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Research Article

Markovian Erlang Non-Constricted Single-Channel with Encouraged Arrival in Steady State with Balking, Feedback Strategy, and Customer Retention

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Abstract: In this article, we aim to provide a solution for the Markovian Erlang non-constricted queue that takes into account encouraged arrival, balking feedback strategy, and customer retention, all in a steady state. Our approach involved using an iterative technique to determine the probability of "n" customers in the system occupying stage "s", the probability of an empty system, and the efficiency of the queuing system. To illustrate the relationship between probability and these additional concepts, we present numerical data.

***Keywords*:** encouraged arrival, balking, non-constricted, feedback strategy, iterative technique

MSC: 60K25, 60K30, 60KM20, 90B22, 90B36

1. Introduction

Mathematical probability theory is a discipline that includes queuing theory. Agner Krarup Erlang, a Danish mathematician, and statistician who lived from 1878 to 1929, is credited with founding it just over a century ago through the publication of his papers and effective practical implementation. The system established by M/M/1 and the group of communication activities is named in his honor. The novel queuing theory has received tremendous encouragement for continued research as a result of its successful practical implementation. In addition, the requirements of the practice require transparent analytical strategies for non-Markovian, non-stationary queueing mechanisms with a permanent impetus in accordance with the Erlang standard.

The M/Er/1/k/N service Erlangian machine interference model with balking and reneging investigations in [1]. A/Ek/1 queuing system with working vacation was created in [2]. The cost and profit analysis of the M/Ek/1 queuing system with a removable service station was looked at in [3]. Regarding the queuing mechanism, M/Er/1/N was investigated in [4]. Performance analysis of a server start-up, unreliable server, and balking in an n-policy M/Ek/1 queuing system was explored in [5]. The most effective management strategy for the M/EK/1 queuing system with heterogeneous arrivals was examined in [6]. An analysis of a M/Ek/1 queuing system with n policies, numerous vacations, and balking was done in [7]. Two service rates for a M/Ek/1/N queuing-inventory system dependent on queue lengths were examined in [8]. The feedback of the M/M/1 queue with catastrophe, repair, and customer retention was

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investigated in [9] using a transient behavior technique.

The effects of reneging and balking on a truncated Erlang queuing system with bulk arrivals were examined in [10]. There has been research on an M/M/1/N queue system with encouraged arrivals [11]. Reduced Waiting Time in an M/M/1/N Encouraged arrival queue with feedback, balking, and reneged customer maintenance examined in [12]. Review of the queuing system investigated in [13].

Basic descriptions of queuing theory may be found in [14]. The study on balking, reneging, and retrial queues is described in [15-16]. Retention of reneged customers and balking in a non-truncated Erlang queuing system were also examined in [17].

My research hypothesis is to determine whether creating the iterative strategy to arrive at the analytical solution in steady-state for $M/E_r/1$ by including encouraged arrival with feedback, balking, and retention of reneged customers, Single servers with encouraged arrival time, finding out the number of increased customers arriving, and increasing system size for this model, this model is to apply for any real-life statistical application increasing profit for customers and the company. Why because considering the Erlang model with a single server in this case, a more effective result was provided compared to other models. Erlang distribution is a particular case of the gamma distribution model. The distribution is used in all engineering fields, queuing models, mathematical biology, and many other fields to model a variety of real-world applications.

The explicit probabilities that there are n customers in the system, that each customer in the service occupies a stage, and that the system is empty are derived using a recurrence relation. There are implicit specific examples and effectiveness metrics. Finally, to demonstrate the model's numerical implementation, a simulation study has been taken into consideration.

The non-truncated single-channel encouraged arrival is studied in this work. We propose the study of an M/Er/1-based analytical solution. An introduction is described in Section 1. Description of the model premises in Section 2. The M/Er/1 creation and evaluation of models are described in Section 3. There are expository examples in Section 4. Section 5 is described in the results. Section 6 is described in the limitations. Section 7 contains in conclusion.

2. Notations and main model premises

The following characteristics are defined to build the paper's mechanism:

p_{ns} = The steady-state condition probability n consumers in the system.

p_0 = The steady-state condition probability 0 consumers in the system.

c = The initial phase of the service.

s = The stages of service.

λ = Entering process.

ϖ = Encouraged (discount or offer) arrival.

μ = Service mechanism.

a = Reneging mechanism.

L_q = Expected number of systems in the queue.

In accordance with the Encouraged arrival process, customers approach the server one at a time $(1 + \varpi)$, where (Encouraged arrival) is the percentage change in the number of customers determined from the prior or clear vision. Encouraged arrival may produce a significant rush in the system, stressing the few available service facilities and leaving customers disappointed, who then rejoin the line to wait for satisfying service completion Assumed that $(1 - b)$ is the likelihood (probability) that a client with balking, $0 \leq b < 1$, $n \geq 1$; and $b = 1$, $n = 0$.

It is obvious that for stability, we must confirm our intuition that the encouraged arrival rate $(\lambda(1 + \varpi)$ must be lower than the service rate $(c\mu)$.

(i, e), $\rho < 1$. (Stability condition satisfied under the steady state condition).

As a result, it is apparent that: $\lambda(1 + \varpi)n = \{\lambda(1 + \varpi), n = 0, b\lambda(1 + \varpi), n \geq 1$.

1. Service time customers are treated in accordance with the FIFO discipline in an Erlangian-encouraged arrival queue with rate $\mu_n = k\mu$ service stages for each stage.

2. Following each service's completion, the customer joins the initial queue at the end as a feedback customer with

probability $(1 - q)$ or exits the system with probability q.

3. With probability, each consumer will wait a specific period after joining the queue before service begins $(1 - p)$. The probability that the customer will become impatient and exit the line before receiving service if service has not started by then is given by $(n - 1)$ a p, for $n \geq 2$.

3. Markovian Erlang- non-truncated single channel creation and evaluation of models

We derive the probability differential-difference equations using the notations and assumptions for the Markovian Erlang non-constricted queue with encouraged arrival with balking, feedback strategy, and customer retention as follows:

$$P_0'(t) = \lambda(1+\varpi)p_0(t) - qc\mu p_{11}(t) = 0,\ n = 0, \tag{1}$$

$$\left.\begin{aligned} &P_1'(t) = \left(qc\mu + b\lambda(1+\varpi)\right)p_{1s}(t) - qc\mu p_{11}(t) = 0,\ 1 \leq s \leq c-1 \\ &\left(qc\mu + b\lambda(1+\varpi)\right)p_{1c}(t) - \lambda(1+\varpi)p_0(t) - (qc\mu + ap)p_{21}(t) = 0,\ s = c \end{aligned}\right\},\ n = 1, \tag{2}$$

$$\left.\begin{aligned} &P_n'(t) = \left[qc\mu + (n-1)ap + b\lambda(1+\varpi)\right]p_{n,s}(t) - b\lambda(1+\varpi)p_{n-1,s}(t) - \\ &\qquad \left[qc\mu + (n-1)ap\right]p_{n,s+1}(t) = 0,\ 1 \leq s \leq c-1 \\ &\left[qc\mu + (n-1)ap + b\lambda(1+\varpi)\right]p_{n,c}(t) - b\lambda(1+\varpi)p_{n-1,c}(t) - \\ &\qquad \left[qc\mu + (n-1)ap\right]p_{n-1,k}(t) = 0,\ s = c \end{aligned}\right\},\ n \geq 2. \tag{3}$$

Steady state solution:

$$\lambda(1+\varpi)p_0 - qc\mu p_{11} = 0,\ n = 0, \tag{4}$$

$$\left.\begin{aligned} &\left(qc\mu + b\lambda(1+\varpi)\right)p_{1s} - qc\mu p_{11} = 0,\ 1 \leq s \leq c-1 \\ &\left(qc\mu + b\lambda(1+\varpi)\right)p_{1c} - \lambda(1+\varpi)p_0 - (qc\mu + ap)p_{21} = 0,\ s = c \end{aligned}\right\},\ n = 1, \tag{5}$$

$$\left.\begin{aligned} &\left[qc\mu + (n-1)ap + b\lambda(1+\varpi)\right]p_{n,s} - b\lambda(1+\varpi)p_{n-1,s} - \left[qc\mu + (n-1)ap\right]p_{n,s+1} = 0,\ 1 \leq s \leq c-1 \\ &\left[qc\mu + (n-1)ap + b\lambda(1+\varpi)\right]p_{n,c} - b\lambda(1+\varpi)p_{n-1,c} - \left[qc\mu + (n-1)ap\right]p_{n-1,k} = 0,\ s = c \end{aligned}\right\},\ n \geq 2. \tag{6}$$

We implement an iterative approach which is a mathematical procedure that employs an initial value to develop a series for enhancing approximate solutions for a set of problems, with the n-th approximation derived from the preceding ones as follows:

$$p_{11} = \delta_0 p_0,\ v_1 = \delta_0, \tag{7}$$

with

$$\delta_n = \begin{cases} \dfrac{\lambda(1+\varpi)}{qc\mu}, n = 0 \\ \dfrac{b\lambda(1+\varpi)}{qc\mu}, n \geq 1 \end{cases} \text{ and } \tau_n = \gamma_n + \frac{b\lambda(1+\varpi)}{qc\mu};\ \gamma_n = 1 + \frac{(n-1)ap}{qc\mu}. \tag{8}$$

From equations (5) & (8), we obtain

$$p_{1s} = \tau_1^{s-1} p_{11}. \tag{9}$$

Substitute equation (7) into (9), one can easily get

$$p_{1s} = \tau_1^{s-1} v_1 p_0,\ 1 \leq s \leq c-1, \tag{10}$$

$$p_{1c} = \tau_1^{c-1} v_1 p_0,\ s = c. \tag{11}$$

From the equation (5) and (8), we get;

$$p_{21} = \frac{1}{\gamma_2}\left(\tau_1^c p_{11} - v_1 p_0\right). \tag{12}$$

Substitute equation. (7) into (9), we get:

$$p_{21} = \frac{v_2}{\gamma_2} p_0, \tag{13}$$

with

$$v_2 = v_1\left(\tau_1^c - 1\right). \tag{14}$$

Using the eqn. (3) with n = 2 and eqn. (8), we find:

$$p_{2s} = \left(\frac{\tau_2}{\gamma_2}\right)^{s-1} p_{21} + \frac{\varepsilon_2}{\gamma_2}\left(\frac{\tau_2}{\gamma_2}\right)^{-1} p_1,\ 1 \leq s \leq c-1. \tag{15}$$

Substitute eqn. (10) and (13) in (15), one gets:

$$p_{2s} = \frac{1}{\gamma_2}\left[\left(\frac{\tau_2}{\gamma_2}\right)^{s-1} v_2 + v_1\varepsilon_2\left(\frac{\tau_2}{\gamma_2}\right)^{-1} \tau_1^{s-1}\right] p_0,\ 1 \leq s \leq c. \tag{16}$$

Let s = 1, 2, 3, …. (c – 1) in equation (16) it can be reduced by recursive manipulations that:

$$p_{2s}=\frac{1}{\gamma_2}\left[\left(\frac{\tau_2}{\gamma_2}\right)^{s-1}v_2-v_1\varepsilon_2\sum_{\eta_1=1}^{s-1}\left(\frac{\tau_2}{\gamma_2}\right)^{s-\eta_1-1}\tau_1^{\eta_1-1}\right]p_0,\ 1\le s\le c. \tag{17}$$

As before, eqn. (6) at $n = 2$ and eqn. (8), we get:

$$p_{31}=\frac{1}{\gamma_3}\left(\tau_2 p_{2c}-\varepsilon_2\tau_{1c}\right). \tag{18}$$

Substitute equation. (11) & (17) into (8) we get,

$$p_{31}=\frac{v_3}{\gamma_3}p_0, \tag{19}$$

with

$$v_3=\left(\frac{\tau_2}{\gamma_2}\right)^{c}v_2-v_1\varepsilon_2\left(\frac{\tau_2}{\gamma_2}\right)^{c-\eta_1}\tau_1^{\eta_1-1}. \tag{20}$$

Also, from equation. (4) to (6) with $n = 3$ and (8), we get,

$$p_{3s}=\left(\frac{\tau_3}{\gamma_3}\right)^{s-1}p_{31}+\frac{\varepsilon_3}{\gamma_3}\left(\frac{\tau_3}{\gamma_3}\right)^{-1}p_{2s},\ 1\le s\le c-1. \tag{21}$$

As previous, substitute eqn. (17) & (19) in (21) with $s = 1, 2, 3, \ldots. (c - 1)$ and by recursive method, we get

$$p_{3s}=\frac{1}{\gamma_3}\left[\left(\frac{\tau_3}{\gamma_3}\right)^{s-1}v_3-\frac{v_2\varepsilon_3}{\gamma_2}\sum\nolimits_{\eta_1=1}^{s-1}\left(\frac{\tau_3}{\gamma_3}\right)^{s-\eta_1-1}\left(\frac{\tau_2}{\gamma_2}\right)^{\eta_1-1}\right.$$

$$\left.+\frac{v_2\varepsilon_2\varepsilon_3}{\gamma_2}\sum\nolimits_{\eta_1=1}^{s-1}\tau_1^{\eta_1-1}\sum\nolimits_{\eta_1=1}^{s-1}\left(\frac{\tau_3}{\gamma_3}\right)^{s-\eta_2-1}\left(\frac{\tau_2}{\gamma_2}\right)^{\eta_2-\eta_1}\right]p_0,\ 1\le s\le c. \tag{22}$$

As well, from 5[nd] eqn. of (6) with $n = 3$ and eqn. (8), (17) & (21), we find,

$$p_{3s}=\frac{v_4}{\gamma_4}p_0, \tag{23}$$

$$u_4=\left[\left(\frac{\tau_3}{\gamma_3}\right)^{c}v_3-\frac{v_2\varepsilon_3}{\gamma_2}\sum_{\eta_1=1}^{c}\left(\frac{\tau_3}{\gamma_3}\right)^{c-\eta_1}\left(\frac{\tau_2}{\gamma_2}\right)^{\eta_1-1}+\frac{v_2\varepsilon_2\varepsilon_2}{\gamma_2}\sum_{\eta_1=1}^{c-1}\tau_1^{\eta_1-1}\sum_{\eta_1=1}^{c-1}\left(\frac{\tau_3}{\gamma_3}\right)^{c-\eta_2-1}\left(\frac{\tau_2}{\gamma_2}\right)^{\eta_2-\eta_1}\right]. \tag{24}$$

Also, from the 4[st] eqn. of (6) with $n = 3$ and eqn. (8), (22) and (23) with $s = 1, 2, 3, \ldots. (c - 1)$ and by iterative method, we get

$$p_{4s}=\frac{1}{\gamma_4}\left[\left(\frac{\tau_4}{\gamma_4}\right)^{s-1}v_4p_0-\frac{v_3\varepsilon_4}{\gamma_3}\sum_{\eta_1=1}^{s-1}\left(\frac{\tau_4}{\gamma_4}\right)^{s-\eta_1-1}\left(\frac{\tau_3}{\gamma_3}\right)^{\eta_1-1}+\frac{v_2\varepsilon_3\varepsilon_4}{\gamma_2\gamma_3}\sum_{\eta_1=1}^{s-2}\left(\frac{\tau_2}{\gamma_2}\right)^{\eta_1-1}\sum_{\eta_2=\eta_1}^{s-2}\left(\frac{\tau_4}{\gamma_4}\right)^{s-\eta_2-2}\left(\frac{\tau_3}{\gamma_3}\right)^{\eta_2-\eta_1}\right.$$

$$\left.+\frac{v_1\varepsilon_2\varepsilon_3\varepsilon_4}{\gamma_2\gamma_3}\sum_{\eta_1=1}^{s-3}\tau_1^{\eta_1-1}\sum_{\eta_2=\eta_1}^{s-3}\left(\frac{\tau_4}{\gamma_4}\right)^{s-\eta_3-1}\left(\frac{\tau_3}{\gamma_3}\right)^{\eta_3-\eta_2-3}\right]p_0. \tag{25}$$

From 5nd eqn. of (6) with $n = 4$ & eqn. (8), (22) and (25), we get

$$p_{5s}=\frac{v_5}{\gamma_5}p_0, \tag{26}$$

$$u_5=\left[\left(\frac{\tau_4}{\gamma_4}\right)^{c}v_4-\frac{v_3\varepsilon_4}{\gamma_3}\sum_{\eta_1=1}^{c}\left(\frac{\tau_4}{\gamma_4}\right)^{c-\eta_1}\left(\frac{\tau_3}{\gamma_3}\right)^{\eta_1-1}+\frac{v_2\varepsilon_3\varepsilon_4}{\gamma_2\gamma_3}\sum_{\eta_1=1}^{c-2}\tau_1^{\eta_1-1}\sum_{\eta_2=\eta_1}^{c-2}\left(\frac{\tau_2}{\gamma_2}\right)^{c-\eta_3-2}\left(\frac{\tau_3}{\gamma_3}\right)^{\eta_3-\eta_2}\right]. \tag{27}$$

Finally, from the 4st eqn. of (6) with $n = 5$ & eqn. (8), (25) & (26) with $s = 1, 2, 3, \ldots. (c - 1)$ and by iterative method, we get

$$p_{5s}=\frac{1}{\gamma_5}\left[\left(\frac{\tau_5}{\gamma_5}\right)^{s-1}v_5\right.$$

$$-\frac{v_4\varepsilon_5}{\gamma_4}\sum_{\eta_1=1}^{s-1}\left(\frac{\tau_5}{\gamma_5}\right)^{s-\eta_1-1}\left(\frac{\tau_4}{\gamma_4}\right)^{\eta_1-1}+\frac{v_3\varepsilon_4\varepsilon_5}{\gamma_3\gamma_4}\sum_{\eta_1=1}^{s-2}\left(\frac{\tau_5}{\gamma_5}\right)^{\eta_1-1}\sum_{\eta_2=\eta_1}^{s-2}\left(\frac{\tau_5}{\gamma_5}\right)^{s-\eta_2-2}\left(\frac{\tau_4}{\gamma_4}\right)^{\eta_2-\eta_1}$$

$$\left.+\frac{v_1\varepsilon_2\varepsilon_3\varepsilon_4\varepsilon_5}{\gamma_2\gamma_3\gamma_4}\sum_{\eta_1=1}^{s-4}\tau_1^{\eta_1-1}\sum_{\eta_2=\eta_1}^{s-4}\left(\frac{\tau_2}{\gamma_2}\right)^{\eta_2-\eta_1}\sum_{\eta_3=\eta_2}^{s-4}\left(\frac{\tau_3}{\gamma_3}\right)^{\eta_3-\eta_2}\sum_{\eta_3=\eta_2}^{s-4}\left(\frac{\tau_5}{\gamma_5}\right)^{s-\eta_4-4}\left(\frac{\tau_4}{\gamma_4}\right)^{\eta_2-\eta_2}\right]p_0,$$

$$1\le s\le c, \tag{28}$$

$$p_{ns}=O_{ns}p_0,\ n\ge 1,\ 1\le s\le c. \tag{29}$$

Where

$$O_{ns}=\frac{1}{\gamma_n}\left\{\left(\frac{\tau_n}{\gamma_n}\right)^{s-1}v_n+\sum_{j=0}^{n-2}(-1)^{j+1}v_{n-j-1}*\right.$$

$$\left[\prod_{i=0}^{j}\left(\frac{\varepsilon_{n-i}}{\gamma_{n-i-1}}\sum_{\eta_{i+1}=\eta_1}^{s-j-1}\left(\frac{\tau_{n-i-1}}{\gamma_{n-i-1}}\right)^{\eta_{i+1}-\eta_i}\right)\right]\left(\frac{\tau_n}{\gamma_n}\right)^{s-j-\eta_{j+1}-1}\Bigg\},\ \eta_0=i,\ 1\le s\le c,\ n\ge 1. \tag{30}$$

With

$$v_n=\begin{cases}\varepsilon_0, & n=1,\\ v_1\left(\tau_1^c-1\right), & n=2,\\ \left(\frac{\tau_{n-1}}{\gamma_{n-1}}\right)^c v_{n-1}+\sum_{j=0}^{n-3}(-1)^{j+1}v_{n-j-2}*\left(\frac{\tau_n}{\gamma_n}\right)^{s-1}v_n & \\ +\sum_{j=0}^{n-2}(-1)^{j+1}v_{n-j-1}*\left[\prod_{i=0}^{j}\left(\frac{\varepsilon_{n-i}-1}{\gamma_{n-i-2}}\sum_{\eta_{i+1}=\eta_1}^{c-j}\left(\frac{\tau_{n-i}-2}{\gamma_{n-i-1}}\right)^{\eta_{i+1}-\eta_i}\right)\right]\left(\frac{\tau_{n-1}}{\gamma_{n-1}}\right)^{c-j-\eta_{j+1}}, & n\ge 3.\end{cases} \tag{31}$$

From eqn. (29) and by the normalization condition i,e., in probability theory, a normalizing constant or normalizing factor is used to transform any probability function to a probability density function with a total probability of one which is given as

$$p_0+\sum_{n=1}^{\infty}p_{ns}=1.$$

The expected number of system and the queue is we obtain,

$$L=\sum_{n=1}^{\infty}nO_{ns}p_0, \tag{32}$$

$$L_q=L+p_0-1. \tag{33}$$

4. An expository example

Table 1. The value of p_0, p_{ns}, L and L_q

B	$\lambda(1+\varpi)$	μ	Q	α	p	p_0	p_{21}	p_{22}	p_{23}	p_{24}	L	L_q
0.1	5.5	8	0.9	1	0.7	0.97	0.0516	0.0784	0.1375	0.2324	0.8577	0.8277
0.3	6.6	9	0.8	2	0.6	0.86	0.1858	0.5637	2.3947	10.7159	4.7310	4.5910

Assume the values $n=2$ units, $c=4$ phase of service. The solution of p_0, p_{ns}, L, L_q for different values of β, λ, ϖ, μ, α,

… q, and p are calculated by using MATLAB software with system configurations 1 TB storage, 16 GB RAM, and 64-bit as shown in Table 1.

The result of the model will be determined most readily by plotting p_0, p_{ns}, L and L_q are drawn in β, $\lambda(1 + \varpi)$, μ, α, … q and p as given in Figures 1, 2, 3, 4, 5 and 6 respectively.

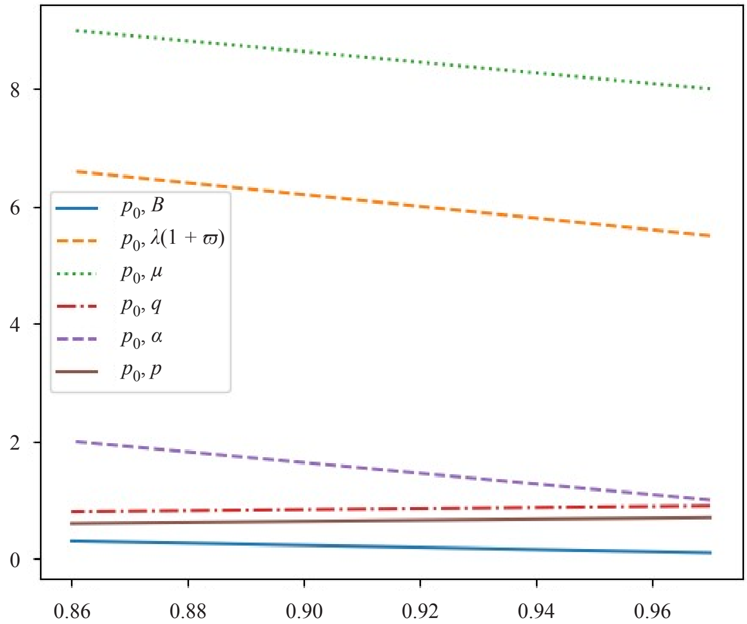

Figure 1. The relation between p_0 & (β $\lambda(1 + \varpi)$, μ, α, … q)

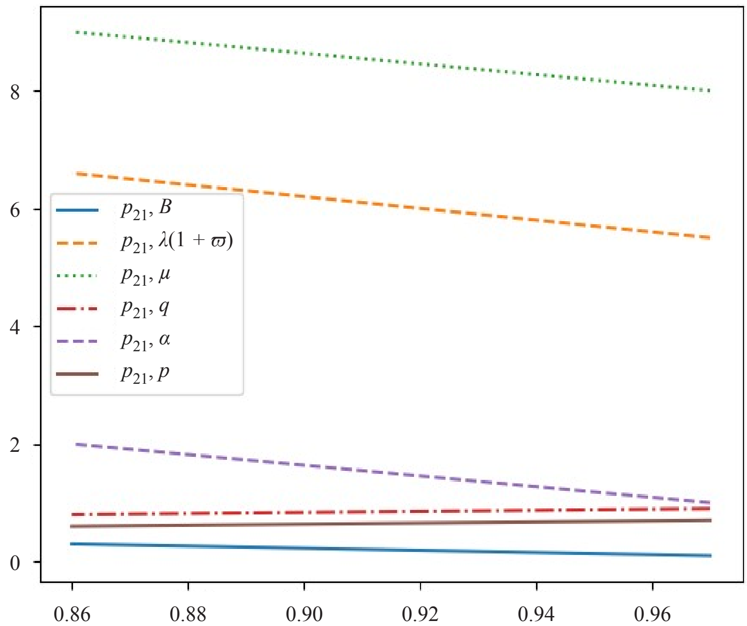

Figure 2. The relation between p_{21} & (β $\lambda(1 + \varpi)$, μ, α, … q)

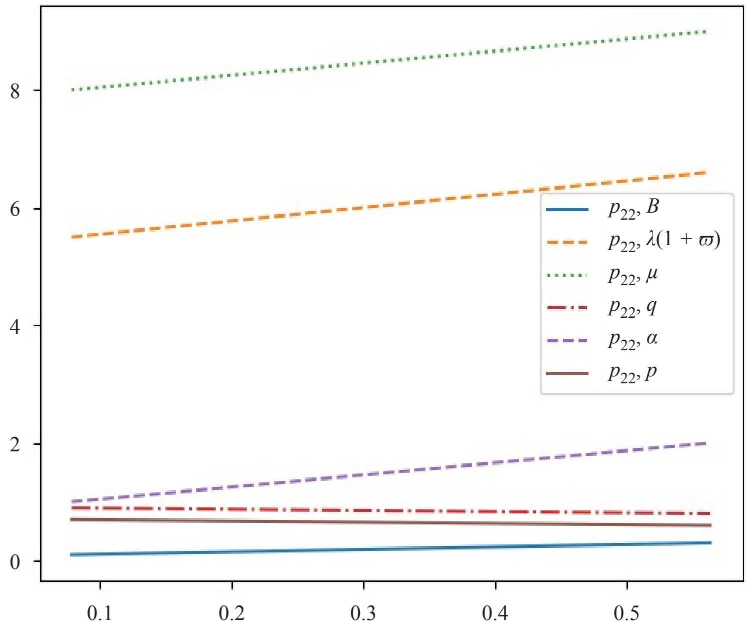

Figure 3. The relation between p_{22} & ($\beta\ \lambda(1+\varpi)$, μ, α, … q)

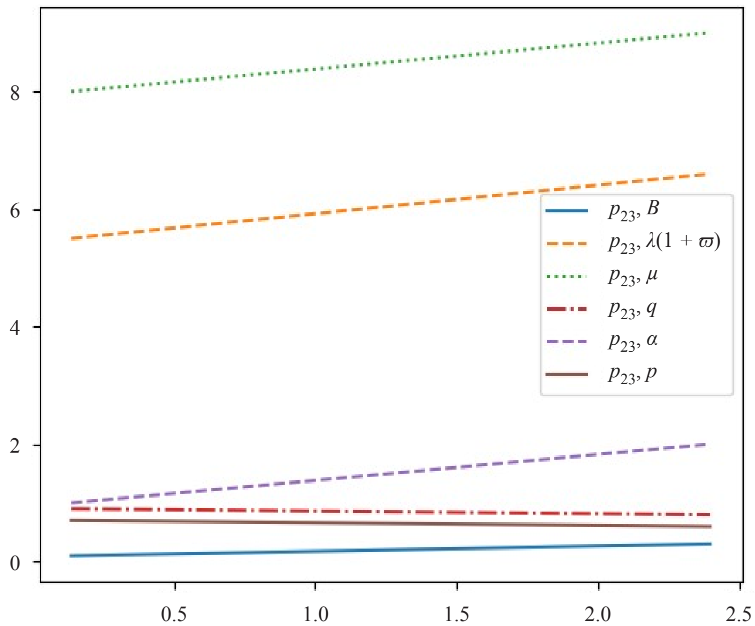

Figure 4. The relation between p_{23} & ($\beta\ \lambda(1+\varpi)$, μ, α, … q)

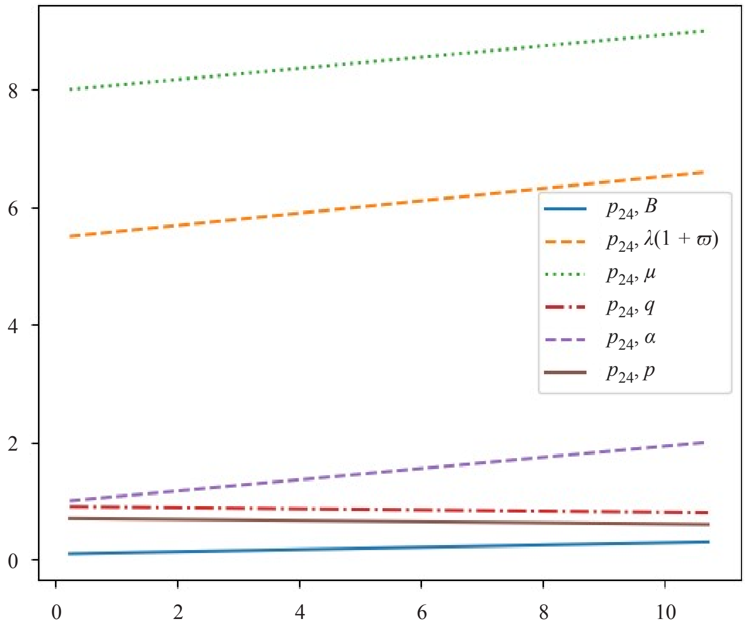

Figure 5. The relation between p_{24} & $(\beta\, \lambda(1+\varpi), \mu, \alpha, \ldots q)$

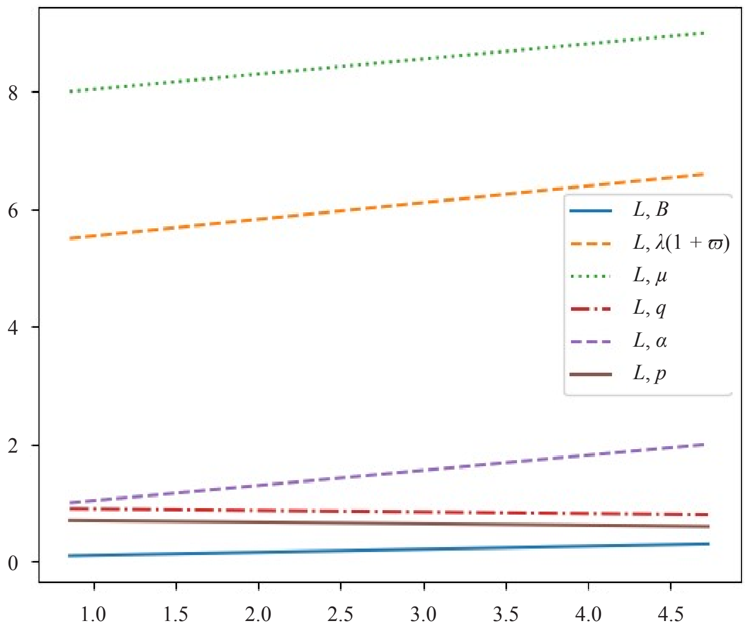

Figure 6. The relation between L & $(\beta\, \lambda(1+\varpi), \mu, \alpha, \ldots q)$

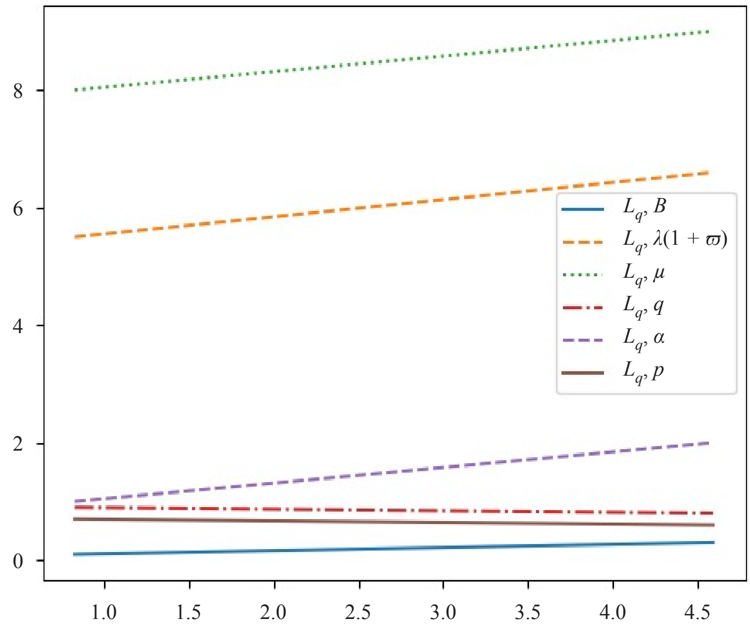

Figure 7. The relation between L_q & $(\beta\, \lambda(1+\varpi), \mu, \alpha, \ldots q)$

Table 2. The comparative of Poisson and Encouraged arrival L and L_q

Poisson arrival (L)	Encouraged arrival (L) under 10% discount	Poisson arrival (L_q)	Encouraged arrival (L_q) under 10% discount
0.061	0.8577	0.031	0.8277
0.274	4.7310	0.137	4.5910

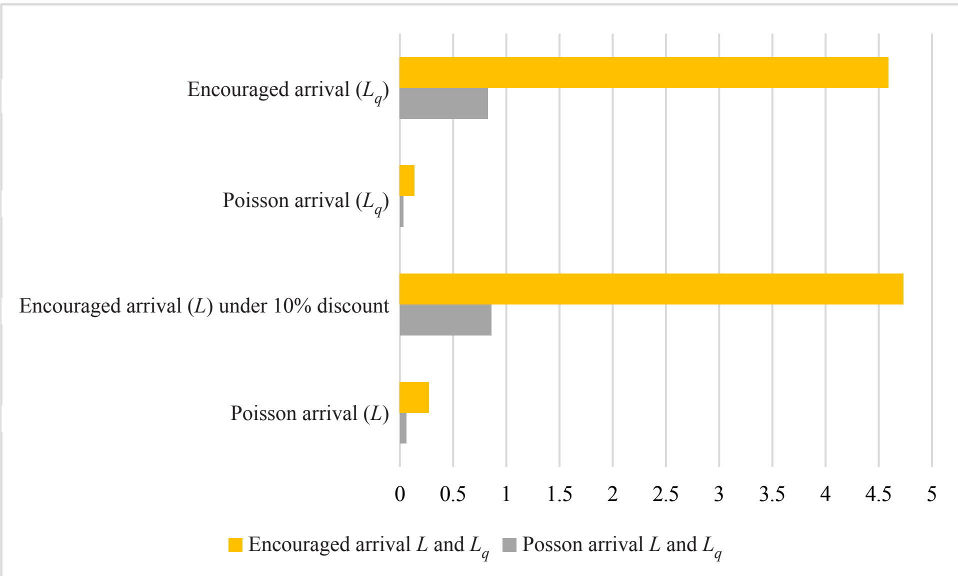

Figure 8. Comparative of Poisson and Encouraged arrival L and L_q

From Table 2, with 10% discounts of encouraged arrival, the number of arrivals is more the Poisson arrival Process.

5. Results

• As shown in Graph 1, increasing both (encouraged arrival rate, service rate, balking, and reneging) and decreasing both decreases the probability that there are no consumers in the system (feedback approach and retention of reneged customers).

• According to graphs 2, 3, 4, and 5 in that order, a rise in the encouraged arrival rate, service rate, balking rate, and reneging rate counteracts a rise in the probability that there are n consumers in the system.

• Graph 6 also demonstrates that the increase in the anticipated number of customers in the system was countered by the more increased encouraged arrival rate, service rate, balking, and reneging as well as the more diminished feedback strategy and retention of reneged clients.

• In Graph 7, it is clear that the increase in the line's projected number of customers was countered by rising encouraged arrival rates, service rates, rates of balking and reneging, as well as declining encouraged feedback strategies and rates of keeping reneged clients.

• In Graph 8, it is clear that the encouraged arrival are more effective comparative of Poisson arrival condition.

• According to Table 2, encouraged arrival, with a maximum 10% discount applicable in this model. Because the system size maximum has increased in this research model comparative of Poisson arrival.

6. Limitations

• This conception is only applicable to the Markovian Erlang non-constricted single-channel queuing model.

• This conception is an increased system size of the $M/E_r/1$ queuing model

• This conception applies to all real-world applications implementing a single service channel.

• Real-world applications are constantly limited in capacity.

7. Conclusion

This study created the iterative strategy to arrive at the analytical solution in steady-state for $M/E_r/1$ by including encouraged arrival with feedback, balking, and retention of reneged customers, and also, we computed the probability (i) that there are n customers in the system. (ii) The consumer is in the service phase (iii) the possibility of no customers in the service department; (iv) the expected number of clients in the system and (v) the expected number of customers in the queue. The introduced model was given a numerical example, which validates it. The probability that there are "n" customers in the system and that the customer in service is in phase S has been determined to be increasing along with the utilization factor, balking, and retention of impatient customers. In this model, encouraged arrival, with a maximum 10% discount applicable. Because the system size maximum has increased in this research model, we can expect a maximum profit. In addition, it was discovered that there would be no units in the system as the utilization factor, balking, and retention of impatient customers grew. When we apply this research concept to all statistical real-life applications. This work may be extended in the future to include a quality control approach and then apply a stock market industry. The difficulty of the work is equations term setup in MATLAB is very complex because all the individual term is different.

Conflict of interest

The authors declare no competing financial interest.

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