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Internally Disclosed Sets of a Real Space

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Abstract: There is no single set in a real space, for which an exact mathematical definition would not exist by the mathematical symmetry laws. We discuss a theory in which a real number axis is defined at the new level, namely, at the level of real space allowing one to formulate and prove the theorem on the basis of its internal disclosure. This new theory of a set makes it possible to introduce a notion of the full compactness of sets of a real space, confirming their availability in the defined symmetry of elements of a definitely symmetrical line.

***Keywords*:** real space, algebraical logic, geometrical logic, mathematically united logic, regular sets, casual sets, real number axis, full compactness of a set, logic of the commutativity law, full finiteness of a set

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1. Introduction

The mathematical notion of sets is undoubtedly based only on the unity of all types of mathematical symmetries and not on the absence of this latently internal unification. Therefore, each set [1, 2] comes forward in the first-initial theory [3] as a unified whole.

A feature of this structure is, as will be seen, such that we can strictly define the mathematical notion of sets if and only if the very space in which they exist, as the structural subspaces separates them into two groups by the symmetry laws. The first group includes internally disclosed sets, each of which corresponds to one pair of algebraical and geometrical objects of latent unification. The second group consists of internally undisclosed sets in which there is no single object of unification. This classification, as we shall see below, gives the exact mathematical definitions of fully regular and casual sets, a real space, and the latent algebraical object of unification.

Thereby, one must follow the mathematical logic of the commutativity law [4] in a real space at the new level, namely, at the level of the suggested theory of a set from the viewpoint of mutually crossing curved lines of images of a selected pair of elements of a set.

It is here that we must, for the first time, derive the two pairs of relations such that we can formulate eight more new definitions, two new axioms, ten new lemmas, and two new theorems, including a discussion of their proof, and everything that is connected to a latent geometrical object of unification within a set of a real space. A logical set of 12 new mathematical concepts thus established reflects so far unknown structural properties of the real objects. Thereby, it

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becomes possible to introduce an Appendix, recognizing that we must not confuse the names. Our purpose is to shed light on what a real space is and its mathematical structure.

2. Preliminaries

One structural feature of investigated types of sets of a real space is, according to our description [5], the mathematical notion of what unites all elements of each of them in a unified whole as an algebraical object of latent unification.

Definition 1 A latent algebraical object of unification of one set is the second set, such that it consists of conserving sizes of the same defined symmetry of elements of both sets.

We must not confuse the sets with and without an object of unification of their elements. A class with some latent object of unification of its elements refers to the set of internal disclosure. A class without any latent object of unification of its elements refers to the set of internal undisclosure. In both sets, an empty class [6] not containing any element is necessarily present.

To express the idea more clearly, one must refer to a set of forest birds of different forms forming an answer signal to the defined tones. However, this class cannot exist at the level of a forest for a long time without permutation of elements. It can individually pass from a casual set of uncertainty into some disclosed sets of defined symmetry. Of course, in each of them, the crucial role of the kind of object of unification of its elements appears.

Definition 2 The sets are called fully regular or internally disclosed ones if each of them corresponds to one pair of the algebraical and geometrical objects of latent unification of its elements.

Definition 3 The sets are called fully casual or internally undisclosed ones if none of them has neither the algebraical nor geometrical objects of latent unification of its elements.

An empty set $\varnothing$, namely, a set $\{\}$ comes forward in a set of the internal disclosure as one single empty subset, confirming that it is fully compatible with an object of unification of elements of each of all the remaining subsets. The existence of an object of unification for $\varnothing$ would indicate that it is not empty.

For example, insofar as a set $\{a\}$ consisting of one element a is concerned, a latent algebraical object of unification for this class may be a set $\{1\}$ consisting of one unity. This correspondence principle, which may symbolically be written as

$$\begin{cases} \varnothing, \\ \nexists, \end{cases} \qquad \begin{cases} \{a\}, \\ \{1\}, \end{cases} \tag{1}$$

does not imply the absence, for each of $\{a\}$ and $\{1\}$, of a kind of internally disclosed set in which it has existed as one of its nonempty subsets.

Lemma 1 (Theorem on the smallest nonempty set) No single fully regular set from the same element exists without an internally disclosed set of a higher cardinality.

We thus have some analogy to the fact that a connection between two or many variables indicates a latent system of equations with the same unknowns. These objects may, according to Lemma 1, be, for example, a set from the two elements and what unites them as a unified whole. They constitute here the united system of two sets from two elements of the same defined symmetry.

One group is exactly the same as the first-initial given class. A good example of this is a linearly ordered set

$$A = \{a_{1B},\ \ b_{1B}\} \tag{2}$$

in which either $a_{1B} < b_{1B}$ or $b_{1B} < a_{1B}$.

Each element here must distinguish itself from another element by an individual number of a conserving size $\mathcal{B}$ of the defined B symmetry

$$\mathcal{B} = \begin{cases} +1 & \text{for} \quad a_{1B}, \\ -1 & \text{for} \quad b_{1B}, \\ \nexists & \text{for} \quad \text{remaining objects}, \end{cases} \tag{3}$$

the violation of which within a set (2) expressed as

$$\sum \mathcal{B} \neq const \tag{4}$$

would imply its internal undisclosure. However, according to a constancy law of the sum

$$\sum \mathcal{B} = const, \tag{5}$$

this would take place only in the case of the presence in a set A of one or more elements from those internally disclosed classes in which there is B symmetry. Therefore, the second set

$$\mathcal{B} = \{1, \quad -1\} \tag{6}$$

corresponds in (2) to a kind of defined B symmetry. In other words, a set (2) and an object of unification of its elements (6) constitute the system of the two classes of two elements of the same B symmetry

$$\begin{cases} A = \{a_{1B}, \quad b_{1B}\}, \\ \mathcal{B} = \{1, \quad -1\}. \end{cases} \tag{7}$$

Thus, only part of a general system of internally disclosed sets among which coexistence appears is obtained. As a consequence, any of the classes of our system (7) indicates the availability of each of sets A and $\mathcal{B}$ even if one of them remains latent. Therefore, to follow the algebraical [7] logic of latent unification of elements within each of internally disclosed sets of a system (7) from the viewpoint of the defined B symmetry, one must present (3) in the form of a set such as

$$\mathcal{B} = \begin{cases} \{+1\} & \text{for} \quad \{a_{1B}\}, \\ \{-1\} & \text{for} \quad \{b_{1B}\}, \\ \varnothing & \text{for} \quad \text{remaining subclasses}. \end{cases} \tag{8}$$

For completeness, we include in this discussion the internally disclosed set of another defined symmetry, namely, a set

$$C = \{a_{2L}, \quad b_{2L}\} \tag{9}$$

of the defined L symmetry with a conserving size

$$\mathcal{L} = \begin{cases} +1 & \text{for} \quad a_{2L}, \\ -1 & \text{for} \quad b_{2L}, \\ \nexists & \text{for} \quad \text{remaining objects}, \end{cases} \tag{10}$$

at which it has a linear order such that either $a_{2L} < b_{2L}$ or $b_{2L} < a_{2L}$.

By following those stages that led to (6) but with a view of the constancy law of the sum

$$\sum \mathcal{L} = const \tag{11}$$

expressing the internal disclosure of a set (9) one can find from (10) that the class

$$\mathcal{L} = \{1, \ -1\} \tag{12}$$

corresponding in (9) to a kind of defined L symmetry is a fully regular algebraical object of unification of its elements. Together, they constitute a system of two sets from two elements of the same L symmetry

$$\begin{cases} C = \{a_{2L}, \ b_{2L}\}, \\ \mathcal{L} = \{1, \ -1\}. \end{cases} \tag{13}$$

As well as in (7), one set here must itself be an object of unification of the elements of the second set. This connection in system (13) will correspond to the coexistence of both of its classes. Thereby, one must follow the algebraical [7] logic of latent unification of elements within each of internally disclosed sets of a system (13) from the viewpoint of the defined L symmetry not presented in (10) in the form of a set

$$\mathcal{L} = \begin{cases} \{+1\} & \text{for} \quad \{a_{2L}\}, \\ \{-1\} & \text{for} \quad \{b_{2L}\}, \\ \varnothing & \text{for} \quad \text{remaining subclasses}. \end{cases} \tag{14}$$

Thus, we have established one more part of a general system of internally disclosed sets, in which the role of unity among all types of symmetries appears. Therefore, it is not surprising that if

$$\sum (\mathcal{B} + \mathcal{L}) = const \tag{15}$$

holds, then, for example, classes (2) and (9) constitute another set

$$D = \{A, \ C\} \tag{16}$$

such that a constancy law of the sum (15) would seem to transform it from (16) into

$$\mathcal{D} = \{\mathcal{A},\ \ \mathcal{C}\} \tag{17}$$

with subsets

$$\mathcal{A} = \{a_{1B},\ \ a_{2L}\},\ \ \mathcal{C} = \{b_{1B},\ \ b_{2L}\}. \tag{18}$$

Of course, in both presentations (16) and (17), as we can expect from the foregoing discussion, neither of the objects (6) and (12) is forced to change its structure. Thus, there exists a set of the two types of symmetries, namely, a set of the two types of conserving sizes

$$\mathcal{E} = \{\mathcal{B},\ \ \mathcal{L}\}. \tag{19}$$

This united class in turn characterizes each pair of elements or subclasses of a set (16), respectively, by an individual number

$$\mathcal{E} = \begin{cases} +1 & \text{for} \quad a_{1B},\ \ a_{2L}, \\ -1 & \text{for} \quad b_{1B},\ \ b_{2L}, \\ \nexists & \text{for} \quad \text{remaining objects} \end{cases} \tag{20}$$

or, according to Lemma 1, by an individual subset

$$\mathcal{E} = \begin{cases} \{+1\} & \text{for} \quad \{a_{1B}\},\ \ \{a_{2L}\}, \\ \{-1\} & \text{for} \quad \{b_{1B}\},\ \ \{b_{2L}\}, \\ \varnothing & \text{for} \quad \text{remaining subclasses} \end{cases} \tag{21}$$

and thereby describes a situation where

$$\mathcal{E} = \{1,\ \ -1\}. \tag{22}$$

The question as to what is a latent object of unification within each subclass from (18) still has no unequivocal answer.

A given circumstance indicates that the same latent algebraical object (22) that unites the elements of each subclass from (2) and (9) does not, by itself, unite the elements of any subset from (18), and the class (17) having a linear order is of those linearly ordered sets, in each of which each pair of elements establishes one pair of relations such that its internal structure encounters the condition of the commutativity law and requires explanation from the point of view of the interratio of the very set and the set of images of its elements. In our case, the expected relations relate one element within any subset from (18) to another of its elements as a latent geometrical object of their unification.

However, we cannot establish them until the very real space is able to formulate the theorems on the internal disclosure and undisclosure algebraical logic of sets.

3. An internal disclosure algebraical logic of sets of a real space

We have presented arguments in favor of the definition of the number axis [8, 9] at the new level, namely, at the level of real space, where it coincides with a real axis, allowing one to formulate and prove the theorem on an internal disclosure of a real number axis. But we can present it if and only if the very notion of real space has an exact mathematical definition expressing the ideas of the symmetry laws about that to each type of positive (negative) number of a real axis corresponds a kind of right (left) point.

Definition 4 A nonempty space is called a real one if and only if it consists of the right and left points of infinitely many selected systems of real axes with a general center of symmetry.

Definition 5 A number axis is called a real one if it has right and left real points relative to its center of symmetry.

Theorem 1 To each pair of right and left real points corresponds one pair of objects such that together they constitute a system of two sets from two elements of a really defined symmetry.

Proof of Theorem 1. A real number axis

$$..., -(n+1),\ \ -n, ..., -3,\ \ -2,\ \ -1,\ \ 0,\ \ 1,\ \ 2,\ \ 3, ..., n,\ \ (n+1), ..., \tag{23}$$

according to Theorem 1, is created such that a kind of object corresponds to each point. This gives the way to characterize any of its right points

$$1,\ \ 2,\ \ 3, ..., n,\ \ (n+1), ... \tag{24}$$

by a corresponding object from

$$a_{1B},\ \ a_{2L},\ \ a_{3X}, ..., a_{nY},\ \ a_{(n+1)Z}, ..., \tag{25}$$

where each index of a distinction from $E = B,\ L,\ X, ...,\ Y,\ Z, ...$ must be considered as one of all types of symmetries of a real space.

In the same way, one can describe the left points

$$-1,\ \ -2,\ \ -3, ..., -n,\ \ -(n+1), ... \tag{26}$$

by their own objects

$$b_{1B},\ \ b_{2L},\ \ b_{3X}, ..., b_{nY},\ \ b_{(n+1)Z}, ... \tag{27}$$

such that any index E distinguishes one pair of objects of (25) and (27) from all others.

However, the fact that the very real number axis does not exclude the symmetry with respect to its middle point 0 indicates the role of the unified principle in all systems of its objects. One object of each pair must therefore be distinguishable from another of its objects by an individual number of a conserving size of one of all types of symmetries, the unity of which constitutes the united symmetry such that its existence in a real space is according to Theorem 1.

To show this, one must refer to one of the possible groupings of (24) and (26), because it suggests a formula

$$1,\quad -1,\quad 2,\quad -2,\quad 3,\quad -3, ..., n,\quad -n,\quad n+1,\quad -(n+1), ... \tag{28}$$

In a similar way, one can group (25) and (27) and find the sequence

$$a_{1B},\quad b_{1B},\quad a_{2L},\quad b_{2L},\quad a_{3X},\quad b_{3X}, ..., a_{nY},\quad b_{nY},\quad a_{(n+1)Z},\quad b_{(n+1)Z}, ... \tag{29}$$

Taking into account that to any set of points

$$\{1,\quad -1\},\quad \{2,\quad -2\},\quad \{3,\quad -3\}, ..., \{n,\quad -n\},\quad \{(n+1),\quad -(n+1)\}, ... \tag{30}$$

corresponds a kind of set of objects

$$\{a_{1B},\quad b_{1B}\},\quad \{a_{2L},\quad b_{2L}\},\quad \{a_{3X},\quad b_{3X}\}, ..., \{a_{nY},\quad b_{nY}\},\quad \{a_{(n+1)Z},\quad b_{(n+1)Z}\}, ... \tag{31}$$

accepting 1 and -1 as the individual numbers of each conserving size $\mathcal{E} = \mathcal{B},\ \mathcal{L},\ \mathcal{X}, ...,\ \mathcal{Y},\ \mathcal{Z}, ...$ of all types of symmetries E corresponding in a real space to the existence of one-to-one correspondence between the series (30) and

$$\{1,\quad -1\},\quad \{1,\quad -1\},\quad \{1,\quad -1\}, ..., \{1,\quad -1\},\quad \{1,\quad -1\}, ..., \tag{32}$$

it is not difficult to establish the following systems of sets:

$$\begin{cases} G = \{a_{3X},\quad b_{3X}\}, \\ \mathcal{X} = \{1,\quad -1\}, \end{cases} \tag{33}$$

$$\begin{cases} K = \{a_{nY},\quad b_{nY}\}, \\ \mathcal{Y} = \{1,\quad -1\}, \end{cases} \tag{34}$$

$$\begin{cases} T = \{a_{(n+1)Z},\quad b_{(n+1)Z}\}, \\ \mathcal{Z} = \{1,\quad -1\}, \end{cases} \tag{35}$$

The logical basis of this presentation is such that all systems of this general system may not be written completely. However, insofar as its first two systems are concerned, they are, of course, exactly the same as (7) and (13), and therefore, according to

$$\sum \mathcal{X} = const, ..., \sum \mathcal{Y} = const,\quad \sum \mathcal{Z} = const, ..., \tag{36}$$

one set comes forward in any system of (33)–(35) as an object of latent unification of elements of the second set.

Thus, we have established a general system of internally disclosed sets

$$\begin{cases} R = \{A,\ \ C,\ \ G, ..., K,\ \ T, ...\}, \\ \mathcal{E} = \{\mathcal{B},\ \ \mathcal{L},\ \ \mathcal{X}, ..., \mathcal{Y},\ \ \mathcal{Z}, ...\} \end{cases} \tag{37}$$

in which it is definitely stated that a set (19) includes only the first and second subsets $\mathcal{B}$ and $\mathcal{L}$ of the united set $\mathcal{E}$ consisting of conserving sizes of all types of symmetries. Their unity constitutes in a real space the same defined symmetry E with the same conserving size $\mathcal{E}$, thus confirming the validity of Theorem 1 and all implications implied from it. □

On this basis, Theorem 1 itself characterizes each element or subset of a set R of a general system (37), respectively, by an individual real number

$$\mathcal{E} = \begin{cases} +1 & \text{for} \quad a_{1B},\ \ a_{2L},\ \ a_{3X}, ..., a_{nY},\ \ a_{(n+1)Z}, \cdots, \\ -1 & \text{for} \quad b_{1B},\ \ b_{2L},\ \ b_{3X}, ..., b_{nY},\ \ b_{(n+1)Z}, \cdots, \\ \nexists & \text{for} \quad \text{remaining objects} \end{cases} \tag{38}$$

or, according to Lemma 1, by an individual subset

$$\mathcal{E} = \begin{cases} \{+1\} & \text{for} \quad \{a_{1B}\},\ \ \{a_{2L}\},\ \ \{a_{3X}\}, ..., \{a_{nY}\},\ \ \{a_{(n+1)Z}\}, \cdots, \\ \{-1\} & \text{for} \quad \{b_{1B}\},\ \ \{b_{2L}\},\ \ \{b_{3X}\}, ..., \{b_{nY}\},\ \ \{b_{(n+1)Z}\}, \cdots, \\ \varnothing & \text{for} \quad \text{remaining subclasses} \end{cases} \tag{39}$$

such that a constancy law of the sum

$$\sum \mathcal{E} = const \tag{40}$$

expressing its internal disclosure is never violated.

This is exactly the same as when a notion of an algebraical disclosure of a set in a real space is based logically on the really defined symmetry of elements. Consequently, a notion of an algebraical undisclosure of a set of a real space is based logically on the really defined antisymmetry of elements. Each of the Definitions 2 and 3 jointly with the Definition 1 says herewith about the existence of a kind of Theorem.

Lemma 2 (Theorem on an internal disclosure algebraical logic) There is no algebraical disclosure in a set without a strictly defined symmetry of elements.

Lemma 3 (Theorem on an internal undisclosure algebraical logic) There is no algebraical undisclosure in a set without a strictly defined antisymmetry of elements.

For completeness, we recall the imaginary number axis [10] strictly defined at the level of imaginary space, allowing one to formulate and prove the theorem on the basis of its internal disclosure such that an imaginary number axis including only the imaginary points with objects must distinguish from a real number axis consisting only of the real points with objects. Therefore, a nonzero point is called an imaginary one if and only if it has an imaginary coordinate [10]. A nonzero point is called a real one if it has a real coordinate [4]. Together they constitute one complex point with coordinates of the structural points, namely, one point with a complex coordinate.

Each complex point corresponds to a kind of complex number. These pairs can constitute the families of the complex numbers as well as of the complex points. On this basis, it becomes possible to unite the imaginary and real number axes in a unified whole. Insofar as the fate of a complex number axis thus formed is concerned, it will be illuminated in our further works.

4. The commutativity images of objects of either the right or the left real points

An algebraically disclosed set of a real space can possess each of the innate properties even at any permutation of elements. This reflects the characteristic features of their spatial structure depending on the space of a real number axis, namely, on the space of a real base.

Turning again to (37), we remark that each of the subsets from R comes forward in it as one of its finite sets. To any of these sets corresponds a kind of ball, within which one can find both points with its elements and points with images of each of them. Under such circumstances, a ball of the same internally disclosed set from R involves both the very set and the set consisting of those subsets, in each of which appear images of one and only one element of the finite set. Formulating more concretely, one can write this consequence of Theorem 1 in a general form:

$$a_{NE},\ b_{NE} \in R,\ \ a_{NE}^{(k)},\ b_{NE}^{(k)} \in R^{(k)},$$

$$f_k\colon R \to R^{(k)},\ \ a_{NE}^{(k)} = f_k(a_{NE}),\ \ b_{NE}^{(k)} = f_k(b_{NE}),$$

where it has been accepted that

$$NE = 1B,\ \ 2L,\ \ 3X,\ ...,\ nY,\ \ (n+1)Z,\ ...,\ \ k \in Z_0,\ \ Z_0 = \{0,\ \ 1,\ \ 2,\ ...\}.$$

Thus, Theorem 1 characterizes not only the very element or subclass but also the corresponding images of any element or subclass of a set R of a general system (37), respectively, by an individual real number

$$\mathcal{E} = \begin{cases} +1 & \text{for} \quad a_{NE}^{(k)}, \\ -1 & \text{for} \quad b_{NE}^{(k)}, \\ \nexists & \text{for} \quad \text{remaining images} \end{cases} \tag{41}$$

or, according to Lemma 1, by an individual subset

$$\mathcal{E} = \begin{cases} \{+1\} & \text{for} \quad \{a_{NE}^{(k)}\}, \\ \{-1\} & \text{for} \quad \{b_{NE}^{(k)}\}, \\ \varnothing & \text{for} \quad \text{remaining subclasses} \end{cases} \tag{42}$$

such that a constancy law of the sum (40) stating its internal disclosure is never violated.

Definition 6 The images of each element of the same internally disclosed set constitute one of its spectra.

Lemma 4 (Theorem on the spectra of a set) There is no single spectrum in an internally disclosed set without a crossing point with another of its spectra.

Definition 6 and Lemma 4 require following of the mathematical logic of the commutativity of elements of a set from the point of view of their mutually crossing spectra. On the other hand, as follows from Definition 6, the spectrum of images of any element of each algebraically disclosed set of real space must be finite from above and below by images of the limited size.

Therefore, there exist the possible maximal $\max\{a_{NE}^{(k)}\},\ \max\{b_{NE}^{(k)}\}$ and minimal $\min\{a_{NE}^{(k)}\},\ \min\{b_{NE}^{(k)}\}$ limits both on the images of a_{NE} and on the images of b_{NE} in their spectra

$$f_0\colon R \to R,\ \ a_{NE} = f_0(a_{NE}),\ \ b_{NE} = f_0(b_{NE}),$$

$$f_1\colon R \to R^{(1)},\ \ a_{NE}^{(1)} = f_1(a_{NE}),\ \ b_{NE}^{(1)} = f_1(b_{NE}),$$

$$f_2\colon R \to R^{(2)},\ \ a_{NE}^{(2)} = f_2(a_{NE}),\ \ b_{NE}^{(2)} = f_2(b_{NE}),$$

Definition 7 The relation

$$a_{1B}a_{2L} = a_{2L}a_{1B} \tag{43}$$

between a_{1B} and a_{2L} is a relation of commutativity only if

1. $a_{1B}^{(k)} \to \max\{a_{1B}^{(k)}\} \Rightarrow a_{2L}^{(k)} \to \min\{a_{2L}^{(k)}\}$,
2. $a_{2L}^{(k)} \to \max\{a_{2L}^{(k)}\} \Rightarrow a_{1B}^{(k)} \to \min\{a_{1B}^{(k)}\}$,
3. $a_{1B}^{(m_1)} \ll a_{1B} \to a_{2L}^{(m_1)} \gg a_{2L} \Rightarrow a_{1B}^{(m_1)} = a_{2L}^{(m_1)} \Longrightarrow [a_{1B}^{(m_1)}]^2 = [a_{2L}^{(m_1)}]^2 = a_{1B}^{(m_1)}a_{2L}^{(m_1)} = a_{1B}^{(k)}a_{2L}^{(k)} = const$,
4. $a_{2L}^{(m_2)} \ll a_{2L} \to a_{1B}^{(m_2)} \gg a_{1B} \Rightarrow a_{2L}^{(m_2)} = a_{1B}^{(m_2)} \Longrightarrow [a_{2L}^{(m_2)}]^2 = [a_{1B}^{(m_2)}]^2 = a_{2L}^{(m_2)}a_{1B}^{(m_2)} = a_{2L}^{(k)}a_{1B}^{(k)} = const$.

The commutativity of a_{1B} and a_{2L} expresses the idea of a principle that each of the two pairs of spectra of their paraimages $a_{1B}a_{2L}$ and $a_{2L}a_{1B}$ is crossed at one of the two points m_1 and m_2 with some objects

$$a_{1B}^{(m_1)} = a_{2L}^{(m_1)},\ \ a_{2L}^{(m_2)} = a_{1B}^{(m_2)}, \tag{44}$$

which takes place if and only if

$$a_{1B}^{(m_1)} \ll a_{1B} \to a_{2L}^{(m_1)} \gg a_{2L},\ \ a_{2L}^{(m_2)} \ll a_{2L} \to a_{1B}^{(m_2)} \gg a_{1B}. \tag{45}$$

These properties correspond in a set $\mathcal{A}$ from (18) to those paraimages of $a_{1B}a_{2L}$ and $a_{2L}a_{1B}$, which can be called the images of commutativity of its elements:

$$[a_{1B}^{(m_1)}]^2 = [a_{2L}^{(m_1)}]^2 = a_{1B}^{(m_1)}a_{2L}^{(m_1)} = a_{1B}^{(k)}a_{2L}^{(k)} = const, \tag{46}$$

$$[a_{2L}^{(m_2)}]^2 = [a_{1B}^{(m_2)}]^2 = a_{2L}^{(m_2)}a_{1B}^{(m_2)} = a_{2L}^{(k)}a_{1B}^{(k)} = const. \tag{47}$$

To establish a relation of commutativity

$$b_{1B}b_{2L} = b_{2L}b_{1B} \tag{48}$$

between b_{1B} and b_{2L} within a set $\mathcal{C}$ from (18), one must refer to the substitutions

$$a_{1B} \to b_{1B}, \quad a_{2L} \to b_{2L} \;\Leftrightarrow\; m_1 \to m_1^*, \quad m_2 \to m_2^* \tag{49}$$

because they can generalize Definition 7 to the case of objects from the left points of the number axis. However, the question of spatial coordinates of points $m_1(m_1^*)$ and $m_2(m_2^*)$ still remains open. We can solve only the question of whether they are of real or imaginary points.

Theorem 2 If a set consists only of objects of either the right or the left points of a real axis, each of the two pairs of spectra of each pair of its elements is crossed at one of the real points.

Proof of Theorem 2. An investigated set of a given Theorem is one of those sets, within which there are objects from only one sequence (25) or (27) and that, consequently, is a subset of an algebraically disclosed set. An example of this is each subset from (18), namely, $\mathcal{A}$ and $\mathcal{C}$, the images of the commutativity of elements of which were already established above.

Returning to (41), (43) and (46)–(49), we now remark that

$$\mathcal{E}_{a_{1B}^{(k)}}\mathcal{E}_{a_{2L}^{(k)}} = 1, \quad \mathcal{E}_{a_{2L}^{(k)}}\mathcal{E}_{a_{1B}^{(k)}} = 1, \tag{50}$$

$$\mathcal{E}_{b_{1B}^{(k)}}\mathcal{E}_{b_{2L}^{(k)}} = 1, \quad \mathcal{E}_{b_{2L}^{(k)}}\mathcal{E}_{b_{1B}^{(k)}} = 1, \tag{51}$$

at which neither of the conserving sizes

$$\mathcal{E}_{a_{1B}^{(m_1)}} = \mathcal{E}_{a_{2L}^{(m_1)}} = [\mathcal{E}_{a_{1B}^{(k)}}\mathcal{E}_{a_{2L}^{(k)}}]^{1/2} = \pm 1, \tag{52}$$

$$\mathcal{E}_{a_{2L}^{(m_2)}} = \mathcal{E}_{a_{1B}^{(m_2)}} = [\mathcal{E}_{a_{2L}^{(k)}}\mathcal{E}_{a_{1B}^{(k)}}]^{1/2} = \pm 1, \tag{53}$$

$$\mathcal{E}_{b_{1B}^{(m_1^*)}} = \mathcal{E}_{b_{2L}^{(m_1^*)}} = [\mathcal{E}_{b_{1B}^{(k)}}\mathcal{E}_{b_{2L}^{(k)}}]^{1/2} = \pm 1, \tag{54}$$

$$\mathcal{E}_{b_{2L}^{(m_2^*)}} = \mathcal{E}_{b_{1B}^{(m_2^*)}} = [\mathcal{E}_{b_{2L}^{(k)}}\mathcal{E}_{b_{1B}^{(k)}}]^{1/2} = \pm 1 \tag{55}$$

contradicts the implications of Theorem 2 that the points $m_1(m_1^*)$ and $m_2(m_2^*)$ undoubtedly refer only to real points and that the images of the commutativity of elements of each subset from (18) exist only in their space. □

To solve the question of why internal disclosure of a set of real space comes forward at the level of real number axis itself as a commutativity of each pair of its objects one must refer to (43) and (48), according to which, there is no single pair of objects in the same real axis, for which a relation of commutativity would not exist. Insofar as the commutativity images of objects of both right and left real points, the implications implied from the consideration of this problem call for special presentation. However, here we have already mentioned that it formulates one more pair of Theorems.

Lemma 5 (Theorem on a commutative pair mathematical logic) There is no mathematical disclosure in a set without commutative pairs of elements.

Lemma 6 (Theorem on an anticommutative pair mathematical logic) There is no mathematical undisclosure in a set without anticommutative pairs of elements.

Each real axis in a real space must distinguish itself from other real axes by the individual objects. Therefore, it should be characterized any commutative pair of any real number axis both by the imaginative curved and straight lines.

Definition 8 A line is called an imaginative curved one if it unites all points with images of one and only one of objects of each commutative pair.

Definition 9. A line is called an imaginative straight one if it unites selected points with images of each of objects of each commutative pair.

If we use the objects 2 and 8 from right points of a real number axis when $2 \cdot 8 = 8 \cdot 2 = (4)^2,$ and the commutativity images 4 of each pair of $2 \cdot 8$ and $8 \cdot 2$ together with 2 and 8 satisfy the inequalities $2 < 4,\ 4 > 2,\ 4 < 8,\ 8 > 4,$ the images of each object of 2 and 8 constitute in whole a kind of spectrum having a defined directionality. An imaginative curved line of each of the real objects of each really commutative pair can therefore be accepted as a curved vector [11].

5. Full finiteness of sets of a real space

If we recall that

$$a_{1B},\ a_{2L} \in \mathcal{A},\ \ a_{1B}^{(k)},\ a_{2L}^{(k)} \in \mathcal{A}^{(k)},$$

$$f_k\colon \mathcal{A} \to \mathcal{A}^{(k)},\ \ a_{1B}^{(k)} = f_k(a_{1B}),\ \ a_{2L}^{(k)} = f_k(a_{2L}),$$

and consequently, that one element from $\mathcal{A}$ has k images in $\mathcal{A}^{(k)},$ then either $\mathcal{A}$ or $\mathcal{A}^{(k)}$ is not in a state to give a categorical answer to the question of whether or not one-to-one correspondence between them exists. However, as will be seen, the answer is still hidden in these sets.

To reveal this feature, it is desirable to present in a real space a compound structure of both elements of a set $\mathcal{A}$ and their images in an explicit form:

$$a_{1B} = \{a_{1B},\ \ a_{1B},\ \ a_{1B},\ \ldots,\ a_{1B},\ \ a_{1B},\ \ldots\},$$

$$a_{2L} = \{a_{2L},\ \ a_{2L},\ \ a_{2L},\ \ldots,\ a_{2L},\ \ a_{2L},\ \ldots\},$$

$$a_{1B}^{(k)} = \{a_{1B},\ \ a_{1B}^{(1)},\ \ a_{1B}^{(2)},\ \ldots,\ a_{1B}^{(n)},\ \ a_{1B}^{(n+1)},\ \ldots\},$$

$$a_{2L}^{(k)} = \{a_{2L},\ \ a_{2L}^{(1)},\ \ a_{2L}^{(2)},\ \ldots,\ a_{2L}^{(n)},\ \ a_{2L}^{(n+1)},\ \ldots\}.$$

This presentation corresponds in $\mathcal{A}$ to the fact that it is, in a set R of a general system (37), one of its finite subsets. The existence for $\mathcal{A}$ of a kind of ball is by no means excluded logically. Its structure may not be geometrically defined regardless of the interratio of the very set $\mathcal{A}$ and the set of images of its elements. In other words, among all points of the ball, there are k points with each element of $\mathcal{A}$ and k points with its images, an equal number of which expresses the full finiteness of the very finite set.

Definition 10 The finite sets of a real space are called full finiteness ones if and only if within a ball of each of them a number of points with each element and a number of points with its images coincide.

It is already clear from the foregoing that within a ball of a full finiteness set there is no single pair of points, for which a single pair of its elements and their images would not appear. In other words, to each pair of points from a space, where the finite set becomes fully finite, corresponds one and only one pair of its elements and their images. This does not require, of course, the rewriting of any element of an investigated set, and a symbolic presentation of a_{1B} and a_{2L} in conformity with $a_{1B}^{(k)}$ and $a_{2L}^{(k)}$ simply implies that $\mathcal{A}$ and $\mathcal{A}^{(k)}$ are connected only with those points, which allow establishing between them of a one-to-one correspondence.

6. An internal disclosure geometrical logic of sets of a real space

Furthermore, if it turns out that the relations (46) and (47) relate one element within a set $\mathcal{A}$ from (18) to another from its elements, we cannot exclude the existence of a geometrical object of their latent unification as a definitely symmetrical line of elements.

Definition 11 A latent geometrical object of unification of one set is the second set, such that it consists of conserving points of the same defined line of elements of both sets.

Finally, for a set $\{a_{1B}\}$ consisting of one element a_{1B}, a latent geometrical object of unification for this class may be a set $\{a_{1B}^{(k)}\}$ consisting of k points with images of the same single element. A set $\{a_{1B}^{(k)}\}$, as stated in Definitions 6, 8 and 10, constitutes in whole a kind of imaginative curved line such that it must be finite from above and below by points of images of the limited size. However, according to Lemma 1, each set from $\{a_{1B}\}$ and $\{a_{1B}^{(k)}\}$ exists in a kind of internally disclosed set of a higher cardinality as one of its nonempty subsets. For example, in sets such as $\mathcal{A}$ and $\mathcal{A}^{(k)}$, the elements are united in a unified whole constituting a curvilinear triangle of both sets.

The existence of the latter reflects just a regularity that a notion of a geometrical disclosure of a set in a real space is based logically on the really symmetrical line of elements. But, as stated in its absence, a notion of a geometrical undisclosure of a set of a real space is based logically on the really antisymmetrical line of elements. Any of the Definitions 2 and 3 together with the Definition 11 says herewith in favor of a kind of Theorem.

Lemma 7 (Theorem on an internal disclosure geometrical logic) There is no geometrical disclosure in a set without a definitely symmetrical line of elements.

Lemma 8 (Theorem on an internal undisclosure geometrical logic) There is no geometrical undisclosure in a set without a definitely antisymmetrical line of elements.

7. An internal disclosure mathematical logic of sets of a real space

To conform with the Lemmas 2, 3, 7 and 8, the Definitions 2 and 3 jointly with the Definitions 1 and 11 express, for each of the two sets from two elements, the idea of a unified logic within their system. They convince that a notion of a mathematical disclosure of a set in a real space is based really on the definitely symmetrical line of elements in their strictly defined symmetry. In contrast to this, a notion of a mathematical undisclosure of a set of a real space is based really on the definitely antisymmetrical line of elements in their strictly defined antisymmetry. Therefore, we not only formulate one more pair of highly important Theorems but also must recognize that mathematical logic [12] comes forward in an internally disclosed set as a united logic.

Lemma 9 (Theorem on an internal disclosure mathematical logic) There is no mathematical disclosure in a set without a definitely symmetrical line of elements of strictly defined symmetry.

Lemma 10 (Theorem on an internal undisclosure mathematical logic). There is no mathematical undisclosure in a set without a definitely antisymmetrical line of elements of strictly defined antisymmetry.

8. Concluding remarks

Definitions 2 and 3 together with the Lemmas 7 and 8 state that the sets in a real space are called internally disclosed ones if each of them has a mathematical disclosure. Conversely the sets of a real space are called internally undisclosed ones if each of them has a mathematical undisclosure. The regularities thus found express, for each of the two forms of sets, the idea of a kind of Axiom.

Axiom 1 An internal disclosure of a set is none other than its mathematical disclosure.

Axiom 2 An internal undisclosure of a set is none other than its mathematical undisclosure.

Such a mathematically united logic in turn makes it possible to introduce a notion of the full compactness of sets of a real space.

Definition 12 The compact sets of a real space are called full compactness ones if and only if they have a mathematical disclosure.

However, we cannot exclude the existence in a real space of geometrically degenerated sets such that each of them with elements of really defined symmetry has a geometrical undisclosure. Exactly the same the very real space is not in force to exclude the definitely symmetrical line of elements in each of algebraically degenerated sets even at its algebraical undisclosure properties. It together with a geometrically degenerated set constitutes in a real space the united family of mathematically degenerated sets.

Of course, to any of the internally disclosed subsets corresponds, in a set R of a general system (37), a kind of geometrical object of unification. The building of each of them would be outside the scope of this work, and therefore, they will be described in detail in our further works.

We recognize that an established set of 10 Lemmas takes place in the case where an investigated set consists of objects of an imaginary space [10]. Such an order, however, can exist regardless of the source of a nonempty space. Thereby, it requires one to follow the logic of proofs of all these 10 Lemmas from the point of view of their generality, allowing one to specially present this.

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Conflict of interest

The authors declare no competing financial interest.

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Appendix A

We recognize that a logical set of 12 new mathematical concepts established above reflects so far unknown latent properties of the real objects.

Their set constitutes *a real space* of Definition 4. An imaginary space including only the imaginary points must distinguish from *a real space* such that it consists of the real points.

A nonzero point not having a real coordinate is called an imaginary one. A nonzero point not having an imaginary coordinate is called a real one. They are of course the structural points of a complex point, noted in the third section.

Theorem 1 characterizes each pair of objects a_{NE} of the right points and b_{NE} of the left points of a real number axis by one pair of individual numbers 1 and -1 of each conserving size $\mathcal{E}$ of all types of symmetries E of a real space. Jointly with (40), their sum does not exclude the strictly defined *symmetry of elements* of each set having an algebraical disclosure. An algebraically disclosed set consisting of conserving sizes of *the symmetry of elements* is none other than *an algebraical object of latent unification* given by Definition 1.

The line of elements relates one element in an internally disclosed set to another of its elements. However, according to Definitions 6 and 8, all images of each element constitute in a set an imaginative curved line one of its spectra. At the same time, Lemma 4 itself states that there is no single imaginative curved line in an internally disclosed set without a crossing point with another of its imaginative curved lines. Thus, if *the line of elements,* the imaginative curved lines of which are crossed at one of the real points, does not change a number of own values then geometrical disclosure of each set of a real space appears in a definitely symmetrical *line of elements*. A geometrically disclosed set consisting of conserving points of this *line of elements* is none other than *a geometrical object of latent unification* of Definition 11.

An internally disclosed (fully regular) set, as follows from Lemmas 2, 7 and 9, has a mathematical disclosure, which unites its algebraical and geometrical disclosures in a unified whole. In other words, Lemma 5 and Axiom 1 express *an internal disclosure* of each *fully regular set* as a commutativity of each pair of objects of this mathematically disclosed set.

An internally undisclosed (fully casual) set, as stated in in Lemmas 3, 8 and 10, has a mathematical undisclosure, which unites its algebraical and geometrical undisclosures in a unified whole. On their basis, Lemma 6 and Axiom 2 explained *an internal undisclosure* of each *fully casual set* as an anticommutativity of each pair of objects of this mathematically undisclosed set.

Lemma 9 including *a definitely symmetrical line of elements of strictly defined symmetry* convinces that *the line of elements* is becoming a definitely symmetrical one owing to a strictly defined *symmetry of elements.* However, *the line of elements* must be a definitely antisymmetrical one at a strictly defined *antisymmetry of elements.* Lemma 10 will, therefore, indicate to the existence of *a definitely antisymmetrical line of elements of strictly defined antisymmetry.*

An implication of *conserving points of the same defined symmetry of elements* arises from a comparison of Definitions 2 and 11, confirming their availability in the logical correspondence. Consequently, both a number of points of *the line of elements* and a number of points with images of each of them will not change if the very set consists only of objects of either the right or the left points of a real axis. In it appears a part of *the same defined symmetry of elements.*

A set consisting of objects of both right and left points of a real axis in a real space is called *a geometrically degenerated set,* because each of the two pairs of spectra of each pair of its elements is crossed at one of the imaginary points. This indicates the role of the crossing of a real space with an imaginary space. However, as noted in the fourth section, the results following from its consideration will be presented in the separate work.