1})}{1-s_2q_1},\quad Q_{42}(t)=\frac{q_1r_2(1-(q_1s_2)^{t+1})}{1-q_1s_2},\quad Q_{45}(t)=\frac{p_1r_2(1-(q_1s_2)^{t+1})}{1-q_1s_2},$$

$$Q_{47}(t)=\frac{p_1s_2(1-(q_1s_2)^{t+1})}{1-q_1s_2},\quad Q_{52}(t)=\frac{ar_1(1-(s_1)^{t+1})}{1-s_2},\quad Q_{56}(t)=\frac{br_1(1-(s_1)^{t+1})}{1-s_1},$$

$$Q_{61}(t)=\frac{ar_1r_2(1-(s_1s_2)^{t+1})}{1-s_1s_2},\quad Q_{63}(t)=\frac{ar_1s_2(1-(s_1s_2)^{t+1})}{1-s_1s_2},\quad Q_{65}(t)=\frac{s_1r_2(1-(s_1s_2)^{t+1})}{1-s_1s_2},$$

$$Q_{67}(t)=\frac{br_1r_2(1-(s_1s_2)^{t+1})}{1-s_1s_2},\quad Q_{68}(t)=\frac{br_1s_2(1-(s_1s_2)^{t+1})}{1-s_1s_2},\quad Q_{72}(t)=\frac{ar_1r_2(1-(s_1s_2)^{t+1})}{1-s_1s_2},$$

$$Q_{78}(t)=\frac{br_1s_2(1-(s_1s_2)^{t+1})}{1-s_1s_2},\quad Q_{87}(t)=\frac{r_2(1-(s_2)^{t+1})}{1-s_2}.$$

Following are the solutions for the probability steady-state transition from S_i to S_j:

$$P_{ij}=\lim_{t\to\infty}\ Q_{ij} \tag{1}$$

where Q_i represents the "cumulative density function" from state i to j. And,

$$P_{01}(t)=\frac{p_1q_1}{1-q_1q_1},\quad P_{02}(t)=\frac{q_1p_1}{1-q_1q_1},\quad P_{05}(t)=\frac{p_1p_1}{1-q_1q_1},$$

$$P_{10}(t)=\frac{ar_1q_1}{1-s_1q_1},\quad P_{12}(t)=\frac{ar_1p_1}{1-s_1q_1},\quad P_{13}(t)=\frac{br_1q_1}{1-s_1q_1},$$

$$P_{15}(t)=\frac{s_1p_1}{1-s_1q_1},\quad P_{16}(t)=\frac{br_1p_1}{1-s_1q_1},\quad P_{20}(t)=\frac{ar_1q_1}{1-s_1q_1},$$

$$P_{21}(t)=\frac{ar_1p_1}{1-s_1q_1},\quad P_{24}(t)=\frac{br_1q_1}{1-s_1q_1},\quad P_{25}(t)=\frac{p_1r_1}{1-s_1q_1},$$
$$P_{27}(t)=\frac{br_1p_1}{1-s_1q_1},\quad P_{31}(t)=\frac{r_2q_1}{1-s_2q_1},\quad P_{35}(t)=\frac{p_1r_2}{1-s_2q_1},$$
$$P_{36}(t)=\frac{p_1s_2}{1-s_2q_1},\quad P_{42}(t)=\frac{q_1r_2}{1-q_1s_2},\quad P_{45}(t)=\frac{p_1r_2}{1-s_2q_1},$$
$$P_{47}(t)=\frac{p_1s_2}{1-q_1s_2},\quad P_{52}(t)=\frac{ar_1}{1-s_1},\quad P_{56}(t)=\frac{br_1}{1-s_1},$$
$$P_{61}(t)=\frac{ar_1r_2}{1-s_1s_2},\quad P_{63}(t)=\frac{ar_1s_2}{1-s_1s_2},\quad P_{65}(t)=\frac{s_1r_2}{1-s_1s_2},$$
$$P_{67}(t)=\frac{br_1r_2}{1-s_1s_2},\quad P_{68}(t)=\frac{br_1s_2}{1-s_1s_2},\quad P_{72}(t)=\frac{ar_1r_2}{1-s_1s_2},$$
$$P_{74}(t)=\frac{ar_1s_2}{1-s_1s_2},\quad P_{75}(t)=\frac{s_1r_2}{1-s_1s_2},\quad P_{76}(t)=\frac{br_1r_2}{1-s_1s_2},$$
$$P_{78}(t)=\frac{br_1s_2}{1-s_1s_2},\quad P_{87}(t)=\frac{r_2}{1-s_2},$$

and,

$$P_{01}+P_{02}+P_{05}=1,\quad P_{10}+P_{12}+P_{13}+P_{15}+P_{16}=1,\quad P_{20}+P_{21}+P_{24}+P_{25}+P_{27}=1$$
$$P_{31}+P_{35}+P_{36}=1,\quad P_{42}+P_{45}+P_{47}=1,\ P_{52}+P_{56}=1,\quad P_{61}+P_{63}+P_{65}+P_{67}+P_{68}=1$$
$$P_{72}+P_{74}+P_{75}+P_{76}+P_{78}=1,\ P_{87}=1$$

2.2. Mean Sojourn Times

Let T_i be the sojourn time in state S_i (i = 0 to 8) and the symbol "μ" the mean sojourn time for state S_i

$$\mu_i=E(T_i)\sum_{t=0}^{\infty}P(T_i>t)$$

therefore,

$$\mu_0=\frac{1}{1-q_1q_1}\quad \mu_1=\frac{1}{1-s_1q_1}\quad \mu_2=\frac{1}{1-s_1q_1}\quad \mu_3=\frac{1}{1-s_2q_1}\quad \mu_4=\frac{1}{1-q_1s_2}$$

$$\mu_5=\frac{1}{1-s_1}\quad \mu_6=\frac{1}{1-s_1s_2}\quad \mu_7=\frac{1}{1-s_1s_2}\quad \mu_8=\frac{1}{1-s_2}$$

When a system is about to transition into state S, its means sojourn time (m_{ij}) in state S is calculated as follows:

$$m_{ij}=\sum_{t=0}^{\infty}t\ q_{ij}(t)$$

$m_{01}+m_{02}+m_{05} = q_1q_2\mu_0$ $m_{10}+m_{12}+m_{13}+m_{15}+m_{16}= s_1q_1\mu_1$
$m_{20}+m_{21}+m_{24}+m_{25}+m_{27} = s_1q_1\mu_2$ $m_{31}+m_{35}+m_{36} = s_2q_1\mu_3$
$m_{42}+m_{45}+m_{47}= q_1s_2\mu_4$ $m_{52}+m_{56} = s_1\mu_5$
$m_{61}+m_{63}+m_{65}+m_{67}+m_{68} = s_1s_2\mu_6$ $m_{72}+m_{74}+m_{75}+m_{76}+m_{78} = s_1s_2\mu_7$
$m_{87}=s_2\mu_8$

3. Mean time to system failure (MTSF)

MTSF is the average length of time a nonrepairable component or system lasts before failing which is measured by the maintenance statistic. The reliability analysis Λ_i at time 't' is therefore achieved by solving the equation shown below:

$$\begin{aligned} &\Lambda_0= Z_0+ q_{01}\,\Lambda_1+q_{02}©\Lambda_2 \\ &\Lambda_1= Z_1+ q_{10}©\,\Lambda_0+q_{12}©\,\Lambda_2+q_{13}©\,\Lambda_3 \\ &\Lambda_2= Z_2+ q_{20}\,©\,\Lambda_0+q_{21}\,©\,\Lambda_1+q_{24}\,©\,\Lambda_4 \\ &\Lambda_3= Z_3+ q_{31}\,©\,\Lambda_1 \\ &\Lambda_4= Z_4+ q_{42}\,©\,\Lambda_2 \end{aligned} \tag{2-6}$$

By taking Geometric transformation and solving the above equations, we obtain

$$\Lambda_i(h)=\frac{N_1(h)}{D_1(h)}$$

$$MTSF=\mu_i=\lim_{h\to 1}\frac{N_1(h)}{D_1(h)}-1=\frac{N_1}{D_1} \tag{7}$$

where

$$N_1= \mu_0[(1-P_{13}P_{31})(-P_{24}P_{42})+(-P_{13}P_{31})+(1-P_{12}P_{21})]+\mu_1[(-P_{01}P_{24}P_{42})+(P_{01}+P_{02}P_{21})]+ \\ \mu_2[(P_{02}P_{13}P_{31})-(P_{01}P_{12}-P_{02})]+\mu_3[P_{13}(P_{01}(1-P_{24}P_{42})+(P_{02}P_{21}))]+\mu_4[P_{24}((P_{01}P_{12})+P_{02}(1-P_{13}P_{31}))] \tag{8}$$

$$D_1=-P_{24}P_{42}(1-P_{13}P_{31})-(P_{13}P_{31})+(1-P_{12}P_{21})-P_{01}P_{10}(1-P_{24}P_{42})-P_{01}P_{12}P_{20}-P_{02}(P_{01}P_{21}+P_{20}(1-P_{13}P_{31})) \tag{9}$$

4. System availability

The availability of a particular system is defined as the likelihood that a repairable system or system component is operating at a specific time and under a specific set of environmental conditions. If Δ_i is the system's availability period at time t, then the following probabilistic relations will be deduced:

$$\begin{aligned} &\Delta_0= Z_0 + q_{01}©\,\Delta_1+q_{02}©\,\Delta_2+q_{05}©\,\Delta_5 \\ &\Delta_1= Z_1+ q_{10}©\,\Delta 0+q_{12}©\,\Delta_2+ q_{13}©\,\Delta_3+q_{15}©\,\Delta_5+q_{16}©\,\Delta_6 \\ &\Delta_2= Z_2+ q_{20}©\,\Delta_0+q_{21}©\,\Delta_1+q_{24}©\Delta_4+q_{25}©\Delta_5+q_{27}©\Delta_7 \\ &\Delta_3=Z_3+q_{31}©\,\Delta_1+q_{35}©\,\Delta_5+q_{36}©\,\Delta_6 \\ &\Delta_4 = Z_4+ q_{42}©\,\Delta_2+q_{45}©\,\Delta_5+q_{47}©\,\Delta_7 \\ &\Delta_5= q_{52}©\,\Delta_2+q_{56}©\,\Delta_6 \\ &\Delta_6= q_{61}©\,\Delta_1+q_{63}©\,\Delta_3+q_{65}©\Delta_5+q_{67}©\Delta_7\; q_{68}©\Delta_8 \\ &\Delta_7= q_{72}©\,\Delta_2+q_{74}©\,\Delta_4+q_{75}©\Delta_5+q_{78}©\Delta_8 \\ &\Delta_8= q_{87}©\,\Delta_7 \end{aligned} \tag{10-18}$$

So, by taking geometric transformation and solving the above equations, we obtain

$$U_0(h) = \frac{N_2(h)}{D_2(h)}$$

and

$$Z_i(h) = \mu_i$$

The steady state availability of system is:

$$U_0 = \lim_{t \to \infty} U_0(t) \tag{19}$$

By 'L' Hospital rule, we obtained

$$U_0 = -\frac{N_2(1)}{D_2(1)},$$

where

$$D_2(1) = (P_{21}P_{52}P_{78}P_{87}+P_{21}P_{52})[(P_{13}(P_{35}+P_{36}P_{65})+(P_{15}+P_{16}P_{65})+P_{63}(P_{15}P_{36}+P_{16}P_{35})] \\ +P_{10}(-P_{01}P_{56}P_{68}P_{87}-P_{01}P_{56}P_{67})[(-P_{75}-P_{45}P_{74})+P_{42}(P_{24}P_{75}-P_{25}P_{74})-P_{72}(P_{24}P_{45}+P_{25})] \\ -P_{20}[P_{12}(1-P_{56}P_{65})+P_{52}(P_{15}+P_{16}P_{65})] \tag{20}$$

$$N_2(1) = \mu_0(P_{21}P_{52}P_{78}P_{87}+P_{21}P_{52})[(P_{13}(P_{35}+P_{36}P_{65})+(P_{15}+P_{16}P_{65})+P_{63}(P_{15}P_{36}+P_{16}P_{35})] \\ +\mu_1(-P_{01}P_{56}P_{68}P_{87}-P_{01}P_{56}P_{67})[(-P_{75}-P_{45}P_{74})+P_{42}(P_{24}P_{75}-P_{25}P_{74})-P_{72}(P_{24}P_{45}+P_{25})]+ \\ \mu_2[P_{12}(1-P_{56}P_{65})+P_{52}(P_{15}+P_{16}P_{65})]+\mu_3(P_{01}P_{13}P_{56}P_{68}P_{87}+P_{01}P_{13}P_{56}P_{67})[(P_{75}+P_{45}P_{74}) \\ +P_{42}(P_{25}P_{74}-P_{24}P_{75})+P_{72}(P_{25}+P_{24}P_{45})]+\mu_4(P_{13}P_{24}P_{52}+P_{13}P_{52}P_{27}P_{74})[P_{01}(P_{35}+P_{36}P_{65})-P_{05}(P_{31}+P_{36}P_{61})] \tag{21}$$

5. Repairman (r_1) and inspection period of System

In order to boost system dependability and ensure customer satisfaction, it is crucial to have the finest repair procedures for all of its goods. However, as we are all aware, there are several potential causes for any mechanically sound system to fail. To reduce time waste and provide accurate information to the client regarding the time and cost of repair, it is increasingly vital to have the failed unit evaluated in order to determine the cause of the failure and proceed with the appropriate repair procedure.

Therefore, the repair process has been divided into two stages: a) inspection of failure or repair of a small failure or regular service by the repairman (r_1), and b) repair of a serious failure indicated by repairman (r_2). If Ω_i denotes the repairman (r_1) period of the system at time 't,' then the resulting relations will be designed as follows:

$$\begin{gathered} \Omega_0 = q_{01}© \Omega_1+q_{02}© \Omega_2+q_{05}© \Omega_5 \\ \Omega_1 = Z_1+q_{10}© \Omega_0+q_{12}© \Omega_2+q_{13}© \Omega_3+q_{15}© \Omega_5+q_{16}© \Omega_6 \\ \Omega_2 = Z_2+ q_{20}© \Omega 0+q_{21}© \Omega_1+q_{24}© \Omega_4+q_{25}© \Omega_5+q_{27}© \Omega_7 \\ \Omega_3 = q_{31}© \Omega_1+q_{35}© \Omega_5+q_{36}© \Omega_6 \\ \Omega_4 = q_{42}© \Omega_2+q_{45}© \Omega_5+q_{47}© \Omega_7 \\ \Omega_5 = Z_5+ q_{52}© \Omega_2+q_{56}© \Omega_6 \\ \Omega_6 = Z_6+ q_{61}© \Omega_1+q_{63}© \Omega_3+q_{65}© \Omega_5+q_{67}© \Omega_7+q_{68}© \Omega_8 \\ \Omega_7 = Z_7+ q_{72}© \Omega_2+q_{74}© \Omega_4+q_{75}© \Omega_5+q_{78}© \Omega_8 \\ \Omega_8 = q_{87}© \Omega_7 \end{gathered} \tag{22-30}$$

By geometric transformation and solving the above equations, we obtain

$$V_0(h) = -\frac{N_3(h)}{D_2(h)},$$

$$\text{and } V_0 = \lim_{t\to\infty} V_0(t)$$

Thus,

$$V_0 = \frac{N_3(1)}{D_2(2)} \quad (31)$$

$$N_3(1)=\mu_1(-P_{01}P_{56}P_{68}P_{87}-P_{01}P_{56}P_{67})[(-P_{75}-P_{45}P_{74})+P_{42}(P_{24}P_{75}-P_{25}P_{74})-P_{72}(P_{24}P_{45}+P_{25})]+\mu_2[P_{12}(1-P_{56}P_{65})+P_{52}(P_{15}+P_{16}P_{65})]+\mu_5[P_{13}(P_{35}+p_{36}p_{65})+P_{15}(1-P_{36}P_{63})+P_{16}(P_{65}+P_{35}P_{63})]+\mu_6[P_{24}(P_{45}+P_{47}P_{75})+(P_{25}+P_{27}P_{75})-P_{74}(P_{25}P_{47}-P_{27}P_{45})]+\mu_7[P_{01}(P_{12}P_{25}+P_{15})+(P_{02}P_{25}+P_{05})+P_{21}(P_{02}P_{15}+P_{05}P_{12})] \quad (32)$$

6. Repairman (r_2) Period of the System

If T_i denotes the repairman (r_2) period of the system at time 't,' then the resulting relations will be designed as follows:

$$\begin{gathered}
T_0 = q_{01}© T_1+q_{02}© T_2+q_{03}© T_3 \\
T_1 = q_{10}© T_0+q_{12}© T_2+q_{13}© T_3+q_{15}© T_5+q_{16}© T_6 \\
T_2=q_{20}© T_0+q_{21}© T_1+q_{24}© T_4+q_{25}© T_5+q_{27}© T_7 \\
T_3=Z_3+q_{31}© T_1+q_{35}© T_5+q_{36}© T_6 \\
T_4= Z_4+ q_{42}© T_2+q_{45}© T_5+q_{47}© T_7 \\
T_5= q_{52}© T_2+q_{56}© T_6 \\
T_6 = Z_6+q_{61}© T_1+q_{63}© T_3+q_{65}© T_5+q_{67}© T_7+q_{68}© T_8 \\
T_7 =Z_7+ q_{72}© T_2+q_{74}© T_4+q_{75}© T_5+q_{78}© T_8 \\
T_8 =Z_8+q_{87}© T_7
\end{gathered} \quad (33\text{-}41)$$

By taking geometric transformation and solving the above equations, we obtain

$$W_0(h) = \frac{N_4(h)}{D_2(h)},$$

$$\text{and } W_0 = \lim_{t\to\infty} W_0(t).$$

After applying 'L' Hospital rule:

$$W_0 = \frac{N_4(1)}{D_2(1)}, \quad (42)$$

$$\begin{gathered}
N_4(1)=\mu_3(P_{01}P_{13}P_{56}P_{68}P_{87}+P_{01}P_{13}P_{56}P_{67})[((P_{75}+P_{45}P_{74})+P_{42}(P_{25}P_{74}-P_{24}P_{75})+P_{72}(P_{25}+P_{24}P_{45})]+ \\
\mu_4(P_{13}P_{24}P_{52}+P_{13}P_{52}P_{27}P_{74})[P_{01}(P_{35}+P_{36}P_{65})-P_{05}(P_{31}+P_{36}P_{61)}]+ \\
\mu_6[P_{24}(P_{45}+P_{47}P_{75})+(P_{25}+P_{27}P_{75})-P_{74}(P_{25}P_{47}-P_{27}P_{45})]+ \\
\mu_7[P_{01}(P_{12}P_{25}+P_{15})+(P_{02}P_{25}+P_{05})+P_{21}(P_{02}P_{15}+P_{05}P_{12})]+ \\
\mu_8[(P_{27}P_{78})(P_{02}P_{56}+P_{05}P_{52})[(1-P_{36}P_{63})-P_{31}(P_{13}+P_{16}P_{63})-P_{61}(P_{13}P_{36}+P_{16})]
\end{gathered} \quad (43)$$

7. Result and Discussions

The system profit function (P) at steady-state has been evaluated using:

$$P = E_1U_0 - E_2V_0 - E_3W_0,$$

where

E_1: System per unit up time revenue.

E_2 and E_3: System per unit down time expenditure.

Results and behavior for system profit have been analyzed by fixing a few specific parameters E_1,E_2,E_3,p_2,and ‘a’ as follows:

$$E_1=10000, E_2=1000, E_3=500, p_2=0.6, \text{ and } a=0.6$$

Table 2, 3 and Figure 2, 3 show how reliability metrics, such as the profit function, change when the failure rate p_1 and repair rate r_1 rise from 0.1 to 0.8.

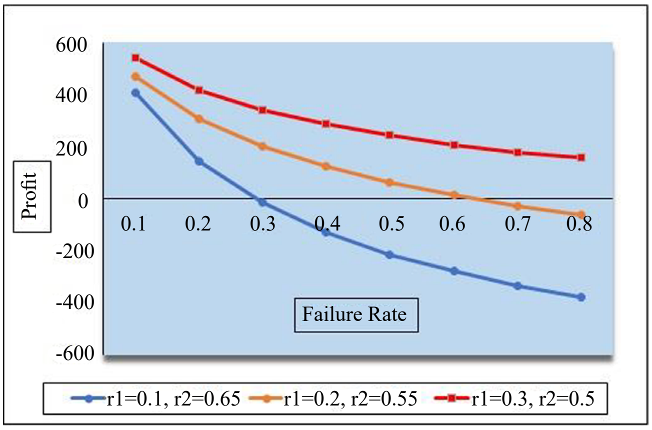

Figure 2. Profit P vs Failure rate p_1

Table 2. Reliability parameter values corresponding to repair rate r_1 and r_2

Repair Rate	MTSF	U_0	V_0	W_0	Profit(P)
	18.41938401	0.117340919	0.644495213	0.238163869	409.8320413
	8.958971095	0.094017952	0.686816446	0.219165602	143.7802693
	5.835730697	0.080325562	0.717604534	0.202069904	-15.38386852
r_1=0.1	4.248222733	0.070758139	0.74148818	0.187753681	-127.7836322
r_2=0.65	3.267932718	0.063586183	0.760861151	0.175552666	-212.7756543
	2.587934919	0.057994816	0.777190721	0.164814462	-279.6497923
	2.076045545	0.053528966	0.791465501	0.155005533	-333.6786057
	1.664798994	0.049911449	0.804413965	0.145674586	-378.1367672
	20.20870039	0.121164354	0.601372459	0.277463187	471.539486
	9.484710004	0.106333434	0.622430474	0.271236091	305.2858211
	6.069925375	0.097340177	0.639301532	0.26335829	202.4210966

$r_1 = 0.2$	4.37270967	0.090681496	0.65361963	0.255698874	125.3458928
$r_2 = 0.55$	3.340850453	0.08543415	0.666169357	0.248396493	63.97390063
	2.633009056	0.081199772	0.677487378	0.24131285	13.85391738
	2.104341688	0.077770129	0.68799675	0.234233121	-27.41201894
	1.681795916	0.075032696	0.698071303	0.226896001	-61.19234564
	22.05899759	0.126834391	0.582490692	0.290674917	540.5157623
	10.06511974	0.115517541	0.594653499	0.28982896	415.6074297
	6.346028343	0.1089683	0.604197605	0.286834095	342.0683492
$r_1 = 0.3$	4.528970677	0.104130033	0.612377	0.283492967	287.1768459
$r_2 = 0.5$	3.437566727	0.100302408	0.619651252	0.28004634	243.349656
	2.695505901	0.097224866	0.626308185	0.276466949	207.7069962
	2.144849426	0.094785834	0.63257295	0.272641216	178.9647815
	1.706600487	0.092948684	0.638649504	0.268401812	156.6364351

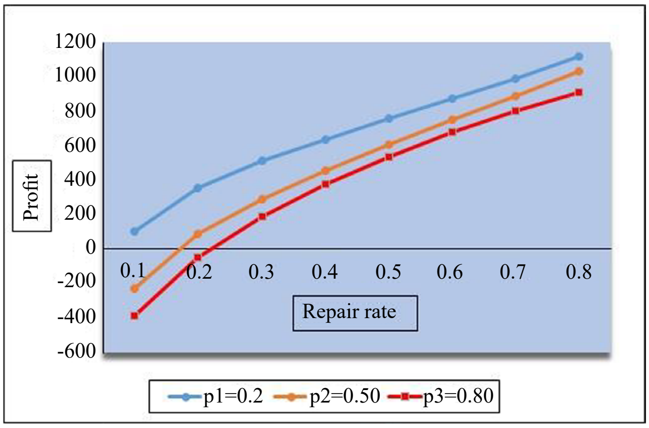

Figure 3. Profit P vs Repair rate r_1

Table 3. Reliability parameters values corresponding to Failure Rate p_1.

Failure Rate	MTSF	U_0	V_0	W_0	Profit(P)
	8.853743852	0.089692833	0.681417923	0.228889244	101.0658
	8.853743852	0.111286928	0.626544876	0.262168195	355.2403
	9.6097673	0.124910338	0.600783284	0.274306378	511.1669
p_1=0.2	10.37151298	0.136206318	0.584070242	0.279723439	638.1312
	11.15405131	0.146771164	0.571335951	0.281892885	755.4293
	11.96796821	0.157302456	0.560725567	0.281971977	871.313
	12.82193851	0.16824144	0.551408516	0.280350044	990.8309
	13.72391927	0.179994459	0.542961365	0.277044176	1118.461
	3.248144538	0.06203077	0.753800646	0.184168584	-225.577
	3.361709817	0.08787833	0.67237733	0.23974434	86.5338
	3.483292221	0.105473389	0.629407046	0.265119565	292.7671
p_1 = 0.5	3.61275125	0.120020244	0.601400155	0.278579602	459.5125
	3.750057056	0.133235004	0.580749313	0.286015683	608.5929
	3.895277906	0.145893407	0.564232931	0.289873662	749.7643

	4.048569257	0.158455289	0.550233391	0.29131132	888.6638
	4.210165593	0.049217042	0.797229847	0.153553112	1029.564
	1.662620664	0.076491169	0.705246801	0.21826203	-381.836
	1.683946304	0.096384479	0.650448939	0.253166582	-49.4661
	1.71094077	0.112690551	0.612802761	0.274506689	186.8126
$p_1 = 0.8$	1.743215084	0.126761071	0.584608466	0.288630463	376.8494
	1.780641954	0.139145073	0.562278913	0.298576014	538.687
	1.823295042	0.150056495	0.54392595	0.306017555	679.8838
	1.871418664	0.159534178	0.528473778	0.311992043	803.6302
	1.925417283	0.049217042	0.797229847	0.153553112	910.872

8. Conclusions

The focus of the current paper is on analyzing the active redundancy system for the food packaging machine and explaining the for different possibilities of repair for distinct failures. The working structure of the discussed problem is well designed through the transition model. The calculated numerical results for MTSF, availability period of the system, maintenance period in Sections 5 and 6 help to calculate the profit function of the system. The increasing and decreasing behavior of the profit function w.r.t increasing repair rate and a decreasing failure rate support the reliability parameters results. The numerical and graphical analysis of the reliability parameters proved that the concepts introduced in the paper proved beneficial, concluding with satisfying the focus of the research. Hence, the study report will demonstrate that its goals of advancing industries through the development of new procedures employing recommended repair methods for various failures.

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