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Research Article

A Mixed Neuro Graph Approach with Gradient Boosting to Hybrid Job-Shop Scheduling to Minimize a Regular Function of Job Completion Times and Numbers of Used Machines

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Abstract: The paper considers a multi-stage processing system including sets of identical (parallel) machines and a set of dedicated machines processing different operations of the given jobs in any sectors of economy. Based on the weighted Mixed Neuro graph model, the paper proposes adaptive algorithms for solving this problem via appropriate Mixed Neuro graph transformations. The main novelty is (1) low demands on the source data-unlike classical machine learning algorithms, the approach can offer stable interpretable results even with a short dataset size; (2) the number of new matrix multiplication operations that make up the main load when training models increases linearly with the number of new data from 0 to 999 time periods; (3) the results of the model are repeatable due to the stability of the coefficients of the model. These algorithms are able to solve (exactly or heuristically) the tested instances with N jobs and W types of parallel identical machines within on the personal computer. The gradient boosting result is in interval 5.9677410-3.4982093.

***Keywords*:** scheduling, flexible job-shop, regular objective function, adaptive algorithm

MSC: 62P20

1. Introduction

The paper describes a multi-stage processing system that comprises parallel machines and dedicated machines for processing specific operations of a given job. The parallel machines are identical and work in sets, while the dedicated machines are designed to handle different tasks. This system is useful for optimizing the processing of large and complex

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datasets, where different operations require different amounts of time and resources. By using parallel machines and dedicated machines, the system can efficiently allocate resources and minimize processing time. The paper likely provides insights on how to design and optimize such a system, including factors such as machine capacity, task allocation, and scheduling algorithms. This research could be useful for industries that require efficient and optimized processing of large datasets, such as manufacturing, logistics, and healthcare.

The objective function used in the paper is a regular, non-decreasing function that takes into account both the job completion times and the number of identical machines of different types used in the schedule. This objective function is likely used to optimize the multi-stage processing system described in the paper, by minimizing the completion times of individual jobs and maximizing the utilization of different machine types. By incorporating both factors into the objective function, the system can balance the trade-offs between completion time and machine usage, and find an optimal schedule that minimizes overall processing time. The paper may provide insights on how to formulate and solve such an objective function, using mathematical optimization techniques such as linear programming or dynamic programming. This research could be useful for industries that require efficient and optimized processing of large datasets, such as manufacturing, logistics, and healthcare.

The main novelty of the approach described in the paper is its ability to offer stable and interpretable results, even with a short dataset size. This is achieved by having low demands on the source data, which is in contrast to classical machine learning algorithms that typically require larger datasets to produce reliable results. By being able to produce stable and interpretable results with smaller datasets, the approach has potential implications for a wide range of applications, particularly in cases where data availability is limited or where the cost of data collection is high. This novelty could have significant implications for researchers in the field of machine learning, particularly in relation to optimizing training models and improving the accuracy and interpretability of their results. The paper may provide insights into how this novelty was achieved and how it can be applied to different types of datasets and problems.

The second novelty of the approach described in the paper is that the number of new matrix multiplication operations required to train the models increases linearly with the number of new data. This is unlike classical machine learning algorithms, where the number of operations required to train the models can increase exponentially with the size of the dataset. By having a linear relationship between the number of new matrix multiplication operations and the size of the dataset, the approach is computationally efficient and can handle larger datasets without requiring significantly more computational resources. This novelty could have significant implications for researchers in the field of machine learning, particularly in relation to optimizing training models and improving the scalability of their algorithms. The paper may provide insights into how this novelty was achieved and how it can be applied to different types of datasets and problems.

The third novelty of the approach described in the paper is that the results of the model are repeatable, due to the stability of the coefficients of the model. This means that the model produces consistent and reliable results, even when applied to different datasets or when trained with different parameters. By having stable coefficients, the approach can offer greater interpretability of the results and better generalization to new data. This is in contrast to classical machine learning algorithms, where the coefficients can be highly sensitive to small changes in the data or training parameters, leading to less reliable and less interpretable results. This novelty could have significant implications for researchers in the field of machine learning, particularly in relation to improving the interpretability and reliability of their algorithms. The paper may provide insights into how this novelty was achieved and how it can be applied to different types of datasets and problems.

Even for simple objective function $\Phi(c) = C_{\max}: = \max\{C_1, C_2, \ldots, C_n\}$ that aims to find a schedule of jobs with minimum length $C_{\max}$, where $c = (C_1, C_2, \ldots, C_n)$ for processing after its arriving time $r_i \geq 0$ [1]. Hereafter, three-field notations $\alpha|\beta|\gamma$ from [1] are used, where field α determines a set of machines, field β job characteristics, and field γ an objective function.

In this paper, the paper investigate the hybrid job-shop scheduling problem $HJ\,|r_i|\,\Phi(c, m)$ with generalized objective function $\Phi(c, m)$, $\cdot$ which is non-decreasing of the following arguments:- job completion times $c = (C_1, C_2, \ldots, C_n)$ and numbers m_k - The real-valued objective function is regular if it is non-decreasing of each argument. The main contribution of this paper can be summarized as follows: (1) the Mixed Neuro graph model is determined for the hybrid job-shop scheduling problem $HJ\,|r_i|\,\Phi(c, m)$ with generalized objective function $\Phi(c, m)$; (2) the adaptive algorithms are developed for solving the problem $HJ\,|r_i|\,\Phi(c,\,m)$ via appropriate Mixed Neuro graph transformations; (3) the developed software tested on $HJ\,|r_i|\,\Phi(c,\,m)$.

The proposed approach for addressing gaps in the scientific literature in relation to machine learning algorithms and short dataset sizes offers several advantages. Firstly, the approach requires low demands on the source data, meaning that stable and interpretable results can be produced even with a short dataset size. This is in contrast to classical machine learning algorithms, which may require larger datasets to produce reliable results. Secondly, the number of new matrix multiplication operations increases linearly with the number of new data, making it a computationally efficient approach. Finally, the stability of the coefficients in the model means that the results are repeatable and produce modified results for the reuse of the same data with greater plasticity in accordance with new conditions. These advantages can have significant implications for researchers in the field of machine learning, particularly in relation to optimizing training models and improving the accuracy and interpretability of their results.

First, the paper the proposed Mixed Neuro graph model for the problem $HJ\,|r_i|\,\Phi(c,m)$ in Section 2. The mathematic background of the Mixed Neuro graph approach to the problem $HJ\,|r_i|\,\Phi(c,\,m)$, alalysis, results and conclusion are presented in Sections 3-7.

2. Related work

The firefly algorithm exhibited significant enhancements over the original firefly algorithm and other metaheuristics in terms of convergence speed and results' quality. It achieved improvement in solving workflow scheduling in a cloud-edge environment by reducing makespan and cost compared to other approaches [1, 2].

The Genetic Operator and Quasi-Reflection-based Learning Firefly Algorithm showed the best performance in terms of makespan, cost, and combined objective in comparative analysis with other metaheuristics. It outperformed the original PSO (Particle Swarm Optimization) and established better results than the original FA (Firefly Algorithm) in terms of exploration power and exploitation procedure [3–5].

In [6–8] a hybrid bee colony algorithm was developed. A genetic algorithm with non-binary encoding was developed [9–15].

The number of published works for the problem $HJ\,|r_i|\,\Phi(c)$ comparing to those for the problem fJSP is smaller [16–18]. It is reported a study on the hybrid job-shop scheduling problem with blocking.

In [19] the algorithm developed in [20] used the scheduling procedure based on learning with two steps. In the first step, a part of the large problem $HJ\,|r_i|\,C_{\max}$ under consideration or several small problems were solved by an exact scheduling algorithm and some features are extracted from the solved instances. Then, an adaptive algorithm used the weighted dispatching rules based on the extracted features to resolve the conflicts between operations processed on the identical machines. In the algorithm proposed [21], optimal solutions were used to train a neural network algorithm. The algorithm provided a heuristic procedure for paralleling the operations on available machines. A fast heuristic algorithm for solving the problem $HJ\,|r_i|\,C_{\max}$ was developed [22]. This algorithm tries to find a proper sequence of the operations and paralleling the operations at the same time. The computational results showed that this algorithm is efficient for the problems $HJ\,|r_i|\,C_{\max}$ especially with the large numbers of identical machines. The authors of the works [23–25] have considered a hybrid job-shop scheduling problem with blocking constraint for train scheduling problem in a multi-track railway system.

The problem $HJ\,|r_i|\,\Phi(c)$ with arbitrary regular objective function was investigated [16–18], where Mixed Neuro graph models have been developed. To handle a large-dimensional problem, the heuristic decomposition algorithm was proposed, i.e., the initial problem $HJ\,|r_i|\,\Phi(c)$ was partitioned into similar sub-problems.

In this paper, the Mixed Neuro graph models used [20, 21] are generalized for the problem $HJ\,|r_i|\,\Phi(c,\,m)$ with arbitrary regular objective function $\Phi(c,\,m)$. In what follows, the monographs for the terminology have been used [22–25].

3. Problem settings and notations

There are given n jobs $\boldsymbol{J} = \{J_1, J_2, \ldots, J_n\}$ and $\boldsymbol{M} = \{M_1, M_2, \ldots, M_m\}$. The machine set $\boldsymbol{M}$ is partitioned into $\boldsymbol{w}$ subsets as follows: $\boldsymbol{M} = \boldsymbol{M}^1 \cup \cdot \boldsymbol{M}^2 \cup \ldots \cup \boldsymbol{M}^w$, where set $\boldsymbol{M}^k \neq \emptyset$ is a subset and $\boldsymbol{M}^k \cap \boldsymbol{M}^u = \emptyset with \{k, u\}$ of indexes with $1 \leq k \neq u \leq w$.

The following equalities hold: $\boldsymbol{M}^k = \{M_k\}$ and $w = m$:

$$c\left(v_{h-1}^i\right) \leq s\left(v_h^i\right) \tag{1}$$

In the problem $HJ\,|r_i|\,\Phi(c, m)$, the set $\boldsymbol{Q}$ of all given operations, $\boldsymbol{Q}\colon = \boldsymbol{O}^1 \cup \boldsymbol{O}^2 \cup \ldots \cup \boldsymbol{O}^n$, is partitioned into w subsets as follows: $\boldsymbol{Q} = O_1 \cup O_2 \cup \ldots \cup O_{\mathrm{w}}$, where sett $O_k \neq \emptyset$ is a subset.

The objective of the problem $HJ\,|r_i|\,\Phi(c, m) has O_k$ to the machines from the set $\boldsymbol{M}^k$ for all types $k \in \{1, 2, \ldots, w\}$ and to determine the completion time $c\left(v_h^i\right) \geq 0$ for each operation $v_h^i \in O_k$, $i \in \{1, 2, \ldots, n\}$, $h \in \{1, 2, \ldots, n(i)\}$, $k \in \{1, 2, \ldots, w\}$:

$$S = \left(c\left(v_1^1\right), c\left(v_2^1\right), \ldots, c\left(v_{n(1)}^1\right); c\left(v_1^2\right), c\left(v_2^2\right), \ldots, c\left(v_{n(2)}^2\right); \ldots; c\left(v_1^n\right), c\left(v_2^n\right), \ldots, c\left(v_{n(n)}^n\right)\right) \tag{2}$$

the objective function $\Phi(c, m)$ reaches a minimal possible value $\Phi(c(S), m(S))$ where $HJ\,|r_i|\,\Phi(c, m)$. Hereafter, $c(S)\colon = \left(c\left(v_{n(1)}^1\right), \quad c\left(v_{n(2)}^2\right), \ldots, c\left(v_{n(n)}^n\right)\right)$ and $m(S)\colon = (m_1(S), m_2(S), \ldots, m_w(S))$, where $m_k(S)$ denotes a number of the identical machines of the type $k \in \{1, 2, \ldots, w\}$, which are used in the schedule S determined by (2).

In the multi-stage processing system is like here:

$$\left|\boldsymbol{M}^1\right| = \left|\boldsymbol{M}^2\right| = \ldots = \left|\boldsymbol{M}^w\right| = 1 \tag{3}$$

since $\left|\boldsymbol{M}^k\right| = 1$ and so the assignment of the operations $\boldsymbol{Q} = O_1 \cup O_2 \cup \ldots \cup O_{\mathrm{w}}$ to the given machines $\boldsymbol{M} = \{M_1, M_2, \ldots, M_m\}$ with $m = w$ is uniquely determined.

4. A modelling

Lemma 1 A circuit-free digraph $G_g = (Q, A \cup A_g, \emptyset) \in \boldsymbol{W}(G)$.

It is easy convincing $J\,|r_i|\,\Phi(c) J\,|r_i|\,\Phi(c)$ based on this lemma.

Lemma 2 A digraph $G_g = (Q, A \cup A_g, \emptyset) \in \boldsymbol{W}^*(G)$.

Thus, Lemma 2 establishes that the digraph $G_g = (Q, A \cup A_g, \emptyset) \in \boldsymbol{W}^*(G)$ constructed for the problem $J\,|r_i|\,\Phi(c)$ determines a semi-active schedule $S(G_g)$ for the corresponding problem $HJ\,|r_i|\,\Phi(c, m)$. Of course, such a schedule $S(G_g)$ could be inefficient for the problem $HJ\,|r_i|\,\Phi(c, m)$ since some machines from the set $\boldsymbol{M}$ are not used in the schedule $S(G_g)$.

Let $\boldsymbol{W}^*\left(G\left(E^\delta\right)\right)$ denote a set of all circuit-free digraphs $G_d\left(E^\delta\right) = \left(Q, A \cup A_d^\delta, \emptyset\right)$ belonging to the set $\boldsymbol{W}\left(G\left(E^\delta\right)\right)$. Thus, in the circuit-free digraph $G_d\left(E^\delta\right) = \left(Q, A \cup A_d^\delta, \emptyset\right) \in \boldsymbol{W}^*\left(G\left(E^\delta\right)\right)$, each edge $\left[v_j^i, v_b^a\right] \in E^\delta \subset E$ of the Mixed Neuro graph $G = (Q, A, E)$ was either replaced by one of the two possible arcs (either arc $\left(v_j^i, v_b^a\right) \in A_d^\delta$ or arc $\left(v_b^a, v_j^i\right) \in A_d^\delta\right)$ or edge $\left[v_j^i, v_b^a\right] \in E^\delta$ was simply deleted from the Mixed Neuro graph $G = (Q, A, E)$.

Let edge $\left[v_j^i, v_b^a\right]$ be deleted from the Mixed Neuro graph $G = (Q, A, E)$ (i.e., the inclusion $\left[v_j^i, v_b^a\right] \in E \backslash E^\delta$ holds). Then, operations $v_j^i \in Q and v_b^a \in Q$ have to be processed on different machines from the set $\boldsymbol{M}^k$, where $k = w(i, j) = w(a, b)$, in the schedule $S(G(E^{\delta}))$ determined by the digraph $G_d\left(E^{\delta}\right) = \left(Q, A \cup A_d^{\delta}, \emptyset\right) \in \boldsymbol{W}^*\left(G\left(E^{\delta}\right)\right)$. It is clear that this is an assignment stage for solving the problem $HJ\,|r_i|\,\Phi(c, m)$.

Let edge $\left[v_j^i, v_b^a\right]$ be replaced by the arc $\left(v_j^i, v_b^a\right) \in A_d^{\delta}$. Then, operation v_j^i is from the set M^k, where $k = w(i, j) = w(a, b)$, in the schedule $S(G(E^{\delta})$. determined by the digraph $G_d\left(E^{\delta}\right) = \left(Q, A \cup A_d^{\delta}, \emptyset\right) \in W^*\left(G\left(E^{\delta}\right)\right)$. Let edge $\left[v_j^i, v_b^a\right]$ be replaced by arc $\left(v_b^a, v_j^i\right) \in A_d^{\delta}$.

The transformation of the Mixed Neuro graph G=(Q, A, E) into the digraph is that $G_d\left(E^{\delta}\right) = \left(Q, A \cup A_d^{\delta}, \emptyset\right) \in \boldsymbol{W}^*\left(G\left(E^{\delta}\right)\right)$ while solving the problem $HJ\,|r_i|\,\Phi(c, m)$.

Theorem 1 Let set E^{δ} be a subset of the edges in the Mixed Neuro graph $G = (Q, A, E)$; $E^{\delta} \subseteq E$. Then, the digraph $G_d\left(E^{\delta}\right) = \left(Q, A \cup A_d^{\delta}, \emptyset\right) \in \boldsymbol{W}^*\left(G\left(E^{\delta}\right)\right)$.

Proof. There exists a circuit $\mu = \left(v_b^a, v_e^c, v_f^g, \ldots, v_y^x, v_b^a\right)$ in the digraph $G_d\left(E^{\delta}\right)$. The inequalities imply the following contradiction: $c\left(v_b^a\right) < c\left(v_b^a\right)$. □

The digraph $G_d\left(E^{\delta}\right) = \left(Q, A \cup A_d^{\delta}, \emptyset\right)$ determines it.

5. Sequencing them for solving the problem

In most algorithms published for the problem $HJ\,|r_i|\,\Phi(c)$ with the objective function $\Phi(c) \in \{C_{\max}, \sum C_i\}$[], an assignment the operations O_k to the machines $\boldsymbol{M}^k$ is realized and where both stages are realized simultaneously. It is demonstrated that using the combinatorial structure of the problem $HJ\,|r_i|\,\Phi(c, m)$, an operation assignment may be realized optimally, if a sequence of the operations O_k is already determined.

The digraph $G_d\left(E^{\delta}\right) = \left(Q, A \cup A_d^{\delta}, \emptyset\right) \in \boldsymbol{W}^*\left(G\left(E^{\delta}\right)\right)$ with $E^{\delta} \subseteq E$ may be constructed from the digraph $G_g = (Q, A \cup A_g, \emptyset) \in \boldsymbol{W}^*(G)$.

The circuit-free digraph $G_d\left(E^{\delta}\right) = \left(Q, A \cup A_d^{\delta}, \emptyset\right) \in \boldsymbol{W}^*\left(G\left(E^{\delta}\right)\right)$ is transformed into the digraph $G_d\left(E^{\delta}, A_r(\alpha)\right) = \left(Q, A \cup A_d^{\delta} \backslash A_r(\alpha), \emptyset\right)$.

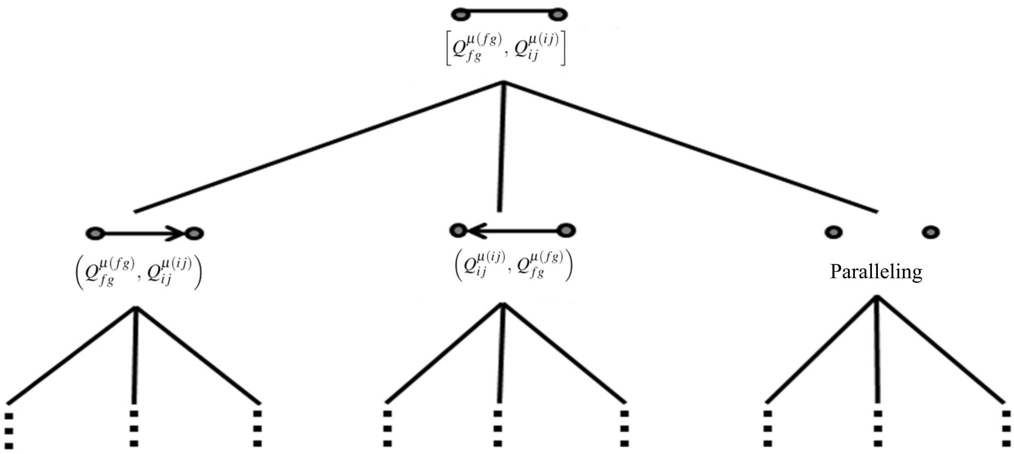

Figure 1. Neuro net with $G = (Q, A, E)$

6. Gradient boosting

The next step is a gradient boosting. The compliance with the technological norm is determined as a performer with the appropriate or higher qualifications and it is assigned for each job:

$$\sum k \in K(j)xj, k + gj = 1 \forall j \in J \tag{4}$$

One active performer is used for each task:

$$\sum k \in Kxj, k \leq 1 \forall j \in J \tag{5}$$

Each performer has enough resources for the work:

$$\sum j \in Jpj \cdot xj, k + \sum i \in L \sum j \in L\tau i, j \cdot yi, j, k \leq Wk \cdot uk \forall k \in K \tag{6}$$

All assigned resources and the contractor are available at the place of work:

$$\sum j \in Lyi, j, k = xi, k \forall i \in J, k \in K \tag{7}$$

All resources that required relocation can be returned to the original center:

$$\sum j \in Jyd, j, k = \beta k, d \cdot uk \forall k \in K, d \in D \tag{8}$$

Time relations is the sequence of operations performed by one executor assumes that the beginning of the next operation does not occur before the end of the previous one:

$$tj \geq ti + pi + \tau i, j - M \cdot \left(1 - \sum k \in Kyi, j, k\right) \forall i \in L, j \in J \tag{9}$$

Each process must fit within the time limits of the constraints:

$$tj \geq aj - xaj \forall j \in J \tag{10}$$

$$tj \leq bj + xbj \forall j \in J \tag{11}$$

The dataset has from 0 to 999 time periods; (3) the results of the model are repeatable due to the stability of the coefficients of the model. These algorithms are able to solve (exactly or heuristically) the tested instances with N jobs and W types of parallel identical machines within on the personal computer. The gradient boosting result is in interval 5.9677410-3.4982093 (Table 1).

Table 1. Gradient boosting statistics

Lenth	Best result	Best time	Time
0	5.9677410	365	365
250	4.7999958	72	215
500	4.1388537	122	121
750	3.7510570	173	574
999	3.4982093	220	0

After these results are compared with the Genetic Operator and Quasi-Reflection-based Learning Firefly Algorithm result and the best Benchmark results (Firefly algorithm).

The benchmark result is in interval 5.9677410-3.4982093 (Table 2).

Table 2. Benchmark statistics (Firefly algorithm)

Lenth	Best result	Best time	Time
0	6.8763569	656	765
250	7.8767757	232	315
500	4.7656546	456	621
750	4.9391765	876	987
999	4.7198876	974	0

The Genetic Operator and Quasi-Reflection-based Learning Firefly Algorithm result is in interval 5.9677410-3.4982093 (Table 3).

Table 3. Genetic Operator and Quasi-Reflection-based Learning Firefly Algorithm statistics

Lenth	Best result	Best time	Time
0	6.8763569	656	765
250	7.8767757	232	315
500	4.7656546	456	621
750	4.9391765	876	987
999	4.7198876	974	0

6.1 *Discussion and conclusion*

The conditional optimization of matrices is that determine the expenditure of resources under the matrices of input conditions by gradient boosting a set of linear and nonlinear models.

The developed hybrid model outperformed other models (Tables 1-4). The developed model showed comparable performance to other hybrid wind speed estimation studies in the literature [26].

For comparison, other algorithms have been selected that are suitable for solving the task and are quite popular in the scientific community [20, 21]. So, the following algorithms were chosen for comparison: neural networks, random forests, linear optimization models, support vector machine. To compare algorithms, parameters of efficiency, speed of work, resources and robustness are used (Table 4).

Table 4. Genetic Operator and Quasi-Reflection-based Learning Firefly Algorithm statistics

Algorithms	Efficiency	Speed	Resources	Robustness
Gradient boosting	Combine linear and non-linear approaches, but has flexibility limitations	Fast develop, train, use	Memory & data efficient	Robust
Genetic Operator and Quasi-Reflection-based Learning Firefly Algorithm	Most flexible model	Slow develop, train, use	Resource inefficient	Non- Robust
Firefly algorithm	Best fits discrete data	Slow develop, fast train & use	Memory efficient	Non- Robust

When comparing machine learning algorithms, it is important to consider the specific application and dataset, as well as the strengths and limitations of each algorithm. Gradient boosting of a set of linear and nonlinear models is a powerful algorithm capable of handling complex data structures and interactions, but may require more computational resources and can be prone to overfitting. Neural networks are highly flexible and can handle large and complex datasets, but may require significant amounts of data and can be challenging to interpret. Random Forests are a popular algorithm for classification and regression tasks, providing good accuracy and interpretability, while also being relatively fast and scalable. Linear models are simple and efficient and can provide good interpretability, but may not be suitable for more complex datasets or non-linear relationships. Support Vector Machines are powerful for separating complex data sets with a clear margin, but may require more computational resources and can be less interpretable.

Conclusions are supported by empirical data (Tables 1-4) which was created with practical experiments. So, it is often useful to compare the performance of multiple algorithms on the same dataset to determine which is the most appropriate for a given problem. The main achievements of the proposed approach close gaps in the scientific literature and provide the following advantages: (1) low demands on the source data-unlike classical machine learning algorithms, the approach can offer stable interpretable results even with a short dataset size; (2) the number of new matrix multiplication operations that make up the main load when training models increases linearly with the number of new data from 0 to 999 time periods; (3) the results of the model are repeatable due to the stability of the coefficients of the model. These algorithms are able to solve (exactly or heuristically) the tested instances with N jobs and W types of parallel identical machines within on the personal computer. The gradient boosting result is in interval 5.9677410-3.4982093.

Results can be implemented in industries, energetic, computer technologies and other fields, where there are planning and scheduling problems.

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Conflict of interest

The authors declare no competing financial interest.

References

[1] Graham RL, Lawler EL, Lenstra JK, Kan AR. Optimization and approximation in deterministic sequencing and scheduling: A survey. *Annals of Discrete Mathematics*. 1979; 5: 287-326.

[2] Chiang CC, Lin HJ. A simple and effective evolutionary algorithm for multi objective flexible job-shop scheduling. *International Journal of Production Economics*. 2012; 141(1): 87-98.

[3] Gholami O, Sotskov YN. Solving parallel machines job-shop scheduling problems by an adaptive algorithm. *International Journal of Production Research*. 2014; 52(13): 3888-3904.

[4] Al-Hinai N, El Mekkawy TY. Robust and stable flexible job shop scheduling with random machine breakdowns using a hybrid genetic algorithm. *International Journal of Production Economics*. 2011; 132(2): 279-291.

[5] Paulli J. A hierarchical approach for the FMS scheduling problem. *European Journal of Operational Research*. 1995; 86: 33-42.

[6] Zhang G, Gao L, Shi Y. An effective genetic algorithm for the flexible job-shop scheduling problem. *Expert Systems with Applications*. 2011; 38: 3563-3573.

[7] Sauvey C, Trabelsi W. Hybrid job shop scheduling with mixed blocking constraints between operations. In *Proceeding of IEEE international Conference on Emerging Technologies and Factory Automatic (ETFA 2015)*, Luxembourg; 2015. p.1-8.

[8] Rossi A, Boschi E. A hybrid heuristic to solve the parallel machines job-shop scheduling problem. *Advances in Engineering Software*. 2009; 40(2): 118-127.

[9] Liu SQ, Kozan E. Scheduling trains as a blocking parallel-machine job shop scheduling problem. *Computers & Operations Research*. 2009; 36: 2840-2852.

[10] Liu SQ, Kozan E. Parallel-identical-machine job-shop scheduling with different stage-dependent buffering requirements. *Computers & Operations Research*. 2016; 74: 31-41.

[11] Thakur A, Goraya MS. RAFL: A hybrid metaheuristic based resource allocation framework for load balancing in cloud computing environment. *Simulation Modelling Practice and Theory*. 2022; 116: 102485.

[12] Özçelik YB, Altan A. Overcoming nonlinear dynamics in diabetic retinopathy classification: A robust AI-based model with chaotic swarm intelligence optimization and recurrent long short-term memory. *Fractal and Fractional*. 2023; 7(8): 598.

[13] An J, Mikhaylov A, Chang T. Relationship between the popularity of a platform and the price of NFT assets. *Finance Research Letters.* 2024; 61(3): 105057. Available from: https://doi.org/10.1016/j.frl.2024.105057.

[14] Mikhaylov A. Understanding the risks associated with wallets, depository services, trading, lending, and borrowing in the crypto space. *Journal of Infrastructure Policy and Development.* 2023; 7(3): 2223. Available from: https://doi.org/10.24294/jipd.v7i3.2223.

[15] An J, Mikhaylov A, Jung S-U. The strategy of South Korea in the global oil market. *Energies*. 2020; 13(10): 2491. Available from: https://doi.org/10.3390/en13102491.

[16] Candila V, Maximov D, Mikhaylov A, Moiseev N, Senjyu T, Tryndina N. On the relationship between oil and exchange rates of oil-exporting and oil-importing countries: From the great recession period to the COVID-19 era. *Energies*. 2001; 14: 8046. Available from: https://doi.org/10.3390/en14238046.

[17] Saqib A, Chan TH, Mikhaylov A, Lean HH. Are the responses of sectoral energy imports asymmetric to exchange rate volatilities in Pakistan? Evidence from recent foreign exchange regime. *Frontiers in Energy Research*. 2021; 9: 614463. Available from: https://doi.org/10.3389/fenrg.2021.614463.

[18] An J, Mikhaylov A, Jung S-U. The strategy of South Korea in the global oil market. *Energies*. 2020; 13(10): 2491. Available from: https://doi.org/10.3390/en13102491.

[19] Mikhaylov A, Dinçer H, Yüksel S, Pinter G, Shaikh ZA. Bitcoin mempool growth and trading volumes: Integrated approach based on QROF Multi-SWARA and aggregation operators. *Journal of Innovation & Knowledge*. 2023; 8(3): 100378. Available from: https://doi.org/10.1016/j.jik.2023.100378.

[20] Mikhaylov A, Dinçer H, Yüksel S. Analysis of financial development and open innovation oriented fintech potential for emerging economies using an integrated decision-making approach of MF-X-DMA and golden cut bipolar q-ROFSs. *Financial Innovation*. 2023; 9(1): 4. Available from: https://doi.org/10.1186/s40854-022-00399-6.

[21] Karlibaeva R, Yousif NBA, Stepanova D. Current analysis of cryptocurrency mining industry. *Journal of Infrastructure Policy and Development.* 2024; 8(5): 4803. Available from: https://doi.org/10.24294/jipd.v8i5.4803.

[22] An J, Mikhaylov A. Russian energy projects in South Africa. *Journal of Energy in Southern Africa*. 2020; 31(3): 58-64. Available from: http://dx.doi.org/10.17159/2413-3051/2020/v31i3a7809.

[23] Mutalimov V, Kovaleva I, Mikhaylov A, Stepanova D. Assessing regional growth of small business in Russia. *Entrepreneurial Business and Economics Review*. 2021; 9(3): 119-133. Available from: https://doi.org/10.15678/EBER.2021.090308.

[24] Yumashev A, Mikhaylov A. Development of polymer film coatings with high adhesion to steel alloys and high wear resistance. *Polymer Composites.* 2020; 41(7): 2875-2880. Available from: https://doi.org/10.1002/pc.25583.

[25] Mikhaylov A. Development of friedrich von hayek's theory of private money and economic implications for digital currencies. *Terra Economicus*. 2021; 19(1): 53-62. Available from: https://doi.org/10.18522/2073-6606-2021-19-1-53-62.

[26] Mikhaylov A, Bhatti IM, Dinçer H, Yüksel S. Integrated decision recommendation system using iteration-enhanced collaborative filtering, golden cut bipolar for analyzing the risk-based oil market spillovers. *Computational Economics*. 2024; 63: 305-338. Available from: https://doi.org/10.1007/s10614-022-10341-8.