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Research Article

A CRITIC-TOPSIS MCDM Technique under the Neutrosophic Environment with Application on Aircraft Selection

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Abstract: This paper develops a CRITIC-TOPSIS "multi-criteria decision-making" method under a neutrosophic environment. The neutrosophic set is a generalized version of the fuzzy and "intuitionistic fuzzy set". The purpose of this paper is the extension of the CRITIC-TOPSIS method using neutrosophic sets to create a more accurate and efficient decision support system for selecting a very light business jet (Bizjet). It provides the ranking to the selected number of Bizjets based on some pre-defined criteria under the neutrosophic CRITIC-TOPSIS method. The results of this study are obtained from the extended version of the previous study on decision-making for the selection of Bizjets. This study obtains results under the neutrosophic environment with a more realistic scenario.

***Keywords*:** Neutrosophic set, CRITIC, TOPSIS, Multi-criteria decision-making

MSC: 90B50, 03E72

1. Introduction

Early in the 1950s, the first business jets were created and flown. Over the past few decades, the demand for bizjets increased, and this industry has expanded a lot. Moreover, small business aircraft is used mainly by government officials, Celebrities, and entrepreneurs to cover many destinations in a single day along with their luxurious features. Nowadays, there is a lot of competition in various airplane manufacturing companies to fulfill the rising demand for small business jets. As a result, designers and manufacturers are constantly improving a few factors, such as aerodynamic features, fuel efficiency, time savings by reducing flight duration, and comfortable surroundings. All these factors are directly or indirectly related to aircraft performance, ultimately enhancing the overall experience of passengers and also improving operational efficiency [1-2]. MCDM methods provide decision-makers with tools and techniques to assess and compare alternatives based on different criteria, taking into account the preferences and priorities of the decision-maker. In multi-criteria decision-making (MCDM), the CRITIC method is a scientifically supported method for calculating the relative weights or importance of criteria based on their correlations. It makes use of statistical methods to examine the correlation matrix, which represents the relationships between pairs of criteria. The

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CRITIC method determines the criteria weights through calculations by assessing the magnitude and direction of these correlations. By incorporating statistical analysis, the CRITIC method provides decision-makers with a quantitative measure of criterion importance, reducing the potential for subjective biases. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a widely employed method within the field of multi-criteria decision-making (MCDM) [3-4]. It offers a scientific approach for evaluating and ranking alternatives based on their proximity to an ideal solution. TOPSIS incorporates both positive and negative ideal solutions to assess the relative performance of alternatives across multiple criteria [5-6]. The motivation for this study lies in the limitations of existing approaches. Many research works have been carried out regarding TOPSIS with crisp, fuzzy, and intuitionistic fuzzy data, whereas TOPSIS with a neutrosophic environment was rarely initiated. In real-life situations, uncertainty is unavoidable, and it is difficult for crisp set theory to handle vagueness and uncertainty. The problem of ambiguous and imprecise information present in real-world decision-making problems can be resolved by using fuzzy TOPSIS [7-9]. Neutrosophic TOPSIS is a valuable tool for decision-makers to manage uncertainties and imprecisions inherent in neutrosophic information effectively. This method systematically incorporates neutrosophic sets and employs suitable distance measures to establish a decision-making framework specifically designed to address situations involving indeterminacy and contradiction. Neutrosophic TOPSIS enables decision-makers to analyze and evaluate alternatives by considering the unique characteristics of neutrosophic information, ensuring a comprehensive and accurate decision-making process. Our paper proposes a framework for evaluating and recommending the best alternative by utilizing the CRITIC-TOPSIS approach in a single-valued neutrosophic environment. The main objectives of this paper can be summarized as follows:

• A framework is proposed that provides the best choice of the bizjet from a list of some predefined alternatives using the neutrosophic MCDM approach.

• A systematic approach is proposed for selecting the essential features and for converting the crisp data into the single-valued neutrosophic set.

• Neutrosophic TOPSIS for performance evaluation and ranking is used to combat vagueness, indeterminacy, inconsistency, and subjectivity in decision-making.

• A case study using the data of aircraft is presented in which objective weights of criteria are calculated under the CRITIC method and ranking of Bizjets is done using the TOPSIS method using neutrosophy.

2. Literature review

Several kinds of research have been done in the last decade to incorporate the ambiguity and vagueness of the initial information into multi-criteria decision-making (MCDM) methods for the solution of complex nature practical problems [10-12]. Fuzzy set theory is the most important tool used to solve decision-making problems that involve uncertainty and vagueness. Zadeh [13] introduced fuzzy set as a class of objects with a continuum of grades of membership. Bellman and Zadeh [14] has used fuzzy set in decision-making problems in 1970. Using fuzzy sets, they managed multistage decision processes where either stochastic or deterministic systems are controlled.

Atanassov [15-16] introduced the degree of non-membership/falsehood and defined the intuitionistic fuzzy set. Daneshvar et al. [17] proposed the intuitionistic fuzzy TOPSIS method for the green supplier selection problem. Beskese et al. [18] has done evaluation of wind turbine using the hesitant fuzzy AHP-TOPSIS method. They demonstrate the proposed method through a case study in Turkey. Zeng et al. [19] proposed the new intuitionistic fuzzy aggregation method for an assessment of digital reforms of the manufacturing industry in China. Akram et al. [20] suggested two novel modified techniques, the Pythagorean fuzzy hybrid ELimination and Choice Translating REality *I* (PFH-ELECTRE I) method and the Pythagorean fuzzy hybrid Order of Preference by Similarity to an Ideal Solution (PFH-TOPSIS) method, to measure risk rankings in failure modes and effects analysis (FMEA). These approaches are intended to address the drawbacks and limitations of conventional crisp risk priority numbers and fuzzy FMEA procedures in risk rankings. Moreover, in recant years many researchers [21-24] have used intuitionistic fuzzy sets to solve several ambiguous and uncertainty-based MCDM problems. In 1998, Smarandache [25] introduced the degree of indeterminacy as an independent component and defined the neutrosophic set on three components (T, I, F) = (truth, indeterminacy, falsehood). Smarandache [26] in 2005 explained that neutrosophic sets are the generalization of intuitionistic fuzzy sets. He has discussed many examples to show the difference between neutrosophic sets and intuitionistic fuzzy sets. Nie et al. [27] solved an MCDM problem of solar-wind power station location using an extended weighted aggregated sum product assessment (WASPAS) technique with interval neutrosophic sets. Rani et al. [28] proposed single valued "neutrosophic-CRITIC-MULTIMOORA" model to resolve the problem of selection of multi-criteria food waste treatment method. In recent research, many researchers have applied the concept of Neutrosophic sets and their properties to resolve some multi-criteria decision-making (MCDM) based problems [29-35].

3. Notations and abbreviations

The various notations used in this study are described in Table 1.

Table 1. Notations and abbreviations

Notation	abbreviation
F_s	Fuzzy set
I_s	Intuitionistic fuzzy set
N_u	Neutrosophic set
N_{S_V}	Single valued neutrosophic set
$\in$, $u \in U$	belongs to
$\notin$, $u \notin U$	not belongs to
$X - Y$	set-theoretic difference of sets
$X \subseteq Y$	set inclusion
$X \subsetneq Y$	proper set inclusion
$X \cup Y$	union of two sets A and B
$X \cap Y$	intersection of two sets A and B
$X \oplus Y$	addition of two sets
$X \otimes Y$	multiplication of two sets
$f : X \to Y$	function (or mapping) with domain X and range in Y
φ_{F_s}	membership function of fuzzy set A
μ_{I_s}	membership function of the intuitionistic fuzzy set F
v_{I_s}	non-membership function of the intuitionistic fuzzy set F
π_{I_s}	hesitation function of the intuitionistic fuzzy set F
T_{N_u}	membership function of neutrosophic set N
I_{N_u}	indeterminacy function of neutrosophic set N

Table 1. (cont.)

Notation	abbreviation
F_{N_u}	non-membership function of neutrosophic set N
$D_N(X, Y)$	separation measure between X and Y
$\rho(A, B)$	correlation coefficient of two sets A and B
σ	standard deviation
$C_N(A, B)$	correlation number of two neutrosophic sets A and B
$V(A)$	informational energy of set A
$\bar{n}_{ij}$	normalized decision value
CC_i	relative closeness coefficient

4. Preliminaries

In this section, we have presented an overview of MCDM, fuzzy set, Intuitionistic fuzzy set, and single-valued neutrosophic sets and related definitions.

4.1 *Multi-criteria decision-making(MCDM)*

Multi-criteria decision-making technique is a handy utility of operations research that is used to provide ranking to a set of alternatives based on some pre-defined criteria [36-37]. Most of the real-life based problems are based on vague and imprecise information which gives rise to conflicting criteria. Fuzzy MCDM helps decision makers to resolve such types of problems and find optimal solutions under fuzzy environment [38]. Many researchers have applied general MCDM and Fuzzy MCDM to resolve decision-making problems [39-40].

4.2 *Fuzzy set*

The fuzzy set theory is the most appropriate tool used by researchers for solving problems in MCDM where the information is vague and imprecise. Fuzzy sets can effectively represent uncertain and imprecise parameters and can be handled through different operations on fuzzy numbers. Since uncertain parameters are treated as imprecise values instead of precise ones, the process will be stronger and the results will be more creditable. The concept of the fuzzy set was introduced by [13] in 1965. He defined a fuzzy set as a class of objects with grades of membership. One way to identify such a set is by its membership function, which gives each object a membership grade between zero and one. Mathematically, the Fuzzy set is defined as

Definition 1. Fuzzy set [13]. Let S be the universal non-empty set. The Fuzzy set F_s in S is defined as

$$F_s = \{\langle s, \varphi_{F_s}(s)\rangle \mid s \in S\} \tag{1}$$

Whereas, the membership function

$$\varphi_{F_s} : S \to [0, 1] \tag{2}$$

signifying the degree of association with each element $s \in S$ with condition

$$0 \le \varphi_{F_s} \le 1 \tag{3}$$

4.3 *Intuitionistic fuzzy set*

In a fuzzy set, every element is associated with membership and non-membership functions. There is always a relation between the degree of non-membership and the degree of membership. Non-membership functions have degrees equal to one minus membership degrees. But in real situations, it is not always true. For example, in some particular location, people want to elect one leader, so the degree of membership will lie between 0 and 1 for those persons who will elect, but for those who will not elect their non-membership value is not equal to one minus membership value. This is because there can be few members who will give invalid voting or who don't want to vote at all.

The intuitionistic fuzzy set was introduced by [15] in 1986 as a way to address the limitations of traditional fuzzy sets in capturing uncertainty. These sets are the generalization of the fuzzy sets. A fuzzy set is associated with membership function only whereas in an intuitionistic fuzzy set each element has a pair of membership and non-membership functions along with a hesitation function. The intuitionistic fuzzy sets are more efficient in dealing with ambiguity and uncertainty. In recent research, intuitionistic fuzzy set theory has been utilized effectively to resolve some MCDM problem-based applications.

Definition 2. Intuitionistic Fuzzy set (IFS) [16]. Let S be the universal non-empty set. The Intuitionistic Fuzzy set I_s in S is defined as

$$I_s = \left\{ \langle x, \mu_{I_s}(x), \nu_{I_s}(x) \rangle \mid x \in S \right\} \tag{4}$$

such that

$$\mu_{I_s} : S \to [0, 1] \text{ and } \nu_{I_s} : S \to [0, 1] \tag{5}$$

are the membership function and non-membership function respectively of the element x which satisfy the condition

$$0 \le \mu_{I_s} + \nu_{I_s} \le 1 \tag{6}$$

The function

$$\pi_{I_s}(x) = 1 - \mu_{I_s}(x) - \nu_{I_s}(x) \tag{7}$$

is called the hesitant function [41] of the element x.

4.4 *Neutrosophic set*

Neutrosophic Fuzzy Sets are a kind of fuzzy set that incorporates the concept of neutrosophy and that deals with indeterminacy, ambiguity, and uncertainty. F Smarandache, provides a framework for handling complex decision-making problems in which uncertainty, ambiguity, and incomplete information are involved [25, 42]. Neither fuzzy nor intuitionistic fuzzy sets can deal with indeterminate or inconsistent data properly, but a "neutrosophic set" can deal with such type of information.

Definition 3. Neutrosophic set [25]. Let S be the universe of the elements, which is an infinite non-empty set. A neutrosophic set N_u in S is defined as

$$N_u = \{\langle s, T_{N_u}(s), I_{N_u}(s), F_{N_u}(s)\rangle \mid s \in S\} \tag{8}$$

where

T_{N_u} : membership/truth function

I_{N_u} : indeterminacy/neutral/unknown function

F_{N_u} : non-membership/falsity

and

$$T_{N_u}(s): S \to]0^-, 1^+[, I_{N_u}(s): S \to]0^-, 1^+[, \text{ and } F_{N_u}(s): S \to]0^-, 1^+[$$

are defined on non-standard subsets of $]0^-, 1^+[$. Also,

$$0^- \leq sup(T_{N_u}(s)) + sup(I_{N_u}(s)) + sup(F_{N_u}(s)) \leq 3^+ \tag{9}$$

here is no restriction on the sum of T_{N_u}(s), I_{N_u}(s), and F_{N_u}(s) in neutrosophic set.

Definition 4. Single-valued neutrosophic set [43]. Let S be a universe of the elements and $s \in S$. A Single-valued neutrosophic set N_{S_V} in S is defined as

$$N_{S_V} = \{\langle s, T_{N_{S_V}}(s), I_{N_{S_V}}(s), F_{N_{S_V}}(s)\rangle \mid s \in S\} \tag{10}$$

where

$T_{N_{S_V}}(s)$: membership function of single-valued neutrosophic set

$I_{N_{S_V}}$(s): indeterminacy of single-valued neutrosophic set

$F_{N_{S_V}}$(s): falsity or non-membership of Single-valued neutrosophic set

and

$$T_{N_{S_V}}(s): S \to [0, 1], I_{N_{S_V}}(s): S \to [0, 1], \text{ and } F_{N_{S_V}}(s): S \to [0, 1]$$

Also

$$0 \leq sup(T_{N_{S_V}}(s)) + sup(I_{N_{S_V}}(s)) + sup(F_{N_{S_V}}(s)) \leq 3. \tag{11}$$

Basically when S comprises single element, it is known as "Single-valued neutrosophic number".

For Simplification,

$$N_{S_V} = \{\langle s, T_{N_{S_V}}(s), I_{N_{S_V}}(s), F_{N_{S_V}}(s)\rangle \mid s \in S\}$$

can be written as

$$N_{S_V} = (u, T_{N_{S_V}}(s), I_{N_{S_V}}(s), F_{N_{S_V}}(s)) \tag{12}$$

4.5 *Set operations on Single-valued neutrosophic set*

1. Let N_X and N_Y be two Single-valued neutrosophic sets in an infinite non-empty set U, then

a. $N_X = N_Y \Leftrightarrow T_{N_X}(s) = T_{N_Y}(s), I_{N_X}(s) = I_{N_Y}(s), F_{N_X}(s) = F_{N_Y}(s)$.

b. $N_X \subseteq N_Y \Leftrightarrow T_{N_X}(s) \le T_{N_Y}(s), I_{N_X}(s) \le I_{N_Y}(s), F_{N_X}(s) \ge F_{N_Y}(s)$.

c. $(N_X)^c = \{\langle s, F_{N_X}(s), 1 - I_{N_X}(s), T_{N_X}(s)\rangle \mid s \in S\}$

d. $N_X \setminus N_Y = \{\langle s, T_{N_X}(s) \wedge F_{N_Y}(s), (I_{N_X}(s) \wedge (1 - I_{N_Y}(s)), F_{N_X}(s) \vee F_{N_Y}(s)\rangle \mid s \in S\}$

e. $N_X \cap N_Y = \{\langle s, T(N_X)(s) \wedge F_{N_Y}(s), I(N_X)(s) \wedge (I_{N_Y}(s)), (F(N_X)(s)) \wedge F_{N_Y}(s)\rangle \mid s \in S\}$

f. $N_X \cup N_Y = \{\langle s, T_{N_X}(s) \vee F_{N_Y}(s), I_{N_X}(s) \vee (I_{N_Y}(s)), (F_{N_X}(s)) \vee F_{N_Y}(s)\rangle \mid s \in S\}$

where $\vee$ denotes maximum value and $\wedge$ denotes minimum value.

2. Suppose N_X, N_Y, N_Z, and N_W are four single neutrosophic sets in an infinite non-empty set S, then

i. If $N_X \subseteq N_Y$ and $N_Y \subseteq N_Z$ then $N_X \subseteq N_Z$.

ii. If $N_X \subseteq N_Y$ then $N_X^C \subseteq N_Y^C$.

iii. If $N_X \subseteq N_Y$ and $N_Z \subseteq N_Y$ then $N_X \cup N_Z \subseteq N_Y$.

iv. If $N_X \subseteq N_Y$ and $N_X \subseteq N_Z$ then $N_X \subseteq N_Y \cap N_Z$.

v. If $N_X \subseteq N_Y$ and $N_Z \subseteq N_W$ then $N_X \cap N_Z \subseteq N_Y \cap N_W$.

Definition 5. Let S be the universe of elements and $s \in S$. A "Single-valued neutrosophic set" N_S in S is said to be a universe neutrosophic set if

$$T_{N_S}(s) = I_{N_S}(s) = 1, F_{N_S}(s) = 0 \quad \forall s \in S \tag{13}$$

Definition 6. Let S be the universe of elements and $s \in S$. Let $N_{S_1} = (T_{S_1}, I_{S_1}, F_{S_1})$ and $N_{S_2} = (T_{S_2}, I_{S_2}, F_{S_2})$ be two "Single-valued neutrosophic numbers", then the sum of N_{S_1} and N_{S_2} can be written as

$$N_{S_1} \oplus N_{S_2} = (T_{S_1}(s) + T_{S_2}(s) - T_{S_1}(s)T_{S_2}(s), I_{S_1}(s)I_{S_2}(s), F_{S_1}(s)F_{S_2}(s)) \quad \forall s \in S \tag{14}$$

Definition 7. Let S be the universe of elements and $s \in S$. Let $N_{S_1} = (T_{S_1}, I_{S_1}, F_{S_1})$ and $N_{S_2} = (T_{S_2}, I_{S_2}, F_{S_2})$ be two "Single-valued neutrosophic numbers" , then the product of N_{S_1} and N_{S_2} can be written as

$$N_{S_1} \otimes N_{S_2} = (T_{S_1}(s) \cdot T_{S_2}(s), I_{S_1}(s) + I_{S_2}(s) - I_1(s) \cdot I_{S_2}(s), F_{S_1}(s) + F_{S_2}(s) -$$

$$F_{S_1}(s) \cdot F_{S_2}(s)) \forall s \in S \tag{15}$$

Definition 8. Let $N_{S_1} = (T_{S_1}, I_{S_1}, F_{S_1})$ be a "Single-valued neutrosophic number", and $t \in R$ is any arbitrary positive real number then

$$tN_{S_1} = 1 - (1 - T_{S_1}{}^t, I_{S_1}{}^t, F_{S_1}{}^t),\ t > 0 \tag{16}$$

Definition 9. Let U be the universe of elements. Let $X = (X_1, X_2, X_3, \ldots X_n)$ and $Y = (Y_1, Y_2, Y_3, \ldots Y_n)$ be two "Single-valued neutrosophic sets", then the separation measure between X and Y is defined as

$$D_N(X, Y) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}\left[\left(T_X(u_i) - T_Y(u_i)\right)^2 + \left(I_X(u_i) - I_Y(u_i)\right)^2 + \left(F_X(u_i) - F_Y(u_i)\right)^2\right)\right]} \tag{17}$$

This separation measure is based upon the formula of normalized Euclidean distance.

Definition 10. Let $N_S = (T_S, I_S, F_S)$ be a "Single-valued neutrosophic number", then the score function $\delta(N_S) : N_S \rightarrow [0, 1]$ is defined as

$$\delta(N_S) = \frac{(T_s - 2I_s - F_s + 3)}{4} \tag{18}$$

Definition 11. Correlation coefficient of two neutrosophic sets [44] Let $N_{\mathscr{A}_1}$ and $N_{\mathscr{A}_2}$ be two neutrosophic sets defined in a universe finite space $X = \{x_1, x_2, x_3, \dots x_n\}$ such that

$$N_{\mathscr{A}_1} = \{\langle x, T_{N_{\mathscr{A}_1}}(x), I_{N_{\mathscr{A}_1}}(x), F_{N_{\mathscr{A}_1}}(x)\rangle \mid x \in X\} \tag{19}$$

and

$$N_{\mathscr{A}_2} = \{\langle x, T_{\mathscr{A}_2}(x), I_{\mathscr{A}_2}(x), F_{\mathscr{A}_2}(x)\rangle \mid x \in X\} \tag{20}$$

then Correlation coefficient of neutrosophic sets $N_{\mathscr{A}_1}$ and $N_{\mathscr{A}_2}$ is written as

$$\rho(N_{\mathscr{A}_1}, N_{\mathscr{A}_2}) = \frac{C_N(N_{\mathscr{A}_1}, N_{\mathscr{A}_2})}{\sqrt{V(N_{\mathscr{A}_1}) \cdot V(N_{\mathscr{A}_2})}} \tag{21}$$

where

$$C_N(N_{\mathscr{A}_1}, N_{\mathscr{A}_2}) = \sum_{i=1}^{n}\left[T_{N_{\mathscr{A}_1}}(x_i) \cdot T_{N_{\mathscr{A}_2}}(x_i) + I_{N_{\mathscr{A}_1}}(x_i) \cdot I_{N_{\mathscr{A}_2}}(x_i) + F_{N_{\mathscr{A}_1}}(x_i) \cdot F_{N_{\mathscr{A}_2}}(x_i)\right] \tag{22}$$

$$V(N_{\mathscr{A}_1}) = \sum_{i=1}^{n}\left[T^2_{N_{\mathscr{A}_1}}(x_i) + I^2_{N_{\mathscr{A}_1}}(x_i) + F^2_{N_{\mathscr{A}_1}}(x_i)\right] \tag{23}$$

and

$$V(N_{\mathscr{A}_2}) = \sum_{i=1}^{n}\left[T^2_{N_{\mathscr{A}_2}}(x_i) + I^2_{N_{\mathscr{A}_2}}(x_i) + F^2_{N_{\mathscr{A}_2}}(x_i)\right] \tag{24}$$

Some important results

1. If $N_{\mathscr{A}_1}$ and $N_{\mathscr{A}_2}$ are two neutrosophic sets, then

$$C_N(\mathscr{A}_1, N_{\mathscr{A}_2}) = C_N(N_{\mathscr{A}_2}, N_{\mathscr{A}_1}) \tag{25}$$

and so

$$\rho(N_{\mathscr{A}_1}, N_{\mathscr{A}_2}) = \rho(N_{\mathscr{A}_2}, N_{\mathscr{A}_1}) \tag{26}$$

2. If $N_{\mathscr{A}_1}$ and $N_{\mathscr{A}_2}$ are two neutrosophic sets then

$$0 \leq \rho(N_{\mathscr{A}_1}, N_{\mathscr{A}_2}) \leq 1 \tag{27}$$

5. Methodology

The goal of this work is to build an efficient ”multi-criteria decision support system. Through the use of the correlation coefficient, this system determines first the weights of criteria by using the CRITIC-TOPSIS approach, and then it calculates the rank of the considered alternatives. The case study discussed here is using the data set of [2] in which ranking of considered bizjets was done using CRITIC-TOPSIS method under crisp set data.

5.1 *Neutrosophic Criteria Importance Through Inter Criteria Correlation (N-CRITIC) method*

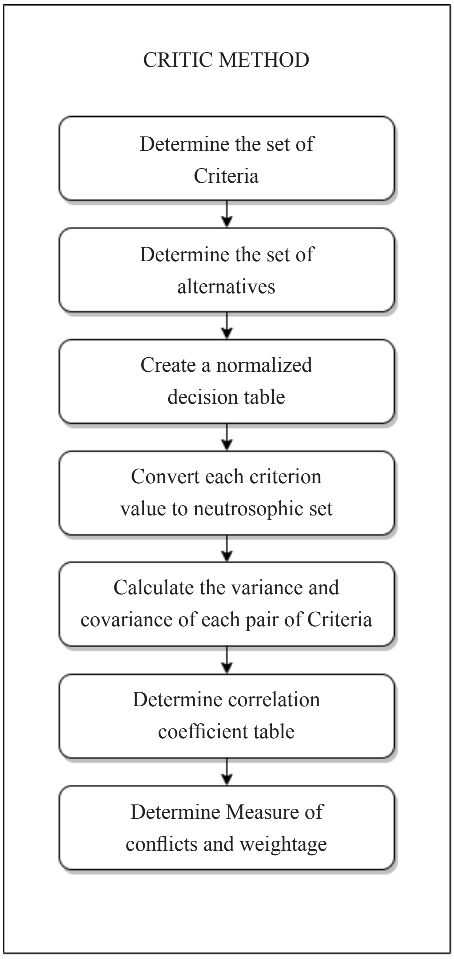

Figure 1. Various steps involved in CRITIC method

To determine inconsistency and indeterminacy, Smarandache [25] introduced the concept of a neutrosophic set. In any MCDM problem, the ranking of alternatives is entirely depending upon the choice of some suitable criteria. Each Criterion has its own importance. The Importance of Criteria can be determined by providing weights to them. One of the weighing approaches for determining objective weights for criteria is the CRITIC (CRiteria Importance Through Intercriteria Correlation) method. Diakoulaki et al. [45] proposed the CRITIC method in 1995. This method identifies the contrast strength of each criterion using standard deviation. A criterion with a higher contrast strength or standard deviation is allocated a higher weight. It employs correlation analysis to determine the variations between criteria. In this method, the normalized decision matrix is determined and correlation coefficients of all possible pairs of columns are used to calculate the criteria contrast [28, 46]. In this research, the CRITIC method is used under neutrosophic environment. The various steps involved in this method are shown in Figure 1.

5.1.1 *Step 1: Determine the table for selection Criteria*

The Decision maker determines the set of selection criteria and set of alternatives for which ranking is to be done. The table of selection criteria is demonstrated in Table 2.

Table 2. Table for Selection criteria in crisp values

Alternatives	Selection Criteria				
	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$		$\mathscr{C}_m$
$\mathscr{A}_1$	$\mathfrak{x}_{11}$	$\mathfrak{x}_{12}$	$\mathfrak{x}_{13}$	$\cdots$	$\mathfrak{x}_{1m}$
$\mathscr{A}_2$	$\mathfrak{x}_{21}$	$\mathfrak{x}_{22}$	$\mathfrak{x}_{23}$	$\cdots$	$\mathfrak{x}_{2m}$
$\mathscr{A}_3$	$\mathfrak{x}_{31}$	$\mathfrak{x}_{32}$	$\mathfrak{x}_{33}$	$\cdots$	$\mathfrak{x}_{3m}$
:	:	:	:	:	:
$\mathscr{A}_n$	$\mathfrak{x}_{n1}$	$\mathfrak{x}_{n2}$	$\mathfrak{x}_{n3}$	$\cdots$	$\mathfrak{x}_{nm}$

5.1.2 *Step 2: Determine the normalized neutrosophic decision table*

Table 3. Normalized decision table

Alternatives	Selection Criteria				
	favorable $\mathscr{C}_1$	favorable $\mathscr{C}_2$	Non-favorable $\mathscr{C}_3$	Non-favorable $\cdots$	favorable $\mathscr{C}_m$
$\mathscr{A}_1$	$\bar{\mathfrak{n}}_{11}$	$\bar{\mathfrak{n}}_{12}$	$\bar{\mathfrak{n}}_{13}$	$\cdots$	$\bar{\mathfrak{n}}_{1m}$
$\mathscr{A}_2$	$\bar{\mathfrak{n}}_{21}$	$\bar{\mathfrak{n}}_{22}$	$\bar{\mathfrak{n}}_{23}$	$\cdots$	$\bar{\mathfrak{n}}_{2m}$
$\mathscr{A}_3$	$\bar{\mathfrak{n}}_{31}$	$\bar{\mathfrak{n}}_{32}$	$\bar{\mathfrak{n}}_{33}$	$\cdots$	$\bar{\mathfrak{n}}_{3m}$
:	:	:	:	:	:
$\mathscr{A}_n$	$\bar{\mathfrak{n}}_{n1}$	$\bar{\mathfrak{n}}_{n2}$	$\bar{\mathfrak{n}}_{n3}$	$\cdots$	$\bar{\mathfrak{n}}_{nm}$

The normalized decision value denoted as $\bar{\mathfrak{n}}_{ij}$ can be determined by using the formula given by equation 28 and is demonstrated in Table 3.

$$\bar{\mathfrak{n}}_{ij} = \frac{\mathfrak{x}_{ij} - (\mathfrak{x}_{ij})_{\text{worst}}}{(\mathfrak{x}_{ij})_{\text{best}} - (\mathfrak{x}_{ij})_{\text{worst}}}; 1 \le i \le m, 1 \le j \le n \tag{28}$$

Here the $(\mathfrak{x}_{ij})_{worst}$ and $(\mathfrak{x}_{ij})_{best}$ values can be calculated after segregating selection criteria into favorable and non-favorable criteria. For favorable criteria, a larger value is better and for non-favorable criteria, a smaller value is better. Now each crisp value can be converted into Neutrosophic sets demonstrated in Table 4.

Table 4. Normalized neutrosophic decision table

Alternatives	Selection Criteria				
	favorable $\mathscr{C}_1$	favorable $\mathscr{C}_2$	Non-favorable $\mathscr{C}_3$	Non-favorable …	favorable $\mathscr{C}_m$
$\mathscr{A}_1$	$\langle T_{\bar{n}_{11}}, I_{\bar{n}_{11}}, F_{\bar{n}_{11}} \rangle$	$\langle T_{\bar{n}_{12}}, I_{\bar{n}_{12}}, F_{\bar{n}_{12}} \rangle$	$\langle T_{\bar{n}_{13}}, I_{\bar{n}_{13}}, F_{\bar{n}_{13}} \rangle$	…	$\langle T_{\bar{n}_{1m}}, I_{\bar{n}_{1m}}, F_{\bar{n}_{1m}} \rangle$
$\mathscr{A}_2$	$\langle T_{\bar{n}_{21}}, I_{\bar{n}_{21}}, F_{\bar{n}_{21}} \rangle$	$\langle T_{\bar{n}_{22}}, I_{\bar{n}_{22}}, F_{\bar{n}_{22}} \rangle$	$\langle T_{\bar{n}_{23}}, I_{\bar{n}_{23}}, F_{\bar{n}_{23}} \rangle$	…	$\langle T_{\bar{n}_{2m}}, I_{\bar{n}_{2m}}, F_{\bar{n}_{2m}} \rangle$
$\mathscr{A}_3$	$\langle T_{\bar{n}_{31}}, I_{\bar{n}_{31}}, F_{\bar{n}_{31}} \rangle$	$\langle T_{\bar{n}_{32}}, I_{\bar{n}_{32}}, F_{\bar{n}_{32}} \rangle$	$\langle T_{\bar{n}_{33}}, I_{\bar{n}_{33}}, F_{\bar{n}_{33}} \rangle$	…	$\langle T_{\bar{n}_{3m}}, I_{\bar{n}_{3m}}, F_{\bar{n}_{3m}} \rangle$
:	:	:	:	:	:
$\mathscr{A}_n$	$\langle T_{\bar{n}_{n1}}, I_{\bar{n}_{n1}}, F_{\bar{n}_{n1}} \rangle$	$\langle T_{\bar{n}_{n2}}, I_{\bar{n}_{n2}}, F_{\bar{n}_{n2}} \rangle$	$\langle T_{\bar{n}_{n3}}, I_{\bar{n}_{n3}}, F_{\bar{n}_{n3}} \rangle$	…	$\langle T_{\bar{n}_{nm}}, I_{\bar{n}_{nm}}, F_{\bar{n}_{nm}} \rangle$

5.1.3 *Step 3: Determine the correlation formula and Variance of pairwise criteria*

In the case of neutrosophic sets, the Correlation coefficient cannot be determined in a single step formula for all criteria [47]. Pairwise analysis has to be done for calculating the overall correlation coefficient table.

5.1.4 *Step 4: Determine the correlation coefficient table*

The correlation coefficient table generated by taking values of all possible pairs of pre-defined criteria is demonstrated in the Table 5.

5.1.5 *Step 5: Determine the measure of conflicts and objective weights to criteria*

This step determines the value of measure of conflicts $(1 - C_{ij})$ for each criterion and then determine the index value (C_j) given by

$$C_j = \sigma \sum_{i=1}^{n} (1 - C_{ij}); \ j = 1, 2, \ldots, n \tag{29}$$

In this equation C_{ij} represents the criterion value for i^{th} alternative and j^{th} criterion and σ represents the standard deviation which is a measure of the amount of variation or dispersion of a set of values.

Objective weights of each criterion can be determined by formula written as

$$w_j = \frac{C_j}{\sum_{j=1}^{n} C_j};\ j = 1, 2, \ldots, n \tag{30}$$

Table 5. Correlation coefficient table

Attributes	Selection Criteria				
	favorable $\mathscr{C}_1$	favorable $\mathscr{C}_2$	Non-favorable $\mathscr{C}_3$	Non-favorable $\ldots$	favorable $\mathscr{C}_m$
$\mathscr{A}_1$	$\mathscr{C}_{11}$	$\mathscr{C}_{12}$	$\mathscr{C}_{13}$	$\ldots$	$\mathscr{C}_{1m}$
$\mathscr{A}_2$	$\mathscr{C}_{21}$	$\mathscr{C}_{22}$	$\mathscr{C}_{23}$	$\ldots$	$\mathscr{C}_{2m}$
$\mathscr{A}_3$	$\mathscr{C}_{31}$	$\mathscr{C}_{32}$	$\mathscr{C}_{33}$	$\ldots$	$\mathscr{C}_{3m}$
:	:	:	:	:	:
$\mathscr{A}_n$	$\mathscr{C}_{n1}$	$\mathscr{C}_{n2}$	$\mathscr{C}_{n3}$	$\ldots$	$\mathscr{C}_{nm}$

5.2 *The Neutrosophic TOPSIS method*

With Neutrosophic TOPSIS, several externally determined alternatives can be ranked and selected using distance measurements. Using this approach, it is assumed that the specified alternatives are closest to the "positive ideal solution" and farthest from the "negative ideal solution". The "ideal solution" is one that maximizes the benefit criteria and minimizes the cost criteria. Let $\mathscr{A} = \{\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3, \ldots, \mathscr{A}_n\}$, $n \geq 1$ be the set of alternatives and $\mathscr{C} = \{\mathscr{C}_1, \mathscr{C}_2, \mathscr{C}_3, \ldots, \mathscr{C}_m\}$, $m \geq 2$ be the set of criteria. A neutrosophic TOPSIS method involves the following steps:

5.2.1 *Step 1: Construction of normalized neutrosophic decision table*

The normalized decision table for various alternatives under pre-selected criteria is given in Table 4 as discussed in the CRITIC method.

5.2.2 *Step 2: Construction of weighted normalized neutrosophic decision table*

The weighted normalized neutrosophic decision table is given by Table 6. where

$$\overline{n}_{ij}^{w_j} = \left(1 - \left(1 - T_{\overline{n}_{ij}}\right)^{w_j}, I_{\overline{n}_{ij}}^{w_j}, F_{\overline{n}_{ij}}^{w_j}\right), w_j > 0, 1 \leq i \leq n, 1 \leq j \leq m \tag{31}$$

here the weights w_j for each criterion are determined by the CRITIC method as given by relation 30.

Table 6. Weighted normalized neutrosophic decision table

Alternatives	Selection Criteria				
	favorable $\mathscr{C}_1$	favorable $\mathscr{C}_2$	Non-favorable $\mathscr{C}_3$	Non-favorable …	favorable $\mathscr{C}_m$
$\mathscr{A}_1$	$\overline{n}_{11}^{w_1}$	$\overline{n}_{12}^{w_2}$	$\overline{n}_{13}^{w_3}$	…	$\overline{n}_{1m}^{w_m}$
$\mathscr{A}_2$	$\overline{n}_{21}^{w_1}$	$\overline{n}_{22}^{w_2}$	$\overline{n}_{23}^{w_3}$	…	$\overline{n}_{2m}^{w_m}$
$\mathscr{A}_3$	$\overline{n}_{31}^{w_1}$	$\overline{n}_{32}^{w_2}$	$\overline{n}_{33}^{w_3}$	…	$\overline{n}_{3m}^{w_m}$
:	:	:	:	:	:
$\mathscr{A}_n$	$\overline{n}_{n1}^{w_1}$	$\overline{n}_{n2}^{w_2}$	$\overline{n}_{n3}^{w_3}$	…	$\overline{n}_{nm}^{w_m}$

5.2.3 *Step 3: Establish the neutrosophic relative positive ideal solution and negative ideal solution*

The “positive ideal solution” and “negative ideal solution” are calculated after deciding which criterion will be considered as a favorable criterion and which one will be considered as non-favorable criterion.

Let us assume

V_1: set of benefit criteria

V_2: set of non-benefit criteria

V_N^+: “neutrosophic relative positive ideal solution”

and

V_N^-: “neutrosophic relative negative ideal solution”

then

$$V_N^+ = [\overline{n}_1^{w^+}, \overline{n}_2^{w^+}, \ldots, \overline{n}_n^{w^+}] \tag{32}$$

where

$$\overline{n}_j^{w^+} = \langle T_j^{w^+}, I_j^{w^+}, F_j^{w^+} \rangle, 1 \leq j \leq m \tag{33}$$

and

$$T_j^{w^+} = \left\{ \left(\max_i \{T_{ij}^{w_j}\} | \ j \in V_1 \right), \left(\min_i \{T_{ij}^{w_j}\} | \ j \in V_2 \right) \right\} \tag{34}$$

$$I_j^+ = \left\{ \left(\min_i \{I_{ij}^{w_j}\} | \ j \in V_1 \right), \left(\max_i \{I_{ij}^{w_j}\} | \ j \in V_2 \right) \right\} \tag{35}$$

$$F_j^{w^+} = \left\{ \left(\min_i \{F_{ij}^{w_j}\} | \ j \in V_1 \right), \left(\max_i \{F_{ij}^{w_j}\} | \ j \in V_2 \right) \right\} \tag{36}$$

and V_N^- can be written as

$$V_N^- = \left[\bar{n}_1^{w^-}, \bar{n}_2^{w^-}, \ldots, \bar{n}_n^{w^-} \right] \tag{37}$$

where

$$\bar{n}_j^{w^-} = \left\langle T_j^{w^-}, I_j^{w^-}, F_j^{w^-} \right\rangle, 1 \le j \le m \tag{38}$$

and

$$T_j^{w^-} = \left\{ \left(\min_i \{T_{ij}^{w_j}\} | \, j \in V_1 \right), \left(\max_i \{T_{ij}^{w_j}\} | \, j \in V_2 \right) \right\} \tag{39}$$

$$I_j^{w^-} = \left\{ \left(\max_i \{I_{ij}^{w_j}\} | \, j \in V_1 \right), \left(\min_i \{I_{ij}^{w_j}\} | \, j \in V_2 \right) \right\} \tag{40}$$

$$F_j^{w^-} = \left\{ \left(\max_i \{F_{ij}^{w_j}\} | \, j \in V_1 \right), \left(\min_i \{F_{ij}^{w_j}\} | \, j \in V_2 \right) \right\} \tag{41}$$

5.2.4 *Step 4: Establish the distance of each alternative from the neutrosophic relative positive ideal solution and the neutrosophic relative negative ideal solution*

The normalized Euclidean distance of each alternative $\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ from the neutrosophic relative positive ideal solution $\langle T_j^{w^+}, I_j^{w^+}, F_j^{w^+} \rangle$ for $1 \le i \le n$ and $1 \le j \le m$ can be described as

$$D_{EU}^{+}(\bar{n}_{ij}^{w_j}, \bar{n}_j^{w^+}) = \sqrt{\frac{1}{3m} \sum_{j=1}^{m} \left[(T_{ij}^{w_j}(u_i) - T_j^{w^+}(u_i))^2 + (I_{ij}^{w_j}(u_i) - I_j^{w^+}(u_i)^2 + (F_{ij}^{w_j} - F_j^{w^+}(u_i))^2) \right]} \tag{42}$$

Similarly, the formula for normalized Euclidean distance of each alternative $\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ from the neutrosophic relative negative ideal solution $\langle T_j^{w^-}, I_j^{w^-}, F_j^{w^-} \rangle$ for $1 \le i \le n$ and $1 \le j \le m$ can be described as

$$D_{EU}^{-}(\bar{n}_{ij}^{w_j}, \bar{n}_j^{w^-}) = \sqrt{\frac{1}{3m} \sum_{i=1}^{m} \left[(T_{ij}^{w_j}(u_i) - T_j^{w^-}(u_i))^2 + (I_{ij}^{w_j}(u_i) - I_j^{w^-}(u_i)^2 + (F_{ij}^{w_j} - F_j^{w^-}(u_i))^2) \right]} \tag{43}$$

5.2.5 *Step 5: Establish the relative coefficient of closeness to the neutrosophic ideal solution*

The relative coefficient of closeness CC_i of each alternative $\mathscr{A}_i$ to the neutrosophic relative positive ideal solution V_N^+ is given as:

$$CC_i = \frac{D_{EU}^{-}(\overline{n}_{ij}^{w_j}, \overline{n}_j^{w^-})}{D_{EU}^{+}(\overline{n}_{ij}^{w_j}, \overline{n}_j^{w^+}) + D_{EU}^{-}(\overline{n}_{ij}^{w_j}, \overline{n}_j^{w^-})} \quad (44)$$

5.2.6 *Step 6: Calculate the rank of each alternative*

The ranking of each alternative is identified by using the value of the relative coefficient of closeness. The larger the value of CC_i more is the importance of alternative $\mathscr{A}_i$, $i = 1, 2, 3, \ldots, n$.

6. Numerical analysis

The approach of neutrosophic CRITIC-TOPSIS can be analyzed by using the case study of [2] in which the five alternatives are $\{\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4, \mathscr{A}_5\}$ = {"Honda Jet HA-420, Cessna Citation Jet M2, Embraer Phenom 100, Eclipse 550, Cessna Citation Mustang"} and the six criteria are $\{\mathscr{C}_1, \mathscr{C}_2, \mathscr{C}_3, \mathscr{C}_4, \mathscr{C}_5, \mathscr{C}_6\}$ = {Speed, Range, Gross weight, Cost, Aesthetics, Comfort}. Out of these criteria, $\mathscr{C}_1$, $\mathscr{C}_2$, $\mathscr{C}_5$ and $\mathscr{C}_6$ are considered as favorable criteria whereas $\mathscr{C}_3$ and $\mathscr{C}_4$ are considered as non-favorable criteria. Based on the proposed neutrosophic CRITIC-TOPSIS approach discussed in section 5.1, the various steps involved to solve the well-chosen problem are following:

6.1 *Determination of Criterion's weights and ranking of light Bizjets using neutrosophic CRITIC-TOPSIS method*

6.1.1 *Step 1: Determination of selection criteria table*

The selection criteria table based on the paper of is given by Table 7.

Single-valued neutrosophic numbers associated with Linguistic Variables are given by Table 8.

Table 7. Table for selection criteria

Attributes		favorable	favorable	Non-favorable	Non-favorable	favorable	favorable
Bizjet	Notations	Speed $\mathscr{C}_1$	Range $\mathscr{C}_2$	Weight $\mathscr{C}_3$	Cost $\mathscr{C}_4$	Aesthetic $\mathscr{C}_5$	Comfort $\mathscr{C}_6$
Honda Jet HA-420	$\mathscr{A}_1$	0.63	2,661	4,854	5.3	10	10
Cessna Citation Jet M2	$\mathscr{A}_2$	0.61	2,871	4,853	4.7	5	5
Embraer Phenom 100	$\mathscr{A}_3$	0.7	2,183	4,800	4.25	8	8
Eclipse 550	$\mathscr{A}_4$	0.59	2,084	2,737	2.7	5	3
Cessna Citation Mustang	$\mathscr{A}_5$	0.63	2,161	3,930	3.2	7	5
Best		0.7	2,871	2,737	2.7	10	10
Worst		0.59	2,084	4,854	5.3	5	3

Table 8. Single-valued neutrosophic numbers associated with Linguistic Variables

Linguistic Variables	Points	Single-valued neutrosophic numbers	Linguistic Variables	Points	Single-valued neutrosophic numbers
Extremely high (EH)	10	(1.00, 0.00, 0.00)	Very Medium (VM)	4	(0.40, 0.60, 0.65)
Very very High (VVH)	9	(0.90, 0.10, 0.10)	Low (L)	3	(0.30, 0.70, 0.75)
Very good (VG)	8	(0.80, 0.20, 0.15)	Very low (VL)	2	(0.20, 0.80, 0.85)
Good (G)	7	(0.70, 0.30, 0.25)	Very very low (VL)	1	(0.10, 0.90, 0.90)
Fair (F)	6	(0.60, 0.40, 0.35)	Extremely Low (EL)	0	(0.00, 1.00, 1.00)
Medium (M)	5	(0.50, 0.50, 0.50)			

6.1.2 *Step 2: Determination of neutrosophic normalized Decision table*

The neutrosophic normalized decision table for alternatives in terms of crisp values is given by Table 9. This table is generated by using equation 28.

Table 9. Neutrosophic normalized decision table for alternatives

Attributes		favorable	favorable	Non-favorable	Non-favorable	favorable	favorable
Bizjet	Notations	Speed $\mathscr{C}_1$	Range $\mathscr{C}_2$	Weight $\mathscr{C}_3$	Cost $\mathscr{C}_4$	Aesthetic $\mathscr{C}_5$	Comfort $\mathscr{C}_6$
Honda Jet HA-420	$\mathscr{A}_1$	(0.36, 0.11, 0.75)	(0.73, 0.35, 0.19)	(0.50, 0.37, 0.82)	(0.67, 1.00, 0.78)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)
Cessna Citation Jet M2	$\mathscr{A}_2$	(0.18, 0.83, 0.45)	(1.00, 0.78, 0.35)	(0.99, 0.01, 0.10)	(0.89, 0.05, 0.20)	(0.50, 0.50, 0.50)	(0.50, 0.50, 0.50)
Embraer Phenom 100	$\mathscr{A}_3$	(1.00, 0.82, 0.53)	(0.13, 0.25, 0.35)	(0.03, 0.97, 0.30)	(0.40, 0.10, 0.10)	(0.80, 0.20, 0.15)	(0.80, 0.20, 0.15)
Eclipse 550	$\mathscr{A}_4$	(0.00, 0.98, 0.32)	(0.00, 0.27, 0.40)	(0.10, 0.25, 0.50)	(0.51, 0.10, 0.25)	(0.50, 0.50, 0.50)	(0.30, 0.70, 0.75)
Cessna Citation Mustang	$\mathscr{A}_5$	(0.36, 0.43, 0.87)	(0.10, 0.38, 0.40)	(0.44, 0.89, 0.45)	(0.81, 0.15, 0.20)	(0.70, 0.30, 0.25)	(0.50, 0.50, 0.50)
Best		(1, 00, 0.98, 0.87)	(1.00, 0.78, 0.40)	(1.00, 0.97, 0.82)	(1.00, 1.00, 0.78)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)
Worst		(0.00, 0.11, 0.32)	(0.00, 0.25, 0.19)	(0.03, 0.37, 0.10)	(0.23, 0.10, 0.10)	(0.50, 0.50, 0.50)	(0.30, 0.70, 0.75)

6.1.3 *Step 3: Determination of pairwise correlation coefficient tables*

There are six pre-defined criteria as Speed, Range, Gross weight, Cost, Aesthetics, and Comfort. The pairwise correlation coefficient determines how any two criteria are related to each other. The pairwise correlation coefficient between speed and range for all alternatives is given by Table 10.

The correlation coefficient of two neutrosophic sets is calculated using the formula in equation 21.

Table 10. The Correlation coefficient table between Speed and Range

Alternatives	$T_{\mathscr{C}_1} \cdot T_{\mathscr{C}_2}$	$I_{\mathscr{C}_1} \cdot I_{\mathscr{C}_2}$	$F_{\mathscr{C}_1} \cdot I_{\mathscr{C}_2}$	$T_{\mathscr{C}_1}^2$	$I_{\mathscr{C}_1}^2$	$F_{\mathscr{C}_1}^2$	$T_{\mathscr{C}_2}^2$	$I_{\mathscr{C}_2}^2$	$F_{\mathscr{C}_2}^2$
$\mathscr{A}_1$	0.2628	0.0385	0.1425	0.1296	0.0121	0.5625	0.5329	0.1225	0.0361
$\mathscr{A}_2$	0.18	0.6474	0.1575	0.0324	0.6889	0.2025	1	0.6084	0.1225
$\mathscr{A}_3$	0.13	0.205	0.1855	1	0.6724	0.2809	0.0169	0.0625	0.1225
$\mathscr{A}_4$	0	0.2646	0.128	0	0.9604	0.1024	0	0.0729	0.16
$\mathscr{A}_5$	0.036	0.1634	0.348	0.1296	0.1849	0.7569	0.01	0.1444	0.16
SUM	0.6088	1.3189	0.9615	1.2916	2.5187	1.9052	1.5598	1.0107	0.6011

Here the value of correlation coefficient for speed ($\mathscr{C}_1$) and range ($\mathscr{C}_2$) is calculated as

$$C_{\mathscr{N}}(\mathscr{C}_1, \mathscr{C}_2) = 0.6088 + 1.3189 + 0.9615 = 2.8892 \tag{45}$$

$$V_{\mathscr{N}}(\mathscr{C}_1, \mathscr{C}_2) = 1.2916 + 2.5187 + 1.9052 = 5.7155 \tag{46}$$

$$V_{\mathscr{N}}(\mathscr{C}_1, \mathscr{C}_2) = 1.5598 + 1.0107 + 0.6011 = 3.1716 \tag{47}$$

Therefore

$$\rho_{\mathscr{N}}(\mathscr{C}_1, \mathscr{C}_2) = \frac{2.8892}{\sqrt{5.7155 \times 3.1716}}$$

$$= 0.6786 \tag{48}$$

Similarly, all other values of neutrosophic correlation coefficients for each possible pair of criteria are calculated.

6.1.4 *Step 4: Determination of correlation coefficient table*

Combining all values of pairwise correlation coefficients of different criteria with respect to all alternatives are given in Table 11.

Table 11. Pairwise Correlation coefficient table

Attributes	Speed ($\mathscr{C}_1$)	Range ($\mathscr{C}_2$)	Weight ($\mathscr{C}_3$)	Cost ($\mathscr{C}_4$)	Aesthetic ($\mathscr{C}_5$)	Comfort ($\mathscr{C}_6$)
Speed ($\mathscr{C}_1$)	1	0.6786	0.7330	0.5149	0.7214	0.8296
Range ($\mathscr{C}_2$)	0.6786	1	0.7660	0.5323	0.7504	0.7894
Weight ($\mathscr{C}_3$)	0.7330	0.7660	1	0.7134	0.6871	0.6941
Cost ($\mathscr{C}_4$)	0.5150	0.5324	0.7135	1	0.6737	0.5869
Aesthetic ($\mathscr{C}_5$)	0.7214	0.7504	0.6871	0.6737	1	0.9670
Comfort ($\mathscr{C}_6$)	0.8296	0.7894	0.6941	0.5869	0.9670	1

6.1.5 *Step 5: Determination of measures of conflicts and weights of criteria*

The measure of conflicts and index values are calculated by using the equation 29 and weights of criteria are calculated by using the equation 30. Measures of conflicts and weights of criteria are summarized by Table 12.

Table 12. Measures of conflicts and weights of criteria

Attributes	Speed ($\mathscr{C}_1$)	Range ($\mathscr{C}_2$)	Weight ($\mathscr{C}_3$)	Cost ($\mathscr{C}_4$)	Aesthetic ($\mathscr{C}_5$)	Comfort ($\mathscr{C}_6$)	$\sum_{j=1}^{n}(1 - C_{ij})$	Index C_j	Objective weights W_i
Speed ($\mathscr{C}_1$)	0	0.3215	0.267	0.4851	0.2786	0.1704	1.5226	3.6401	0.1954
Range ($\mathscr{C}_2$)	0.3215	0	0.234	0.4677	0.2496	0.2106	1.4834	3.7109	0.1992
Weight ($\mathscr{C}_3$)	0.267	0.234	0	0.2866	0.3129	0.3059	1.4064	2.5047	0.1345
Cost ($\mathscr{C}_4$)	0.4851	0.4677	0.2866	0	0.3264	0.4131	1.9789	4.0478	0.2173
Aesthetic ($\mathscr{C}_5$)	0.2786	0.2496	0.3129	0.3264	0	0.033	1.2005	2.5025	0.1343
Comfort ($\mathscr{C}_6$)	0.1704	0.2106	0.3059	0.4131	0.033	0	1.133	2.221662	0.1193
							$\sum_{j=1}^{m} C_j = 18.62766$		1

6.1.6 *Step 6: Determination of neutrosophic normalized weighted decision table*

To determine the weighted decision table , the weights of each criterion get multiplied with corresponding neutrosophic set of Table 9. The weighted decision table is given by Table 13.

The first neutrosophic weighted normalized value of the decision table for speed ($\mathscr{C}_1$)

Table 13. The Neutrosophic normalized weighted decision table

Alternatives	favorable	favorable	Non-favorable	Non-favorable	favorable	favorable
	Speed $\mathscr{C}_1$	Range $\mathscr{C}_2$	Weight $\mathscr{C}_3$	Cost $\mathscr{C}_4$	Aesthetic $\mathscr{C}_5$	Comfort $\mathscr{C}_6$
$\mathscr{A}_1$	(0.0836, 0.6497, 0.9454)	(0.2296, 0.8113, 0.7184)	(0.0891, 0.8749, 0.9737)	(0.2141, 1, 0.9475)	(1.0000, 0.0000, 0.0000)	(1.0000, 0.0000, 0.0000)
$\mathscr{A}_2$	(0.0381, 0.9642, 0.8556)	(1.000, 0.9518, 0.8113)	(0.2324, 0.9638, 0.7337)	(0.0553, 0.7399, 0.7049)	(0.0889, 0.9112, 0.9112)	(0.0794, 0.9207, 0.9207)
$\mathscr{A}_3$	(1.000, 0.9620, 0.8834)	(0.0274, 0.7587, 0.8113)	(0.0041, 0.996, 0.8505)	(0.1051, 0.6064, 0.6064)	(0.1944, 0.8057, 0.7751)	(0.1747, 0.8254, 0.7975)
$\mathscr{A}_4$	(0.000, 0.9960, 0.8004)	(0.000, 0.7705, 0.8332)	(1.000, 0.9476, 0.8982)	(1.000, 0.6064, 0.7399)	(0.0889, 0.9112, 0.9112)	(0.0417, 0.9584, 0.9663)
$\mathscr{A}_5$	(0.0836, 0.8480, 0.9732)	(0.0208, 0.8247, 0.8332, 1)	(0.0751, 0.9845, 0.8982)	(0.303, 0.6622, 0.7049)	(0.1493, 0.8508, 0.8302)	(0.0794, 0.9207, 0.9207)

w.r.t alternative $\mathscr{A}_1$ is calculated by using equation 31 and is given as

$$\bar{n}_{11}^{w_1} = \left\langle T_{\bar{n}_{11}}^{w_1}, I_{\bar{n}_{11}}^{w_1}, F_{\bar{n}_{11}}^{w_1} \right\rangle$$

$$= \left(1-\left(1-T_{\bar{n}_{11}}\right)^{w_1}, I_{\bar{n}_{11}}^{w_1}, F_{\bar{n}_{11}}^{w_1}\right)$$

$$= (1-(1-0.36)^{0.1954}, (0.11)^{0.1954}, (0.75)^{0.1954})$$

$$= (0.0836, 0.6497, 0.9454) \tag{49}$$

Similarly, all other values are calculated.

6.1.7 *Step 7: Determination of neutrosophic relative positive ideal solution and neutrosophic negative ideal solution*

The neutrosophic relative positive ideal solution and neutrosophic negative ideal solution are calculated on the basis of type of criteria i.e., benefit type or non-benefit type by using equations 32 to 41 and given by Table 14 where

$$\bar{n}_1^{w^+} = \left\langle T_1^{w^+}, I_1^{w^+}, F_1^{w^+} \right\rangle, 1 \le j \le m \tag{50}$$

and

$$T_1^{w^+} = \max\{0.0835, 0.0380, 1, 0, 0.0835\} = 1$$

$$I_1^{w^+} = \min\{0.6497, 0.9642, 0.9620, 0.9961, 0.8480\} = 0.6497$$

$$F_1^{w^+} = \min\{0.9453, 0.8555, 0.8833, 0.8004, 0.9732\} = 0.8004$$

Table 14. Neutrosophic relative positive ideal solution

Criteria	Type of criteria set	V_N^+ (NRPIS)
$\mathscr{C}_1$	favorable	(1.0000, 0.6497, 0.8004)
$\mathscr{C}_2$	favorable	(1.0000, 0.7587, 0.7183)
$\mathscr{C}_3$	Non-favorable	(0.0041, 0.9960, 0.9737)
$\mathscr{C}_4$	Non-favorable	(0.0552, 1.0000, 0.9474)
$\mathscr{C}_5$	favorable	(1.0000, 0.0000, 0.0000)
$\mathscr{C}_6$	favorable	(1.0000, 0.0000, 0.0000)

and other values can be calculated on the same way. Similarly, values of V_N^- can be calculated and the table representing values of neutrosophic negative ideal solution for each criterion is given by Table 15.

Table 15. Neutrosophic relative negative ideal solution

Criteria	Type of criteria set	V_N^+ (NRPIS)
$\mathscr{C}_1$	favorable	(0.0000, 0.9960, 0.9732)
$\mathscr{C}_2$	favorable	(0.0000, 0.9518, 0.8331)
$\mathscr{C}_3$	Non-favorable	(1.0000, 0.8749, 0.7337)
$\mathscr{C}_4$	Non-favorable	(1.0000, 0.6063, 0.6063)
$\mathscr{C}_5$	favorable	(0.0889, 0.9111, 0.9111)
$\mathscr{C}_6$	favorable	(0.0416, 0.9583, 0.9663)

$$V_N^- = \left[\overline{n}_1^{w^-}, \overline{n}_2^{w^-}, \ldots, \overline{n}_n^{w^-}\right] \tag{51}$$

where

$$\overline{n}_1^{w^-} = \left\langle T_1^{w^-}, I_j^{w^-}, F_1^{w^-} \right\rangle, 1 \le j \le m \tag{52}$$

and

$$T_1^{w^-} = \min\{0, 0, 1, 1, 0.08889, 0.04166\} = 0.0000$$

$$I_1^{w^-} = \max\{0.9960, 0.9517, 0.8748, 0.6063, 0.9111, 0.9583\} = 0.9960$$

$$F_1^{w^-} = \max\{0.9732, 0.8332, 0.7336, 0.6063, 0.9111, 0.9663\} = 0.9732$$

6.1.8 *Step 8: Determination of neutrosophic normalized Euclidean distance and relative closeness coefficient*

Measures of neutrosophic normalized euclidean distance as described in equation 42 and equation 43 are used to determine the euclidean distance of each alternative form the neutrosophic relative positive ideal solution and the neutrosophic relative negative ideal solution and finally relative closeness coefficient is calculated by using Equation 44 and shown in Table 16. The maximum value of the relative closeness coefficient

Table 16. Relative Euclidean distance and relative coefficient closeness

Alternatives	D_{EU}^{+}	D_{EU}^{-}	CC_i	Ranking
$\mathscr{A}_1$	0.6151	0.5891	0.4892	4
$\mathscr{A}_2$	0.2892	0.5455	0.6536	1
$\mathscr{A}_3$	0.5937	0.5805	0.4944	3
$\mathscr{A}_4$	0.5398	0.5553	0.5071	2
$\mathscr{A}_5$	0.726	0.5931	0.4497	5

indicates that $\mathscr{A}_2$ is the most efficient and desirable alternative.

7. Conclusions

The present study of this paper described a novel CRITIC-TOPSIS method under a neutrosophic environment. The neutrosophic set extends the general intuitionistic fuzzy set where membership, non-membership, and hesitation simultaneously can vary between 0 and 1. This generalization makes the intuitionistic fuzzy set more flexible. The CRITIC-TOPSIS method is analyzed with a neutrosophic environment and a numerical analysis was performed. The same data from several aircraft from the previous studies were used in this paper. The results show that Cessna Citation

Jet M2 was chosen as the most preferable aircraft based on the criteria considered in this study. Meanwhile, Cessna Citation Mustang was selected as the least preferable aircraft. Rank-wise arranged aircraft are summarized as Cessna Citation Jet M2, Eclipse 550, Embraer Phenom 100, Honda Jet HA-420, and Cessna Citation Mustang. The CRITIC-TOPSIS (Criteria Importance Through Intercriteria Correlation and Technique for Order of Preference by Similarity to Ideal Solution) method is a multi-criteria decision-making (MCDM) technique used to evaluate alternatives based on multiple criteria. In addition to its merits, it has some limitations worth considering. Here are a few key limitations of this study.

• The CRITIC-TOPSIS method assigns weights to criteria based on their relative importance. In reality, the decision-maker's preferences and biases often affect the determination of these weights.

• In order to properly analyze the CRITIC-TOPSIS data, it must be normalized first to a common scale. There can be a significant impact on the final ranking and outcome based on the choice of normalization technique. There is no universally accepted approach to normalization, so different methods may produce different results.

• No comprehensive sensitivity analysis is provided for CRITIC-TOPSIS to assess the robustness of its results. It fails to assess how alternate rankings or changing the weights of the criteria would affect the outcome. This limitation makes it challenging to understand the stability and reliability of the decision-making process.

• For large-scale decision problems with lots of criteria and alternatives, the CRITIC-TOPSIS method can become time-consuming and computationally intensive. As the number of criteria and alternatives increases, the analysis becomes more complex, which could render it challenging to use in such situations.

7.1 Future scope

While neutrosophic fuzzy sets are helpful in handling complex decision-making problems, challenges exist in terms of computational complexity and the interpretation of the indeterminacy function. Further research is needed to develop efficient methods and algorithms for effectively applying neutrosophic fuzzy sets in various domains such as healthcare and medical diagnosis system, data analysis and data mining, Internet of Things (IoT), and control systems. The current study can also be extended by using spherical fuzzy information [48-50].

Conflict of interest

The authors declare no competing financial interest.

References

[1] Agarwal I, Gowda KR. The effect of airline service quality on customer satisfaction and loyalty in India. *Materials Today: Proceedings*. 2021; 37: 1341-1348.

[2] Dhara A, Kaur G, Kishan PM, Majumder A, Yadav R. An efficient decision support system for selecting very light business jet using CRITIC-TOPSIS method. *Aircraft Engineering and Aerospace Technology*. 2022; 94(3): 458-472.

[3] Dhiman HS, Deb D. Fuzzy TOPSIS and fuzzy COPRAS based multi-criteria decision making for hybrid wind farms. *Energy.* 2020; 202: 117755.

[4] Zhang K, Zhan J, Wang X. TOPSIS-WAA method based on a covering-based fuzzy rough set: an application to rating problem. *Information Sciences*. 2020; 539: 397-421.

[5] Akram M, Kahraman C, Zahid K. Extension of TOPSIS model to the decision-making under complex spherical fuzzy information. *Soft Computing*. 2021; 25(16): 10771-10795.

[6] Duong TTT, Thao NX. TOPSIS model based on entropy and similarity measure for market segment selection and evaluation. *Asian Journal of Economics and Banking*. 2021; 5(2): 194-203. Available from: https://doi.org/10.1108/AJEB-12-2020-0106.

[7] Akram M, Garg H, Zahid K. Extensions of ELECTRE-I and TOPSIS methods for group decision-making under complex Pythagorean fuzzy environment. *Iranian Journal of Fuzzy Systems*. 2020; 17(5): 147-164.

[8] Akram M, Shumaiza, Arshad M. Bipolar fuzzy TOPSIS and bipolar fuzzy ELECTRE-I methods to diagnosis.

Computational and Applied Mathematics. 2020; 39: 1-21.
[9] Lin SS, Zhou A, Shen SL. Safety assessment of excavation system via TOPSIS-based MCDM modelling in fuzzy environment. *Applied Soft Computing*. 2023; 138: 110206.
[10] Kaya T, Kahraman C. Multicriteria decision making in energy planning using a modified fuzzy TOPSIS methodology. *Expert Systems with Applications*. 2011; 38(6): 6577-6585.
[11] Wang CN, Hsueh MH, Lin DF. Hydrogen power plant site selection under fuzzy multicriteria decision-making (FMCDM) environment conditions. *Symmetry*. 2019; 11(4): 596.
[12] Chen Y, Li B. Dynamic multi-attribute decision making model based on triangular intuitionistic fuzzy numbers. *Scientia Iranica*. 2011; 18(2): 268-274.
[13] Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8(3): 338-353. Available from: https://doi.org/10.1016/S0019-9958(65)90241-X.
[14] Bellman RE, Zadeh LA. Decision-making in a fuzzy environment. *Management Science*. 1970; 17(4): B141-B164.
[15] Atanassov KT, Stoeva S. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*. 1986; 20(1): 87-96.
[16] Atanassov KT. *Intuitionistic fuzzy sets*. Springer; 1999.
[17] Rouyendegh BD, Yildizbasi A, Arikan ÜZB. Using intuitionistic fuzzy TOPSIS in site selection of wind power plants in Turkey. *Advances in Fuzzy Systems*. 2018; 2018: 6703798. Available from: https://doi.org/10.1155/2018/6703798.
[18] Beskese A, Camci A, Temur GT, Erturk E. Wind turbine evaluation using the hesitant fuzzy AHP-TOPSIS method with a case in Turkey. *Journal of Intelligent & Fuzzy Systems*. 2020; 38(1): 997-1011.
[19] Zeng S, Zhou J, Zhang C, Merigó JM. Intuitionistic fuzzy social network hybrid MCDM model for an assessment of digital reforms of manufacturing industry in China. *Technological Forecasting and Social Change*. 2022; 176: 121435.
[20] Akram M, Luqman A, Alcantud JCR. Risk evaluation in failure modes and effects analysis: hybrid TOPSIS and ELECTRE I solutions with Pythagorean fuzzy information. *Neural Computing and Applications*. 2021; 33: 5675-5703.
[21] Kaur G, Yadav R, Majumder A. An efficient intuitionistic fuzzy approach for location selection to install the most suitable energy power plant. *Journal of Physics: Conference Series*. 2020; 1531(1): 012057. Available from: https://doi.org/10.1088/1742-6596/1531/1/012057.
[22] Alao MA, Popoola OM, Ayodele TR. A novel fuzzy integrated MCDM model for optimal selection of waste-to-energy-based-distributed generation under uncertainty: A case of the City of Cape Town, South Africa. *Journal of Cleaner Production*. 2022; 343: 130824. Available from: https://doi.org/10.1016/j.jclepro.2022.130824.
[23] Hoang TN, Ly TTB, Do HTT. A hybrid approach of wind farm site selection using Group Best-Worst Method and GIS-Based Fuzzy Logic Relations. A case study in Vietnam. *Environmental Quality Management*. 2022; 32(2): 251-267. Available from: https://doi.org/10.1002/tqem.21839.
[24] Kaur G, Majumder A, Yadav R. An efficient generalized fuzzy TOPSIS algorithm for the selection of the hybrid energy resources: A comparative study between single and hybrid energy plant installation in Turkey. *RAIRO-Operations Research*. 2022; 56(3): 1877-1899.
[25] Smarandache F. *Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis*. Rehoboth, NM: American Research Press; 1998.
[26] Smarandache F. Neutrosophic set-a generalization of the intuitionistic fuzzy set. *International Journal of Pure and Applied Mathematics*. 2005; 24(3): 287.
[27] Nie RX, Wang JQ, Zhang HY. Solving solar-wind power station location problem using an extended weighted aggregated sum product assessment (WASPAS) technique with interval neutrosophic sets. *Symmetry*. 2017; 9(7): 106.
[28] Rani P, Mishra AR, Krishankumar R, Ravichandran K, Kar S. Multi-criteria food waste treatment method selection using single-valued neutrosophic-CRITIC-MULTIMOORA framework. *Applied Soft Computing*. 2021; 111: 107657.
[29] Abdel-Basset M, Saleh M, Gamal A, Smarandache F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*. 2019; 77: 438-52.
[30] AbdelMouty AM. Decision making model for strategy choice in higher education under neutrosophic environment. *Journal of Neutrosophic and Fuzzy Systems*. 2023; 1(2): 99-106. Available from: https://doi.org/10.54216/JNFS.010206.
[31] Mahmood A, Abbas M, Murtaza G. Multi-Valued Multi-Polar Neutrosophic Sets with an application in Multi-Criteria Decision-Making. *Neutrosophic Sets and Systems*. 2023; 53(1): 32.

[32] Ouallane AA, Broumi S, Ayele ET, Bakali A, Bahnasse A, Talea M. Towards intelligent road traffic management based on neutrosophic logic: a brief review. *Neutrosophic Sets and Systems*. 2022; 51: 107-122.
[33] Voskoglou M, Broumi S, Smarandache F. A combined use of soft and neutrosophic sets for student assessment with qualitative grades. *Journal of Neutrosophic and Fuzzy Systems (JNFS)*. 2022; 4(1): 15-20.
[34] Zaher MA, Eldakhly NM. Integrated multi-criteria decision making via trapezoidal neutrosophic sets to evaluate the risks of failure mode. *Journal of Neutrosophic and Fuzzy Systems (JNFS)*. 2023; 5(1): 23-29. Available from: https://doi.org/10.54216/JNFS.050103.
[35] Debnath S. Single-valued neutrosophic covering-based rough set model over two universes and its application in MCDM. *Neutrosophic Sets and Systems*. 2023; 53(1): 29.
[36] Chen CT, Hung WZ. A MCDM method with linguistic variables and intuitionistic fuzzy numbers to evaluate product development projects. *International Journal of Computational Intelligence Systems.* 2021; 14(1): 935-945.
[37] Namin FS, Ghadi A, Saki F. A literature review of Multi Criteria Decision-Making (MCDM) towards mining method selection (MMS). *Resources Policy*. 2022; 77: 102676.
[38] Jahan A, Edwards K, Bahraminasab M. Multi-attribute decision-making for ranking of candidate materials. *Multi-criteria Decision Analysis for Supporting the Selection of Engineering Materials in Product Design*. Elsevier Ltd.; 2016. p.43-82. Available from: https://doi.org/10.1016/B978-0-08-100536-1.00005-9.
[39] Sotoudeh-Anvari A. The applications of MCDM methods in COVID-19 pandemic: A state of the art review. *Applied Soft Computing*. 2022; 126: 109238. Available from: https://doi.org/10.1016/j.asoc.2022.109238.
[40] Zolfani SH, Görener A, Toker K. A hybrid fuzzy MCDM approach for prioritizing the solutions of resource recovery business model adoption to overcome its barriers in emerging economies. *Journal of Cleaner Production*. 2023; 413: 137362. Available from: https://doi.org/10.1016/j.jclepro.2023.137362.
[41] Jamkhaneh EB, Nadarajah S. A new generalized intuitionistic fuzzy set. *Hacettepe Journal of Mathematics and Statistics*. 2015; 44(6): 1537-1551.
[42] Smarandache F. *A unifying field in Logics: Neutrosophic Logic*. American Research Press; 1999.
[43] Wang H, Smarandache F, Zhang Y, Sunderraman R. Single valued neutrosophic sets. *Infinite Study*. 2010; 12.
[44] Broumi S, Smarandache F. Correlation coefficient of interval neutrosophic set. *Neutrosophic Theory and Its Applications. Collected Papers*. 2013; 436: 511-517.
[45] Diakoulaki D, Mavrotas G, Papayannakis L. Determining objective weights in multiple criteria problems: The critic method. *Computers & Operations Research*. 1995; 22(7): 763-770.
[46] Yilmaz B, Harmancioglu N. Multi-criteria decision making for water resource management: a case study of the Gediz River Basin, Turkey. *Water SA*. 2010; 36(5): 563-576.
[47] Karaaslan F. *Correlation coefficient of neutrosophic sets and its applications in decision-making*. Springer; 2019.
[48] Masmali I, Ahmad A, Azeem M, Koam AN. MADM and assessment of pilot health projects based on spherical fuzzy information. *Neural Computing and Applications*. 2023: 35: 16619-16632. Available from: https://doi.org/10.1007/s00521-023-08533-w.
[49] Azeem M, Imran M, Nadeem MF. Sharp bounds on partition dimension of hexagonal Möbius ladder. *Journal of King Saud University-Science*. 2022; 34(2): 101779.
[50] Azeem M, Jamil MK, Shang Y. Notes on the localization of generalized hexagonal cellular networks. *Mathematics*. 2023; 11(4): 844.