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Research Article

Numerical Simulation of Slip Flow and Heat Transfer of Biomagnetic Fluid over a Stretching Sheet in the Presence of a Magnetic Dipole with Temperature Dependent Viscosity

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Abstract: The study of flow and heat transfer of biomagnetic fluid past a stretching sheet has a significant importance in a number of bio-medical and engineering applications including cancer treatment, drug delivery, magnetic resonance imaging, reducing blood flow during surgeries etc. Owing to these applications, the aim of the present paper is to study an electrically non-conducting Newtonian biomagnetic fluid in the presence of a magnetic dipole over an extendable sheet subject to velocity slip. The governing steady boundary layer equations with the help of similarity transformations were converted into a set of highly non-linear ordinary differential equations which are then computationally solved by applying the bvp4c function technique in MATLAB software. The results show that blood velocity and temperature can remarkably be influenced by the variation of ferromagnetic parameter. As the ferromagnetic parameter increases, the rate of heat transfer of blood is increased while coefficient of skin friction is reduced. We hope that this study could be useful in cancer treatment as well as in drug administration.

***Keywords*:** biomagnetic fluid, magnetic dipole, stretching sheet, slip parameter, temperature dependent viscosity

MSC: 76-10

Nomenclature

(u, v)	Velocity components [m/s]
(x, y)	Cartesian coordinates [m]
L	Slip length [m]
c	Stretching parameter [-]
d	Distance between dipole and sheet [m]
M	Fluid magnetization [A/m]
H	Magnetic field of strength [A/m]

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C_p	Specific heat at constant pressure [J Kg^{-1} K^{-1}]
ρ	Density [Kg/m^3]
T	Fluid temperature [K]
T_w	Temperature of the sheet [K]
T_∞	Ambient fluid temperature [K]
μ	Dynamic viscosity [Kg/ms]
μ^*	Constant values of the coefficient of the fluid viscosity [Kg/ms]
μ_0	Magnetic fluid permeability [NA^{-2}]
κ	Thermal conductivity [J/m s K]
$\mathrm{P}r$	Prandtl number [-]
β	Ferromagnetic interaction parameter [-]
λ	Viscous dissipation parameter [-]
A	Viscosity variation parameter [-]
ε	Dimensionless Curie temperature [-]
Re	Local Reynolds number [-]
δ	Slip parameter [-]
f'	Dimensionless velocity
η	Dimensionless coordinates [-]
Nu	Local Nusselt number [-]
C_f	Skin friction coefficient [-]
$\theta'(0)$	Wall heat transfer gradient [-]
θ	Dimensionless temperature [-]

1. Introduction

A fluid is found in a living being that is affected by the presence of a magnetic field is referred to as a biomagnetic fluid. One typical example of a biomagnetic fluid is blood. Because of the intricate interactions between proteins, cell membranes, and hemoglobin, which is made of iron oxides, blood can be also considered as a magnetic fluid. An enormous amount of research has been done over the past few decades on the dynamics of biological fluids in the presence of magnetic fields. The development of magnetic devices for cell separation, minimizing bleeding during operations, employing magnetic particles to carry drugs for the treatment of cancer tumors, and hyperthermia are just a few of the numerous uses that have been suggested in bioengineering and medical sciences.

Crane [1] was the first who described the boundary layer flow resulting from a stretching sheet that moves with linear variable velocity. Carragher et al. [2] extended the work of Crane [1] with considering heat transmission term in that sense that the difference between ambient fluid and surface temperature are in proportional to a power of the distance from a fixed point. Later on, Pop et al. [3] examined the nature of flow and heat transfer of fluid under the variations of fluid viscosity. The term that we familiar with Biomagnetic Fluid Dynamics (BFD) was first explained in mathematical point of view by Haik et al. [4] with the help of concepts of ferroHydrodynamics and considered that in flow domain magnetization force is the most dominant force. The effect of non-linear temperature dependent magnetization on biomagnetic fluid due to stretching sheet was examined by Tzirtzilakis et al. [5]. Later on Tzirtzilakis et al. [6] proposed a mathematical model of BFD which is constituted by Magnetohydrodynamics (MHD) and Ferrohydrodynamic (FHD) principles for the flow over a stretching sheet. Using a finite difference scheme, they observed that fluid flow is significantly affected. In particular, flow is reduced by up to 40% due to the presence of strength and gradient of applied magnetic field. The flow of Newtonian biomagnetic fluid under the combined effect of polarization and electrical conductivity occurred in a channel was examined by Tzirtzilakis et al. [7]. A steady, two dimensional turbulent flow of Newtonian biomagnetic fluid in the appearance of localized magnetic field between two parallel plates was studied by Tzirtzilakis et al. [8]. Using viscoelastic property of biomagnetic fluid, Misra et al. [9] investigated the heat and flow characteristics of bio fluid past a stretching sheet. The heat and flow characteristics of electrically conducting blood past a stretching sheet was presented by Misra et al. [10]. Recently, Murtaza et al. [11]

studied the variations of BFD, MHD and FHD over a stretching sheet for biomagnetic fluid flow problem. They found that in the case of BFD, fluid velocity is appreciably reduced compared to that of MHD and/or FHD cases and in that case the ferromagnetic parameter plays a vital role. The role of signum function on blood flow due to a nonlinear stretched sheet is presented by Murtaza et al. [12].

With the help of Message Authentication Code (MAC) algorithm in MATLAB package, Devi et al. [13] studied the unsteady, 2-dimensional buoyancy-driven non-Newtonian Casson Viscoplastic fluid flow in a square cavity. They observed that for any value of Rayleigh number, the heat transfer rate is improved with boosting values of Casson viscoplastic fluid parameter. Devi et al. [14] examined the impact of external magnetic field on unsteady Casson viscoelastic non-Newtonian fluid flow in a square enclosure and observed that for large number of buoyancy force temperature gradient of fluid is accelerated. Venkatadri [15] investigated the significance of transverse magnetic flux using finite differences based, vorticity-stream function approach on laminar magneto-convective flow through a differentially heated square in porous medium. The impact of thermal radiation and magnetic dipole on unsteady biomagnetic fluid flow past a stretching sheet investigated by Alam et al. [16]. With computational outcomes they seen that with increasing values of ferromagnetic interaction parameter, fluid velocity fall down while temperature profile improved in this case. The combined effect of magnetohydrodynamics and ferrohydrodynamics for blood flow with magnetic particles in cylindrical geometries was studied by Alam et al. [17] and with help of finite difference method they noted that blood flow can remarkably influenced for the case of BFD compared to that of MHD and/or FHD. A mathematical and computational simulation of biomagnetic fluid model in hyperthermia applications was proposed by Faruk et al. [18]. They see that uniform temperature enhanced temperature locally but reduced the overall temperature in the considered flow domain. The simulation of biomagnetic fluid flow due to curved stretching sheet affected by a strong uniform magnetic field is recently studied by Murtaza et al. [19] and looked that both skin friction and Nusselt number improved with the variation of the curvature parameter.

Considering all the aforementioned studies, to the authors' knowledge, very few studies are on the flow and heat transfer of blood that passed through a stretching sheet in the presence of a magnetic dipole subject to partial slip effects. The intention of this study is to mathematically and computationally analyze the characteristics of heat and flow of biomagnetic fluid whose viscosity varies linearly with temperature. The influence of pertinent parameters such as ferromagnetic number, partial slip, and viscosity variation parameter on the flow domain are also fetched into consideration. The governing Partial Differential Equations (PDEs) are transformed into non-linear Ordinary Differential Equations (ODEs) by applying appropriate similarity transformations along with the corresponding boundary conditions and which are finally solved in MATLAB package via bvp4c technique.

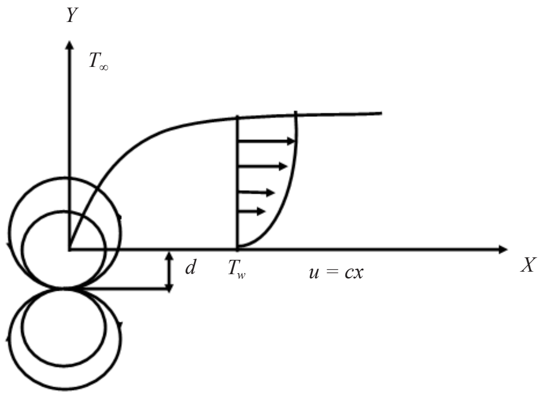

Figure 1. Geometry of the problem

2. Model analysis

For the present flow model, we considered the following assumptions:

i. We consider steady, incompressible, electrically non-conducting biomagnetic fluid namely blood flow, over a two-dimensional stretching sheet.

ii. It is also assumed that the sheet is linearly stretched with of velocity $u = cx$ along x-axis, where c is a constant and $c > 0$. Y-axis is taken perpendicular to the sheet.

iii. The temperature of surface is T_w while T_∞ is the ambient fluid temperature with $T_w < T_\infty$.

iv. A magnetic dipole is positioned under at distance, say d from the sheet. This dipole produces magnetic field strength of enough intensity to attain equilibrium magnetization. Also, the magnetic field gradient is steep enough so as the polarization force dominates in the flow field (see Figure 1).

v. The viscosity of blood is significantly dependent on the temperature.

Therefore, the flow and heat equations for steady biomagnetic fluid problem under the above assumptions are given by following expressions [4, 6, 20]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) + \frac{\mu_0}{\rho}M\frac{\partial H}{\partial x} \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) + \mu_0 T\frac{\partial \mathrm{M}}{\partial T}\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} \tag{3}$$

With appropriate boundary conditions:

$$\begin{gathered} u = cx + L\left(\frac{\partial u}{\partial y}\right), v = 0, T = T_w \quad at \quad y = 0 \\ u \to 0, T \to T_\infty \quad as \quad y \to \infty \end{gathered} \tag{4}$$

Where the fluid components are (u, v) along the (x, y) directions, respectively. Furthermore, the symbols ρ, μ, ϑ, κ, C_p, M, H, μ_0 represent the blood density, dynamic viscosity, kinematic viscosity, thermal conductivity, specific heat at constant pressure, magnetization, magnetic field strength and magnetic permeability, respectively. Also, c indicates stretching parameter where, L is the slip length.

The second term of momentum equation in right hand side of (2) i.e. $\mu_0 M\frac{\partial H}{\partial x}$ represents the magnetic force per unit volume of the applied magnetic field along x-direction and this will depends on the appearance of gradient magnetic and prior that this force disappear/vanish when magnetic gradient is absent. Also noted that due to electromagnetic effect in heat transmission, $\mu_0 T\frac{\partial \mathrm{M}}{\partial T}\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right)$ terms also appeared in energy equation and known as adiabatic magnetization term. These two terms are produced due to FHD [5-7].

Following [5-7, 21], the scalar potential of applied magnetic field is given by $V = \frac{\alpha}{2\pi}\frac{x}{x^2 + (y+d)^2}$, where α is a constant and at source point the strength of magnetic field is $\gamma = \alpha$. Therefore, the components of magnetic field along respective directions written as after some calculations:

$$H_x(x, y) = -\frac{\partial V}{\partial x} = \frac{\gamma}{2\pi}\frac{x^2 - (y+d)^2}{\left[x^2 + (y+d)^2\right]^2} \tag{5}$$

$$H_y(x,y) = -\frac{\partial V}{\partial y} = \frac{\gamma}{2\pi}\frac{2x(y+d)}{\left[x^2+(y+d)^2\right]^2} \tag{6}$$

As a result, the magnitude of H is given by the relation:

$$H(x,y) = \left[H_x^2 + H_y^2\right]^{1/2} = \frac{\gamma}{2\pi}\left[\frac{1}{(y+d)^2} - \frac{x^2}{(y+d)^4}\right] \tag{7}$$

Now the rate of change of H can be expressed as:

$$\frac{\partial H}{\partial x} = \frac{\gamma}{2\pi}\frac{-2x}{(y+d)^4}$$

$$\frac{\partial H}{\partial y} = \frac{\gamma}{2\pi}\left[\frac{-2}{(y+d)^3} + \frac{4x^2}{(y+d)^5}\right]$$

The linear relation of magnetization with temperature as mentioned by [22], can be given in the following form:

$$M = \mathrm{K}\left(T - T_\infty\right)$$

Where pyromagnetic coefficient is a constant defined by the symbol K.

According to [20] fluid viscosity can be considered to change linearly with temperature and the following relation of fluid viscosity is provided:

$$\mu = \mu^*\left[a + b\left(T_w - T\right)\right] \tag{8}$$

At free stream the value of viscosity coefficients is constant and represented by μ^* and a, b are symbol of constants with $b > 0$.

Here, according to the study [23], following mathematical relations of viscosity $\mu = a^* - b^*T$ which follows $\mu = e^{-a^*T}$ possible only when 2nd order terms are neglected from the above expansion.

Additionally, the mathematical form of kinematic viscosity is also provided by the relation $\vartheta = \vartheta^*\left[a + b\left(T_w - T\right)\right]$ where $\vartheta^* = \frac{\mu^*}{\rho}$.

In mathematic analysis, we introduce the corresponding non-dimensional variables:

$$u = cxf'(\eta),\ v = -\sqrt{cv^*}f(\eta),\ \eta(y) = \sqrt{\frac{c}{v^*}}y,\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

Therefore, the continuity equation (1) is automatically satisfied and equation (2)-(3) along with (4) are reduced in following set of equations:

$$\left(a + A - A\theta\right)f''' + ff'' - f'^2 - Af''\theta' - \frac{2\beta\theta}{(\eta+\alpha)^4} = 0 \tag{10}$$

$$\theta'' + \Pr f \theta' - 2\beta\lambda(\varepsilon + \theta)\frac{1}{(\eta+\alpha)^3} f = 0 \tag{11}$$

The corresponding boundary conditions are:

$$\begin{gathered} f' = 1 + \delta f'', \; f = 0, \; \theta = 1, \; at \; \eta = 0 \\ f' \to 0, \; \theta \to 0, \; as \; \eta \to \infty \end{gathered} \tag{12}$$

Where, $\Pr = \frac{\mu^* C_p}{k_\infty}$ is the Prandtl number, $\lambda = \frac{c\mu^{*2}}{\rho\kappa(T_w - T_\infty)}$ is the viscous dissipation parameter, $\beta = \frac{\gamma}{2\pi}$ $\frac{\mu_0 K(T_w - T_\infty)\rho}{\mu^{*2}}$ is the ferromagnetic interaction parameter, $\varepsilon = \frac{T_\infty}{T_w - T_\infty}$ is the dimensionless Curie temperature, $\alpha =$ $\sqrt{\frac{c}{\vartheta^*}}d$ is the dimensionless distance, $R_e = \frac{xU_w}{\vartheta^*}$ is the local Reynolds number, $\delta = L\sqrt{\frac{c}{\vartheta^*}}$ is the slip parameter, $A =$ $b(T_w - T_\infty)$ is the viscosity parameter.

From engineering perspective view to calculate the two important physical aspects are skin friction coefficient and the rate of heat transfer defined as follows:

$$C_f = \frac{\tau_w}{\rho_\infty u_w^{\;2}} \text{ and } Nu = \frac{xq_w}{k(T_w - T_\infty)} \tag{13}$$

Where,

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{14}$$

Hence, C_f and Nu reduce to:

$$C_f = \left(a + A - A\theta\right) R_e^{\frac{-1}{2}} f''(0) \tag{15}$$

$$Nu = -R_e^{\frac{-1}{2}} \theta'(0) \tag{16}$$

3. Numerical method for solution and program validation

For solving the highly non-linear system of equations (10) and (11) subject to the boundary conditions (12), we applied a numerical computation scheme. This technique is well known as bvp4c technique which was built in MATLAB package. To do that we need to reduce the highly non-linear equations to a system of first order ordinary differential equations and for that we have considered the following relations:

$$f = y_1, f' = y_2, f'' = y_3, \; \theta = y_4, \; \theta' = y_5.$$

As a result, equations are converted to the following system of 1st order as given below:

$$\left.\begin{aligned}
&f' = y_2\\
&f'' = y_2' = y_3\\
&f''' = y_3' = \frac{1}{\left(a + A - Ay_4\right)}\left(-y_1 y_3 + y_2^2 + \left(\frac{2\beta}{\left(\eta+\alpha\right)^4}\right) y_4 + Ay_3 y_5\right)\\
&\theta' = y_5\\
&\theta'' = y_5' = -\mathrm{P}ry_1 y_5 + \frac{2\beta\lambda\left(y_4+\varepsilon\right)y_1}{(\eta+\alpha)^3}
\end{aligned}\right\} \tag{17}$$

With the boundary conditions:

$$y_1(0) = 0,\ y_2(0) = 1 + \delta y_3(0),\ y_4(0) = 1,\ y_2(\infty) = 0,\ y_4(\infty) = 0 \tag{18}$$

System (17) and (18) are integrated numerically as an initial value problem until the required convergence is attained. By trial and error, we have fixed $\eta_\infty = 5$.

For accuracy purposes of the applied code, a validation of the numerical analysis has been made for $f''(0)$ with the existing work of Bhattacharyya et al. [20] for the different values of δ with $\beta = 0$ and comparison shows an excellent agreement (see Table 1).

Table 1. Comparison values of the skin friction coefficient $f''(0)$ with Bhattacharyya et al. [20] for various values of δ

δ	Bhattacharyya et al. [20]	Present results
0	-1.000000	-1.000529
0.1	-0.872083	-0.872187
0.2	-0.776377	-0.776481
0.5	-0.591195	-0.591397
1	-0.431060	-0.430163
2	-0.283980	-0.283237
5	-0.283980	-0.283237
10	-0.081242	-0.081450
20	-0.043789	-0.044813
50	-0.0818597	-0.019117
100	-0.009551	-0.00975

4. Results and discussion

The effect of various physical parameters, including the Prandtl number, the ferromagnetic interaction parameter, the slip parameter, the viscosity parameter on the velocity and temperature distributions as well as skin friction coefficient and the heat transfer rate is depicted in various graphical representations.

The regulating parameters values are selected to be physically realistic of the blood flow model. The parameters used for the obtained results pictured from Figure 2 through 15 are the following:

1. Ferromagnetic interaction parameter $\beta = 0 - 10$ as in [9, 24, 25].
2. Dimensionless distance $\alpha = 1$ as in [12].
3. Viscosity parameter A = -0.6, -0.4, -0.2 as in [26].
4. Prandtl number Pr = 21, 23, 25 as in [26-28].
5. Slip parameter δ = 0.0, 0.1, 0.2, 0.3 as in [20].

For the above assignment of the parameters an indicative case flow scenario is adopted. Human body is T_w = 37 °C while body Curie temperature can be considered T_∞ = 41 °C according to [27]. Therefore, the dimensionless Curie temperature is $\varepsilon = \frac{T_\infty}{T_\infty - T_w} = \frac{314}{314-310} = 78.5$. We also consider that blood density ρ = 1,050 kgm^{-3}, dynamical viscosity μ^* = 3.2 × 10^{-3} kgm^{-1} s^{-1}, specific heat at constant pressure is C_p = 3.9 × 10^3 J kg^{-1} K^{-1} and thermal conductivity is κ = 0.5 $J\ (msK)^{-1}$ [27-28]. Using these values, we have the viscous dissipation parameter $\lambda = \frac{c\mu^{*2}}{\rho\kappa(T_\infty - T_w)} = \frac{1.28\times10^{-5}\times\left(3.2\times10^{-3}\right)^2}{1,050\times0.5\times(314-310)} = 6.14\times10^{-14}$. Also, by taking the above values, Prandtl number for human blood is found $Pr = \frac{\mu C_p}{\kappa} = \frac{3.2\times10^{-3}\times3.9\times10^3}{0.5} \cong 25$. All other physical parameters values also allocated in a similar way.

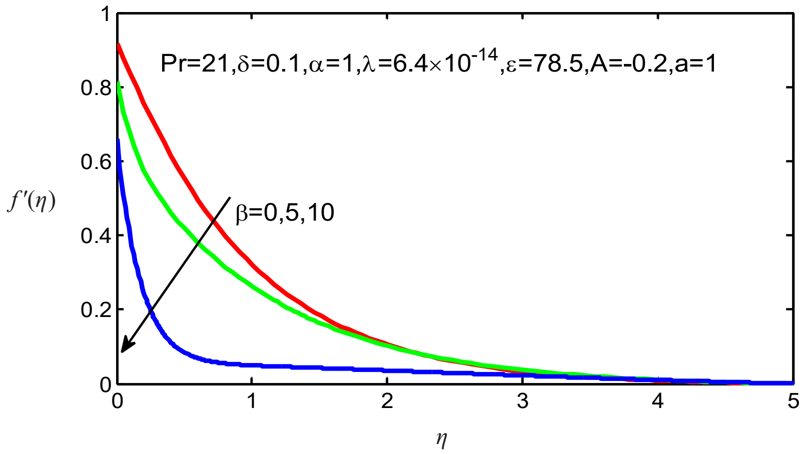

Figure 2. Variations of β on $f'(\eta)$

The variations of velocity $f'(\eta)$ and temperature $\theta(\eta)$ under ferromagnetic interaction parameter effects are shown in Figure 2 and Figure 3. It is seen from figures that, fluid velocity decreases and temperature profile is enhanced as the values of β increase. This is happening because of enhancement in the resistance force that is Kelvin force, which also known as drag force. As a result, velocity slowdown in the flow domain and the corresponding thermal boundary layer is enchased. Thus, the reverse behavior of fluid is observed for the temperature in Figure 3 which is enhanced with the increment of β.

The velocity $f'(\eta)$ and temperature $\theta(\eta)$ variations for various values of Prandtl number are shown at Figure 4 and 5, respectively. The graph indicates that $f'(\eta)$ decreases but $\theta(\eta)$ increases for increasing values of the Prandtl number. This is an expected behavior, for a fixed value of magnetic field i.e. β constant, since, the Prandtl number expresses the ratio of momentum to the thermal transport in the boundary layer. For greater values of Prandtl number the momentum transport is getting greater related to that of the thermal transport and consequently in the boundary layer the velocity is getting greater and the temperature is reduced with increasing values of Pr.

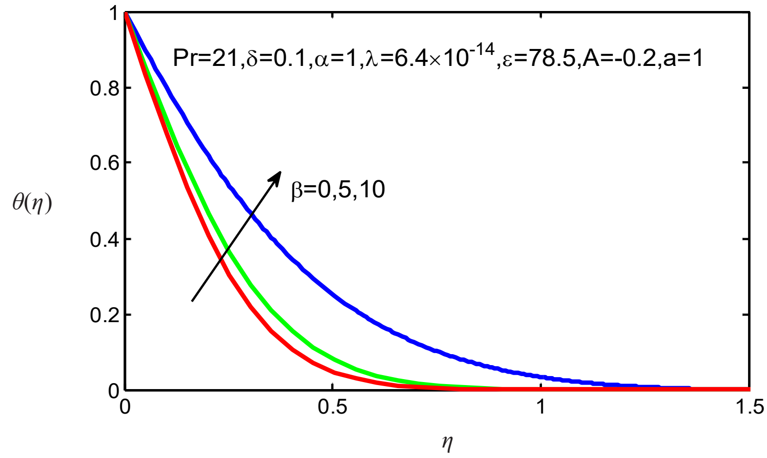

Figure 3. Variations of β on $\theta(\eta)$

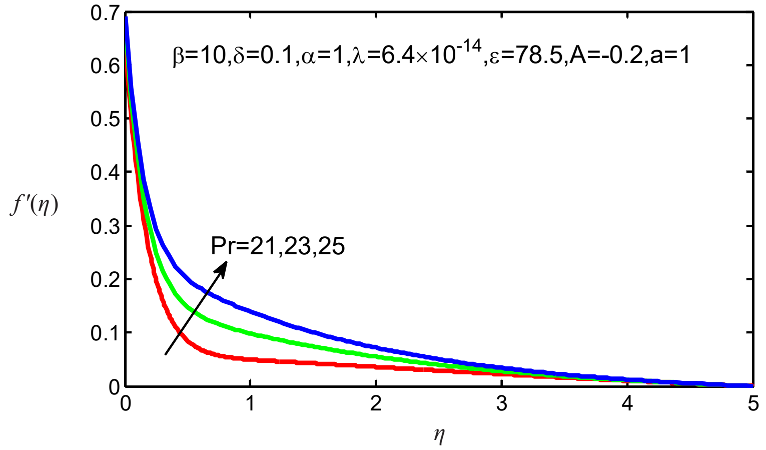

Figure 4. Variations of Pr on $f'(\eta)$

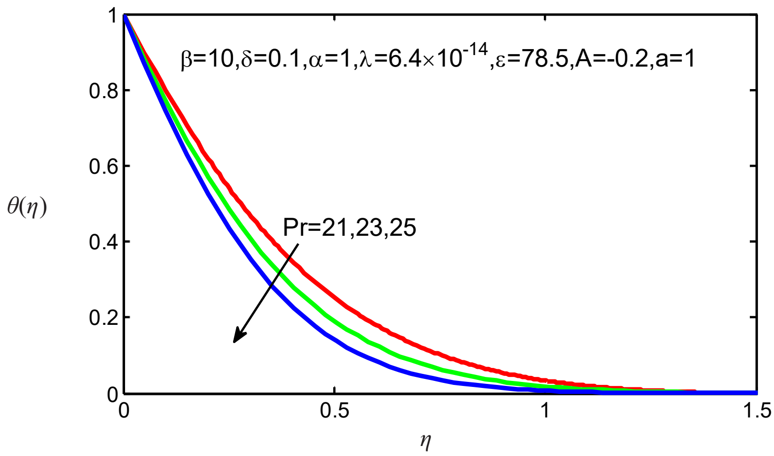

Figure 5. Variations of Pr on $\theta(\eta)$

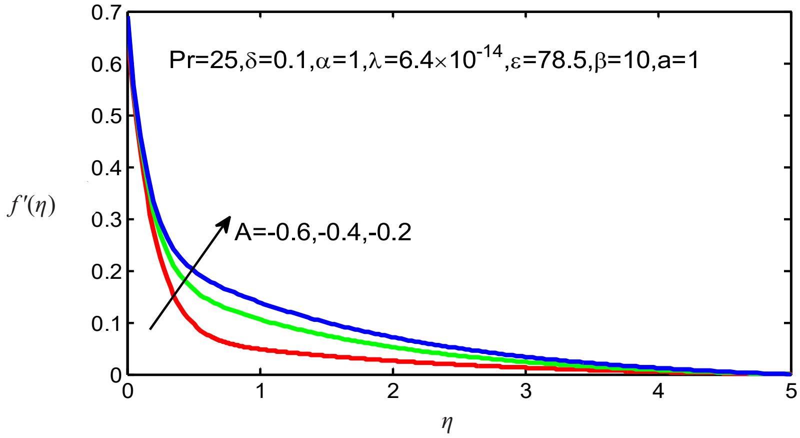

Figure 6. Variations of A on $f'(\eta)$

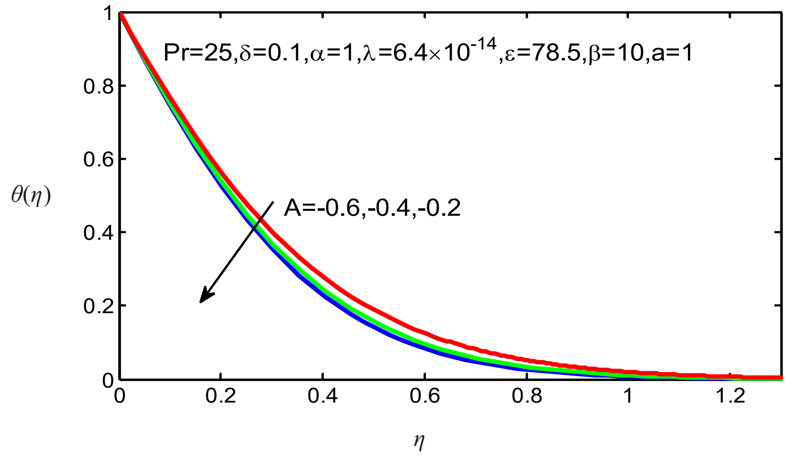

Figure 7. Variations of A on $\theta(\eta)$

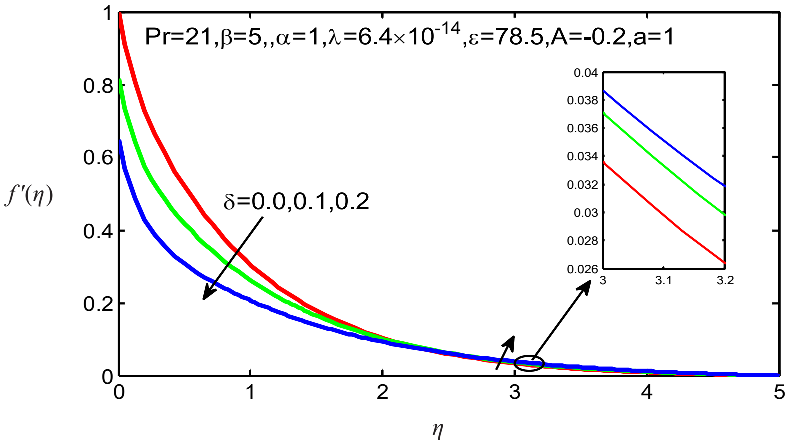

Figure 8. Variations of δ on $f'(\eta)$

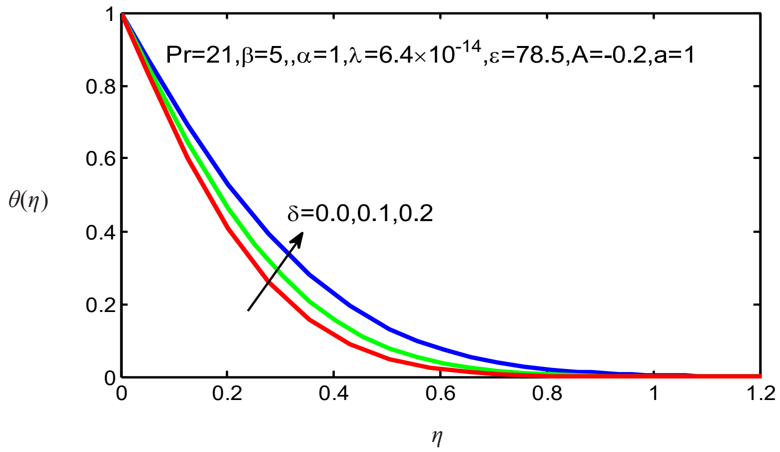

Figure 9. Variations of δ on $\theta(\eta)$

Figure 6 and 7 represent the impacts of viscosity parameter A on $f'(\eta)$ and $\theta(\eta)$ distributions, respectively. From Figure 6 it is stated that with increasing values of A, velocity $f'(\eta)$ increases. Because increasing values of viscosity parameter decreases the fluid viscosity which causes increase of boundary layer thickness. On other hand we see that with rising values of A, temperature profile decreases. It is apparent that the variation of A affects much more the velocity profile than the corresponding temperature one.

The impact of the slip parameter on velocity and temperature profiles is depicted in Figures 8 and 9. The figures show that as the slip parameter values grow, the velocity profile first decreases and then marginally increases. Where the temperature profile rises as the slip parameter values rise. Due to slip, the flow velocity close to the sheet is no longer equal to the sheet's stretching velocity. As a result, as the slip parameter values rise, so does the slip velocity, and consequently, fluid velocity falls as under the condition of slip.

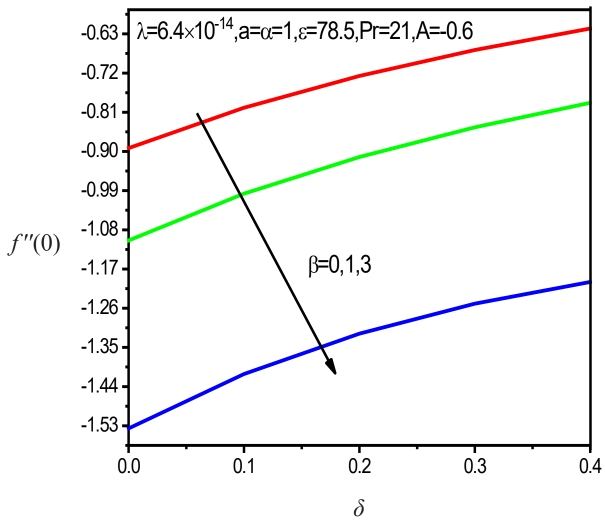

Figure 10. Skin friction coefficient $f''(0)$ with δ different values β

Figures 10 to 15 demonstrate the variations of the skin friction coefficient $f''(0)$ and rate of heat transfer $\theta'(0)$ with regard to δ for various values of the β, A, Pr, respectively. It is seen that $f''(0)$ decreases with increasing values of β and A; while reverse phenomena are observed for the $\theta'(0)$ case. From Figures 12 and 13 it is obtained that the skin friction coefficient as well as the rate of heat transfer are increasing with the increment of δ. Furthermore, the variation of A plays an important role since the skin friction, for specific values of δ, is decreased with the increment of A, whereas the corresponding rate of heat transfer is increased. This indicates that the variation of viscosity is a significant factor in the formation of critical flow characteristics like the heat transfer rate and the skin friction coefficient of the plate and cannot be ignored even for magnetic flows.

Finally, an expected result is depicted at Figures 14 and 15 where the skin friction coefficient and rate of heat transfer are increased with the slip parameter δ. For this case, the change of Pr has significant effect, for specific values of δ, on the variation of heat transfer rate which is reduced for increasing values of Pr.

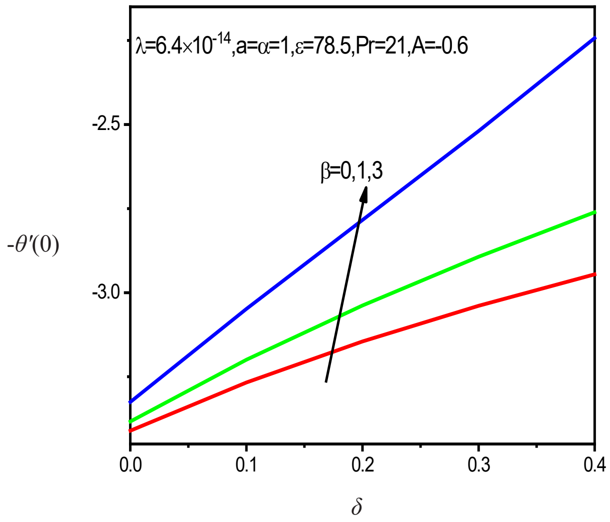

Figure 11. Local Nusselt number -$\theta'(0)$ with δ for for different values of β

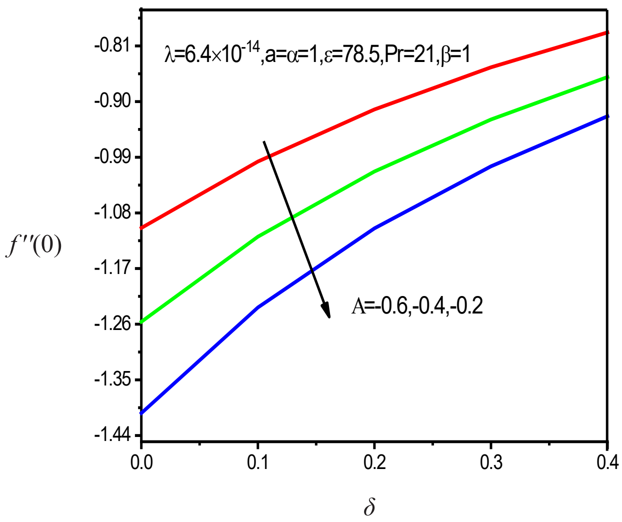

Figure 12. Skin friction coefficient $f''(0)$ with δ for different values of A

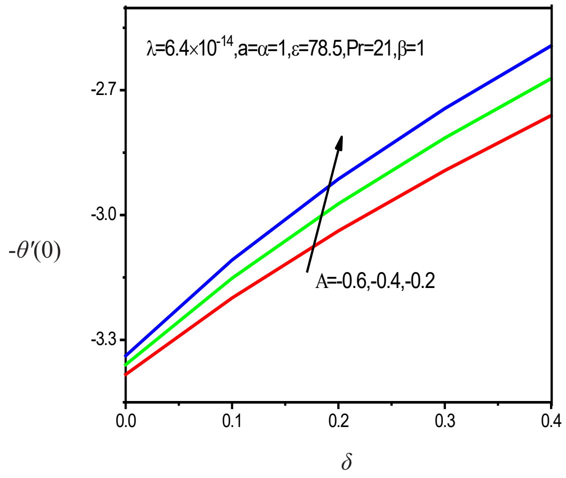

Figure 13. Local Nusselt number $-\theta'(0)$ with δ for different values of A

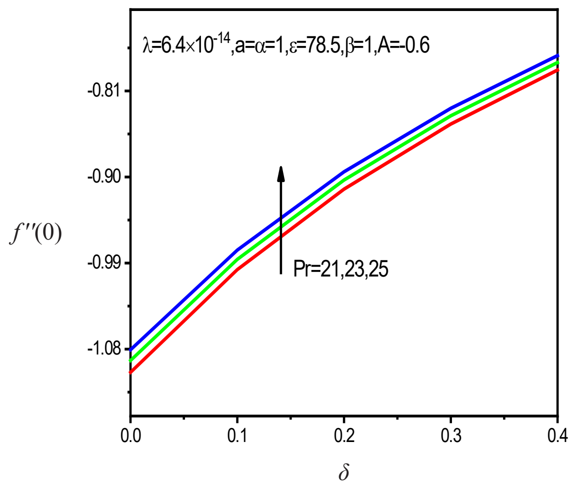

Figure 14. Skin friction coefficient $f''(0)$ with δ for different values of Pr

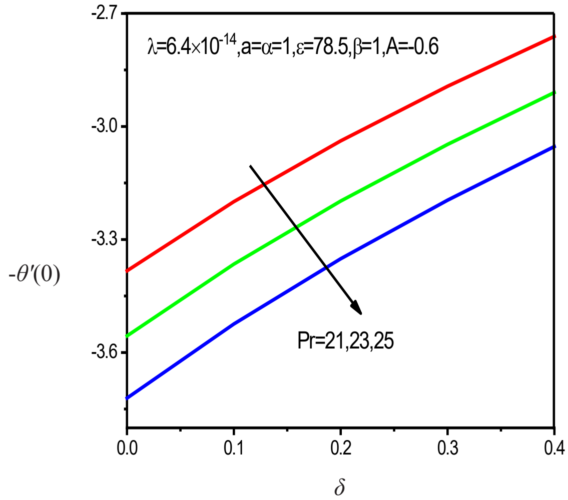

Figure 15. Local Nusselt number $-\theta'(0)$ with δ for different values of Pr

5. Conclusions

In this study, we quantitatively examined how the slip and viscosity parameters affected heat transfer and the flow of biomagnetic fluid (blood) through a stretching sheet when a magnetic dipole was present. Before being numerically solved using the bvp4c function approach in the MATLAB software, the governing PDEs were transformed into ODEs using appropriate transformations. The results are presented graphically for various values of the parameters. Additionally, comparisons are done to ensure that the methodology is valid, and show an acceptable agreement. Major findings from the present investigations are:

• An increment values of ferromagnetic number, slip parameter, fluid velocity decreases but temperature distribution is boosting up.

• Temperature distributions fall down with rising values of Prandtl number and viscosity variation parameter, whereas reverse phenomena occurred for velocity.

• The skin friction coefficient decreases with ferromagnetic number and viscosity parameter.

• For Prandtl number, the rate of heat transfer of blood is decreased.

• Increment of the viscosity parameter results to significant increment of the rate of heat transfer and decrement of the skin friction coefficient.

6. Future outlooks

Much more research is required to better understand the process of blood flow under various conditions.

i. In the presence of thermal radiation and chemical reaction models.

ii. To provide a mathematical representation of a non-Newtonian fluid model.

iii. This model can be expanded to include the concept of nanofluid, hybrid nanofluid, allowing one to distinguish between blood that is pure and blood that contains particles.

iv. The 3-D geometries applied in medical applications.

v. More accurate mathematical models relating to simulation-based engineering applications are required.

Author contributions

MG and JA carried out the formal analysis, investigation, and conceptualization and drafted the manuscript. MF and ET carried out analysis, review and editing. MG, MF, JA carried out methodology, review and editing. All authors read and approved the final manuscript.

Conflict of interest

The authors declare that they have no competing interests.

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