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Research Article

# Analysis of Charge and Current Flow in the LCR Series Circuit via Chebyshev Wavelets

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**Abstract:** In this research paper, an investigation of charge and current flow in an LCR series circuit has been presented. For this purpose, basis functions of Chebyshev wavelets of the second kind have been utilized. The proposed method involves representing the highest-order derivatives as a series of basis functions using Chebyshev wavelets. In order to demonstrate the effectiveness of this approach, numerical examples have been provided, and their results are presented to show the accuracy of the proposed scheme.

***Keywords*:** Chebyshev wavelets, LCR circuit, function approximation, numerical examples

**MSC:** 65N99, 65H10

## 1. Introduction

Wavelet analysis is a mathematical tool used for analyzing signals or data in both the time and frequency domains. It involves decomposing a signal into a set of wavelets, which are small waveforms that capture different frequency components of the original signal at different scales. Wavelet analysis has applications in various fields, including signal processing, image compression, feature extraction, time series analysis, and pattern recognition. It provides a flexible and powerful framework for analyzing and understanding complex signals with both local and global features. Wavelets are used to solve differential equations and integral equations in engineering and science. They have been applied to several applications, and various wavelets such as Haar, Hermite, Chebyshev, Legendre, Bernoulli, etc. have been utilized. Many algorithms utilizing Haar and Hermite wavelets have been created to tackle integral and differential equations and are discussed in Cattani [1, 2], Chen and Hsiao [3, 4], Lepik [5], Singh [6], Ali et al. [7], Pirim and Ayaz [8], and Singh and Kumar [9, 10]. The Chebyshev wavelet is a highly effective mathematical tool for solving a range of scientific and engineering problems. Its simplicity and accuracy have garnered attention from researchers, scientists, and engineers alike. In numerous works, the Chebyshev wavelet has been used to provide precise, reliable, and applicable solutions in a minimal time interval.

Chebyshev polynomials have been utilized to solve both linear and nonlinear two-dimensional integral equations of the second kind (Avazzadeh & Heydari [11]). Zhu and Fan [12] utilized Chebyshev wavelets of the second kind to solve integro-differential equations of the fractional nonlinear Fredholm type. A novel spectral algorithm has been created that utilizes second-kind Chebyshev wavelets to solve second-order differential equations of both linear and nonlinear nature, including those that involve singular and Bratu-type equations (Abd-Elhameed et al. [13]). In Babolian


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and Fattahzadeh [14], numerical solutions of the differential equations with the aid of a Chebyshev wavelet-based operational matrix of integration have been discussed. A second-kind Chebyshev wavelet-based technique has been developed for solving fractional differential equations in Wang and Fan [15]. The application of second-kind Chebyshev wavelets to obtain numerical solutions for convection-diffusion equations has been explored (Zhou and Xu [16]). A method based on Chebyshev wavelets has been employed to solve delay differential equations (Ali et al. [17]). An operational matrix based on Chebyshev wavelets has been utilized to solve differential equations of the Lane-Emden type in Doha et al. [18]. A technique based on Chebyshev wavelets has been developed to solve partial differential equations subject to telegraph-type boundary conditions (Heydari et al. [19]). The Chebyshev wavelet-based technique has been discussed for the solution of problems related to differentiation (Singh & Preeti [20]). A spectral solution of fractional Riccati differential equations has been obtained using a shifted second-kind Chebyshev wavelet method (Abd-Elhameed & Youssri [21]). A numerical method that employs the second-kind Chebyshev wavelet has been proposed for solving the Fredholm and Volterra integral equations (Abd-Elhameed & Youssri [22]). The spectral technique based on shifted second-kind Chebyshev wavelet collocation has been introduced and utilized for solving integro-differential equations by Sweliam et al. [23]. A Chebyshev wavelet-based method has been presented for improved milling stability prediction in Qin et al. [24]. Numerical solutions based on Hermite wavelets have been introduced for the solution of oscillatory electrical circuit equations by Kaur and Singh [25]. Two new spectral algorithms have been presented for the solution of linear and nonlinear fractional-order differential equations (Abd-Elhameed & Youssri [26]). The solution of fractional-order differential equations for an electrical RLC circuit has been introduced (Gomez et al. [27]). An overview of the Haar wavelet method for the solution of differential and integral equations has been discussed (Hariharan and Kannan [28]). For the numerical solution of differential equations, the higher-order Haar wavelet method has been introduced by Majak et al. [29]. The Hermite wavelet method for solving systems of differential equations has been presented by Singh and Kaur [30].

The LCR circuit equation, a second-order differential equation, finds applications in diverse fields due to its ability to describe the behavior of LCR circuits. In circuit analysis, the equation allows engineers to predict and analyze the response of LCR circuits to different inputs, enabling the design and optimization of circuit parameters for specific applications. After solving the equation, engineers can determine the circuit's transient and steady-state behavior. The equation is particularly valuable in frequency response analysis, enabling the design of filters, resonant circuits, and oscillators for applications in audio systems, communication systems, and power electronics. Additionally, the LCR circuit equation plays a vital role in signal processing applications, helping engineers design circuits for accurate and efficient signal manipulation. Overall, the equation serves as a fundamental tool for circuit analysis, design, and signal processing, contributing to advancements in multiple fields.

In this research work, we investigate the charge and current flow in the LCR circuit. Suppose that $I(t)$ is the current in the LCR series electrical circuit as depicted in Figure 1, where $L$, $C$, and $R$ represent the inductance, capacitance and resistance in the circuit, respectively. The voltage drop across inductor is $L\frac{dI}{dt}$, across resistor is $RI$ and across capacitor is $\frac{Q}{C}$, where $Q$ is the charge of the capacitor and $\frac{dQ}{dt} = I$. Applying Kirchoff's law with the voltage drops yields the differential equations

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = E(t)$$

where $E(t)$ represents the voltage impressed on the circuit. Substituting $I = \frac{dQ}{dt}$, we obtain

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

with initial conditions $Q(0) = q_0$, $Q'(0) = q_1$.

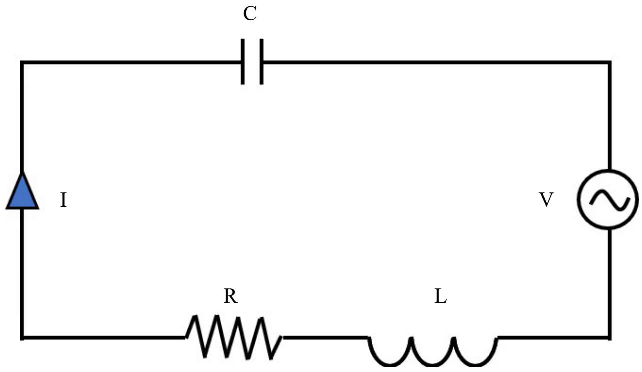


**Figure 1.** LCR circuit diagram

## 2. Chebyshev wavelets of the second kind

Wavelet-based numerical techniques have been widely employed in solving various problems in science, engineering, and technology over the past few decades. Wavelets are a family of functions constructed by dilating and translating a single function known as the mother wavelet. When the dilation parameter (a) and the translation parameter (b) vary continuously, we obtain a continuous family of wavelets as discussed in [14]:

$$\varphi_{a,b}(t) = |a|^{-1/2}\varphi\left(\frac{t-b}{a}\right),\ \ a,b \in R,\ \ a \neq 0$$

The Chebyshev wavelet of second kind, denoted as $\varphi(n, m) = \varphi(k, n, m, t)$, and has four parameters: a positive integer denoted by $k$, $n$ takes values from 1, 2, 3,...,$2^{k-1}$, the degree of Chebyshev polynomials of second kind denoted by $m$, and $t$ denoting the normalized time. These wavelets are defined on the interval [0,1) as follows:

$$\varphi_{n,\,m}(t) = \begin{cases} 2^{\frac{k}{2}}\tilde{U}\left(2^k t - 2n + 1\right), & \frac{n-1}{2^{k-1}} \le t \le \frac{n}{2^{k-1}} \\ 0, & \text{otherwise} \end{cases}$$

where

$$\tilde{U}_m(t) = \sqrt{\frac{2}{\pi}}U_m(t), \tag{1}$$

where $m$ = 0, 1, 2, 3,..., $M-1$ and $M$ denotes the integer, which is fixed. Equation (1) represents the condition of orthonormality. Here, $U_m(t)$ is the Chebyshev polynomials of second-kind of degree denoted by $m$, and is orthogonal with respect to the weight function $\omega(t) = \sqrt{1-t^2}$ on the interval [−1,1], and the following recursive formula is satisfied:

$$U_0(t) = 1,\quad U_1(t) = 2t,\quad U_{m+1}(t) = 2tU_m(t) - U_{m-1}(t),\qquad m = 1,2,3,4,\ldots\ldots$$

It should be noted that for Chebyshev wavelets of the second kind, the weight function needs to be dilated and translated as follows: $\omega_n(t) = \omega(2^k t - 2n + 1)$.

A function $f(x) \in L^2(\mathbb{R})$ defined on the interval [0,1) may be expanded by Chebyshev wavelets of second kind as

$$f(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\varphi_{n,m}(x), \tag{2}$$

where

$$c_{n,m} = \; < f(x), \varphi_{n,m}(x) >_{L^2_\omega[0,1)} = \int_0^1 f(x)\varphi_{n,m}(x)\omega_n(x),$$

in which the inner product denoted by $< .,. >_{L^2_\omega[0,1)}$ in $L^2_\omega\left[0,1\right)$. If the infinite series is truncated (as detailed in [14]) in equation (2), then we write it as

$$f(x) \cong \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\varphi_{n,m}(x) = C^T\varphi(x),$$

where $C$ and $\varphi(x)$ are $2^{k\text{-}1}\, M \times 1$ matrices are given as

$$C = (c_{1,0}, c_{1,1}, \ldots, c_{1,M-1}, c_{2,0}, c_{2,1}, \ldots, c_{2,M-1}, \ldots, c_{2^{k-1},0}, c_{2^{k-1},1}, \ldots, c_{2^{k-1},M-1})^T$$

and

$$\varphi(x) = (\varphi_{1,0}, \varphi_{1,1}, \ldots, \varphi_{1,M-1}, \varphi_{2,0}, \varphi_{2,1}, \ldots, \varphi_{2,M-1}, \varphi_{2^{k-1},0}, \varphi_{2^{k-1},1}, \ldots, \varphi_{2^{k-1},M-1})^T. \tag{3}$$

To find the integral of the second-kind Chebyshev wavelet functions with $k = 1$ and $M = 6$ in the interval [0,1], we first need to determine the six basis functions. The basis functions for the Chebyshev wavelets of the second kind with $k = 1$ and $M = 6$ in the interval [0,1] are given by:

$$\varphi_{1,0}(t) = \frac{2}{\sqrt{\pi}},$$

$$\varphi_{1,1}(t) = \frac{2}{\sqrt{\pi}}(4t - 2),$$

$$\varphi_{1,2}(t) = \frac{2}{\sqrt{\pi}}(16t^2 - 16t + 3),$$

$$\varphi_{1,3}(t) = \frac{2}{\sqrt{\pi}}(64t^3 - 96t^2 + 40t - 4),$$

$$\varphi_{1,4}(t) = \frac{2}{\sqrt{\pi}}(256t^4 - 512t^3 + 336t^2 - 80t + 5),$$

$$\varphi_{1,5}(t) = \frac{2}{\sqrt{\pi}}(1024t^5 - 2560t^4 + 2304t^3 - 896t^2 + 140t - 6)$$

## 3. Proposed methodology

Consider the differential equation

$$LQ'' + RQ' + \frac{1}{C}Q = E(t) \tag{4}$$

with initial conditions $Q(0) = q_0$, $Q'(0) = q_1$.

Assume that

$$Q''(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \varphi_{n,m}(t) \tag{5}$$

Integrating (5) twice with respect to $t$, we obtain

$$Q'(t) = Q'(0) + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \varphi_{n,m}(t)dt \tag{6}$$

and

$$Q(t) = Q(0) + t.Q'(0) + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \int_0^t \varphi_{n,m}(t)dtdt. \tag{7}$$

Applying initial conditions, we obtain

$$Q'(t) = q_0 + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \varphi_{n,m}(t)dt \tag{8}$$

and

$$Q(t) = q_1 + t.q_0 + \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^t \int_0^t \varphi_{n,m}(t)dtdt. \tag{9}$$

Substitute the values from equations (5), (8), and (9) in (4), we obtain

$$\sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \left\{ L.\varphi_{n,m}(t) + R.\int_0^t \varphi_{n,m}(t)dt + \frac{1}{C}\int_0^t \int_0^t \varphi_{n,m}(t)dtdt \right\} = f(t) \tag{10}$$

where

$$f(t) = E(t) - R.q_0 - C.\{q_1 + t.q_0\}.$$

After discretizing equation (10), we obtain a system of linear equations that can be solved using any classical scheme. The solution to this type of system of equations provides us with the wavelet coefficients. We can then substitute the values of these wavelet coefficients into equation (9) to obtain the required solution.

## 4. Numerical observations

Here, in this section, we will illustrate the accuracy of the charge and current using the proposed scheme through some numerical experiments:

**Case 1:** Letting $L = 1H$, $R = 2\Omega$, $C = 1F$, and $E(t) = \sin t$. Equation (4) becomes

$$Q'' + 2Q' + Q = \sin t$$

with initial conditions $Q(0) = 0 = Q'(0)$. The exact solution is

$$Q(t) = \frac{1}{2}(1+t)e^{-t} - \frac{1}{2}\cos t$$

and

$$Q'(t) = -\frac{1}{2}te^{-t} + \frac{1}{2}\sin t.$$

Table 1 shows the comparison of charge flow in the LCR circuit.

**Table 1.** Comparison of charge with exact and Chebyshev wavelet method

| Points | Exact | Chebyshev wavelets | Absolute errors |
|---|---|---|---|
| 1/12 | 9.2498e-005 | 9.2517e-005 | 1.9284e-008 |
| 3/12 | 2.2943e-003 | 2.2943e-003 | 5.2002e-008 |
| 5/12 | 9.7406e-003 | 9.7407e-003 | 7.0700e-008 |
| 7/12 | 2.4462e-002 | 2.4462e-002 | 8.2703e-008 |
| 9/12 | 4.7476e-002 | 4.7476e-002 | 9.0357e-008 |
| 11/12 | 7.8955e-002 | 7.8955e-002 | 8.8148e-008 |

Table 2 shows the comparative study of current flow in the LCR circuit.

**Table 2.** The current flow in the LCR circuit is compared between the exact and Chebyshev wavelet methods

| Points | Exact | Chebyshev wavelets | Absolute errors |
|---|---|---|---|
| 1/12 | 3.2833e-003 | 3.2836e-003 | 3.0582e-007 |
| 3/12 | 2.6352e-002 | 2.6352e-002 | 1.1550e-007 |
| 5/12 | 6.5015e-002 | 6.5016e-002 | 1.0601e-007 |
| 7/12 | 1.1264e-001 | 1.1264e-001 | 4.1530e-008 |
| 9/12 | 1.6368e-001 | 1.6368e-001 | 5.1812e-008 |
| 11/12 | 2.1352e-001 | 2.1352e-001 | 8.9218e-008 |

Figures 2 and 3 show the comparative study of exact and wavelet-based technique.

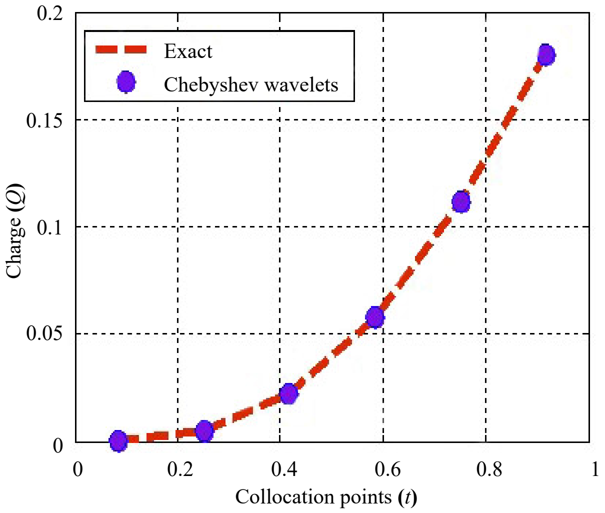


**Figure 2.** Comparison of charge in LCR circuit

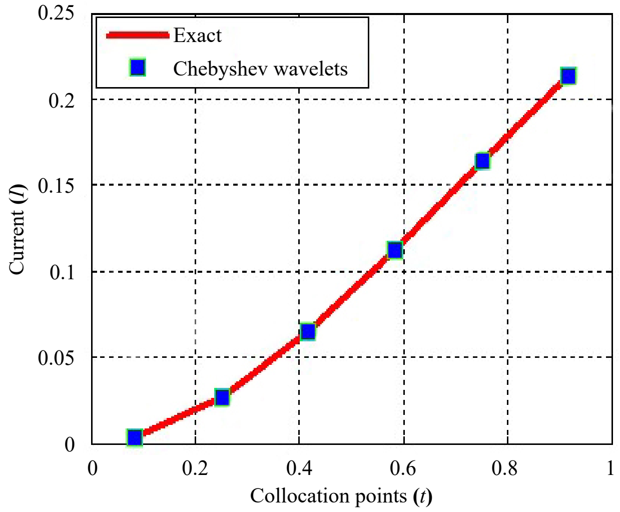


**Figure 3.** Comparison of current in LCR circuit

**Case 2:** Letting $L = 2H$, $R = 0\Omega$, $C = 4F$, and $E(t) = 2 \sin 2t$. Equation (4) becomes

$$Q'' + 4Q = \sin 2t$$

with initial conditions $Q(0) = 0 = Q'(0)$. The exact solution is

$$Q(t) = \frac{1}{8}\sin 2t - \frac{1}{4}t.\cos 2t$$

and

$$Q'(t) = \frac{1}{2}t.\sin 2t$$

Table 3 shows the comparison of charge flow in the LCR circuit.

**Table 3.** A comparison is made between the exact charge and the charge obtained by the Chebyshev wavelet method

| Points | Exact | Chebyshev wavelets | Absolute errors |
|---|---|---|---|
| 1/12 | 1.9237e-004 | 1.9401e-004 | 1.6446e-006 |
| 3/12 | 5.0793e-003 | 5.0844e-003 | 5.1373e-006 |
| 5/12 | 2.2479e-002 | 2.2487e-002 | 7.6645e-006 |
| 7/12 | 5.7586e-002 | 5.7596e-002 | 9.4026e-006 |
| 9/12 | 1.1142e-001 | 1.1143e-001 | 1.0171e-005 |
| 11/12 | 1.8019e-001 | 1.8020e-001 | 9.3638e-006 |

Table 4 shows the comparison of current flow in the LCR circuit.

**Table 4.** A comparison is made between the exact current flow and obtained by Chebyshev wavelet method

| Points | Exact | Chebyshev wavelets | Absolute errors |
|---|---|---|---|
| 1/12 | 6.9123e-003 | 6.9398e-003 | 2.7444e-005 |
| 3/12 | 5.9928e-002 | 5.9944e-002 | 1.5517e-005 |
| 5/12 | 1.5420e-001 | 1.5422e-001 | 1.4331e-005 |
| 7/12 | 2.6817e-001 | 2.6818e-001 | 6.3265e-006 |
| 9/12 | 3.7406e-001 | 3.7406e-001 | 3.0352e-006 |
| 11/12 | 4.4263e-001 | 4.4261e-001 | 1.4285e-005 |

Figure 4 shows the comparison study of exact solutions and wavelets-based solutions in terms of charge flows in LCR circuit. Figure 5 shows the comparison study of exact solutions and wavelets-based solutions in terms of current flows in LCR circuit.

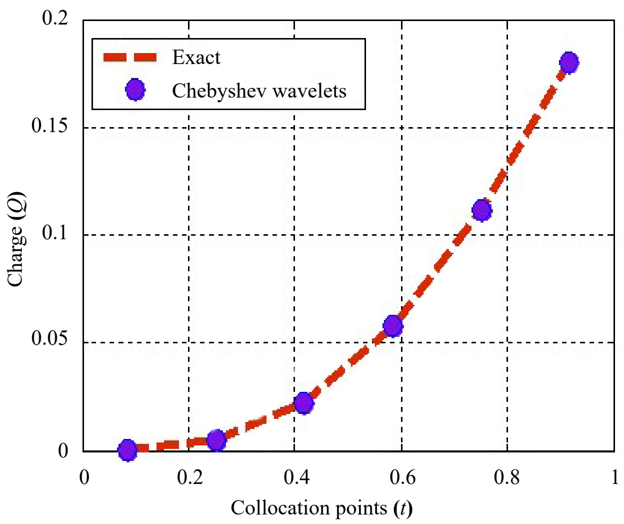


**Figure 4.** Comparison of charge in LCR circuit

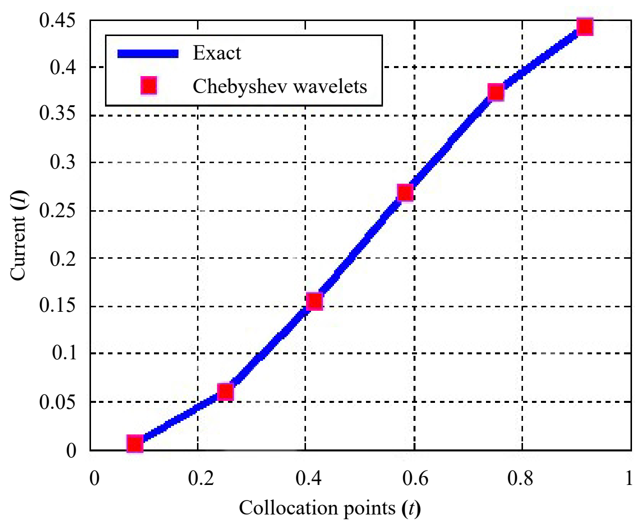


**Figure 5.** Comparison of current in LCR circuit

If the exact solution is piecewise continuous, it may not be sufficiently differentiable at some points. As a result, the wavelet approximation may not be able to accurately capture sharp transitions in the solution. In such cases, it is necessary to consider other basis functions that can better represent the piecewise nature of the solution. One option is to use function approximation through integration, which can provide more accurate results in certain cases.

## 5. Conclusions

The success of the second-kind Chebyshev wavelet method in computing accurate numerical solutions for the LCR series circuit equation suggests that this technique has potential for use in various other areas of engineering and technology. Moreover, the Chebyshev wavelet method can also be used in other areas of engineering, such as civil and mechanical engineering, for solving differential equations that arise in the modeling of various physical systems. For example, this method can be applied to problems related to the mechanics of materials, fluid dynamics, heat transfer, and other areas of engineering where numerical methods are often required to obtain solutions. Furthermore, this technique has the potential to be applied to various other areas of engineering and technology for further investigation and analysis. This technique gives high accuracy when the number of collocation points varies.


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## Conflict of interest

There is no conflict of interest in this study.

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