

## Research Article

# Heat Transfer and Flow of Natural Convection Past a Semi-Infinite Vertical Plate

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**Abstract:** This manuscript presents a numerical investigation of the flow of an electrically conducting, incompressible, viscous fluid past a heated vertical channel with thermal radiation influence. The study uses the Rosseland approximation to describe the radiative heat flux and employs Crank Nicolson's implicit finite difference method utilizing the MATLAB programming tool for solving the non-dimensional system of equations. The results demonstrate that an increase in the thermal radiation parameter leads to a decrease in both the velocity and temperature of the fluid. The main findings of the study highlight the impact of thermal radiation on natural convection and heat transfer processes. This research is significant as it contributes to the understanding of convective heat transfer phenomena and provides valuable insights for applications in science and technology.

**Keywords:** natural convection, thermal radiation, crank Nicolson's method, heat transfer

## 1. Introduction

Natural convection phenomena play a significant role in various heat transfer processes and have important implications for applications in science and technology. Understanding the flow and heat transfer characteristics of natural convection is crucial for optimizing thermal management systems, such as the cooling of electronic devices, energy-efficient building design, and industrial processes. In this study, we focus on investigating the heat transfer and flow behavior of natural convection past a semi-infinite vertical plate.

Numerical simulations have become an indispensable tool for studying complex fluid dynamics problems. The use of numerical methods allows for the efficient and accurate analysis of convective heat transfer processes. Several researchers have investigated natural convection phenomena in different configurations to explore the underlying mechanisms and provide insights for practical applications.

Free convection flow and mass transfer over a vertical plate with radiation and uniform transpiration effects was

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studied by Rouzbahani and Khoramshahi [1]. Anwar et al. [2] investigated an exact analysis of unsteady MHD free convection flow of some nanofluids with ramped wall velocity and ramped wall temperature accounting heat radiation and injection/consumption. Thermal radioactive influence on MHD free convection flow across a porous medium in a vertical surface with temperature was considered by Ahmed and Dutta [3]. Chamuah et al. [4] studied Oscillatory Flow of a Viscous Conducting Fluid Through a Uniformly Moving Vertical Circular Cylinder Under Pressure Gradient. In the work of Omamoke et al. [5], Radiation and heat source effects on MHD free convection flow over an inclined porous plate in the presence of viscous dissipation.

Hellums and Churchill [6] used an explicit finite difference method to solve natural convection flow through a semi-infinite vertical plate. Aydın and Kaya [7] explores mixed convection heat transfer around a breathable vertical plate in the presence of magneto and thermal radiation effects; the set of governing equations is solved using similarity variables. Aydın and Kaya [8] numerically solve the problem of steady laminar magnetohydrodynamic (MHD) mixed convection heat transfer about a vertical plate, taking ohmic heating and viscous dissipation into account. Tania and Samad [9] studied the effect of radiation, heat generation, and viscous dissipation on MHD-free convection flow along a stretching sheet using an explicit finite difference scheme and computer-generated programs.

Using a spectral relaxation approach, Alao et al. [10] investigated heat and mass transfer in a chemically reacting fluid. Oyelami and Falodun [11] used spectral homotopy analysis to investigate the heat and mass transfer of hydrodynamic boundary layer flow along a flat plate under the influence of variable temperature and viscous dissipation (SHAM).

Motivated by these previous works, the present study aims to numerically investigate the flow and heat transfer characteristics of natural convection past a semi-infinite vertical plate. The study considers an electrically conducting, incompressible, viscous fluid flowing past a heated vertical channel with thermal radiation influence. The Rosseland approximation is utilized to describe the radiative heat flux, and Crank-Nicolson's implicit finite difference method is employed to solve the non-dimensional system of equations.

By examining the influence of thermal radiation on fluid velocity and temperature profiles, this research contributes to the understanding of convective heat transfer phenomena. The findings from this study are expected to provide valuable insights for the design and optimization of thermal management systems, such as heat exchangers, solar collectors, and electronic cooling devices.

The choice of this problem, specifically the flow of natural convection past a semi-infinite vertical plate, is motivated by its complexity and the need for advanced numerical methods to solve the governing equations. Previous studies have explored similar problems, but the present research aims to incorporate thermal radiation effects, which adds a new dimension to the analysis. By focusing on the interaction between thermal radiation and natural convection, the researchers aim to enhance their understanding of these phenomena and their implications for practical applications.

## 2. Statement of problem

An unsteady two-dimensional laminar natural convection flow of a viscous, incompressible, electrically conducting, and radiating fluid past an impulsively started semi-infinite vertical plate is considered in this study. The  $\bar{y}$  axis is taken perpendicular to the plate at the leading edge, while the  $\bar{x}$  axis is chosen along the plate in a vertical upward direction. The origin of the  $\bar{x}$  axis is located at the leading edge of the plate. At time  $\bar{t} = 0$ , the plate and the fluid are assumed to be at the same ambient temperature  $\bar{T}_\infty$ . However, when  $\bar{t} > 0$ , the plate is maintained at a higher temperature  $\bar{T}_w$  which is greater than  $\bar{T}_\infty$ . Thermal radiation is assumed to be present in the form of a uni-directional flux in the  $\bar{y}$  direction i.e.,  $q_r$  is transverse to the vertical surface.

The Rosseland diffusion flux is used and defined following Dada and Adefolaju [12] as follows:

$$q_r = \frac{-4\sigma\bar{T}^4}{3k^1\bar{\partial}\bar{y}} \quad (1)$$

where  $\sigma$  is the Stefan-Boltzmann constant and  $k^1$  is the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to an optically thick fluid.

This problem is governed by the momentum, energy, and continuity equations, which can be simplified for a two-dimensional, incompressible, unsteady flow with temperature-dependent buoyancy effects. The dimensional versions of the momentum, energy, and continuity equations, respectively, for the given problem of a semi-infinitely heated vertical plate in an infinite fluid are given as follows:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \bar{T} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \quad (2)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \alpha \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial \bar{y}} \quad (3)$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (4)$$

with the following initial and boundary conditions:

$$\bar{t} \leq 0: \bar{u} = 0, \bar{v} = 0, \bar{T} = \bar{T}_\infty$$

$$\bar{t} \geq 0: \bar{u} = \bar{u}_o, \bar{v} = 0, \bar{T}_w = \bar{T}_\infty \text{ at } \bar{y} = 0$$

$$\bar{u} = 0, \bar{T} = \bar{T}_\infty \text{ at } \bar{x} = 0$$

$$\bar{u} \rightarrow 0, \bar{T} \rightarrow \bar{T}_\infty \text{ as } \bar{y} \rightarrow \infty \quad (5)$$

To transform the dimensional governing equations, the following non-dimensional quantities are defined:

$$X = \frac{\bar{x} \bar{u}_o}{\nu}, Y = \frac{\bar{y} \bar{u}_o}{\nu}, U = \frac{\bar{u}}{\bar{u}_o}, V = \frac{\bar{v}}{\bar{u}_o}, t = \frac{\bar{t} \bar{u}_o^2}{\nu}, T = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, Pr = \frac{\nu}{\alpha}, N = \frac{k^1 k}{4\sigma \bar{T}_\infty^3} \quad (6)$$

The dimensionless forms of equations (2), (3), and (4) are

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = T + \frac{\partial^2 U}{\partial Y^2} \quad (7)$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \left( 1 + \frac{4}{3N} \right) \frac{\partial^2 T}{\partial Y^2} \quad (8)$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (9)$$

where  $Pr$  stands for the Prandtl number, which indicates heat conduction's relative importance and fluid viscosity,  $N$  signifies thermal radiation parameter, and  $t$  is time without dimensions. Equations' initial and boundary conditions are as

follows:

$$t \leq 0 : U = 0, V = 0, T = 0 \text{ for all } X \text{ and } Y$$

$$t > 0 : U = 1, V = 0, T = 1 \text{ at } Y = 0$$

$$U = 0, T = 0 \text{ at } X = 0$$

$$U \rightarrow 0, T \rightarrow 0 \text{ as } Y \rightarrow \infty \tag{10}$$

Now, we must solve the three concurrent non-linear partial differential equations (7), (8), and (9) in (10) for the dependent variables  $U$ ,  $V$ , and  $T$  as functions of  $X$ ,  $Y$ , and  $t$ .

### 3. Method of solution

The numerical method employed in this study is Crank-Nicolson's implicit finite difference method. This method is known for its accuracy, stability, and fast convergence. It is used to solve the system of three non-linear partial differential equations that govern the flow of an electrically conducting, incompressible, viscous fluid past a heated vertical plate with thermal radiation influence.

To apply Crank-Nicolson's method, the governing equations are first discretized in both space and time. The spatial domain is discretized into a grid with evenly spaced points in the  $X$  and  $Y$  directions. The time domain is also discretized into time steps. The discretization step sizes are denoted as  $\Delta X$ ,  $\Delta Y$ , and  $\Delta t$ , respectively.

The discretized equations are then rearranged into a set of finite difference equations, as shown below.

$$\begin{aligned} & \frac{U_{i,j}^{k+1} - U_{i,j}^k}{\Delta t} + U_{i,j}^k \frac{(U_{i,j}^{k+1} - U_{i-1,j}^{k+1} + U_{i,j}^k - U_{i-1,j}^k)}{2\Delta X} + V_{i,j}^k \frac{(U_{i,j+1}^{k+1} - U_{i,j-1}^{k+1} + U_{i,j+1}^k - U_{i,j-1}^k)}{4\Delta Y} \\ & = \left( \frac{T_{i,j}^{k+1} + T_{i,j}^k}{2} \right) + \frac{U_{i,j-1}^{k+1} - 2U_{i,j}^{k+1} + U_{i,j+1}^{k+1} + U_{i,j-1}^k - U_{i,j}^k + U_{i,j+1}^k}{2(\Delta Y)^2} \end{aligned} \tag{11}$$

$$\begin{aligned} & \frac{T_{i,j}^{k+1} - T_{i,j}^k}{\Delta t} + U_{i,j}^k \frac{(T_{i,j}^{k+1} - T_{i-1,j}^{k+1} + T_{i,j}^k - T_{i-1,j}^k)}{2\Delta X} + V_{i,j}^k \frac{(T_{i,j+1}^{k+1} - T_{i,j-1}^{k+1} + T_{i,j+1}^k - T_{i,j-1}^k)}{4\Delta Y} \\ & = \frac{1}{Pr} \left( 1 + \frac{4}{3N} \right) \frac{T_{i,j-1}^{k+1} - 2T_{i,j}^{k+1} + T_{i,j+1}^{k+1} + T_{i,j-1}^k - T_{i,j}^k + T_{i,j+1}^k}{2(\Delta Y)^2} \end{aligned} \tag{12}$$

$$\frac{U_{i,j}^{k+1} - U_{i-1,j}^{k+1} + U_{i,j}^k - U_{i-1,j}^k + U_{i,j+1}^{k+1} - U_{i-1,j+1}^{k+1} + U_{i,j+1}^k - U_{i-1,j+1}^k}{4\Delta X} + \frac{V_{i,j}^{k+1} - V_{i,j-1}^{k+1} + V_{i,j}^k - V_{i,j-1}^k}{2\Delta Y} = 0 \tag{13}$$

For example, equation (11) represents the discretized form of the momentum equation. Similarly, equation (12) represents the discretized form of the energy equation, and equation (13) represents the discretized form of the continuity equation.

In the discretized equations, the variables  $U$ ,  $V$ , and  $T$  are represented at different grid points  $(i, j)$  at a specific

time step ( $k$ ). The superscripts  $(k + 1)$  and  $(k)$  denote the values at the next and current time steps, respectively. The coefficients of the finite difference equations involve neighboring grid points in both spatial directions.

The resulting set of finite difference equations forms a system of equations that is solved at each time step using the Thomas algorithm, which is a tridiagonal matrix algorithm. The Thomas algorithm efficiently solves the tridiagonal system of equations, considering the initial and boundary conditions as the right-hand side of the equation.

To ensure convergence, a convergence criterion is applied. Typically, convergence is checked by comparing the current solution with the previous solution and monitoring the changes in the dependent variables ( $U$ ,  $V$ , and  $T$ ). The calculations are repeated iteratively until the convergence criterion is met, indicating that the solution has reached a steady state.

In terms of boundary conditions, the manuscript mentions the initial and boundary conditions for the dependent variables  $U$ ,  $V$ , and  $T$ . These conditions specify the values of  $U$ ,  $V$ , and  $T$  at different locations and times. The boundary conditions play a crucial role in determining the behavior of flow and heat transfer. The Thomas algorithm, as discussed by Carnahan et al. [13], is used to effectively resolve these simultaneous equations.

Points were evenly spaced after  $X$  and  $Y$  were partitioned into  $P$  and  $Q$  grid spaces, respectively. Mesh sizes were set such that  $X = 0.05$ ,  $Y = 0.25$ , and  $t = 0.01$ , and the area of evaluation considered has sides with a maximum of  $X$  corresponding to 1.05 and a maximum of  $Y$  corresponding to 5.25, where the maximum of  $Y$  slips well outside boundary layers of momentum and energy.

The calculations for  $T$  at time  $(k + 1)$  were used in equation (5) to obtain  $U$  at time  $(k + 1)$ .

As a result, the calculations of  $T$  and  $U$  were obtained at a specific  $i$ -level. This procedure was repeated many times at various  $i$ -levels. This method yielded the calculations of  $T$  and  $U$  at all grid points.

The calculations of  $V$  were also obtained explicitly at each nodal point using equation (7) on a specific  $i$ -level at the  $(k + 1)$  time level.

## 4. Results and discussion

For time  $t = 80$ , the velocity and temperature fields are recreated. After  $t = 40$ , the values of  $U$ ,  $V$ , and  $T$  exhibit little change, according to the investigation. As a result, the numbers for  $t = 80$  are basically steady-state values.

The default values in the computations are  $Pr = 0.733$  and  $N = 3.0$ , and all graphs correspond to these values unless otherwise specified on the graph.

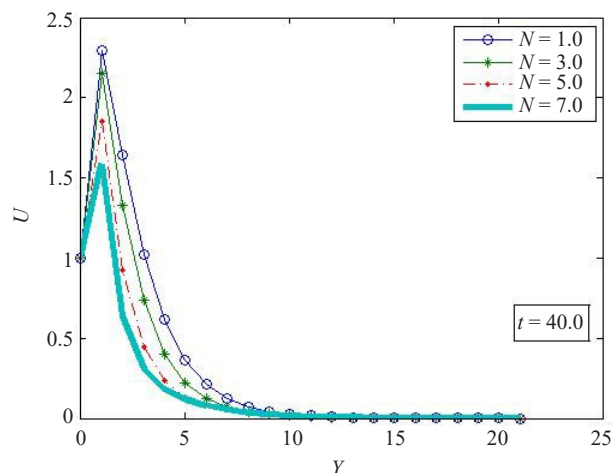
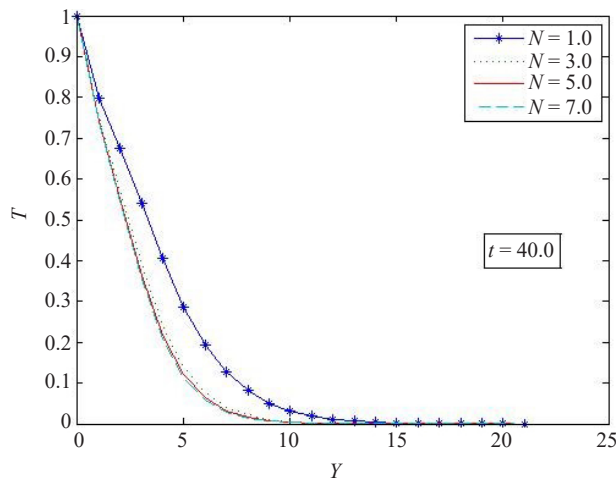


Figure 1. Velocity profile for various values of thermal radiation parameter  $N$

The thermal radiation parameter ( $N$ ) has an impact on the distribution of fluid velocity and temperature, which can be observed in Figures 1 and 2. When the value of  $N$  increases, significant changes occur in the velocity profile,

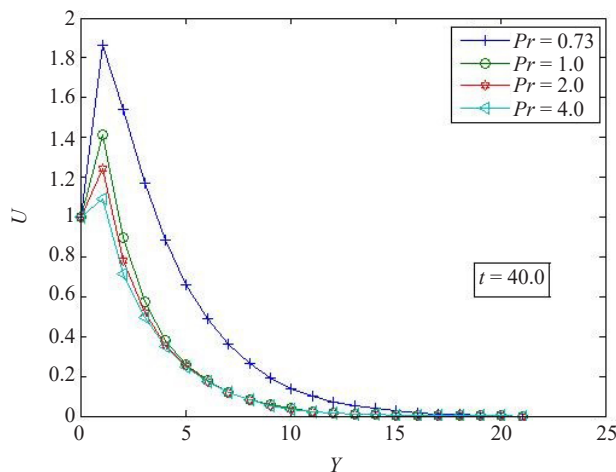
resulting in a decrease in the overall velocity and a reduction in the velocity layers present in the fluid. Additionally, the transmission of energy to the fluid is also diminished. As  $N$  becomes larger, the temperature profile along the stream decreases, indicating a decrease in the temperature of the fluid.



**Figure 2.** Temperature distribution for various values of thermal radiation parameter  $N$

The  $Pr$  has an effect on the distribution of velocity and temperature, which is illustrated in Figures 3 and 4. An increase in the value of  $Pr$  leads to a decrease in the thickness of the thermal boundary layer and the fluid velocity. This is because the  $Pr$  represents the ratio of the thickness of the viscosity boundary layer to the thickness of the thermal boundary layer. As the  $Pr$  increases, the thermal boundary layer becomes thinner, and the fluid velocity decreases.

Furthermore, the fluid's thermal conductivity increases as the  $Pr$  decreases. A lower  $Pr$  indicates that the fluid has a higher thermal conductivity, meaning it can conduct heat more effectively. Conversely, a higher  $Pr$  implies that the fluid has a lower thermal conductivity and heat transfer is less efficient. In this case, a greater amount of heat reaches zero more quickly, indicating a more rapid decrease in temperature.



**Figure 3.** Velocity distribution for various values of  $Pr$

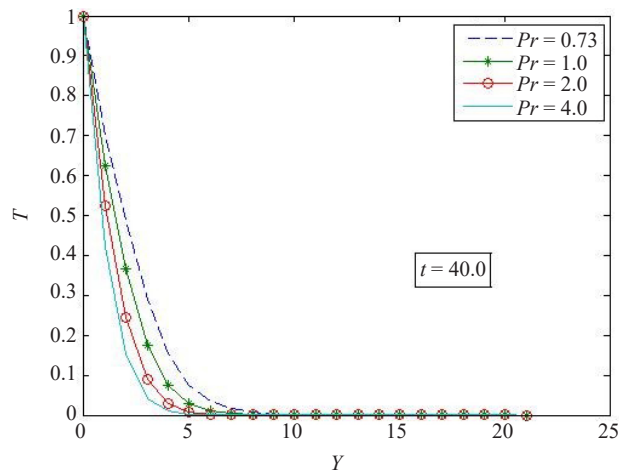


Figure 4. Temperature distribution for various values of  $Pr$

## 5. Conclusion

In conclusion, this study investigated the unsteady natural convection flow and heat transfer characteristics of a heated vertical plate. The governing equations were formulated, simplified, and transformed into non-dimensional form. By employing the Crank-Nicolson implicit finite difference method and utilizing the MATLAB programming tool, a steady-state solution was obtained at  $t = 80$ , indicating the attainment of a stable flow and temperature distribution.

The results of the study demonstrated that an increase in the  $N$  and the  $Pr$  led to changes in the velocity and temperature distributions. Specifically, higher values of  $N$  resulted in a significant decrease in the velocity profile, as well as a reduction in the velocity layers and energy transfer to the fluid. Moreover, larger  $N$  values caused a drop in the temperature profile along the flow direction. Similarly, increasing  $Pr$  values led to a decrease in the thermal boundary layer thickness and fluid velocity, as they represent the ratio of viscosity thickness to thermal boundary layer thickness. Additionally, a lower  $Pr$  indicated a higher thermal conductivity of the fluid.

These findings contribute to the understanding of convective heat transfer phenomena, particularly in the context of natural convection and thermal radiation influence. The study highlights the importance of considering thermal radiation effects in predicting fluid behavior and heat transfer processes in various scientific and technological applications. Further research can build upon these findings by exploring more complex geometries, considering additional parameters, and investigating the effects of other physical phenomena on the flow and heat transfer characteristics.

The future direction of this research lies in expanding the scope, validating the findings, exploring optimization strategies, considering multi-dimensional analyses, applying the research to practical systems, and utilizing advanced numerical techniques to further advance the understanding of fluid dynamics and heat transfer in the studied system.

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## Conflict of interest

There is no conflict of interest in this study.

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