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Research Article

# Dynamic Programming, Neuro-Dynamic Programming, Rollout Method and Model Predictive Control to Optimal Control of a Fermentation Process

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**Abstract:** This work presents the use of dynamic programming (*DP*), neuro-dynamic programming (*NDP*), rollout algorithm (*RA*) and model predictive control (*MPC*) for optimal control of batch cultivation of the yeast *Kluyveromyces marxianus var. lactis* MC5. *DP* is a widespread method for solving problems related to optimization and optimal process control. To reduce the "*curse of dimensionality*", *NDP* has been implemented as an alternative. In *NDP*, a neural network is used to solve the dimensionality problem. A simpler *NDP* method, called *RA*, is used to approximate the optimal cost through the cost of a relatively good suboptimal policy, called the baseline policy. *RA* is a suboptimal method for deterministic and stochastic problems that can be solved by *DP*. In this paper we also present off-line *MPC* technique for tracking of constrained fermentation systems and it overcomes the problem by off-line optimizations prior to implementation. *MPC* is used to provide perturbation feedback and it is developed theoretical on base a controller as an illustration how we can avoid disturbances in the process optimisation. The developed control algorithm-combined *NDP* and *MPC* ensures maximum biomass production at the end of the process and feedback during disturbances and process stability and shows that robust stability can be ensured.

***Keywords*:** optimal control, dynamic programming, neuro-dynamic programming, rollout algorithm, model predictive control, biotechnological processes

**MSC:** 49L20, 90C39

## 1. Introduction

Dynamic programming involves step-wise calculation of the cost-to-go function to arrive at the solution, not only for a specific initial state, but also for the general state. Once obtained, the cost-to-go function is a convenient means of obtaining a solution for the total condition. In very few cases, the stepwise optimisation to analytically obtain a closed-form expression for the cost-to-go function has been solved. The conventional approach to the problem involves a state space grid, computing and storing the traversal cost for each grid point as one backward march from the first stage to the last. Such an approach is rarely practical due to the exponential growth of computing. This is called the "*curse of*


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*dimensionality*", which must be removed for this approach to be widely used [1].

Neuro-dynamic programming [2] is a relatively new class of *DP* method for optimal control and further decision making under uncertainty. These methods have the potential to address the main problem of *DP*-the "*curse of dimensionality*". The name *NDP* expresses the methods dependence on both *DP* and neural network concepts. In this case, the name reinforcement learning is also used in the artificial intelligence community, where the methods come from [3].

A simpler type of *DP* method called *RA* [4-5] is used to approximate the optimal price by the price of a relatively good suboptimal policy called the base policy. *RA* is a suboptimal control method for deterministic and stochastic problems that can be solved by dynamic programming. *RA* will also produce a feasible solution whose cost is not less than the cost corresponding to the baseline heuristic.

*MPC* is a general methodology for solving time domain control problems [6-9]. More than 25 years after the advent of *MPC*, a theoretical basis for this technique has begun to emerge in the industry as an effective means of dealing with multivariable constrained control problems. A lot of progress has been made in nonlinear systems during these years. Many questions remain for practical application, including the reliability and efficiency of the online computational scheme. This dynamic programming problem must be solved in order to "rigorously" deal with model uncertainty. Approximation techniques proposed for this purpose are very often at a conceptual stage. Among the broader research needs, the following areas have been identified: multivariable system identification, performance monitoring and diagnostics, nonlinear condition estimation, and batch system control are essential. *MPC* can be used to provide a maximum/minimum value of the chosen optimisation criteria and ensure perturbation feedback. In this way, the resistance of the process to disturbances is improved.

The successful application of *DP*, *NDP*, *RA* and *MPC* for optimal management of biotechnological (fermentation) processes has been shown in publications [10-28].

This work presents the use of *DP*, *NDP*, *RA* and *MPC* methods for optimal control of fed-batch cultivation of yeast *Kluyveromyces marxianus var. lactis* MC5.

## 2. Materials and methods

The formulation of each optimisation problem focuses on the number of variables and the range of their permissible values to be determined. Other compulsory component of the description is the scalar criterion "*quality*", the so called purpose function. The decision of the optimal control problem concerns the permissible composition of the values of variables, defines the optimal meaning of the purpose function.

### 2.1 *Formulation of optimisation problem of fermentation processes*

Multitude problems connected to the optimisation and optimal control (*OC*) of dynamic objects and in particular to the fermentation process (*FP*) can be examined in the following way:

$$\frac{d\mathbf{X}}{dt} = f\left[\mathbf{X}(t), \mathbf{U}(t), t\right] \tag{1}$$

where $\mathbf{X}(t)$ is $n$-dimensionally vector of the systems state, $\mathbf{U}(t)$ is $m$-dimensionally vector of the control, $f$ is a known function of the system state $\mathbf{X}(t)$ and the control variables $\mathbf{u}(t)$, $\mathbf{X}$ is a continuous and differentiate vector-function, $t$ is time.

Let $\mathbf{U}(t)$ be the multitude of the permissible values of the control variable $\mathbf{u}(t)$:

$$\mathbf{u}(t) = \mathbf{U}(t) \text{ for each } t \tag{2}$$

Control variables that satisfy the conditions (2) are called admissible. $t_0$ marks the starting point in time regarding the start of the system check. The state of (1) at time $t = t_0$ can be known:

$$\mathbf{X}(t_0) = \mathbf{X}_0 \tag{3}$$

where $\mathbf{X}_0$ is a known vector of the initial conditions.

It is useful in solving an optimal control problem to include this vector ($\mathbf{X}_0$) in the control effects vector. In this way, the optimal values of the initial conditions can be determined off-line.

The examination of the system has finished at $t_f$-marked as final time moment. $t_f$ can be known predictably or to be unknown. For the other hand $t_f$ can be one of the optimisation variables. It has been supposed that:

$$t_f \in M(t) \tag{4}$$

where the multitude $M(t)$ contains the permissible values of $t$ and it is usually given by algebraic equations and inequalities.

The constrains can also be imposed on the vector of state:

$$\mathbf{X}(t) \in \theta(\mathbf{X}) \tag{5}$$

in particular: $\mathbf{X}(t_f) \in \theta(\mathbf{X})$.

$J(\mathbf{u})$ the criterion of quality has been noted by the demanded aim as follows:

$$J(\mathbf{u}) = \int_{t_0}^{t_f} L[\mathbf{X}(t), \mathbf{u}(t), t]dt \tag{6}$$

where $L[\mathbf{X}(t), \mathbf{u}(t), t]$ is a scalar, continuous and continuously differentiates from the function of $\mathbf{X}(t)$, $\mathbf{u}(t)$ and $t$.

The optimal control problem can be formulated as follows: to find such an optimal control vector $\mathbf{u}^*(t)$ that maximizes (6) while satisfying (2)-(6).

## 2.2 *Optimisation criteria*

The main criteria of quality that are used for optimal control of biotechnological/fermentation processes are shown in [28-31].

In this work, we use only one of these criteria, namely productiveness of biomass [29]:

$$\max_{\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}} J_1 = \int_{t_0}^{t_f} F(t)\, X(t)\, dt \tag{7}$$

where: $t_0$-initial time, $t_f$-final time of the process, $\mathbf{u}_{\min}$ and $\mathbf{u}_{\max}$-vectors with minimal and maximal values of control variables.

## 2.3 *Constrains and final conditions*

The total substrate quantity $Q_S$ in the bioreactor has been imposed and it should be restricted [29]. For the main substrate, quantity in the bioreactor (regulated for the fermentation) can be written:

$$Q_S = S_0 V_0 + S_0 V_I \tag{8}$$

where $V_0$ is the volume of the fermentation medium at the moment $t_0$; $V_I$ is the volume of the feeding solution at the beginning of the process.

If the *FP* is a fed-batch and the feeding solution has a feeding flow rate ($F$) then the following can be written:

$$\int_{t_0}^{t_f} F(t)dt = V_I \tag{9}$$

According to the fact that $F(t) = dV/dt$, than:

$$\int_{t_0}^{t_f} F(t)dt = \int_{t_0}^{t_f} \frac{dV}{dt} dt = \int_{V_o}^{V_f} dV = V_f - V_0 \tag{10}$$

where: $V_f$ is the volume of the fermentation medium at the end of the process; $V_I = V_f - V_0$.

If the final conditions are known then:

$$X(0) = X_0, S(0) = S_0, P(0) = P_0, V(0) = V_0, V(t_f) = V_f \tag{11}$$

# 3. Methods for optimal control of biotechnological processes

## 3.1 *Dynamic programming*

*DP* algorithm block-scheme is shown on Figure 1.

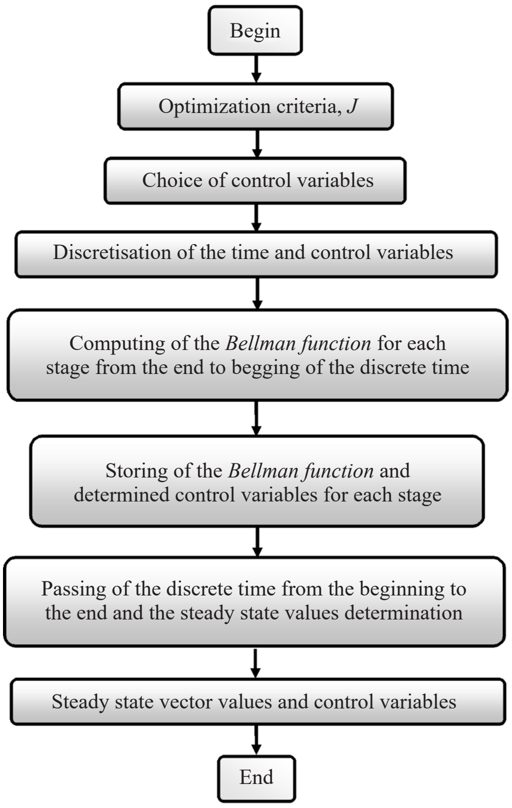


**Figure 1.** *DP* algorithm block-scheme

In general, the optimal control task includes the following elements [1, 4]:

-Selection of the optimisation criteria $J$;
-Mathematical model of the process;
-Selection of control variables and their limits;
-Constraints on phase coordinates.
The choice of optimal control criteria and the choice of optimisation model determine each case.

### 3.2 *Neuro-dynamic optimal control*

The objective of the *NDP* for optimal control is to bring the bioreactor from a steady state of low product concentration to a desired high product concentration. *NDP* uses a neural network to approximate the cost-to-go function. The method is found to be robust to approximation errors. Both deterministic (step changes in kinetic parameters) and stochastic problems (random variations in kinetic parameters and substrate composition) are investigated [2].

The *NDP* algorithm block-scheme is shown in Figure 2 [12, 13].

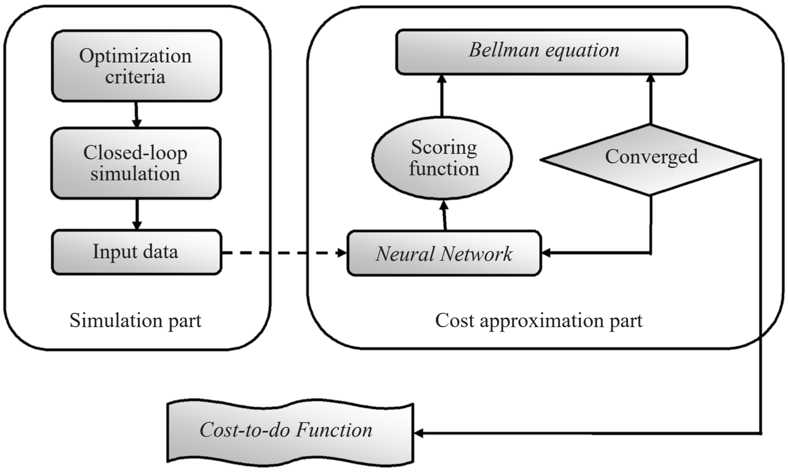


**Figure 2.** Block-scheme of *NDP* algorithm

The simulation involves computing the converged approximation of the profit to move offline. The scheme of the *NDP* algorithm is as follows [2]:

1. Process simulation with selected suboptimal policies for all representative operating conditions. Starting with a given policy (some rule for choosing a vector of decision $\mathbf{u}$ at each possible state $i$). Approximate assessment of this policy.

2. The solution of *one-stage-ahead cost* plus *cost-to-go* problem results in the improvement of the cost values [11].

3. *Cost-to-go* function is calculated using the simulation data for each state made during the simulation, as well as for each closed loop simulation (simulation part).

4. A new policy is determined by minimizing the Bellman equation. The optimal cost is replaced by the calculated point function and the process is repeated.

5. Approximate the neural network function to the data to approximate the *cost-to-go* function to a smooth state function.

6. Sometimes a policy update may be needed to increase state space coverage.

### 3.3 *Rollout optimal control algorithm*

Rollout algorithms were first proposed for the approximate solution of discrete optimisation problems [3-5].

Generally, rollout algorithms are capable of magnifying the effectiveness of any given heuristic algorithm through sequential application. This is due to the policy improvement mechanism of the underlying policy iteration process. Scheme of the algorithm is shown in Figure 3 [5].

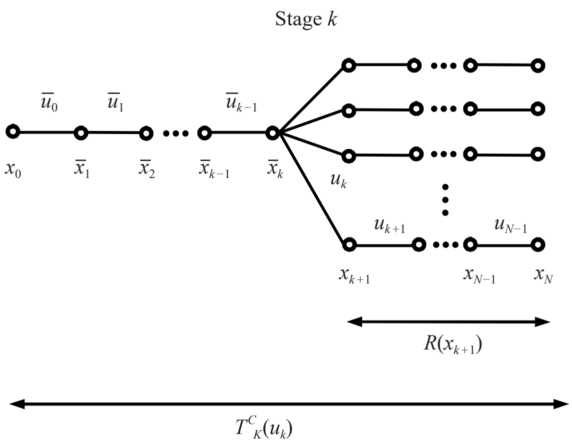


**Figure 3.** *Rollout algorithm* scheme

## 3.4 *Model predictive control*

*MPC* is used to forecast process output within a forecast horizon [6-9]. The control impacts are calculated for a control horizon in such a way that the predicted result is as close as possible to the desired one and the first control action is successively applied in each step (Figure 4).

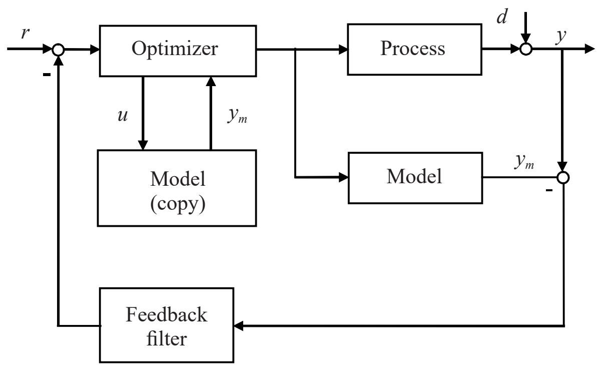


**Figure 4.** Internal model predictive control

The Figure 5 the notations used in the description are adapted from [6, 7].

The first part of the *MPC* algorithm is the specification of the reference trajectory which may be as simple as a step change to a new set point or as it is common for batch processes-a trajectory that the system must follow. At the present time $k$, the reference trajectory has a value $r(k)$.

Also at $k$, consider the predicted process output over a future prediction horizon $p$. A suitable controller model of the process is used to obtain the projected behavior of the output over the prediction horizon by simulating the effects of the past inputs applied to the actual process (value $\hat{y}(k)$ at the current time) [8].

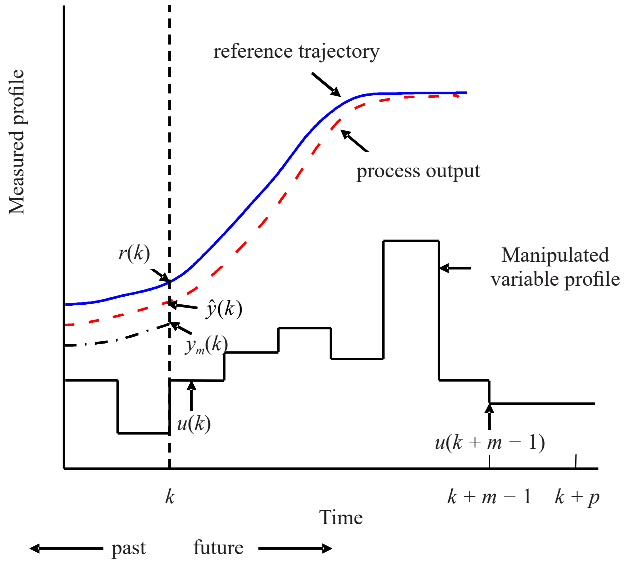


**Figure 5.** *MPC* algorithm scheme

The same controller model is used to compute a sequence of *m* current and future manipulated variable motions to satisfy a specified objective function. Here *m* is the motion horizon. The objective function is to minimize the sum of the squares of the deviations of the predicted values of a controlled variable from a time-varying reference trajectory over the forecast horizon based on the available system information at the current time *k*, subject to imposed constraints. At the next time instant *k* + 1, the process measurement is taken again and the horizon is shifted forward by one step. The optimisation is carried out again based on this new horizon and using the updated systems information and the process continues. Since the horizon recedes at the next time step, it is also known as a receding horizon control problem [6].

# 4. Results and discussion

## 4.1 *Process model of yeast Kluyveromyces marxianus var. lactis MC5*

The process of cultivation the yeast *Kluyveromyces marxianus var. lactis* MC5 from a natural substrate (whey) is not well studied. The system of nonlinear differential equations includes the biomass concentration depending on the two main substrates: lactose and oxygen. The experimental studies were carried out in a laboratory bioreactor with a magnetic drive and a maximum volume of 2 L. The cultivation conditions of the process are described in detail in [34].

$$\frac{dX}{dt} = \mu(S, C_L)X - \frac{F}{V}X \tag{12}$$

$$\frac{dS}{dt} = \frac{F}{V}(S_{in} - S) - \frac{1}{Y_1}\mu(S, C_L)X \tag{13}$$

$$\frac{dC}{dt} = \frac{k_l a}{(100 - \phi_G)}(C_L^* - C) - Y_2\mu(S, C)X - \frac{F}{V}C_L \tag{14}$$

$$\frac{dV}{dt} = F \tag{15}$$

$$\mu(S, C_L) = \mu_m \frac{S^2}{(k_S + S^2)} \frac{C_L}{(k_C + C_L + C_L^2 / k_i)} \tag{16}$$

where:

$X$-biomass concentration, g/L;

$S$-substrate concentration, g/L;

$C_L$-oxygen concentration, g/L;

$C_L^*$-equilibrium oxygen concentration, g/L;

$Sin$-feed concentration of substrate g/L;

$V$-working volume, L;

$k_l a$-volumetric mass-transfer coefficient, $h^{-1}$;

$\varphi_G$-volume fraction of gas in the bioreactor, vol. %;

$Y_1$, $Y_2$-yield coefficients, g/g;

$\mu$-specific grown rate of the biomass, $h^{-1}$;

$\mu_m$-maximal grown rate of the biomass, $h^{-1}$;

$k_S$, $k_C$-Monod saturation constants, g/L;

$k_i$-inhibition constant.

The mass-transfer coefficient ($k_l a$), power ($P_G$ and $P_L$) and gas-hold up ($\varphi_G$) are determined by the following dependences [35]:

$$k_l a = 52 \left(\frac{P_G}{V}\right)^{0.38} \left(\frac{4Q_G}{\pi D^2}\right)^{0.23} \tag{17}$$

$$\varphi_G = 0.2 \left(\frac{P_G}{V}\right)^{0.7} \left(\frac{4Q_G}{\pi D^2}\right)^{0.2} \tag{18}$$

$$P_G = 0.21 \left(\frac{Q_G}{nd^3}\right)^{-0.1} P_L^{0.8} \tag{19}$$

$$P_L = 60.9 \rho n^3 d^5 \mathrm{Re}^{-0.4} \tag{20}$$

$$\mathrm{Re} = \frac{\rho n d^2}{\upsilon} \tag{21}$$

where:

$P_G$-power input with aeration, W;

$P_L$-power input without aeration, W;

$D$-bioreactor diameter, m;

$d$-impeller diameter, m;

$n$-rotation speed, $s^{-1}$;

$Q_G$-gas flow rate, $m^3/s$;

Re-Reynolds number;

$\rho$-liquid density, $kg/m^3$;

$\upsilon$-liquid dynamic viscosity, Pa/s.

The initial conditions of the model (12)-(21), constructive and regime parameters and the kinetics model coefficients have the following values (mean values):

$$X(0)=0.2 \text{ g/L};\; S(0)=S_{in}=44.0 \text{ g/L};\; C_L(0)=6.5\times10^{-3} \text{ g/L};$$

$$F(0)=0.0 \text{ L/h};\; V(0)=1.164 \text{ L};\; n=13.3 \text{ s}^{-1};\; d=0.057 \text{ m};\; D=0.114 \text{ m}.$$

$$\mu_m=0.89 \text{ h}^{-1};\; k_S=1.64 \text{ g/L};\; k_C=1.74\times10^{-3} \text{ g/L};\; k_i=0.91;$$

$$Y_1=0.45 \text{ g/g};\; Y_2=308.64 \text{ g/g}.$$

## 4.2 *NDP for optimal control of yeast Kluyveromyces marxianus var. lactis MC5*

In this part of our work, by applying *NDP*, the optimal feed flow rate ($F$) of a fed-batch process was determined for the yeast *Kluyveromyces marxianus var. lactis* MC5 (12-21) to increase biomass at the end of the process, criteria $J_1$. See eq. (7).

The goal of optimal control is to bring the bioreactor from a low product (biomass) state to a desired high level.

The simulation-based approach involves computation of the converged *profit-to-go* approximation *off-line*.

Further values of $F$ are examined: $F \in [0.01, ..., 0.8] \times 10^{-3}$ g/L which can cover the possible rang of variations.

Improvement to the *cost-to-go* function is obtained through the iterations of the Bellman equation. This method is known as a value iteration.

The converged *cost-to-go* function is used in solving the *one-stage-ahead* problem. The choice for switch over the one-stage-ahead is calculated by:

$$\mathbf{u}(k)=\arg\max_{\mathbf{u}(k)}\left\{f\left(\frac{J(t_k)}{t_k},\mathbf{u}\right)+\tilde{B}^6\left(\frac{J(t_k)}{t_k},\mathbf{u}(k)\right)\right\} \tag{22}$$

where: $\mathbf{u}$ is the vector of control variables, $k$ is the optimisation stages; $B$ is Bellman equation; $J(t_k)$ represents *cost-to-go* values for a stage $k$; $\tilde{J}$ represents functional approximation of *cost-to-go*, the superscript $j$ represents the iteration index.

A MATLAB 7.0 program was developed to solve the problem of optimal process control in the cultivation of yeast *Kluyveromyces marxianus var. lactis* MC5.

The optimal profile of feed flow rate is shown in Figure 6. The biomass production before and after the optimisation are shown in Figure 7.

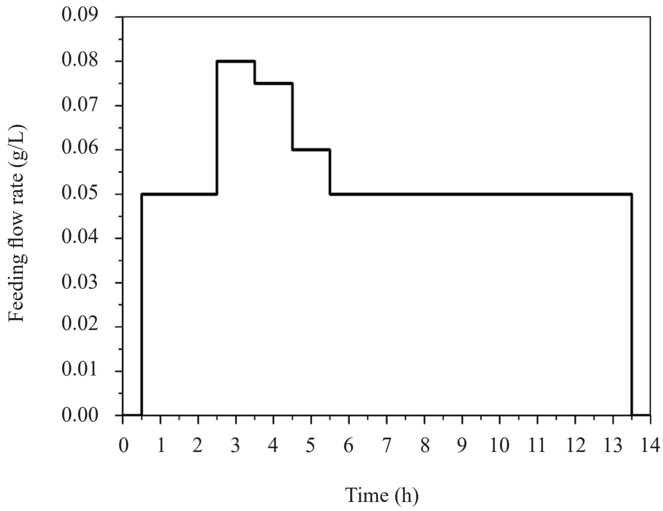


**Figure 6.** Optimal profile of feeding flow rate received through *NDP*

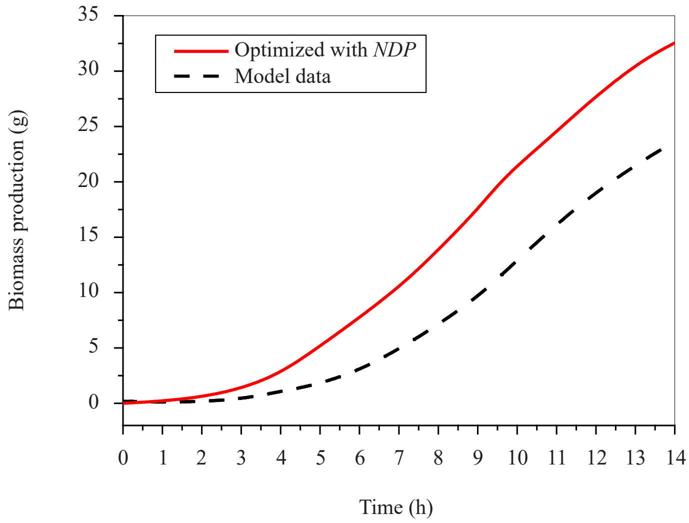


**Figure 7.** Model and optimised biomass production with *NDP*

Through the determined optimal feeding rate profile (Figure 6) an increase in biomass production by 39.41% was obtained.

## 4.3 *Rollout optimal control of yeast Kluyveromyces marxianus var. lactis MC5*

The task of optimal control by rollout method is the same as for optimal control by *NDP*. Control variable is feeding flow rate ($F$), and the criterion is maximum biomass production (7).

The process duration is divided in 14 time intervals:

$$t_0, t_1, t_2, \ldots, t_N(t_f),\ \Delta t = \frac{t_N - t_0}{N} = 1\text{ h},\ N = 14\text{ h}.$$

Admissible values for feeding flow rate are taken in the interval $0 \leq F \leq 10 \times 10^{-3}$, (L/h) with discrete steps $\Delta F = 0.01 \times 10^{-3}$, (L/h).

The optimal biomass production profile is shown in Figure 8.

Using this profile 40% increasing of biomass quantity in the end of the process (Figure 8) is achieved in comparison to the fermentation with constant feeding rate.

However, the optimisation algorithm (*NDP* and *Rollout*) does not have a feedback and it does not guarantee robustness to process disturbances. Therefore a *MPC* will be developed that will guarantee robustness to process disturbances.

## 4.4 *MPC of yeast Kluyveromyces marxianus var. lactis MC5*

In the previously investigation methods did not guarantee robustness of the process disturbances. Because of that, we developed a method, based on *MPC*. The optimisation is off-line in theoretical investigations in our process. The main investigations in this point is to develop an off-line *MPC* algorithm to a non-liner fermentation systems that can deal with both persistent disturbance and time-varying scheduling parameter.

An algorithm for application of *MPC* for the investigated process is developed for *MPC* off-line optimized control strategy. The results are shown in Figure 9.

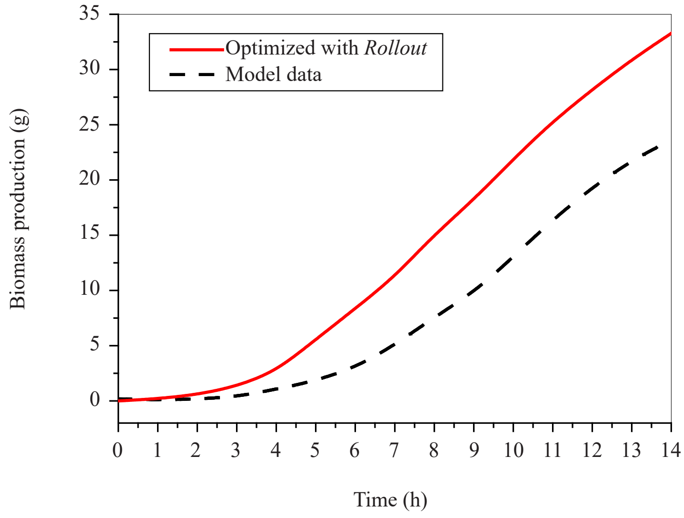


**Figure 8.** Model and optimised biomass with *Rollout method*

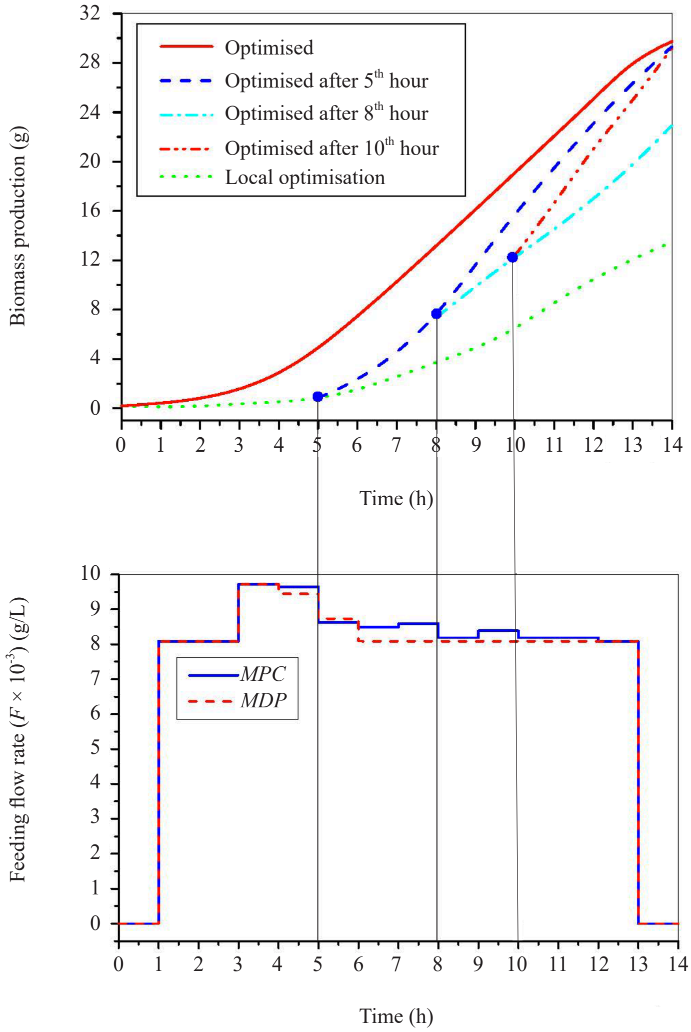


**Figure 9.** *MPC* to yeast *Kluyveromyces marxianus var. lactis* MC5

The 5th hour is chosen as a first control point. As it may be noted that there is a diversion from the reference profile marked on the figure by "-", accordingly the optimal profile is changed-*NDP* profile. The second point is at the 8th hour. The third point is at the 10th hour. The obtained control guarantees the robustness and stability of the biomass production.

The control of fermentation processes focuses on an open-loop operation owing to their highly nonlinear and inherently difficult dynamic behaviour. The optimisation is carried out *off-line* and the bioreactor is fed by the determined optimal feed profile.

Updating the disturbance estimate and solving the optimisation at each time step compensates the unmeasured disturbances and model inaccuracy (which causes actual system outputs to be different from the model outputs). Usually the problem is formulated so that the objective is minimized subject to certain system constraints, for example bounds on the magnitude of current and future inputs or outputs. This ability to handle constraints in an optimal way is the primary advantage of the model predictive control over other design schemes. The primary disadvantage of this *MPC* off-line optimized control strategy relative to other techniques is its inability to deal with model uncertainty.

## 5. Conclusions

In this paper, we have presented some approaches for optimal control of fermentation processes for a whey fermentation by yeast *Kluyveromyces marxianus var. lactis* MC5 fed-batch fermentation.

At the first time, we developed and applied a method based on the optimal control approach based on *NDP* for the examined biotechnological process. The results showed that the quality of biomass was increased at the end of the process.

Second, a method for application of *DP* with the purpose of optimisation of fermentation process named *Rollout* was also developed and presented. Using *Rollout control* method an effective algorithm for process optimisation was synthesized. An optimal profile of feeding rate was obtained. Investigation showed that in this case, it was particularly simple to implement a *Rollout approach* and it should be applied for an *on-line* replanting and in situations when the problem parameters changed the fermentation processes over time.

These developed optimisation methods did not have a feedback and it did not guarantee robustness to process disturbances. *MPC* was developed to guarantee robustness of the process disturbances. This optimisation was carried out off-line, theoretical on base developed investigations an illustration how we can control of disturbance of the optimal control variable (feeding rate). Also this off-line *MPC* was technique for tracking of constrained fermentation systems. The *NDP* algorithm was applied for local optimisation of choice optimisation hour in order to find an optimal profile of the control variable. The developed control algorithm-combined *NDP* and *MPC* ensured maximal biomass production at the end of the process and guaranteed a feedback on disturbance as well as robustness to process disturbances. This off-line *MPC* optimisation to a non-linear fermentation system can lead with both persistent disturbance and time-varying parameters. Also this presented *MPC* optimal control strategy overcame the problem by off-line optimizations prior to implementation and shown that robust stability can be ensured.

## Conflict of interest

The authors declare no conflict of interest.

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