Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

Research Article

A Type-2 Fuzzy *u*-Control Chart Considering Probability-Based Average Run Length

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Received: 10 April 2023; **Revised:** 9 June 2023; **Accepted:** 6 July 2023

Abstract: Fuzzy sets are an emerging trend in shaping the development of control charts for statistical process control. The sets are germane to vague data that comes from incomplete or inaccurate measurements. Nevertheless, fuzzy sets are inadequate in some areas of industry since their membership functions are crisp numbers. The fuzzy sets are not fully able to compute higher levels of uncertainty, which might degrade the performance of the analysis. Therefore, type-2 fuzzy sets are proposed to be merged with control charts since these sets are hypothesized to be more capable of detecting a defect in process control. This paper aims to develop interval type-2 fuzzy *u* (IT2F*u*) charts as a new approach to detecting defects. In addition, this paper presents a comparative analysis of performances between traditional *u*-control charts, type-1 fuzzy *u*-control charts, and type-2 fuzzy *u*-control charts. 23 samples of lubricant data with 48 subgroups were examined to identify the defects. The output showed that all of the control charts produced almost similar results except for data 14, which is "out of control" in IT2F*u*-control charts but "in control" in traditional *u*-control charts and "rather in control" in type-1 fuzzy *u*-control charts. Furthermore, the performances of the charts were compared using a probability-based average run length (ARL), where probability type 1 error is computed. It was found that the ARL value of the IT2F*u*-control chart showed the lowest value among the three types of charts. The analysis indicated that the IT2F*u*-control chart outperformed the traditional *u*-control chart and the type-1 fuzzy *u*-control chart. The results obtained seem to support the idea that IT2F*u*-control charts are more sensitive compared to type 1 fuzzy *u*-control charts and traditional *u*-control charts, so that IT2F*u*-control charts are able to adequately support incomplete and vague data on process control.

***Keywords*:** interval type-2 fuzzy set, quality control, type-1 fuzzy set, *u*-control chart, ARL

MSC: 62C86, 90B50, 68U01

1. Introduction

Statistical process control (SPC) is very important in achieving process stability and improving capability through

DOI: https://doi.org/10.37256/cm.5120242810

the reduction of variability. In measuring SPC, control charts have been widely used in defect analysis. Santos [1], for example, used a new *dpmo* (defects per million opportunities) control chart to track defects in attribute data in the electronic industry. For the purpose of monitoring the process, various control charts have been proposed. Conditionally, expected value control charts [2] and copula-based cumulative sum control charts [3] are among the latest publications about control charts. The term control chart was originally developed in the 1930s by Shewhart [4] to predict the expected range of the results of the process analysis and to determine the improvement of the quality process in order to prevent any problems in the product's variation. However, sometimes the uncertainty and vagueness of the data or human subjectivity in the quality characteristics would cause the process of traditional control charts to not be expressed appropriately. To deal with this issue, fuzzy set theory [5] was employed to deal with incomplete data or inaccurate measurements. Years later, Gülbay et al. [6] established the α-cut fuzzy control charts to be used in attribute control charts. This enabled multiple quality characteristics, particularly attribute control charts, to be proposed. Fraction conforming chart (p chart) [7, 8], fraction non-conforming chart (np chart) [9], non-conformities chart (c chart) [10, 11], or average non-conformities per unit chart (u-chart) [12, 13] were just a few examples of attribute control charts. Erginel [9] used p control charts and np fuzzy control charts using α-cuts in monitoring the process. Şentürk [10] proposed fuzzy c control charts and tested them with data from an incorporated company. Faraz and Moghadam [14], on the other hand, proved that the fuzzy variables control chart with a warning line has more sensitivity to a small process shift. All these charts are more sensitive than the traditional charts and would help researchers improve the sensitivity of the traditional control chart in detecting abnormal occurrences in the process.

Apart from this list of attribute control charts, there are other types of control charts that fully utilize fuzzy variables. Darestani and Nasiri [15], for example, used process capability indices of fuzzy $\overline{X}$ and s control charts in monitoring environmental data. Zabihinpour et al. [16] introduced triangular fuzzy variables to develop fuzzy $\overline{X}$ and s control charts. They proved that the fuzzy chart could improve the detection of abnormal shifts in the process mean. Shu et al. [17] also established fuzzy set theories using s control charts. Other than s charts, Sabahno et al. [18] studied the $\overline{X}$ and R fuzzy control charts, and they concluded that the proposed method was quicker in detecting the process shift compared to the traditional charts.

Fuzzy control charts are widely used in many fields, including the sociological, medical, engineering, service, and management fields. Fuzzy set theory has contributed to the systematic process of evaluating fuzzy data. Very recently, Fadaei and Pooya [19] also used fuzzy u-control charts based on fuzzy rules in the manufacturing industry. The performance of the chart was evaluated using a fuzzy operating characteristic curve. The researcher concluded that the fuzzy chart's efficiency was higher than that of the crisp chart. These related researchers shed some light on the development of fuzzy charts where type-1 fuzzy variables or fuzzy rules were considered. In another study of fuzzy u-chart, Truong et al. [20] monitored process mean in the textile dyeing industry in Vietnam. They suggested that the industry can apply fuzzy charts to reduce operational costs and potential losses.

Darestani et al. [12] investigated the fuzzy non-conformities per unit control chart towards the steering hydraulic oil tank in the automotive industry. The researcher concluded that the proposed method performed better than the traditional method. On the other hand, Aslangiray and Akyuz [21] compared the fuzzy u-chart using fuzzy mode, fuzzy median, fuzzy midrange, and direct fuzzy approach towards the textile company in Istanbul. At the end of the study, they indicate that the fuzzy u-chart found more defects than the conventional method. Şentürk et al. [22] studied the fuzzy u-chart using the α-cut method. They illustrated the proposed method for truck engine manufacturing in the manufacturing process. As a result, they conclude that the proposed approach is effective in monitoring the quality of the products. Apart from these case studies from the previous journal, it can be concluded that many kinds of data from various industries can be applied to fuzzy u-chart.

In line with the development of fuzzy theory where type-1 fuzzy sets were extended to type-2 fuzzy sets, the knowledge of control charts was also developed. In an attempt to include type-2 fuzzy sets in control charts, Teksen and Anagün [23] studied type-2 fuzzy mean ($\overline{X}$) and range (R) control charts. They tested various techniques, such as defuzzification, distance, ranking, and likelihood methods. From their investigation, they summarized that the results from all the techniques showed similarity regarding the "in control" and "out of control" points. Using a type-2 fuzzy control chart, Erginel et al. [24] developed a p control chart to handle uncertainty in the process mean. Şentürk and Antucheviciene [11] developed the interval type-2 fuzzy (IT-2F) c control charts using 18 data points, and they noticed that traditional charts were not appropriate to be applied when the data collected were incomplete and vague. It is true to say that type-2 fuzzy charts can model such uncertainties since their membership functions (MFs) provide additional degrees of freedom in a three-dimensional way. The flexibilities and sensitivities of the type-2 chart are more realistic when analyzing imprecise data with uncertainties. Moreover, type-1 fuzzy sets, occasionally acknowledged as fuzzy sets, are inadequate in some of the areas in the industries since their MFs are crisp. Therefore, type-2 fuzzy sets are more appropriate since they give essential alerts and are more capable of detecting process shifts. However, Mendel et al. [25] posited that the type-2 fuzzy set was very difficult to translate to real problems. Therefore, they proposed IT-2F sets where memberships were given as intervals of upper membership and lower membership. Since then, many application studies have been using IT-2F sets owing to the fact that they are easy to compute and manageable. For instance, research by Mohd Razali et al. [26] compared the IT-2F Cumulative Sum (CUSUM) control chart with the type-1 fuzzy CUSUM chart and the traditional CUSUM chart. The average run length (ARL) was also computed at the end of the study to confirm the performance of the three charts. In a nutshell, the researcher summarized that the proposed method performed better in analyzing the crisp data as it has the lowest ARL compared to other charts.

Nevertheless, previous research indicates that little discussion can be found about the use of IT-2F sets in *u*-control chart studies. This study seeks to propose a new *u*-control chart based on IT-2F sets that will help address these research gaps. Specifically, the research aims to evaluate the interval type-2 fuzzy *u* (IT2F*u*) control charts. In addition, this study also aims to provide comparative 'in control-out control' results. Moreover, the performances of each control chart are computed using ARL, in which the ARLs between traditional *u*-control charts, type-1 fuzzy *u* (T1F*u*) control charts, and IT2F*u*-charts are compared. This work contributes to the existing knowledge control charts by providing an ensemble of IT-2F sets and *u*-control charts. The new control charts are tested on the lubricant data of the oil and gas company. This paper is structured as follows: Section 2 presents the theoretical structure of IT-2F sets. The proposed method, which is the IT2F*u*-control chart, is discussed in Section 3. In Section 4, the illustrative example with the lubricant data is presented. A comparative analysis based on ARLs is discussed in Section 5. Finally, the conclusions of this study are revealed in Section 6.

2. Preliminary

In this section, we provide the definition of the IT-2F set and discuss some of its arithmetic operations related to the trapezoidal IT-2F fuzzy set.

The uniqueness of studies of fuzzy sets lies in their MF. MF plays an important role in the performance of fuzzy techniques. They are the building blocks of fuzzy set theory, as the fuzziness of a fuzzy set is determined by its MF. There are different forms of MF, namely triangular, trapezoidal, piecewise, linear, gaussian, and singleton MFs. All these MFs must vary between 0 and 1 [27]. The most widely accepted MFs are the triangular and trapezoidal MFs [28]. Most researchers that analyze the fuzzy control chart rely on both of these MFs in their studies. Triangular MFs represent fuzzy numbers, whereas trapezoidal MFs represent fuzzy intervals [29]. Both methods are simple to implement and fast to compute. The empirical success of both MFs clearly shows that they represent fuzzy values accurately [28]. As such, most researchers intuitively use these MFs, and a trapezoidal MF will be applied in this study as it investigates fuzzy intervals.

The MFs of type-2 fuzzy sets are three-dimensional. The third dimension of type-2 fuzzy sets makes it possible to model uncertainties [30]. There are several versions of type-2 fuzzy sets that have been proposed, and one of the most popular among researchers is IT-2F sets. Many theoretical and application studies have been using IT-2F sets since they provide straight-forward computation and are manageable [25].

The definition of type-2 fuzzy sets, IT-2F sets, and their arithmetic operators is explained as follows.

Definition 1 [31]. A type-2 fuzzy set $\tilde{A}$ in the universe of discourse X is characterized by a type-2 MF $\mu_{\tilde{A}}(x,u)$, where

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_X\}, \tag{1}$$

$x \in X, u \in J_x$, J_x denote the primary membership of x, $J_x \in [0,1]$, $\mu_{\tilde{A}}(x,u)$ denotes the secondary grade of (x, u) and $0 \le \mu_{\tilde{A}}(x,u) \le 1$. The type-2 fuzzy set $\tilde{A}$ also can be represented as:

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u), \tag{2}$$

where $x \in X, u \in J_x, J_x \subseteq [0,1]$ and $\iint$ denote the union over all admissible x and u. A type-2 MF is three-dimensional, and the third dimension (i.e., $\mu_{\tilde{A}}(x,u)$) provides a degree of freedom in handling uncertainties.

Definition 2 [31]. Assume that $\tilde{A}$ is a type-2 fuzzy set in the universe of discourse. X refers to the type-2 MF $\mu_{\tilde{A}}$. Let's say, if all the secondary grades $\mu_{\tilde{A}}(x,u)$ of $\tilde{A}$ are equal to 1, then $\tilde{A}$ is known as an IT-2F set, obtained as:

$$\tilde{A} = \int_{x\in X}\int_{u\in J} 1/(x,u), \tag{3}$$

while $x \in X, u \in J_x, J_x \subseteq [0,1]$ and $\iint$ refer to the union over all admissible x and u.

Definition 3 [30]. An IT-2F set is defined as

$$I_x = \left\{u \subseteq [0, 1] \,|\, (\mu_{\tilde{A}}(x,u) > 1)\right\}. \tag{4}$$

IT-2F sets are also known as closed IT-2F sets when I_x is a closed interval for every $x \subseteq X$. The upper MFs and lower MFs for an IT-2F set of type-1 MFs, respectively.

Figure 1 shows the upper (U) and lower MFs (L) of IT-2F sets [32].

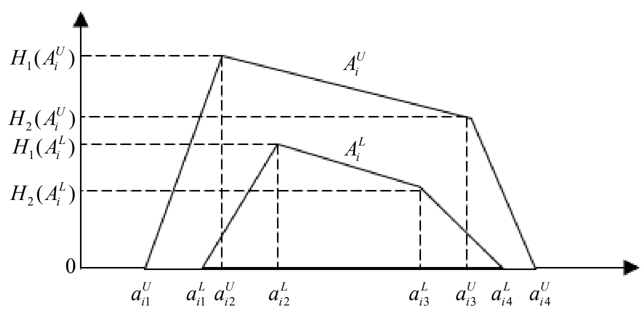

Figure 1. The MFs of IT-2F set $\tilde{A}$

For a special case of IT-2F sets, a trapezoidal IT-2F number is defined as follows:

$$\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = \begin{pmatrix} (a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U \,;\, H_1(A_i^U), H_2(A_i^U)), \\ (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L \,;\, H_1(A_i^L), H_2(A_i^L)) \end{pmatrix} \tag{5}$$

where A_i^U and A_i^L denote as type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U$, $a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L$, are the reference points of the IT-2F $\tilde{A}_i$. $H_j(A_i^U)$ and $H_j(A_i^L)$ are the membership values for the upper and lower A_i^U and A_i^L in between $1 \le j \le 2$.

$$1 \le j \le 2, H_1(A_i^U), H_2(A_i^U), H_1(A_i^L), H_2(A_i^L) \subseteq [0,1], 1 \le i \le n.$$

Let $\tilde{A}_1$ and $\tilde{A}_2$ be two trapezoidal IT-2F sets:

$$\tilde{A}_1 = (A_1^U, A_1^L) = \begin{pmatrix} (a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U \,;\, H_1(A_1^U), H_2(A_1^U)), \\ (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L \,;\, H_1(A_1^L), H_2(A_1^L)) \end{pmatrix} \tag{6}$$

$$\tilde{A}_1 = (A_2^U, A_2^L) = \begin{pmatrix} (a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U\ ;\ H_1(A_2^U), H_2(A_2^U)), \\ (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L\ ;\ H_1(A_2^L), H_2(A_2^L)) \end{pmatrix} \tag{7}$$

Definition 4. Arithmetic operations for given two trapezoidal interval type-2, fuzzy sets are as follows [33]:

i. Addition operation:

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 &= (A_1^U, A_1^L) \oplus (A_2^U, A_2^L) \\ &= \begin{pmatrix} (a_{11}^U + a_{21}^U,\ a_{12}^U + a_{22}^U,\ a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U)\ ; \\ \min(H_1(A_1^U); H_1(A_2^U)), \min(H_2(A_1^U); H_2(A_2^U)), \\ (a_{11}^L + a_{21}^L,\ a_{12}^L + a_{22}^L,\ a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L)\ ; \\ \min(H_1(A_1^L); H_1(A_2^L)), \min(H_2(A_1^L); H_2(A_2^L)) \end{pmatrix} \end{aligned} \tag{8}$$

ii. Subtraction operation:

$$\begin{aligned} \tilde{A}_1 \ominus \tilde{A}_2 &= \left(A_1^U, A_1^L\right) \ominus \left(A_2^U, A_2^L\right) \\ &= \begin{pmatrix} (a_{11}^U - a_{24}^U,\ a_{12}^U - a_{23}^U,\ a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U)\ ; \\ \min(H_1(A_1^U); H_1(A_2^U)), \min(H_2(A_1^U); H_2(A_2^U)), \\ (a_{11}^L - a_{24}^L,\ a_{12}^L - a_{23}^L,\ a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L)\ ; \\ \min(H_1(A_1^L); H_1(A_2^L)), \min(H_2(A_1^L); H_2(A_2^L)) \end{pmatrix} \end{aligned} \tag{9}$$

iii. Multiplication operation:

$$\begin{aligned} \tilde{A}_1 \otimes \tilde{A}_2 &= (A_1^U, A_1^L) \otimes (A_2^U, A_2^L) \\ &= \begin{pmatrix} (a_{11}^U x\, a_{21}^U,\ a_{12}^U x\, a_{22}^U,\ a_{13}^U x\, a_{23}^U, a_{14}^U x\, a_{24}^U)\ ; \\ \min(H_1(A_1^U); H_1(A_2^U)), \min(H_2(A_1^U); H_2(A_2^U)), \\ (a_{11}^L x\, a_{21}^L,\ a_{12}^L x\, a_{22}^L,\ a_{13}^L x\, a_{23}^L, a_{14}^L x\, a_{24}^L)\ ; \\ \min(H_1(A_1^L); H_1(A_2^L)), \min(H_2(A_1^L); H_2(A_2^L)) \end{pmatrix} \end{aligned} \tag{10}$$

iv. Arithmetic operations with crisp value q:

$$qx\ \tilde{A}_1 = \begin{pmatrix} (q \times a_{11}^U, q \times a_{12}^U, q \times a_{13}^U, q \times a_{14}^U)\ ; H_1(A_1^U), H_2(A_1^U)), \\ (q \times a_{11}^L, q \times a_{12}^L, q \times a_{13}^L, q \times a_{14}^L)\ ; H_1(A_1^L), H_2(A_1^L)) \end{pmatrix} \tag{11}$$

$$\frac{\tilde{A}_1}{q} = \begin{pmatrix} \left(\frac{1}{q} \times a_{11}^U,\ \frac{1}{q} \times a_{12}^U,\ \frac{1}{q} \times a_{13}^U,\ \frac{1}{q} \times a_{14}^U\right); H_1(A_1^U), H_2(A_1^U)), \\ \left(\frac{1}{q} \times a_{11}^L,\ \frac{1}{q} \times a_{12}^L,\ \frac{1}{q} \times a_{13}^L,\ \frac{1}{q} \times a_{14}^L\right); H_1(A_1^L), H_2(A_1^L)) \end{pmatrix} \tag{12}$$

where $q > 0$.

The above definitions and arithmetic operations are used in the implementation of the proposed work.

3. IT2F*u*-control charts

The fuzzy *u*-control chart was first proposed by Darestani et al. [12]. They used fuzzy *u*-charts to monitor the average number of defects in devices in the automotive industry. In this section, a new notion of an IT2F*u*-control chart is proposed.

Fundamentally, a *u*-control chart is used to monitor the number of defects per unit for each item since it can have multiple defects. It is also used to identify the instabilities in a process over time. If non-conformities are detected, then their occurrence is found in a sampled subgroup. Examples of occurrences such as scratches, dents, bubbles, blemishes, or missing buttons are just a few. In traditional *u*-control charts, the average defects are distinctly classified as "conformities" or "non-conformities" when determining the average number of non-conformities per unit. With a similar tone, *u*-control charts are extended to IT2F*u*-charts, where the input data or samples are transformed into intervals. The samples used are defined as interval type-2 trapezoidal numbers (a, b, c, d). The average of non-conformities per unit of *u*-chart is now expressed in IT-2F numbers and written as follows:

$$\tilde{u}_i = \begin{bmatrix} (a_{i1}^U,\ a_{i2}^U, a_{i3}^U, a_{i4}^U\ ;\ H_1(A_1^U), H_2(A_1^U)), \\ (a_{i1}^L,\ a_{i2}^L, a_{i3}^L, a_{i4}^L\ ;\ H_1(A_1^L), H_2(A_1^L)) \end{bmatrix} \tag{13}$$

The center line is the mean of the IT-2F samples, and it is given in the following equations:

$$\overline{u}_{a_1^U} = \frac{\sum_{i=1}^{m} a_{i1}^U}{n},\quad \overline{u}_{a_2^U} = \frac{\sum_{i=1}^{m} a_{i2}^U}{n},\quad \overline{u}_{a_3^U} = \frac{\sum_{i=1}^{m} a_{i3}^U}{n},\quad \overline{u}_{a_4^U} = \frac{\sum_{i=1}^{m} a_{i4}^U}{n} \tag{14}$$

$$\overline{u}_{a_1^L} = \frac{\sum_{i=1}^{m} a_{i1}^L}{n},\quad \overline{u}_{a_2^L} = \frac{\sum_{i=1}^{m} a_{i2}^L}{n},\quad \overline{u}_{a_3^L} = \frac{\sum_{i=1}^{m} a_{i3}^L}{n},\quad \overline{u}_{a_4^L} = \frac{\sum_{i=1}^{m} a_{i4}^L}{n} \tag{15}$$

The control limits of IT2F*u*-control charts are written based on the trapezoidal IT-2F numbers. Equation (16) is used to find the upper control limit of the IT2F*u*-control chart.

Upper control limit (UCL) =

$$\begin{bmatrix} \overline{u}_{a_1^U} + 3\sqrt{\frac{\overline{u}_{a_1^U}}{n}}, \overline{u}_{a_2^U} + 3\sqrt{\frac{\overline{u}_{a_2^U}}{n}}, \overline{u}_{a_3^U} + 3\sqrt{\frac{\overline{u}_{a_3^U}}{n}}, \overline{u}_{a_4^U} + 3\sqrt{\frac{\overline{u}_{a_4^U}}{n}}; \min\,(H_1(A_1^U), H_2(A_1^U)), \\ \overline{u}_{a_1^L} + 3\sqrt{\frac{\overline{u}_{a_1^L}}{n}}, \overline{u}_{a_2^L} + 3\sqrt{\frac{\overline{u}_{a_2^L}}{n}}, \overline{u}_{a_3^L} + 3\sqrt{\frac{\overline{u}_{a_3^L}}{n}}, \overline{u}_{a_4^L} + 3\sqrt{\frac{\overline{u}_{a_4^L}}{n}}; \min\,(H_1(A_1^L), H_2(A_1^L)) \end{bmatrix} \tag{16}$$

Similarly, center line and lower control limits of the IT2F*u*-charts are calculated using equations (17) and (18), respectively.

Center line (CL) =

$$\begin{bmatrix} \overline{u}_{a_1^U},\ \overline{u}_{a_2^U},\ \overline{u}_{a_3^U},\ \overline{u}_{a_4^U}\ ;\min\,(H_1(A_1^U), H_2(A_1^U)) \\ \overline{u}_{a_1^L},\ \overline{u}_{a_2^L},\ \overline{u}_{a_3^L},\ \overline{u}_{a_4^L}; \min\,(H_1(A_1^U), H_2(A_1^U)) \end{bmatrix} \tag{17}$$

Lower control limit (LCL) =

$$\left[\begin{array}{l} \overline{u}_{a_1^U} - 3\sqrt{\frac{\overline{u}_{a_4^U}}{n}}, \overline{u}_{a_2^U} - 3\sqrt{\frac{\overline{u}_3}{n}}, \overline{u}_{a_3^U} - 3\sqrt{\frac{\overline{u}_{a_2^U}}{n}}, \overline{u}_{a_4^U} - 3\sqrt{\frac{\overline{u}_{a_1^U}}{n}}; \min\left(H_1(A_1^U), H_2(A_1^U)\right), \\ \overline{u}_{a_1^L} - 3\sqrt{\frac{\overline{u}_{a_4^L}}{n}}, \overline{u}_{a_2^L} - 3\sqrt{\frac{\overline{u}_3}{n}}, \overline{u}_{a_3^L} - 3\sqrt{\frac{\overline{u}_{a_2^L}}{n}}, \overline{u}_{a_4^L} - 3\sqrt{\frac{\overline{u}_{a_1^L}}{n}}; \min\left(H_1(A_1^U), H_2(A_1^U)\right) \end{array}\right] \quad (18)$$

These three control limits are written in IT2 trapezoidal fuzzy numbers, where these numbers must be transformed into crisp numbers to facilitate result interpretation. This transformation process is called defuzzification, which will be explained further in the following subsection.

3.1 *Defuzzification method for IT2Fu-control chart*

Data in the form of type-1 fuzzy sets is transformed into crisp numbers using various defuzzification methods. It has been extensively surveyed and applied in many fields of research to encapsulate a fuzzy number to get a typical value from a given fuzzy set [34]. Central tendencies in descriptive statistics like fuzzy average, fuzzy median, fuzzy mode, and α-level fuzzy midrange are just a few examples of defuzzification methods for type-1 fuzzy sets. Every method mentioned has its own advantage, and there is no indication that one method is better than the others. Şentürk and Antucheviciene [11] concluded that the selection of the method is based on the preferences of the users since the results of all of the methods are similar. Likewise, the data transformation of type-2 fuzzy sets is made using various methods. The centroid method [30], the indices method [35], and the best non-fuzzy performance (BNP) method [36], just to mention a few, are defuzzification methods in type-2 fuzzy control charts. Kahraman et al. [37] proposed BNP with an adjusted centroid method as a defuzzification method for IT-2F sets, where memberships of interval type-2 triangular fuzzy sets were calculated using a mean operator. In our approach, we adapted the centroid's BNP approach so it is compatible with interval type-2 trapezoidal fuzzy sets. Instead of using the defuzzified triangular type-2 fuzzy set approach, we proposed the centroid defuzzified interval type-2 trapezoidal fuzzy set ($CDIT2_{Trap}$). The centroid of the interval type-2 trapezoidal fuzzy set is reflected by the division operator of its memberships. The defuzzification equations for the three control limits are listed as:

$$CDIT2^U_{Trap(i)} = \frac{(a^U_{i4} - a^U_{i1}) + (H_2(A^U_1)a^U_{i2} - a^U_{i1}) + (H_1(A^U_1)\ a^U_{i3} - a^U_{i1})}{4} + a^U_{i1} \quad (19)$$

$$CDIT2^L_{Trap(i)} = \frac{(a^L_{i4} - a^L_{i1}) + (H_2(A^L_1)a^L_{i2} - a^L_{i1}) + (H_1(A^L_1)\ a^L_{i3} - a^L_{i1})}{4} + a^L_{i1} \quad (20)$$

$$CDIT2_{Trap(i)} = \frac{CDIT2^U_{Trap(i)} + CDIT2^L_{Trap(i)}}{2};\ i = 1, 2, \ldots n \quad (21)$$

where $H_1(A_1^U)$ and $H_2(A_1^U)$ are the maximum membership in the upper MFs. a^U_{i4} and a^U_{i1} are the highest and the lowest parameter in the upper MF while a^U_{i2} and a^U_{i3} are the second and third parameters of the upper MFs. Vice versa to, a^L_{i1}, a^L_{i2}, a^L_{i3}, and a^L_{i4} are the parameters for the lower MFs. $CDIT2_{Trap(i)}$ is the defuzzification value of control limits of IT-2F sets on average of non-conformities per unit. The best non-fuzzy performances ($DIT2_{Trap}$) of three control limits are calculated using the following equations:

$$DIT2^U_{Trap(i)}, = \frac{(\overline{u}_{a_4^U} - \overline{u}_{a_1^U}) + (H_2(A^U_1)\overline{u}_{a_2^U} - \overline{u}_{a_1^U}) + (H_1(A^U_1)\ \overline{u}_{a_3^U} - \overline{u}_{a_1^U})}{4} + \overline{u}_{a_1^U} \quad (22)$$

$$DIT2^L_{Trap(i)}, = \frac{(\overline{u}_{a_4^L} - \overline{u}_{a_1^L}) + (H_2(A^L_1)\overline{u}_{a_2^L} - \overline{u}_{a_1^L}) + (H_1(A^L_1)\ \overline{u}_{a_3^L} - \overline{u}_{a_1^L})}{4} + \overline{u}_{a_1^L} \quad (23)$$

$$DIT2_{Trap(i)} = \frac{DIT2^{U}_{Trap(i)} + DIT2^{L}_{Trap(i)}}{2} \tag{24}$$

where $DIT2_{Trap(i)}$ is the defuzzification value of every sample in the study.

It can be seen that the IT2F*u*-control chart is developed through a succession of computational steps inspired by the concepts of fuzzification, control limits, and defuzzification of IT2 trapezoidal fuzzy sets. Figure 2 shows the flowchart for the development of the IT2F*u*-control chart.

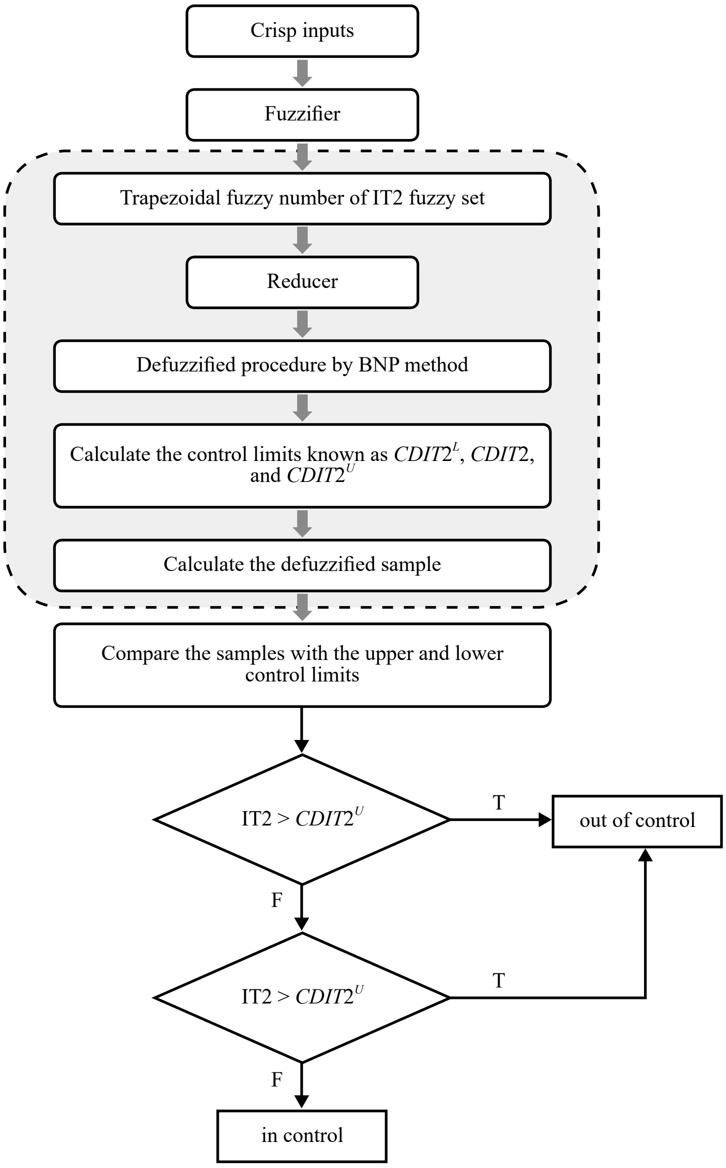

Figure 2. Flowchart of the IT2F*u*-control chart

The algorithm of the proposed method is presented as follows:

Begin

Step 1: Fuzzify the data.

Step 2: Compute fuzzification inputs

Step 3: Compute fuzzy average non-conformities per unit (u) samples.
Step 4: Defuzzify the data using the BNP method
Defuzzify each sample
Defuzzify the control limits
Step 5: Compute mean of defuzzified sample
Step 6: Compute defuzzified control limits
Step 7: If mean < lower control limit or mean > upper control limit, then ‘out of control’
Step 8: Otherwise ‘in control’
End

The final output of the proposed IT2F*u*-control charts leads us to make a decision on whether it is “in control” or “out of control”. This performance of IT2F*u*-control charts will be compared to traditional charts and T1F*u*-control charts.

3.2 *The performance of control chart*

The best performance of control charts is estimated based on the least value of the ARL [38, 39]. The expected number of samples to be obtained before the process goes above the boundaries is known as the ARL, and more importantly, it serves as a performance indicator for control charts. The ARL is the expectation time before the control chart gives a false alarm, which is “in control” or “out of control”. For the ARLs, we need to determine the probability of type 1 error, α, which refers to the probability of an incorrect decision that a process is “out of control” when it is really “in control”. The expression for α can be determined as follows:

$$\begin{aligned}\alpha &= P\{x < UCL \mid u\} - P\{x \le LCL \mid u\} \\ &= P\{x < nUCL \mid u\} - P\{x \le nLCL \mid u\} \\ &= P\{nLCL < x \le nUCL \mid u\}\end{aligned} \tag{25}$$

where x is a Poisson random variable with parameter u. The symbol $nLCL$ denotes the smallest integer greater than or equal to LCL, and $nUCL$ denotes the largest integer less than or equal to UCL. Then, the ARL is computed as:

$$ARL = ARL_0 + ARL_1 = \frac{1}{\alpha} + \frac{1}{1-\beta} \tag{26}$$

where $\beta = 1 - \alpha$. The proposed IT2F*u*-control chart and its performance are illustrated in the next section.

4. Application to lubricant data

Lubricant data are employed to provide a better understanding of the implementation of the proposed works. The application is based on the secondary data of one of the oil and gas companies in Malaysia. It produces petroleum, natural gas, and petrochemicals. The activities of the company include upstream exploration and production of oil and gas, downstream oil refining, marketing of liquefied natural gas, petrochemical manufacturing, and network operations. In this research, we only focus on lubricants used in the auxiliary engines of ships. For an auxiliary engine to perform its function, the lubricant must have key chemical and physical features, including adequate viscosity, excellent thermal and oxidative stability, being non-corrosive, and being able to resist air, water, and solid contaminant entrainment. Lubricant is checked based on its viscosity, neutralization number, water contamination, flash point, pentane insolubleness, and wear metals. If one of the variables does not conform to acceptable standards, the product will be defined as defective. The data were collected from 23 samples, and 48 subgroups were examined to identify the defects. The quality of the lubricants was checked every month and measured using a dipstick to compute the level of oil, then one liter of oil was pumped into the bottle of the sampling tube using a sampling gun. Soon after that, the quality of the oil was checked to ensure the safety of the sailor. If one of the variables is “out of control”, the researcher needs to change to the new lubricant’s oil.

Notwithstanding the good characteristics of lubricants, some uncertainty in data collection might emerge due to human judgment or even mechanical errors in managing and operating the lubricant oil. This vagueness and ambiguity make the data unclear. Henceforth, this study considers the average number of non-conformities and monitors them with IT2Fu-control charts as the best technique for dealing with the vagueness data. The same data sets are also iteratively calculated using traditional charts and T1Fu-control charts.

4.1 *Monitoring defects using IT2Fu-control chart*

The lubricant's production of data is shown in the Appendix. The samples were collected as IT-2F numbers and demonstrated as IT-2F numbers for the average number of non-conformities per unit. The IT-2F MF was used in constructing the control chart.

Table 1 presents data that is given in trapezoidal IT-2F numbers.

Table 1. IT-2F number for 23 subgroups

No.	IT-2F number representation of 23 subgroups											
	a_{1U}	a_{2U}	a_{3U}	a_{4U}			a_{1L}	a_{2L}	a_{3L}	a_{4L}		
1	[(14.6,	16,	18,	20;	1,	1)	(13,	15,	17,	19;	0.9,	0.7)]
2	[(4.0,	5.0,	6,	7;	1,	1)	(3,	4,	5	6;	0.5,	0.4)]
3	[(2.0,	3.0,	4,	5;	1,	1)	(1,	2,	3	4;	0.4,	0.3)]
4	[(26.0,	27.0,	28,	29;	1,	1)	(25.2,	26,	27	28;	0.9,	0.7)]
5	[(3.0,	4.0,	5,	6;	1,	1)	(2,	3,	4	5;	0.5,	0.4)]
6	[(1.5,	2.5,	4.5,	4.5;	1,	1)	(1,	2,	3	4;	0.4,	0.3)]
7	[(15.9,	17,	18,	19;	1,	1)	(15,	16,	17	18;	0.9,	0.8)]
8	[(15.5,	17,	18,	19;	1,	1)	(14,	15,	16	17;	0.8,	0.7)]
9	[(14.5,	16,	18,	20;	1,	1)	(13.1,	14,	15	16;	0.7,	0.6)]
10	[(2.0,	3.0,	4,	5;	1,	1)	(1,	2,	3	4;	0.5,	0.4)]
11	[(3.0,	4.0,	5,	6;	1,	1)	(2,	3,	4	5;	0.5,	0.4)]
12	[(11.8,	14,	16,	18;	1,	1)	(11,	13,	15	17;	0.7,	0.5)]
13	[(15.0,	16.3,	17,	18;	1,	1)	(14,	15,	16	17;	0.8,	0.6)]
14	[(22.0,	23.0,	24,	25;	1,	1)	(21,	22,	23	24;	0.9,	0.8)]
15	[(14.5,	16,	17,	19;	1,	1)	(13,	14,	15	16;	0.9,	0.7)]
16	[(2.0,	3.0,	4,	5;	1,	1)	(1,	2,	3	4;	0.4,	0.3)]
17	[(15.7,	17,	18,	19;	1,	1)	(14,	15,	16	17;	0.8,	0.6)]
18	[(2.0,	3.0,	4,	6;	1,	1)	(1.5,	3.5,	4.5	5.5;	0.4,	0.3)]
19	[(15.0,	16.0,	17,	18;	1,	1)	(14,	15,	16	17;	0.8,	0.7)]
20	[(15.3,	16.3,	17.3,	19.3;	1,	1)	(14,	15,	16	17;	0.8,	0.7)]
21	[(20.0,	21.0,	22,	23;	1,	1)	(19,	20,	21	22;	0.9,	0.7)]
22	[(14.2,	16.2,	18.2,	20.2;	1,	1)	(13,	14,	15	16;	0.8,	0.7)]
23	[(2.0,	3.0,	4,	5;	1,	1)	(1,	2,	3	4;	0.4,	0.3)]

The above IT-2F numbers were used to find the average number of non-conformities per unit. Each sample from Table 1 was divided into 48 subgroups.

For example, in sample 1 (a_{1U}), the number that represents the subgroup is 14.6/48 = 0.304.

Whereas, for sample 2 (a_{2U}), the number that represents the subgroup is 4/48 = 0.083.

The other averaged numbers are calculated similarly. Table 2 shows the interval type 2 fuzzy numbers for the average defects of 48 subgroups.

Table 2. Averaged IT-2F numbers

No.	Averaged IT-2F numbers											
1	[(0.304,	0.333,	0.375,	0.417;	1,	1)	(0.271,	0.313,	0.354,	0.396;	0.9,	0.7)]
2	[(0.083,	0.104,	0.125,	0.146;	1,	1)	(0.063,	0.083,	0.104,	0.125;	0.5,	0.4)]
3	[(0.042,	0.063,	0.083,	0.104;	1,	1)	(0.021,	0.042,	0.063,	0.083;	0.4,	0.3)]
4	[(0.542,	0.563,	0.583,	0.604;	1,	1)	(0.525,	0.542,	0.563,	0.583;	0.9,	0.7)]
5	[(0.063,	0.083,	0.104,	0.125;	1,	1)	(0.042,	0.063,	0.083,	0.104;	0.5,	0.4)]
6	[(0.031,	0.052,	0.094,	0.094;	1,	1)	(0.021,	0.042,	0.063,	0.083;	0.4,	0.3)]
7	[(0.331,	0.354,	0.375,	0.396;	1,	1)	(0.313,	0.333,	0.354,	0.375;	0.9,	0.8)]
8	[(0.324,	0.354,	0.375,	0.396;	1,	1)	(0.292,	0.313,	0.333,	0.354;	0.8,	0.7)]
9	[(0.303,	0.333,	0.375,	0.417;	1,	1)	(0.273,	0.292,	0.313,	0.333;	0.7,	0.6)]
10	[(0.042,	0.063,	0.083,	0.104;	1,	1)	(0.021,	0.042,	0.063,	0.083;	0.5,	0.4)]
11	[(0.063,	0.083,	0.104,	0.125;	1,	1)	(0.042,	0.063,	0.083,	0.104;	0.5,	0.4)]
12	[(0.247,	0.292,	0.333,	0.375;	1,	1)	(0.229,	0.271,	0.313,	0.354;	0.7,	0.5)]
13	[(0.313,	0.340,	0.354,	0.375;	1,	1)	(0.292,	0.313,	0.333,	0.354;	0.8,	0.6)]
14	[(0.458,	0.479,	0.500,	0.521;	1,	1)	(0.438,	0.458,	0.479,	0.500;	0.9,	0.8)]
15	[(0.302,	0.333,	0.354,	0.396;	1,	1)	(0.271,	0.292,	0.313,	0.333;	0.9,	0.7)]
16	[(0.042,	0.063,	0.083,	0.104;	1,	1)	(0.021,	0.042,	0.063,	0.083;	0.4,	0.3)]
17	[(0.328,	0.354,	0.375,	0.396;	1,	1)	(0.292,	0.313,	0.333,	0.354;	0.8,	0.6)]
18	[(0.042,	0.063,	0.083,	0.125;	1,	1)	(0.031,	0.073,	0.094,	0.115;	0.4,	0.3)]
19	[(0.313,	0.333,	0.354,	0.375;	1,	1)	(0.292,	0.313,	0.333,	0.354;	0.8,	0.7)]
20	[(0.319,	0.340,	0.360,	0.402;	1,	1)	(0.292,	0.313,	0.333,	0.354;	0.8,	0.7)]
21	[(0.417,	0.438,	0.458,	0.479;	1,	1)	(0.396,	0.417,	0.438,	0.458;	0.9,	0.7)]
22	[(0.296,	0.338,	0.379,	0.421;	1,	1)	(0.271,	0.292,	0.313,	0.333;	0.8,	0.7)]
23	[(0.042,	0.063,	0.083,	0.104;	1,	1)	(0.021,	0.042,	0.063,	0.083;	0.4,	0.3)]

The mean samples of IT-2F charts for the average number of non-conformities per unit are calculated by using equations (14) and (15) as follows:

$$\bar{u}_{a_1^U} = 0.2279 \qquad \bar{u}_{a_2^U} = 0.2530 \qquad \bar{u}_{a_3^U} = 0.2781 \qquad \bar{u}_{a_4^U} = 0.3043$$

$$\bar{u}_{a_1^L} = 0.2054 \qquad \bar{u}_{a_2^L} = 0.2287 \qquad \bar{u}_{a_3^L} = 0.2514 \qquad \bar{u}_{a_4^L} = 0.2740$$

The control limits of IT-2F are calculated using equations (16) - (18);

LCL = [(-0.011, 0.025, 0.060, 0.098 ;1,1), (-0.021, 0.012, 0.044, 0.078 ;0.678,0.548)]
CL = [(0.228, 0.253, 0.278, 0.304 ;1,1), (0.205, 0.229, 0.251, 0.274 ;0.678,0.548)]
UCL= [(0.435, 0.471, 0.506, 0.543 ;1,1), (0.402, 0.436, 0.468, 0.501 ;0.678,0.548)]

Then, the *u*-control limits are defuzzied using equations (19) - (21). Firstly, the lower control limit (LCL_CDIT2_{Trap}) is calculated as follows:

$$CDIT2^{L}_{Trap} = \frac{(0.098-(-0.011))+(1\times 0.025-(-0.011))+(1\times 0.06-(-0.011))}{4}+(-0.011) = 0.0429$$

$$CDIT2^{L}_{Trap} = \frac{(0.078-(-0.021))+(0.548\times 0.012-(-0.021))+(0.678\times 0.044-(-0.021))}{4}+(-0.021) = 0.0232$$

$$LCL\_CDIT2_{Trap} = \frac{0.0429+0.0232}{2} = 0.0331$$

The lower control limit for these data is 0.0331. Further on, the center line (CL_CDIT2_{Trap}) is calculated as:

$$CDIT2^{U}_{Trap} = \frac{(0.304-0.228)+(1\times 0.253-0.228)+(1\times 0.278-0.228)}{4}+0.228 = 0.2658$$

$$CDIT2^{L}_{Trap} = \frac{(0.274-0.205)+(0.548\times 0.229-0.205)+(0.678\times 0.251-0.205)}{4}+0.205 = 0.1938$$

$$CL\_CDIT2_{Trap} = \frac{0.2658+0.1938}{2} = 0.2298$$

The center line is 0.2298. The upper control limit (UCL_CDIT2_{Trap}) is computed as:

$$CDIT2^{U}_{Trap} = \frac{(0.543-0.435)+(1\times 0.471-0.435)+(1\times 0.506-0.435)}{4}+0.435 = 0.4888$$

$$CDIT2^{L}_{Trap} = \frac{(0.501-0.402)+(0.548\times 0.436-0.402)+(0.678\times 0.468-0.402)}{4}+0.402 = 0.3647$$

$$UCL\_CDIT2_{Trap} = \frac{0.4888+0.3647}{2} = 0.4267$$

The upper control limit is 0.4267. Next, each sample of the average of the non-conformities per unit had also been defuzzied based on the BNP methods using equations (22) - (24). For example, the calculation for the first sample is:

$$DIT2^{U}_{Trap} = \frac{(0.417-0.304)+(1\times 0.333-0.304)+(1\times 0.375-0.304)}{4}+0.304 = 0.3570$$

$$DIT2^{L}_{Trap} = \frac{(0.396-0.271)+(0.7\times 0.313-0.271)+(0.9\times 0.354-0.271)}{4}+0.271 = 0.3010$$

$$DIT2_{Trap} = \frac{0.3570+0.3010}{2} = 0.3290$$

The results for all the defuzzified samples are presented in Table 3. Then, defuzzified samples ($DIT2_{Trap}$) will be compared with the UCL_CDIT2_{Trap} and LCL_CDIT2_{Trap} from the above calculations, and the outcomes are obtained as follows: (Note that the outcomes are referred to as IC = in control and OFC = out of control.)

Table 3. Defuzzification result of the average number of non-conformities per unit

No.	$DIT2^{U}_{Trap}$	$DIT2^{L}_{Trap}$	$DIT2_{Trap}$	Outcomes	No	$DIT2^{U}_{Trap}$	$DIT2^{L}_{Trap}$	$DIT2_{Trap}$	Outcomes
1	0.357	0.301	0.329	IC	13	0.345	0.275	0.310	IC
2	0.115	0.068	0.091	IC	14	0.490	0.434	0.462	OFC
3	0.073	0.035	0.054	IC	15	0.346	0.272	0.309	IC
4	0.573	0.498	0.536	OFC	16	0.073	0.035	0.054	IC
5	0.094	0.053	0.073	IC	17	0.363	0.275	0.319	IC
6	0.068	0.035	0.052	IC	18	0.078	0.051	0.065	IC
7	0.364	0.318	0.341	IC	19	0.344	0.283	0.313	IC
8	0.362	0.283	0.322	IC	20	0.355	0.283	0.319	IC
9	0.357	0.250	0.303	IC	21	0.448	0.385	0.416	IC
10	0.073	0.038	0.055	IC	22	0.358	0.265	0.311	IC
11	0.094	0.053	0.073	IC	23	0.073	0.035	0.054	IC
12	0.312	0.234	0.273	IC					

From the table above, there are two "out of control" and 21 "in control" out of 23 samples. The results were obtained after implementing the samples data using the proposed IT2F*u*-control chart. The next section investigates the performance comparison of the proposed works against two other control charts.

5. Comparative analysis

This section provides a comparative result between three types of control charts. As aforementioned, the IT2F*u*-control chart was developed with the understanding that the samples were converted into IT-2F numbers. This conversion is made as a measure to deal with inaccuracy and uncertainty in the manufacturing process. For comparison purposes, the IT2F*u*-control chart is compared to the traditional *u*-control chart and the T1F*u*-control chart. Table 4 presents the outcome of the comparative analysis. (Note that process control is referred to as IC = in control and OFC = out of control.)

Table 4. Comparative results under traditional *u*-control chart, T1F*u*-control chart, and IT2F*u*-control chart

No.	Process control			No.	Process control		
	Traditional chart	**Type 1**	**Type 2**		**Traditional chart**	**Type 1**	**Type 2**
1	IC	IC	IC	**13**	IC	IC	IC
2	IC	IC	IC	**14**	IC	Rather IC	OFC
3	IC	IC	IC	**15**	IC	IC	IC
4	OFC	OFC	OFC	**16**	IC	IC	IC
5	IC	IC	IC	**17**	IC	IC	IC
6	IC	IC	IC	**18**	IC	IC	IC
7	IC	IC	IC	**19**	IC	IC	IC
8	IC	IC	IC	**20**	IC	IC	IC
9	IC	IC	IC	**21**	IC	IC	IC
10	IC	IC	IC	**22**	IC	IC	IC
11	IC	IC	IC	**23**	IC	IC	IC
12	IC	IC	IC				

To further visualize the number of "in control" and "out of control" for each chart, line graphs against the upper limit and lower limit are plotted. Figure 3 shows the line graphs of the control charts.

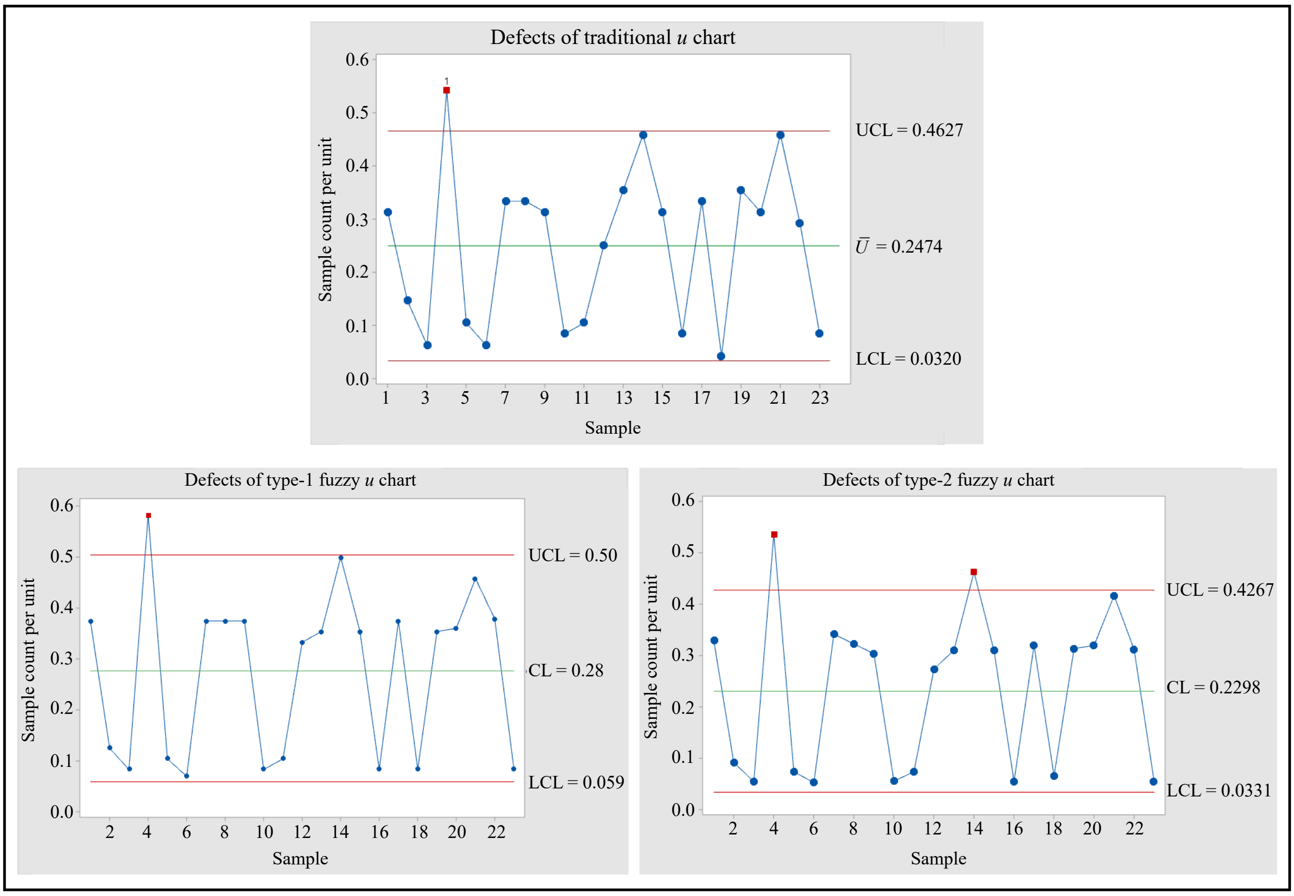

Figure 3. Line graphs of "in control" and "out of control" of traditional u-chart, T1Fu-chart, and T2Fu-chart

As shown in Figure 3, all charts show that data 14 is "out of control". However, the results obtained from traditional u-chart and T1Fu-chart for data 14 are "in control" and "rather in control," respectively. Contrarily, the result of IT2Fu-charts for data 14 is "out of control". It seems that the proposed work, IT2Fu, is more sensitive compared to other control charts.

In addition, a performance comparison of the three control charts is also measured. The ARLs are calculated using equations (25) and (26). For example, the ARL for traditional u-control charts is calculated as follows:

$$\begin{aligned} \alpha &= P(D < nLCL \mid u) + P(D > nUCL \mid u) \\ &= P(D < 23(0.032) \mid u = 5.6) + P(D > 23(0.4627) \mid u = 5.6) \\ &= P(D < 0.736 \mid u = 5.6) + P(D > 10.642 \mid u = 5.6) \\ &= P(D \geq 0 \mid u = 5.6) + P(D \geq 11 \mid u = 5.6) \end{aligned}$$

$$\begin{aligned} &= 1 - P(D \geq 0) + P(D \geq 11) \\ &= (1-1) + 0.0282 \\ &= 0.0282 \end{aligned}$$

Hence, $ARL = ARL_0 + ARL_1 = \dfrac{1}{0.0282} + \dfrac{1}{1-0.9718} = 35.4610 + 35.4610 = 70.9220.$

ARLs for other control charts are calculated similarly. Table 5 highlights the ARLs for the respective control charts.

Table 5. ARLs of traditional chart, T1F*u*-control chart, and IT2F*u*-control chart

Type of chart	Probability type I error, α	ARL_0	ARL
Traditional *u*-chart	0.0282	35.4610	70.9220
T1F*u*-chart	0.0324	30.8642	61.7284
IT2F*u*-chart	0.0397	25.1889	50.3778

It can be seen that the ARL of the IT2F*u*-chart has the lowest value. Hence, we can suggest that the IT2F*u*-chart is able to detect shifts in the process control faster than other control charts.

6. Conclusion

Customarily, a traditional control chart is used to identify defects using crisp numbers. Nevertheless, we are unable to ensure the accuracy of the data in the manufacturing settings all the time. At this point, fuzzy set theory is the best theory to be implemented to deal with the uncertainty and inaccuracy of the data. This is directed at the creation of fuzzy charts where type-1 fuzzy MFs are embedded in the control charts. However, there is a case where type-1 fuzzy numbers are not appropriate to be analyzed in the analysis since their MFs cannot be fully represented by crisp numbers. Therefore, it is a good alternative to introduce IT-2F numbers to the control charts. Formerly, journals from other studies indicate that the analysis of various type-1 fuzzy charts is in great abundance. However, the use of T1F*u*-charts is a handful, and to the best of the author's knowledge, apparently, there is no study that merges the concept of IT-2F numbers and control charts. Therefore, this paper presents the development of IT2F*u*-control charts as a new approach and tests it with lubricant oil manufacturing data. In addition, this paper also highlights performance comparisons between traditional *u*-charts, T1F*u*-charts, and IT2F*u*-charts.

The present study makes several noteworthy contributions to the knowledge of fuzzy statistical process control. Firstly, the proposed IT2F*u*-chart can model a higher level of uncertainty than T1F*u*-charts and traditional *u*-charts. The superiority of intervals in type-2 fuzzy numbers in dealing with uncertainty and inaccuracy motivates the integration of IT-2F sets and *u*-control charts. Secondly, the proposed method is validated by a comparative analysis. We compared the process analysis between the traditional *u*-chart, the T1F*u*-chart, and the proposed IT2F*u*-chart in order to ascertain the best method for controlling the product's quality. The results of the comparative analysis enhance our understanding of the superiority of the proposed charts over the benchmark methods. Lastly, the computations of ARLs at the end of the study are implemented to confirm the performance of the proposed IT2F*u*-chart against the T1F*u*-chart and traditional *u*-chart.

Based on the illustrative example, the results show that the IT2F*u*-chart is more sensitive than the T1F*u*-chart and traditional *u*-chart in monitoring the variations of the lubricant's characteristics since it has the highest samples that are "out of control," which are 14 defects. On the other hand, the results of traditional *u*-chart and T1F*u*-chart for data 14 are "in control" and "rather in control". This means that if the company does not apply the IT2F*u*-chart, they might include the defect in their production. Research by Darestani et al. [12], Fadaei and Pooya [19], Truong et al. [20], Aslangiray and Akyuz [21], and Şentürk et al. [22] studied T1F*u*-chart and did not prove the research by using any method like ARL. Nevertheless, in this study, we extend the knowledge of fuzzy towards the IT2F*u*-chart, and at the end of the study, it proves that the proposed method is much more sensitive and better at finding good-quality products. Furthermore, the comparative analysis of ARL also proves that IT2F*u* is the best chart compared to T1F*u* and the traditional chart since it has the lowest value.

Future studies can explore the IT2F*u*-control chart using a variable sample size. Other than that, control charts based on the definitions of sets such as hesitant fuzzy sets, intuitionistic fuzzy sets, or neutrosophic sets could be a good notion in future works.

Acknowledgments

The authors wish to thank to the Centre of Research and Innovation Management of University Malaysia Terengganu for their help throughout the course of this research.

Conflict of interest

The authors declare that they have no conflict of interest.

Informed consent

Informed consent does not apply for studies that do not involve human participants and/or animals.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Funding

The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

Data availability

Enquiries about data availability should be directed to the authors.

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Appendix

Table A. 23 samples of lubricant's data

No.	Viscosity
1	14.6
2	4-6
3	2.0
4	26.0
5	3-6
6	1.5
7	15.9
8	15.5
9	14.5
10	-
11	3-6
12	11.8
13	15.0
14	22-25
15	14.5
16	2-5
17	15.7
18	-
19	15
20	15.3
21	15-18
22	15.3
23	20.0