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Research Article

Performance Study of an M/M/1 Retrial Queueing System with Balking, Dissatisfied Customers, and Server Vacations

Ismailkhan Enayathulla Khan, Rajendran Paramasivam*

Department of Mathematics, School of advanced sciences, Vellore Institute of Technology, Vellore-632014, India
E-mail: prajendran@vit.ac.in

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Abstract: This study considers a single server retrial queueing system with dissatisfied customers, server vacations, and balking customers. When new arrivals clients see an idle server available, they immediately enter service. If a new arrival sees the server busy, that arrival may balk and leave the system. Otherwise, the new arrival will enter a retrial orbit and try again later. Customers who are dissatisfied with the service will be served again until the service is completed successfully. The server begins a vacation when the current customer completes service. Birth and death state transitions are given. Steady-state distributions for queue length are found. Furthermore, a recursive technique is used to evaluate probabilities. Several performance measures of the system are identified and effects of changes to parameters are observed.

***Keywords*:** retrial queue, balking, server break, dissatisfied customer

MSC: 60K25, 60G20, 60J05, 60J10, 60J20

1. Introduction

In several actual lifetime circumstances, everyone experiences waiting in a queue to get their requirements fulfilled. This can be seen in several situations such as banks, superstores, etc. Such kinds of systems involving clients experiencing a waiting line or queue for service are interpreted as queueing systems. Most queueing disciplines involve arriving clients waiting in a separate waiting space to get their service.

The client finds a busy service channel, then they will move into the waiting space known as orbit. These blocked clients who are deprived of the provision will retry their request from orbit over a random time. This occurrence of clients making a repetitive stream of requests to occupy the busy server is referred to as a retrial queueing system.

For example, in an Simple Mail Transfer Protocol (SMTP) protocol, the messages that are unable to reach the busy mail server will keep on buffering until it reach the destination node. Recently, the queueing theory has been expanding in the field of understanding client impatience. It not only indicates that potential clients will be harmed but also that money will be lost and the organization's brand image will be destroyed, which is a major danger to corporate society.

Sometimes the clients hopeless about the service and quits the system. Balking refers to the behavior of this sort of client who abandons the system after visualizing a larger queue size. For instance, in a healthcare system, a patient who arrives and sees that more patients are waiting outside the clinic would leave without seeing it.

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Another type of client will move into orbit when the server is busy or unavailable. Due to the lengthier waiting period, some of the clients will lose their patience and leave the organization which is named as reneging. For instance, incoming passengers may drop out of the queue at a train reservation counter because of the extended wait time.

In several queueing concepts, there occurs a situation where some of the clients are served repeatedly. This is because when a consumer is dissatisfied with the caliber of service provided by the service provider, he or she joins the back of the queue and requests another service until the service is completed. This behavior of a client who comes back to re-initiate his demand is referred to as feedback in queueing terminology.

For example, this scenario can be found in Automatic Repeat Re-Quest (ARQ) protocol that operates in an excessive frequency transmission network. In ARQ, if the dispatcher from an input port does not receive an acknowledgment about the routed packet before timeout, then it will not reach its proper destination. It usually retransmits the packets until the sender receives an acknowledgment.

The busy server in the service facility will keep on providing service to the arriving clients regularly. Once the service of the last existing client gets completed, then the server will take a break period. At the break end instant, the idle server will be waiting for the latest clients if no clients are waiting in the system. For example, a client care representative in an IVR system will look after the subscriber details, complaint registrations, etc. during his vacation period.

Under the concept of retry queueing representations, many fresh investigations are investigated. Jeongsim Kim and Bara Kim [1] presented an analysis of retrial queues using various queueing designs that focused on queue size delivery and the probability of waiting times for incoming clients.

A concept of communication errors in a single server classical retrial approach was created by Lakaour et al. [3], and stationary probability was reached by using the generating function technique. Additionally, Arivudainambi et al. stochastic decomposition law for the retrial queue is explicit for a single server below the working break period [2].

In their analysis of a retrial system with a Markovian arrival process, D'Arienzo et al. [4] found that a single server admits a cluster of clients together with their impatient behavior. For a single server queue with a retrial strategy and working break time, Tao Li et al. [5] designed the traditional retrial approach and established several display guidelines.

Ke et al. [6] have also examined the single and multiple server break times for multi-server queues with retry strategies using rate transitions and matrix geometric methods. To create a stochastic model of an unsatisfied retrial queueing system, Ke et al. [9] used impatient clients who refused to participate in the scheme.

In a retrial queueing organization with balking and optional service in addition to server break time, Arivudainambi and Godhandaraman [7] urbanized, incoming clients change the orbit or balk from the location where they never enter the queue future. Chang et al.'s analysis of the retrial queue for a single unreliable server that is prone to unanticipated breakdowns with input from consumers [8] examined the shorter conventional, constant retrial queue solutions.

A discrete period retrial queueing technique with geometric distributions and Bernoulli feedback was taken into consideration by Upadhyaya and Shweta [10]. The single-server constant retrial queues with Bernoulli vacations and equilibrium joining procedures were examined in [11]. The server may have unforeseen problems that have the impact of commencing failures.

2. Model description

Consider a single server queue with a retry policy operating during a server break time, an unhappy client, and a client that balked at performing. First-In-First-Out (FIFO) discipline will be used to collect services single from entering clients. Clients' Poisson arrival is interpreted using the mean λ.

The arriving clients will move into the service station when seeing an idle server and take service with exponentially distributed mean μ. Otherwise, these clients will move into the orbit to reattempt their demand in an arbitrary interval of period that is exponentially distributed with mean γ.

The arriving clients on visualizing the lengthier waiting queue will leave without entering the system. The balking clients will receive their probability of β and those who are joining will choose the probability $1 - \beta$. Clients who leave the waiting area due to a prolonged wait will have a likelihood of σ, and those who resubmit their request will have a probability of $1 - \sigma$.

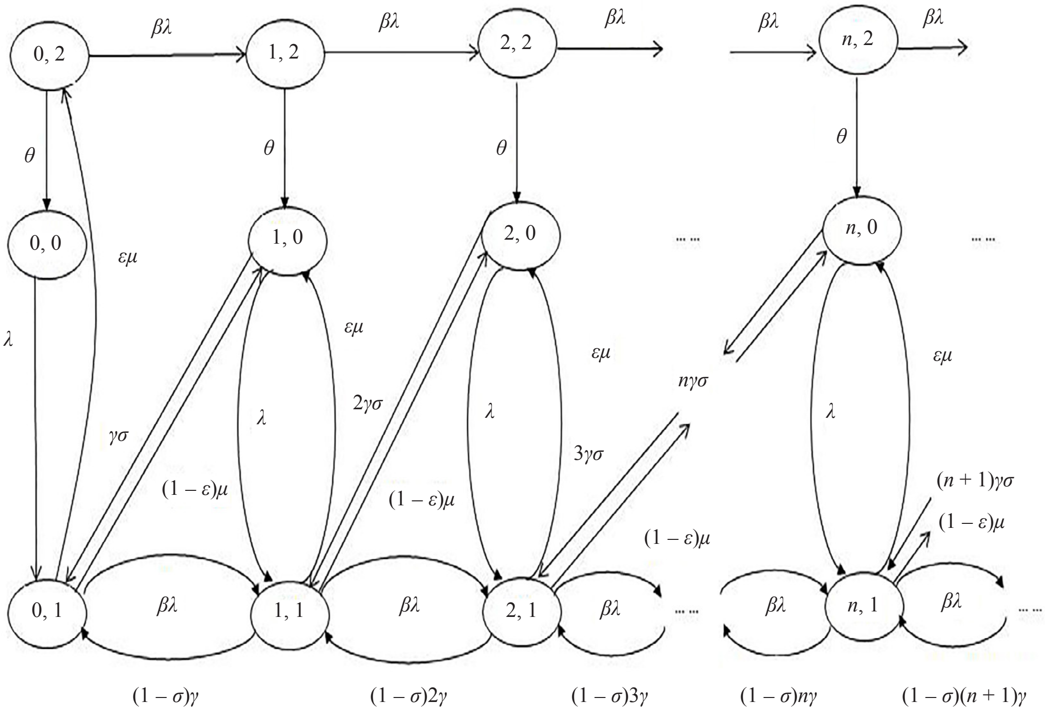

Figure 1. Steady state transition diagram

The clients with successful service completion will leave the system with probability ε and those who are not satisfied with the service will go to reinitiate for another service with probability $1 - \varepsilon$. The vacationing server follows exponential distribution after the service's last existing client with rate θ. Further, all the arrival time distributions, service time distributions, and vacation times are independent of one another.

Let us study the random variables $\zeta(t)$ and $N(t)$ that discusses the status of the server and the total count of clients in the waiting zone at any period t individually. The arbitrary variable representing eminence of the server $\xi(t)$ is given by,

$$\xi(t) = \begin{cases} 0, \text{ The arriving clients will enter into the retrial orbit} \\ 1, \text{ Clients arriving into the system during busy period} \\ 2, \text{ The server is on vacation after serving all the clients} \end{cases}$$

At time t for node (n, s), $P_{n,s}(t) = Prob\{\xi(t) = s, N(t) = n\}$ be the steady-state probabilities. The Markov process of bivariate $\{\xi(t), N(t) : t \geq 0\}$ is discrete and continuous with respect to state space and time respectively. Markov model is analyzed at steady state i.e., when $t \to \infty$ and the state probability is represented by $P_{n,s} = \lim_{t\to\infty} P_{n,s}(t)$.

The Chapman-Kolmogorov equations that control steady-state probability for a single server queue with retrial policy, server break duration, impatience, and dissatisfied clients are as follows. These birth-death birth-death Markov chain equations are encapsulated and described using the recursive technique, where $\xi(t)$ has values of 0, 1, and 2 accordingly.

$$\lambda P_{0,0} = \theta P_{0,2}. \tag{1}$$

$$\left(\lambda + \eta\gamma\sigma\right) P_{n,0} \; = \; \varepsilon\mu P_{n,1} + \theta P_{n,2} + \left(1 \; - \; \varepsilon\right) \mu P_{n-1,0}; \; n \geq 1. \tag{2}$$

$$\left(\beta\lambda + \left(1-\varepsilon\right)\mu + \varepsilon\mu\right) P_{0,1} = \lambda P_{0,0} + \gamma\sigma P_{1,0} + \left(1-\sigma\right)\gamma P_{1,1}. \tag{3}$$

$$\left(\beta\lambda + \left(1-\varepsilon\right)\mu + \varepsilon\mu + \left(1-\sigma\right) n\gamma\right) P_{n,1} = \lambda P_{n,0} + \beta\lambda P_{n-1,1} + \left(1-\sigma\right)\left(n+1\right)\gamma P_{n+1,1} + \left(n+1\right)\gamma\sigma P_{n+1,0}; \; n \geq 1. \tag{4}$$

$$\left(\beta\lambda + \theta\right) P_{0,2} = \varepsilon\mu P_{0,1}. \tag{5}$$

$$\left(\beta\lambda + \theta\right) P_{n,2} = \beta\lambda P_{n-1,2}; \; n \geq 1. \tag{6}$$

From equation (1), the probability for the server is on vacation can be obtained as,

$$P_{0,2} = \frac{\lambda}{\theta} P_{0,0}. \tag{7}$$

Taking equation (6) and substituting (7), after setting $n = 1, 2, \ldots$ the probabilities for the number of clients arriving during server vacation is given by,

$$P_{1,2} = \frac{\beta\lambda^2}{\theta\left(\beta\lambda + \theta\right)} P_{0,0}. \tag{8}$$

$$P_{2,2} = \frac{\beta^2\lambda^3}{\theta\left(\beta\lambda + \theta\right)^2} P_{0,0}. \tag{9}$$

In general, the probabilities of the total amount of clients arriving during server vacation are derived

$$P_{n,2} = \frac{\beta^n\lambda^{n+1}}{\theta\left(\beta\lambda + \theta\right)^n} P_{0,0}. \tag{10}$$

Consider equation (5), and using (7), the following probability can be obtained as,

$$P_{0,1} = \frac{\lambda\left(\beta\lambda + \theta\right)}{\omega} P_{0,0}; \text{ Where } \omega = \theta\varepsilon\mu. \tag{11}$$

Taking equation (3) and using the above-derived equation (11), the probability of the first client retrying to get service is obtained as,

$$P_{1,0} = \frac{\varphi_0}{\gamma\sigma\omega} P_{0,0} - \frac{\left(1-\sigma\right)}{\sigma} P_{1,1}, \tag{12}$$

where

$$\varphi_0 = \lambda\left[(\beta\lambda+\theta)^2 - \omega\right].$$

Substituting n = 1 in equation (2) and using (12) & (8), the probability of leading client arrival during a usually busy time is obtained as,

$$P_{1,1} = \frac{\psi_1}{\gamma\omega\zeta_1} P_{0,0}, \tag{13}$$

where

$$\psi_1 = \varphi_0(\lambda+\gamma\sigma)(\beta\lambda+\theta) - \beta\lambda^2\omega - (1-\varepsilon)\mu\omega(\beta\lambda+\theta).$$

$$\zeta_1 = (\beta\lambda+\theta)\left[(1-\sigma)(\lambda+\gamma\sigma)+\varepsilon\mu\sigma\right].$$

Substituting n = 1 in (4) and using the above-derived equations (11), (12), and (13), the probabilities are given as,

$$P_{2,0} = \frac{\varphi_1}{2\gamma^2\sigma^2\omega\zeta_1} P_{0,0} - \frac{(1-\sigma)}{\sigma} P_{2,1}, \tag{14}$$

where

$$\varphi_1 = \left(\beta\lambda+\mu+(1-\sigma)\gamma\right)\psi_1\sigma - \lambda\varphi_0\zeta_1 + \lambda(1-\sigma)\psi_1 - \beta\lambda^2(\beta\lambda+\theta)\gamma\zeta_1\sigma.$$

Similarly setting n = 2 in equation (2) and using the above-derived equations (14), (9), (12), and (13), the stationary distributions obtained by,

$$P_{2,1} = \frac{\psi_2}{2\gamma^2\sigma\omega\zeta_1\zeta_2} P_{0,0}, \tag{15}$$

where

$$\psi_2 = \varphi_1(\lambda+2\gamma\sigma)(\beta\lambda+\theta)^2 - \beta^2\lambda^3 2\gamma^2\sigma^2\omega\zeta_1 - (1-\varepsilon)\mu\varphi_0 2\gamma\sigma\zeta_1(\beta\lambda+\theta)^2 + (1-\varepsilon)\mu(1-\sigma)\psi_1 2\gamma\sigma(\beta\lambda+\theta)^2.$$

$$\zeta_2 = (\beta\lambda+\theta)^2\left[(1-\sigma)(\lambda+2\gamma\sigma)+\varepsilon\mu\sigma\right].$$

Similarly, setting n = 2 in (4) and using (15), (14) & (13) and further using the above-derived equations after substituting n = 3 in (2), we obtained the probabilities as,

$$P_{3,0} = \frac{\varphi_2}{6\gamma^3\sigma^3\omega\zeta_1\zeta_2} P_{0,0} - \frac{(1-\sigma)}{\sigma} P_{3,1}, \tag{16}$$

where

$$\varphi_2 = (\beta\lambda + \mu + (1-\sigma)2\gamma)\psi_2\sigma - \lambda\varphi_1\zeta_2 + \lambda(1-\sigma)\psi_2 - \beta\lambda\psi_1 2\gamma\sigma^2\zeta_2.$$

$$P_{3,1} = \frac{\psi_3}{6\gamma^3\sigma^2\omega\zeta_1\zeta_2\zeta_3} P_{0,0}, \tag{17}$$

and

$$\psi_3 = \varphi_1(\lambda + 3\gamma\sigma)(\beta\lambda + \theta)^3 - \beta^3\lambda^4 6\gamma^3\sigma^3\omega\zeta_1\zeta_2 - (1-\varepsilon)\mu\varphi_1 3\gamma\sigma\zeta_2(\beta\lambda + \theta)^3 + (1-\varepsilon)\mu(1-\sigma)\psi_2 3\gamma\sigma(\beta\lambda + \theta)^3.$$

$$\zeta_3 = (\beta\lambda + \theta)^3\left[(1-\sigma)(\lambda + 3\gamma\sigma) + \varepsilon\mu\sigma\right].$$

Further setting $n = 3$ in (4) and using the above-derived probabilities (15), (16) & (17) the following equation is obtained as,

$$P_{4,0} = \frac{\varphi_3}{24\gamma^4\sigma^4\omega\zeta_1\zeta_2\zeta_3} P_{0,0} - \frac{(1-\sigma)}{\sigma} P_{4,1}, \tag{18}$$

where

$$\varphi_3 = (\beta\lambda + \mu + (1-\sigma)3\gamma)\psi_3\sigma - \lambda\varphi_2\zeta_3 + \lambda(1-\sigma)\psi_3 - \beta\lambda\psi_2 3\gamma\sigma^2\zeta_3.$$

Therefore, the state of the server during the arrival of clients at the initial stage is given by,

$$P_{0,s} = \begin{cases} \frac{\varphi_0}{\gamma\sigma\omega} P_{0,0} - \frac{(1-\sigma)}{\sigma} P_{1,1} & ;\ \text{for } s = 0 \\ \frac{\lambda(\beta\lambda + \theta)}{\omega} P_{0,0} & ;\ \text{for } s = 1 \\ \frac{\lambda}{\theta} P_{0,0} & ;\ \text{for } s = 2\ . \end{cases}$$

The generalized forms for the above-derived probabilities of client arrival during idle, busy, and vacation times can be expressed in the following manner,

$$P_{n,s}=\begin{cases}\dfrac{1}{n!\gamma^n\sigma^n\omega}\prod_{i=1}^{n-1}\dfrac{1}{\zeta_i}\left[\varphi_{n-1}-(1-\sigma)\psi_n\right]P_{0,0} & ;\ \text{for } s=0\\[2ex] \dfrac{\psi_n}{n!\gamma^n\sigma^{n-1}\omega}\prod_{i=1}^{n-1}\dfrac{1}{\zeta_i}P_{0,0} & ;\ \text{for } s=1\\[2ex] \dfrac{\beta^n\lambda^{n+1}}{\theta(\beta\lambda+\theta)^n}P_{0,0} & ;\ \text{for } s=2,\end{cases}$$

where

$$\varphi_n=\left(\beta\lambda+\mu+(1-\sigma)n\gamma\right)\psi_n\sigma-\lambda\varphi_{n-1}\zeta_n+\lambda(1-\sigma)\psi_n-\beta\lambda\psi_{n-1}n\gamma\sigma^2\zeta_n.$$

$$\psi_n=\varphi_{n-1}(\lambda+n\gamma\sigma)(\beta\lambda+\theta)^n-\beta^n\lambda^{n+1}n!\gamma^n\sigma^n\omega\prod_{i=1}^{n-1}\zeta_i-(1-\varepsilon)\mu\varphi_{n-2}n\gamma\sigma\zeta_{n-1}(\beta\lambda+\theta)^n$$

$$+(1-\varepsilon)\mu(1-\sigma)\psi_{n-1}n\gamma\sigma(\beta\lambda+\theta)^n.$$

$$\zeta_n=(\beta\lambda+\theta)^n\left[(1-\sigma)(\lambda+n\gamma\sigma)+\varepsilon\mu\sigma\right].$$

After performing a few algebraic manipulations, the normalized condition to obtain the value of $P_{0,0}$ is given by, $\sum_{n=0}^{\infty}\sum_{s=0}^{2}P_{n,s}=1$.

$$P_{0,0}=\left(\frac{1}{1+A+\sum_{n=1}^{\infty}B_nZ+\sum_{n=1}^{\infty}Y_n}\right);$$

Where

$$A=\frac{\lambda(\beta\lambda+\theta)}{\omega}+\frac{\lambda}{\theta};$$

$$B_n=\frac{1}{n!\gamma^n\sigma^n\omega}\prod_{i=1}^{n-1}\frac{1}{\zeta_i};$$

$$Z=\left[\varphi_{n-1}-(1-\sigma)\psi_n\right]+\frac{\psi_n}{\sigma};$$

$$Y_n=\frac{\beta^n\lambda^{n+1}}{\theta(\beta\lambda+\theta)^n}.$$

3. Performance measures

Probability of the number of clients arriving during idle server (P_0).

$$P_0 = \sum_{n=0}^{\infty} P_{n,0}.$$

$$P_0 = \left(\frac{1}{1+A+\sum_{n=1}^{\infty} B_n Z + \sum_{n=1}^{\infty} Y_n}\right)\left\{1+\sum_{n=1}^{\infty}\frac{1}{n!\gamma^n\sigma^n\omega}\prod_{i=1}^{n-1}\frac{1}{\zeta_i}\left[\varphi_{n-1}-(1-\sigma)\psi_n\right]\right\}.$$

Probability of number of clients arriving during normal busy period (P_B).

$$P_B = \sum_{n=0}^{\infty} P_{n,2} = \left(\frac{1}{1+A+\sum_{n=1}^{\infty} B_n Z + \sum_{n=1}^{\infty} Y_n}\right)\left[\frac{\lambda(\beta\lambda+\theta)}{\omega}+\sum_{n=1}^{\infty}\frac{\psi_n}{n!\gamma^n\sigma^{n-1}\omega}\prod_{i=1}^{n-1}\frac{1}{\zeta_i}\right].$$

Probability of number of clients arriving during vacation state (P_V).

$$P_V = \sum_{n=0}^{\infty} P_{n,2} = \left(\frac{1}{1+A+\sum_{n=1}^{\infty} B_n Z + \sum_{n=1}^{\infty} Y_n}\right)\left[\frac{\lambda}{\theta}\left(1+\sum_{n=1}^{\infty}\frac{\beta^n\lambda^n}{(\beta\lambda+\theta)^n}\right)\right].$$

The average number of clients in the queueing system $E(N_s)$.

$$E(N_s) = 1+\left(\frac{1}{1+A+\sum_{n=1}^{\infty} B_n Z + \sum_{n=1}^{\infty} Y_n}\right)\left[\frac{\lambda(\beta\lambda+\theta)}{\omega}+\sum_{n=1}^{\infty}\frac{\psi_n}{n!\gamma^n\sigma^{n-1}\omega}\prod_{i=1}^{n-1}\frac{1}{\zeta_i}\right].$$

The average amount of clients in the retrial orbit waiting for service $E(N_o)$.

$$E(N_o) = \left(\frac{n}{1+A+\sum_{n=1}^{\infty} B_n Z + \sum_{n=1}^{\infty} Y_n}\right)\left\{1+\sum_{n=1}^{\infty}\frac{1}{n!\gamma^n\sigma^n\omega}\prod_{i=1}^{n-1}\frac{1}{\zeta_i}\left[\varphi_{n-1}-(1-\sigma)\psi_n\right]\right\}.$$

The expected waiting period of arriving clients in the queueing system W_s.

$$W_s = \frac{1}{\lambda_{eff}}\left\{1+\left(\frac{1}{1+A+\sum_{n=1}^{\infty} B_n Z + \sum_{n=1}^{\infty} Y_n}\right)\left[\frac{\lambda(\beta\lambda+\theta)}{\omega}+\sum_{n=1}^{\infty}\frac{\psi_n}{n!\gamma^n\sigma^{n-1}\omega}\prod_{i=1}^{n-1}\frac{1}{\zeta_i}\right]\right\}.$$

The expected waiting period of arriving clients in an orbit to get service W_o.

$$W_o = \frac{1}{\lambda_{eff}}\left(\frac{n}{1+A+\sum_{n=1}^{\infty} B_n Z + \sum_{n=1}^{\infty} Y_n}\right)\left\{1+\sum_{n=1}^{\infty}\frac{1}{n!\gamma^n\sigma^n\omega}\prod_{i=1}^{n-1}\frac{1}{\zeta_i}\left[\varphi_{n-1}-(1-\sigma)\psi_n\right]\right\}.$$

4. Numerical illustration

Table 1 and Table 2 clearly show that λ and μ increase and the average number of clients in the queue also increases. In Table 3 and Table 4, γ and ε increase but the average number of clients in the queue increases for Exponential distribution, Erlang 2 distribution, and Hyper Exponential distribution.

Table 1. λ versus $E(N_s)$ for Exponential, Erlang 2, and Hyper Exponential distribution

Arrival Rate In order to see the effect of the parameters λ, μ, γ, ε on various characteristics such as the server's idle time, the system's utilization factor, the average number of clients in the queue, we arbitrarily choose values of λ, μ, γ, ε such that the steady state condition is satisfied	Exponential Distribution	Erlang 2 Distribution	Hyper Exponential Distribution
1	0.193	0.208	0.118
1.5	0.213	0.231	0.129
2	0.233	0.254	0.141
2.5	0.255	0.279	0.153
3	0.278	0.306	0.165
3.5	0.302	0.335	0.178
4	0.328	0.367	0.192
4.5	0.355	0.4	0.206
5	0.385	0.437	0.221
5.5	0.416	0.466	0.234

In Tables 5 and 6, Probability values P_0, P_B and P_V decreases as λ and μ decreases. Probability values P_0, P_B, and P_V increases for increasing values of γ and ε in Tables 7 and 8. Figures 1 and 2 depict the experimental results of the different retrial time distributions and different values of λ and μ on $E(N_s)$ respectively.

Table 2. μ versus $E(N_s)$ for Exponential, Erlang 2, and Hyper Exponential distribution

Service Rate	Exponential Distribution	Erlang 2 Distribution	Hyper Exponential Distribution
3	0.35	0.766	0.12
6	0.363	0.825	0.122
9	0.371	0.893	0.123
12	0.39	0.973	0.125
15	0.406	1.061	0.126
18	0.422	1.183	0.128
21	0.44	1.324	0.13
24	0.459	1.502	0.131
27	0.481	1.73	0.133
30	0.504	2.036	0.135

Table 3. γ versus $E(N_s)$ for Exponential, Erlang 2, and Hyper Exponential distribution

γ	Exponential Distribution	Erlang 2 Distribution	Hyper Exponential Distribution
3	0.385	0.437	0.221
3.5	0.347	0.389	0.197
4	0.317	0.351	0.178
4.5	0.291	0.32	0.162
5	0.269	0.294	0.149
5.5	0.251	0.272	0.138
6	0.235	0.253	0.129
6.5	0.221	0.237	0.12
7	0.208	0.222	0.113
7.5	0.198	0.21	0.107

Table 4. Probability (ε) versus $E(N_s)$ for Exponential, Erlang 2, and Hyper Exponential distribution

ε	Exponential Distribution	Erlang 2 Distribution	Hyper Exponential Distribution
1	0.454	1.703	0.125
2	0.435	1.481	0.123
3	0.418	1.308	0.122
4	0.401	1.17	0.12
5	0.386	1.058	0.119
6	0.372	0.965	0.117
7	0.359	0.886	0.116
8	0.347	0.818	0.115
9	0.336	0.76	0.113
10	0.325	0.71	0.112

Table 5. λ versus probability for varying P_0, P_B, P_V

λ	P_0	P_B	P_V
1	0.621	0.419	0.582
1.5	0.596	0.429	0.571
2	0.569	0.438	0.562
2.5	0.543	0.447	0.553
3	0.516	0.455	0.545
3.5	0.489	0.463	0.536
4	0.463	0.472	0.528
4.5	0.436	0.48	0.522
5	0.408	0.495	0.52
5.5	0.38	0.499	0.52

Table 6. μ versus probability for varying P_0, P_B, P_V

μ	P_0	P_B	P_V
3	0.244	0.221	0.118
6	0.221	0.197	0.129
9	0.198	0.178	0.14
12	0.188	0.162	0.152
15	0.179	0.149	0.165
18	0.172	0.138	0.178
21	0.166	0.129	0.191
24	0.155	0.12	0.206
27	0.151	0.114	0.223
30	0.147	0.107	0.243

Table 7. γ versus probability for varying P_0, P_B, P_V

γ	P_0	P_B	P_V
3	0.408	0.24	0.372
3.5	0.444	0.326	0.342
4	0.475	0.389	0.318
4.5	0.502	0.437	0.298
5	0.526	0.476	0.282
5.5	0.557	0.507	0.267
6	0.582	0.533	0.255
6.5	0.597	0.555	0.24
7	0.621	0.574	0.227
7.5	0.633	0.589	0.212

From Figures 1 and 2, the comparison of the $E(N_s)$ and λ, μ of the three retrial time distributions such as Exponential distribution, Erlang 2 distribution, and Hyper Exponential distribution. When λ and μ increases then $E(N_s)$ also increases.

Table 8. ε versus probability for varying P_0, P_B, P_V

ε	P_0	P_B	P_V
1	0.035	0.403	0.596
2	0.059	0.432	0.562
3	0.079	0.448	0.548
4	0.092	0.462	0.537
5	0.102	0.472	0.534
6	0.11	0.482	0.532
7	0.119	0.488	0.53
8	0.124	0.492	0.529
9	0.129	0.499	0.528
10	0.134	0.502	0.527

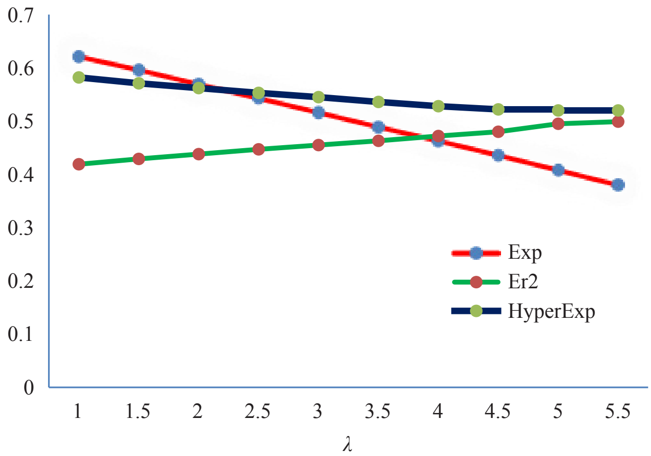

Figure 2. λ versus $E(N_s)$

When γ and ε increases then the average number of clients in the queue decreases in Figure 3 and 4. In Figure 5 and 6, λ and μ increases then the probability values P_B increases and P_0, and P_V decreases. When γ increases then the probability values P_0, P_V decreases and P_B increases in Figure 7. In Figure 8, when ε increases then the probability values P_0, P_B and P_V decreases.

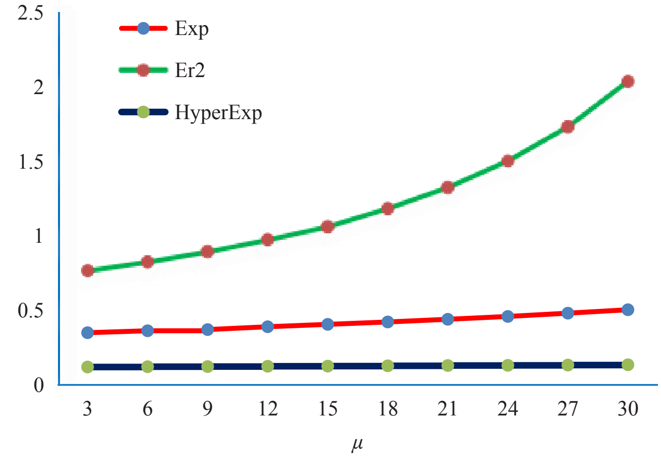

Figure 3. μ versus $E(N_s)$

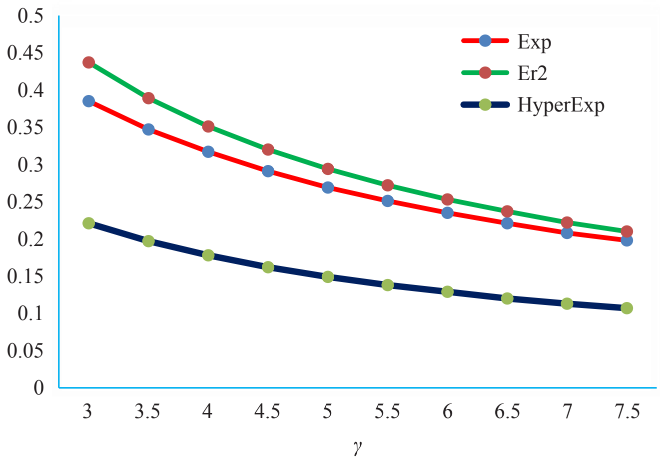

Figure 4. γ versus $E(N_s)$

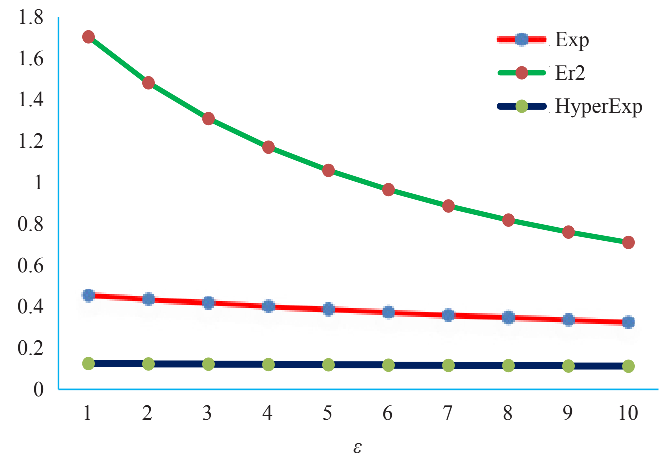

Figure 5. ε versus $E(N_s)$

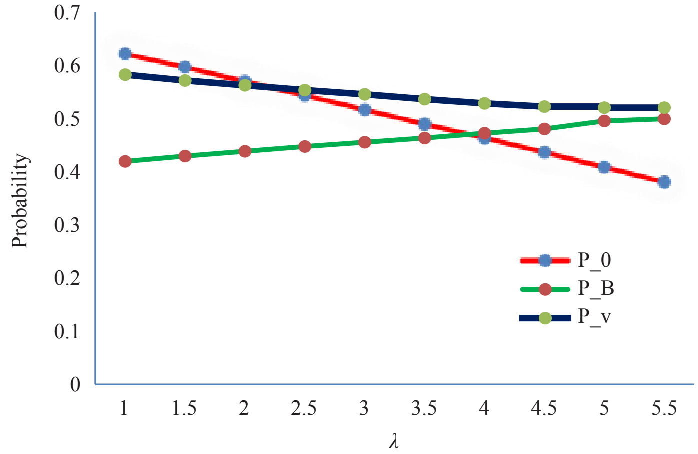

Figure 6. λ versus probability

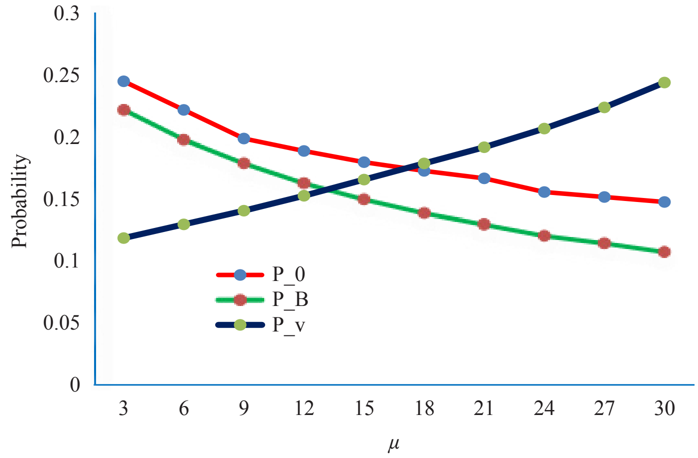

Figure 7. μ versus probability

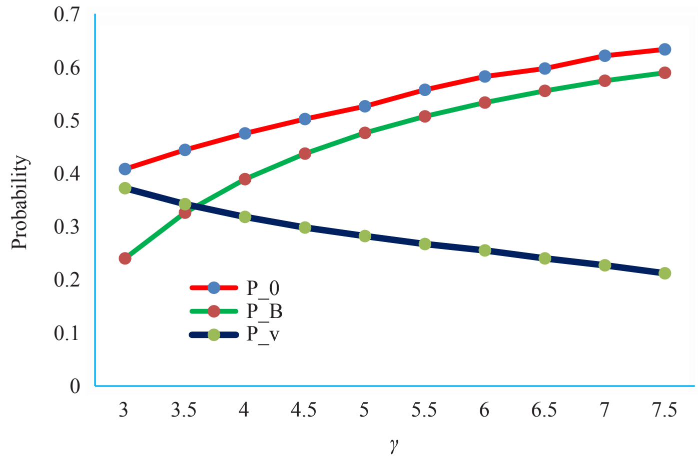

Figure 8. γ versus probability

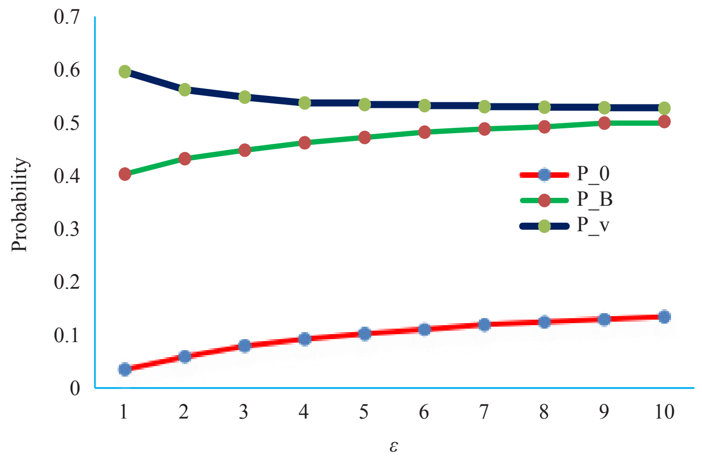

Figure 9. ε versus probability

5. Conclusion

The proposed model of a single server retrial queueing system is investigated under server break period, balking, and unsatisfied clients. The work is generalized to frame the process of clients initiating their repeat requests for service due to unsatisfied service completion. The clients who have successfully completed their service will leave the system with at most satisfaction. Recursive methods are employed to solve the balancing equations. Using explicit formulations, several performance measurements that represent the properties of the system are developed.

Author contributions

Conceptualization, writing-review and editing, resources I.E.K, and methodology, supervision, validation R.P.

Conflict of interest

The authors declare no competing financial interest.

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