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Research Article

An Imperfect Production Inventory Model for Instantaneous Deteriorating Items with Preservation Investment Under Inflation on Time Value of Money

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Abstract: Effective inventory management is crucial for businesses dealing with deteriorating items, where demand is influenced not only by traditional factors but also dependent on the selling price. This research paper proposes a comprehensive inventory model that considers the dynamic nature of demand for deteriorating items and incorporates the impact of inflation on the time value of money, all within the context of an imperfect production and screening process. To reduce the waste of products, the proposed model uses a rework process to convert imperfect items into perfect ones. Additionally, this study incorporates preservation technology investment to alleviate the rate of deterioration under partially backlogged shortages. The incorporation of an imperfect production process further enhances the model's practical applicability by addressing real-world scenarios where production may not always be flawless. The study employs mathematical modeling and optimization techniques to derive optimal ordering policies that balance the conflicting objectives of minimizing costs. To validate the model, numerical analysis has been performed, and the convexity of the total cost function has been shown graphically and analytically using MATHEMATICA software. Sensitivity analyses are conducted to assess the robustness of the proposed model under varying parameters, providing valuable insights for decision-makers.

***Keywords*:** imperfect items, deterioration, preservation investment, shortage, time value of money, inflation

MSC: 90B05, 90B06, 90B30

Notations

Dependent variables

K	Production rate (unit/time)
D	Demand rate

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θ_o	Initial deterioration rate ($0 \leq \theta_o << 1$)
θ	Deterioration rate after preservation investment ($0 \leq \theta << 1$)
ξ	Preservation investment
M	Sensitive parameter of preservation investment
B	Backlogging rate ($0 < B < 1$)
h_c	Holding cost ($/unit/time)
ρ	Fraction (imperfect items)
C_s	Backorder cost ($/unit/time)
R_w	Rework cost ($/unit/time)
L_s	Lost sale cost ($/unit/time)
S	Setup cost ($/setup)
r	Difference between the discount rate and the inflation rate
p	Selling price per unit ($)
K_c	Production cost ($/unit/time)
I_c	Inspection cost per unit per unit time ($/unit/time)
T_1	Production time (months)
T	Inventory cycle length (months)
$I_1(t)$	Level of inventory at time t in time interval $[0, T_1]$
$I_2(t)$	Level of inventory at time t in time interval $[T_1, T_2]$
$I_3(t)$	Level of inventory at time t in time interval $[T_2, T_3]$
$I_4(t)$	Level of inventory at time t in time interval $[T_3, T]$
R	Backlogged quantity
Decision/independent variables	
T_2	Time at which inventory becomes zero (months)
T_3	Time at which production again starts (months)
ξ	Preservation investment ($/unit/time)

1. Introduction

In the dynamic field of supply chain management, efficient administration and optimization of inventory systems are critical to the success of a business. The integration of preservation investment techniques and the effect of inflation on the time value of money are the main topics of this research article, which explores the complicated intricacies of such inventory systems. Any company's financial assets heavily depend on its inventory. Market fluctuations have a big impact on this job, especially in situations like inflation. One way to conceptualize inflation is as a disequilibrium in which rising prices are caused by rising purchasing power. Long-term inflation has an impact on society's morals, politics, social structures, and economy. It is evident from a literature study of earlier inventory studies that the time value of money has not gotten much attention, which is an unrealistic and unfeasible condition. The time value of money, which is connected to return on investment, plays a major role in determining an enterprise's resources. Inflation and the time value of money have proved their utility in today's economic scenario and have attracted the attention of researchers. Buzacott [1] presented an Economic Ordered Quantity (EOQ) model in an inflationary environment in which they assumed a uniform inflation rate for all the associated costs, and by doing so, the model was designed to not only optimize the ordering quantity but, more importantly, to minimize the overall total cost. Further, Bose et al. [2] introduced an EOQ model with a discounting of time policy. Their model revolutionized traditional economic order quantity approaches by incorporating nuanced discounts over time. Ray and Chaudhuri [3] presented an inventory model for items with a stock level-dependent demand rate, a time discounting policy, and inflation. Moon and Lee [4] discussed an EOQ model with inflation, in which they tried to find out the effect of inflation on replenishment policies. Their model offers a valuable resource for businesses seeking to optimize their replenishment policies amidst the complexities of inflationary environments. Jindal and Solanki [5] discussed a Supply Chain Model (SCM) for the vendor and buyer under the condition of controllable lead time. Hemapriya and Uthayakumar [6] discussed a SCM with regard to cost reduction and inflation.

Because of imperfections in manufacturing procedures and the quick deterioration of products, inventory modelling requires a more comprehensive methodology. The unique characteristics of real-world situations, where variables like preservation investments and inflation dynamics have a significant impact on decision-making, are frequently missed by traditional models. In order to close this gap, this study suggests an extensive and intricate inventory model that takes into consideration both the rapid deterioration of products and manufacturing process flaws. The necessity of proactive efforts in minimizing the consequences of deterioration is acknowledged by the inclusion of preservation investment as a fundamental element in the suggested model. The overall performance of inventory systems can be significantly impacted by preservation techniques, whether they involve the strategic allocation of resources or the use of modern technology for storage. In order to increase the robustness of the inventory system, this research aims to determine the ideal amount of preservation investment that reduces expenses while optimizing preservation benefits. The study also looks at how inflation affects the temporal value of money, which is an important topic that is sometimes disregarded in inventory models that follow conventional methods. The goal of the research is to create a more accurate picture of the economic climate in which inventory decisions are made by incorporating inflation dynamics into the framework. This factor is crucial for guaranteeing the sustainability of the supply chain and coordinating inventory strategies with the organization's larger financial objectives.

The paper is structured as follows: In Section 1, an introduction to this paper is presented. Section 2 explores the realm of literature, offering a comprehensive survey that navigates through existing knowledge. Section 3 constitutes a pivotal component of this paper, elucidating the notation, assumptions, and problem definition. Section 4 elucidates the sophisticated techniques underpinning the mathematical formulation of our model, emphasizing the intricate processes involved. Section 5 outlines the solution methodology. A numerical example validating the model is provided in Section 6. Section 7 discloses the results of sensitivity analysis utilizing various parameters and presents a graphical representation of sensitivity. Section 8 further unveils the outcomes of sensitivity analysis, offering deeper insights. Finally, in Section 9, we conclude this paper, delineate the findings, discuss the model's limitations, and set the stage for future research, inviting further exploration.

2. Literature review

2.1 *Inventory models based on imperfect production*

The unrealistic assumption of the modern machine system is that during the production process, machines always produce perfect-quality items, but sometimes machines may produce a perfect type of product. Generally, the machine produces two types of imperfect items: reworkable imperfect items and non-reworkable imperfect items. Reworkable, imperfect items are reworked at some cost and taken back into the system; on the other hand, non-reworkable objects are removed from the system. Porteus [7] was the first to investigate the effect of imperfection on items during the production process. Salmeh et al. [8] discussed an economic production quantity (EPQ) model where items are not of perfect quality. Chiu et al. [9] discussed the effect of imperfect rework processes on the EPQ model. Singh and Singh [10] presented an inventory model (IM) in an inflationary environment by considering exponential demand and Weibull deterioration. Imperfect production has been considered by many researchers while developing their inventory models. Uthayakumar and Palanivel [11] presented an inventory model where demand is dependent upon the selling price. The whole model is studied under the effect of inflation on trade credit policies for imperfect quality items. Jain et al. [12] presented an imperfect production model where demands for items are dependent on time. In addition, volume flexibility and inflation were also considered by them in this model. Khanna et al. [13] developed an imperfect production model with a rework process, inspection errors, and a poor sales return. Ahmed et al. [14] studied the synergic effect of reworking and delay in payment with partially backlogged shortages of items under the effect of an imperfect production process. Singh and Rani [15] developed an EPQ model with multi-variable demand for lifetime items under the effect of inflation and shortages with a markdown policy. Padiyar et al. [16] proposed a multi-echelon supply chain inventory model for deteriorating items with a fuzzy deterioration rate and imperfect production, considering two warehouses under an inflationary environment, involving addressing various complexities.

2.2 *Inventory models based on time value of money*

The money that is there today is likely to increase through investment, so the more invested, the greater the value of the money. When a person wishes to receive money tomorrow instead of today, it implies that he is lending money. The money lent also carries risks like inflation and default risk, so even the money received tomorrow cannot remain unaffected. Understanding the dynamics of investment, the time value of money, and the associated risks is crucial for making informed financial decisions. Diversification, risk management strategies, and staying informed about economic conditions are essential elements of a sound financial approach. Chandra and Bahner [17] examined the effect of inflation on optimal policies. For two different costs, they considered two different inflation rates. Later, Singh et al. [18] presented a three-stage integrated inventory model for imperfect items with limited storage capacity and a variable demand rate. Sarkar et al. [19] discussed the effect of reliability and inflation on the economic manufacturing quantity (EMQ) model. Yadav et al. [20] discussed a lot-sizing model with finite planning horizons and carbon tax regulation under the learning effect. Slobodnyak and Sidorov [21] discussed the time value of money applications in asymmetric payment distributions and real-world economic dynamics in their inventory model. Taheri et al. [22] presented an inventory model considering financial issues. They tried to manage the problems related to dairy products using this model.

2.3 *Inventory models based on deterioration*

When a substance leaves its original state due to decay or loss in its original quality, it is defined as deterioration. Like some chemicals, medicines, food items, etc., can spoil after a certain time. Apart from this, it is the natural tendency of any substance to deteriorate after a certain period. While developing inventory models, the deterioration of items plays a very important role; therefore, the deterioration of items should not be ignored. Tiwari et al. [23] presented a sustainable model in which they focused on deteriorating items under imperfect production processes. Khurana et al. [24] discussed an inventory model where the demand for an item is considered variable under shortages of items, and the model is derived for deteriorating items. Mahapatra et al. [25] described an EOQ model for deteriorating items with a learning effect; the entire model is studied under a fuzzy environment. Tayal et al. [26] presented a two-storage model under shortages for deteriorating items with different demands. Saren et al. [27] discussed discount policies in inventory. Halim et al. [28] presented an inventory model for deteriorating items considering two types of demands, where the first demand is a nonlinear price demand, and the second is a linear stock-dependent demand. Sharma et al. [29] proposed an inventory model for deteriorating items with time-dependent demand from the initial stage to the final stage, i.e., the end of the stock level. For profit maximization and control of the deterioration rate, they assumed different demands at different stages of the model. Malumfashi et al. [30] assumed an exponential demand pattern and linear holding cost in an EPQ model for deteriorating items. Mallick et al. [31] introduced an EOQ model for deteriorating items with stock-dependent demand under the effect of inflation. Their proposed inventory model is analyzed under the finite time horizon.

2.4 *Inventory models based on selling price dependent demand and inflation*

When evaluating inventory models with selling price-dependent demand rates, it is crucial to consider the dynamic nature of consumer behaviour and market conditions. Moreover, it is essential to address the impact of inflation on the time value of money in the context of inventory models. Inflation erodes the purchasing power of currency over time, influencing both the costs associated with holding inventory and the potential revenue generated from sales. Businesses must incorporate inflation-adjusted metrics into their inventory models to account for changes in the value of money over time. In summary, the incorporation of selling price-dependent demand rates and the effect of inflation on the time value of money in inventory models provides a more comprehensive and realistic framework for businesses to make informed decisions about their inventory management. This approach acknowledges the complexities of the market environment, allowing for strategic adaptations that enhance overall efficiency and profitability. Shaikh et al. [32] developed an innovative EPQ model that considers the dual challenges of managing deteriorating items and incorporating a partial trade credit policy under the impact of inflation and reliability on production and inventory decisions. Rahman et al. [33] presented a parametric approach for addressing interval differential equations in an inventory model for deteriorating items where the demand is influenced by selling prices. Indrajitsingha et al. [34] developed an EOQ model that integrates selling-price-dependent demand with considerations for non-instantaneous deteriorating items, taking into account the impact of the COVID-19 pandemic on supply chains and demand uncertainty. Sekar et al. [35] proposed a comprehensive inventory model that incorporates sustainability considerations at three levels for items susceptible to defects and deterioration with rework under marketplace selling prices.

2.5 *Inventory models based on preservation technology*

Preservation technology refers to the methods, techniques, and tools used to protect and extend the lifespan of various objects, materials, or information. The goal of preservation technology is to prevent deterioration, damage, or loss over time, ensuring that items of historical, cultural, scientific, or personal significance are maintained for future generations. Preservation can apply to a wide range of fields, including cultural heritage, digital information, food, and more. Preservation technology includes techniques such as perishable goods storage, quality preservation methods, and advanced tracking systems. Das et al. [36] discuss the application of preservation technology within inventory control systems, specifically addressing scenarios characterized by price-dependent demand and partial backlogging. Padiyar et al. [37] presented an integrated inventory model considering imperfect production processes, preservation facilities, fuzziness, under an inflationary environment. Developing such a model requires a combination of operations research, fuzzy logic, and inventory management principles. Sindhuja and Arathi [38] invented an inventory management model for deteriorating products under preservation technology with time-dependent quality demand, optimizing the ordering and preservation policies to minimize costs and meet customer demand. Bhadoriya et al. [39] discussed the complex relationship between advertising strategies, inventory management, deteriorating goods, preservation technology, customer returns, and trade credit policies in the retail industry.

2.6 *Contribution of this research*

This research work makes several significant contributions to the field of inventory management and supply chain optimization, particularly in the context of deteriorating items with selling price dependent demand with the influence of inflation on the time value of money under imperfect production processes. The key contribution of this study is as follows:

• The propose inventory model that integrates deteriorating items, preservation technology, selling price dependent demand, and imperfect production processes. This holistic approach addresses the complexity of real-world scenarios, providing a more accurate representation of the challenges faced by businesses in managing their inventory systems.

• Explore the impact of inflation on the time value of money within the context of inventory management. Recognizing the interplay between inflation and the financial aspects of inventory control, our study provides valuable insights into the challenges posed by changing economic conditions, assisting businesses in crafting robust strategies for long-term sustainability.

• Incorporating the imperfections inherent in production processes, our model goes beyond idealized assumptions to capture the uncertainties and variations encountered in real-world manufacturing environments. This consideration enhances the practical applicability of our research findings, enabling businesses to develop more resilient inventory management practices.

• The model explicitly incorporates the selling price dependence of demand, recognizing the dynamic relationship between pricing strategies and consumer behaviour. By accounting for this nuanced aspect, our research offers a more realistic portrayal of market dynamics, aiding practitioners in making informed decisions regarding pricing and inventory management.

2.7 *Motivation of research*

In today's economic and financial landscape, the time value of money is a crucial concept, closely intertwined with inventory management. It is rooted in the well-established principle that receiving an amount now is more profitable than receiving an equal amount later. This principle is vital for generating accounting and analytical information in financial management within the context of inventory management. The integration of various methods facilitates a more accurate assessment of economic facts, enhancing the quality of financial management. The concept of the time value of money has ancient roots, with the basic principles of modern call options originating from the works of the ancient Greek philosopher Thales. Many researchers have subsequently developed relevant ideas in their works. Mishra [40] presented a cost model akin to the EOQ model but with a different inflation rate. Datta and Pal [41] introduced an inventory model based on a finite horizon policy and the time value of money, incorporating shortages with time-dependent demand. In a similar vein, Mondal et al. [42] proposed a model considering imperfect production processes and shortages with a variable production cost of items. Chakraborty et al. [43] discussed an EPQ model for imperfect production processes with shortages of items under inflation. In summary, this research paper contributes to the evolving field of inventory management by presenting an imperfect production inventory model tailored for instantaneous deteriorating items. By integrating preservation investment strategies and accounting for the impact of inflation on the time value of money, the proposed model provides a more accurate and practical tool for decision-makers seeking to optimize their inventory systems in a dynamic and uncertain business environment.

2.8 *Research gap*

Differences between this study and previous in the relevant literature can be seen with the help of Table 1. It helps to understand that in previous studies researchers adequately considered the time value of money, but little attention was paid to the imperfect production process. Our study fills the above research gap. The research gap with the existing literature is depicted in Table 1.

Table 1. Research gap table

References	Imperfect items	Demand	Time value of money	Shortages	Deterioration	Preservation investment
Rosenblatt & Lee [44]	Yes	Constant	No	No	No	No
Sarkar & pan [45]	No	Constant	Yes	Yes	No	No
Bose et al. [2]	No	Time dependent	Yes	Yes	Yes	No
Salameh & Jaber [46]	Yes	SPD	No	No	No	No
Chung et al. [47]	No	Constant	Yes	No	Yes	No
Chen [48]	No	Time proportion	Yes	Yes	Yes	No
Lin [49]	Yes	Constant	No	No	No	No
Wee & Law [50]	Yes	SPD	Yes	No	Yes	No
Roy & Chaudhari [51]	No	SD	Yes	Yes	No	No
Mondal et al. [44]	Yes	SPD	No	No	No	No
Maiti et al. [52]	No	SPD	No	Yes	No	No
Chakrabarty et al. [43]	Yes	SPD	Yes	Yes	No	No
Singh & Rani [15]	No	Multi variable demand	No	Yes	Yes	No
Mallick et al. [31]	No	SD	No	No	Yes	No
Present paper	Imperfect	SPD	Yes	Yes	Yes	Yes

SPD: Selling Price Dependent, SD: Stock Dependent

3. Problem description, assumptions, and notations

3.1 *Problem description*

The current study is based on an economic production quantity model of instantaneous deteriorating items. In this production model, producer considers such a production system which produces perfect as well as imperfect quality items. Perfect items are ready to be sold in the market, on the other hand imperfect items are reworked at some cost to make them perfect. The graphical representation of the proposed model is shown in Figure 1.

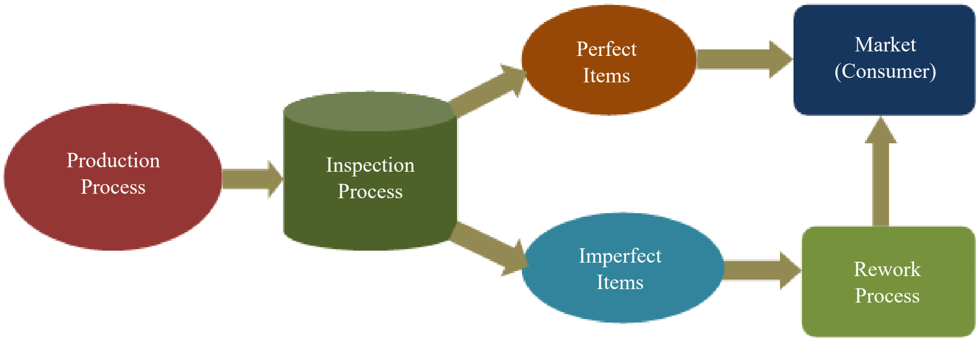

Figure 1. Graphical framework of imperfect production inventory model

3.2 *Assumptions*

Following are the assumptions for the development of this model:

1. This model assumes production of imperfect items. Imperfect items can affect customer satisfaction and demand. Customers may be less willing to purchase imperfect items or may demand discounts. Incorporating imperfect items into the model a more accurate representation of the impact on demand and revenue becomes possible.

2. Making imperfect items reworkable at some cost provides an opportunity to enhance the overall quality of the product. This iterative process leads to better customer satisfaction and loyalty.

3. The primary purpose of incorporating selling price-dependent demand into models is to provide businesses with valuable insight into consumer behaviour, market dynamics, and the financial implications of pricing decisions. Due to this reason, assumption of selling price dependent demand for the items is considered here. Therefore $D = ap^{-b}$, where p is selling price per unit, a and $b > 0$ (Chakraborty et al. [43]).

4. In inventory management, a constant production rate can have many effects on the overall system. With its help, businesses streamline their inventory management processes, so, they can more accurately forecast demand, optimize stock levels and reduce the possibility of stock outs or excess inventory.

5. A partially backlogged shortage occurs after the production and consumption period.

6. Inflation is also taken into consideration along time value of money. By considering inflation and the time value of money, businesses can make more informed decisions about inventory levels, pricing strategies, and overall financial planning within their supply chain operations.

7. Efficient lead time management is essential for avoiding stock outs and maintaining optimal inventory levels. In this model zero lead time has been taken.

8. Deterioration arises in intervals $[0, T_1]$ and $[T_1, T_2]$, where the deterioration rate ($0 \le \theta_o << 1$) is very small.

9. To control rate of deterioration preservation technology is used and the reduced deterioration rate is $\theta = \theta_o e^{-M\xi}$. Preservation technology effectively extends the shelf life of deteriorating items, due to which the rate of spoilage reduces.

4. Mathematical formulation

In this model, producer considers a production system which produces perfect as well as imperfect quality items. Perfect items are ready to be sold in the market, on the other hand imperfect items are reworked at some cost to make them perfect. The production starts at time $t = 0$, continuous up to $t = T_1$ andhere level of inventory increases at its maximum level due to production and decreases due to the combined effect of deterioration and demand. In the time interval $[T_1, T_2]$ the level of inventory decreases only due to the combined effect of demand and deterioration and finally reaches at zero at time $t = T_2$. After this shortage occurs and level of inventory decreases in the time interval $[T_2, T_3]$ only due to the demand. At $t = T_3$ production starts again and backlogged cleared at the time $t = T$. The pictorial representation of producer's inventory model is shown in Figure 2. The inventory system is depicted with the help of the first order differential equation given below.

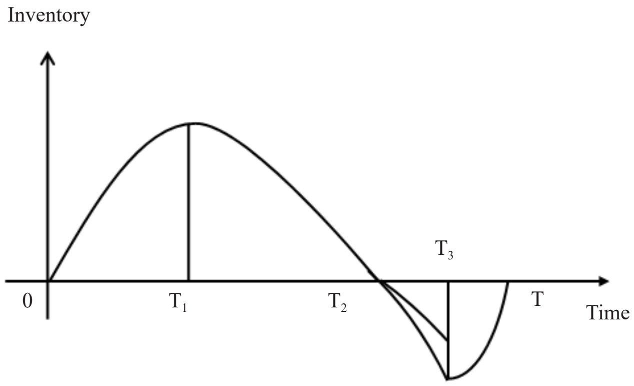

Figure 2. Representation of production inventory system

$$\frac{dI_1(t)}{dt} = K - ap^{-b} - \theta I_1(t), \quad 0 \le t \le T_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -ap^{-b} - \theta I_2(t), \quad T_1 \le t \le T_2$$

$$\frac{dI_3(t)}{dt} = -Bap^{-b}, \quad T_2 \le t \le T_3 \tag{2}$$

$$\frac{dI_4(t)}{dt} = -ap^{-b}, \quad T_3 \le t \le T \tag{3}$$

With boundary conditions

$$I_1(0) = 0,\ I_2(T_1) = Q,\ I_2(T_2) = 0,\ I_3(T_2) = 0,\ I_3(T_3) = -R,\ I_4(T) = 0 \tag{4}$$

Solution of equation (1), (2), (3) and (4) are

$$I_1(t) = \frac{K - ap^{-b}}{\theta}\left(1 - e^{-\theta t}\right) \tag{5}$$

$$I_2(t) = \frac{ap^{-b}}{\theta}\left[e^{\theta(T_2-t)} - 1\right] \tag{6}$$

$$I_3(t) = Bap^{-b}(T_2 - t) \tag{7}$$

$$I_4(t) = \left(K - ap^{-b}\right)(t - T) \tag{8}$$

Using $I_1(T_1) = I_2(T_1)$

$$T_1 = \frac{1}{\theta}\log\left[1 - \frac{ap^{-b}}{K}\left(1 - e^{\theta T_2}\right)\right] \tag{9}$$

Using $I_3(T_3) = I_4(T_3)$

$$T_3 = \frac{KT + ap^{-b}(BT_2 - T)}{K + ap^{-b}(B-1)} \tag{10}$$

Now the inventory costs are:

(a) **Production Cost:** Production cost includes the sum of all direct and indirect costs that are involved in the production of services or goods in the process of production. It also includes cost of materials, cost of labor and cost of energy etc. Hence cost of production in this case is:

$$P.C. = K_c\left[\int_0^{T_1} Ke^{-rt}dt + \int_{T_3}^{T} Ke^{-rt}dt\right]$$

$$= \frac{K_c K}{r}\left[\left(1 - e^{-rT_1}\right) + \left(e^{-rT_3} - e^{-rT}\right)\right] \tag{11}$$

(b) **Inspection Cost:** It includes the cost for separating imperfect and perfect items from the items to be produced during production. Hence inspection cost for inventory system is:

$$I.C. = I_c\int_0^{T_1} Ke^{-rt}dt$$

$$= I_c K\left(\frac{1 - e^{-rT_1}}{r}\right) \tag{12}$$

(c) **Holding Cost:** The first problem that arises after placing of an order is related to handling of inventory and services. For this, selection of store or warehouse is required to keep these products. Holding cost is an important factor in the cost incurred by the buyer. Therefore, holding cost in this case is:

$$H.C. = h_c\left[\int_0^{T_1} I_1(t)e^{-rt}dt + \int_{T_1}^{T_2} I_2(t)e^{-rt}dt\right]$$

$$= h_c \left[\begin{array}{l} \dfrac{K - ap^{-b}}{\theta}\left(\dfrac{1 - e^{-rT_1}}{r} + \dfrac{e^{-(\theta+r)T_1} - 1}{\theta + r}\right) \\ + \dfrac{ap^{-b}}{\theta}\left\{ e^{\theta T_2}\left(\dfrac{e^{-(\theta+r)T_1} - e^{-(\theta+r)T_2}}{\theta + r}\right) + \left(\dfrac{e^{-rT_2} - e^{-rT_1}}{r}\right)\right\} \end{array} \right] \tag{13}$$

(d) **Shortage Cost:** Shortage costs are those costs incurred by an organization when there is a shortage of stock. Shortage of stock can result in loss of customer confidence, and can also lead to cancellation of orders, which cause in reduced sales of goods or services, which may result in loss of business. Therefore, shortage cost is:

$$S.C. = C_s\left[\int_{T_2}^{T_3} -I_3(t)e^{-rt}dt + \int_{T_3}^{T} I_4(t)e^{-rt}dt\right]$$

$$= C_s \left[\begin{array}{l} Bap^{-b}\left\{(T_2 - T_3)\dfrac{e^{-rT_3}}{r} + \dfrac{1}{r^2}(e^{-rT_2} - e^{-rT_3})\right\} \\ +(K - ap^{-b})\left\{(T - T_3)\dfrac{e^{-rT_3}}{r} + \dfrac{1}{r^2}(e^{-rT} - e^{-rT_3})\right\} \end{array} \right] \tag{14}$$

(e) **Lost Sale Cost:** In today's competitive market environment backordering cost is the most sensitive cost. In case of shortage or non-availability of products at retail outlets, customers cannot wait long for them, In this situation customers can shift elsewhere. When unfulfilled demand is completely lost, it is called lost sale cost.

$$L.S.C. = L_S\left[\int_{T_2}^{T_3}(1 - B)ap^{-b}e^{-rt}dt\right]$$

$$= L_S(1 - B)ap^{-b}\left(\frac{e^{-rT_2} - e^{-rT_3}}{r}\right) \tag{15}$$

(f) **Setup Cost:** Smart production of goods or services requires improvement in mechanical infrastructure. So that one-time investment by the manufacturer can get the production in full cycle time. It is one of the major costs involved in production. So, setup cost is:

$$S.T.P. = S + S(1 - rT) \tag{16}$$

(g) **Rework Cost:** Rework cost is the cost incurred to separate the defective items and make them functional again after the process of inspection. This cost helps reduce raw material consumption, reduce energy use, and keep the environment healthy. In this case the rework cost is:

$$R.C. = R_w\left[\int_0^{T_1}\rho K^{\sigma-1}e^{-rt}dt\right]$$

$$= \rho K^{\sigma-1} R_w \left(\frac{1-e^{-rT_1}}{r} \right) \tag{17}$$

Hence the Total Cost of the production inventory system

$$TC = P.C + I.C + H.C + S.C + L.S.C + S.T.P + R.W$$

$$TC = \left[\begin{array}{l} \frac{K_c K}{r}\left[(1-e^{-rT_1}) + (e^{-rT_3} - e^{-rT}) \right] + I_c K \left(\frac{1-e^{-rT_1}}{r} \right) \\ +h_c \left[\begin{array}{l} \frac{K - ap^{-b}}{\theta} \left(\frac{1-e^{-rT_1}}{r} + \frac{e^{-(\theta+r)T_1} - 1}{\theta + r} \right) \\ + \frac{ap^{-b}}{\theta} \left\{ e^{\theta T_2} \left(\frac{e^{-(\theta+r)T_1} - e^{-(\theta+r)T_2}}{\theta + r} \right) + \left(\frac{e^{-rT_2} - e^{-rT_1}}{r} \right) \right\} \end{array} \right] \\ +C_S \left[\begin{array}{l} Bap^{-b} \left\{ (T_2 - T_3)\frac{e^{-rT_3}}{r} + \frac{1}{r^2}(e^{-rT_2} - e^{-rT_3}) \right\} \\ +(K - ap^{-b}) \left\{ (T - T_3)\frac{e^{-rT_3}}{r} + \frac{1}{r^2}(e^{-rT} - e^{-rT_3}) \right\} \end{array} \right] \\ +L_S (1-B) ap^{-b} \left(\frac{e^{-rT_2} - e^{-rT_3}}{r} \right) + \left[S + S(1 - rT) \right] \\ +\rho K^{\sigma-1} R_w \left(\frac{1-e^{-rT_1}}{r} \right) \end{array} \right] \tag{18}$$

At the time of planning horizon, we have N production cycles. Time value of money and inflation are eliminated in each replenishment cycle so effect of these are taken over time horizon NT. Equation (16) represent the expected total cost at the starting of first cycle. Suppose TC_P represents the current value of the expected total cost for all cycles. The time value of money affects the net profit except the first setup cost. Therefore TC_P is given by:

$$TC_P = TC \sum_{i=0}^{N} e^{irT} - S$$

$$TC_P = TC \left(\frac{1-e^{-rNT}}{1-e^{-rT}} \right) - S$$

$$TC_P = \left[\left\{ \begin{array}{l} \frac{K_c K}{r}\left[(1-e^{-rT_1})+(e^{-rT_3}-e^{-rT})\right]+I_c K\left(\frac{1-e^{-rT_1}}{r}\right) \\ +h_c\left[\begin{array}{l} \frac{K-ap^{-b}}{\theta}\left(\frac{1-e^{-rT_1}}{r}+\frac{e^{-(\theta+r)T_1}-1}{\theta+r}\right) \\ +\frac{ap^{-b}}{\theta}\left\{e^{\theta T_2}\left(\frac{e^{-(\theta+r)T_1}-e^{-(\theta+r)T_2}}{\theta+r}\right)+\left(\frac{e^{-rT_2}-e^{-rT_1}}{r}\right)\right\} \end{array} \right] \\ +C_S\left[\begin{array}{l} Bap^{-b}\left\{(T_2-T_3)\frac{e^{-rT_3}}{r}+\frac{1}{r^2}(e^{-rT_2}-e^{-rT_3})\right\} \\ +(K-ap^{-b})\left\{(T-T_3)\frac{e^{-rT_3}}{r}+\frac{1}{r^2}(e^{-rT}-e^{-rT_3})\right\} \end{array} \right] \\ +L_S(1-B)ap^{-b}\left(\frac{e^{-rT_2}-e^{-rT_3}}{r}\right)+\left[S+S(1-rT)\right] \\ +\rho K^{\sigma-1}R_w\left(\frac{1-e^{-rT_1}}{r}\right) \end{array} \right\} \left(\frac{1-e^{-rNT}}{1-e^{-rT}}\right)-S \right] \tag{19}$$

5. Solution methodology

There are three independent variables in the total cost function, preservation investment ξ, time T_2 and T_3. To optimize the total cost function of inventory model, the following steps are followed:

Step 1 Calculate the first order partial derivative w. r. to ξ, T_2, and T_3

$$\text{i.e, } \frac{\partial TC_P}{\partial \xi}, \frac{\partial TC_P}{\partial T_2} \text{ and } \frac{\partial TC_P}{\partial T_3}.$$

Step 2 Solve

$$\frac{\partial TC_P}{\partial \xi}=0, \frac{\partial TC_P}{\partial T_2}=0, \frac{\partial TC_P}{\partial T_3}=0, \text{ and find } \xi, T_2, \text{ and } T_3.$$

Step 3 Again find

$$\frac{\partial^2 TC_P}{\partial \xi^2}, \frac{\partial^2 TC_P}{\partial T_2^2}, \frac{\partial^2 TC_P}{\partial T_3^2}, \frac{\partial^2 TC_P}{\partial T_2 \partial \xi}, \frac{\partial^2 TC_P}{\partial \xi \partial T_2}, \frac{\partial^2 TC_P}{\partial T_3 \partial \xi}, \frac{\partial^2 TC_P}{\partial \xi \partial T_3}, \frac{\partial^2 TC_P}{\partial T_2 \partial T_3}, \text{ and } \frac{\partial^2 TC_P}{\partial T_3 \partial T_2}.$$

Step 4 Derive Hessian matrix G

$$G=\begin{bmatrix} \frac{\partial^2 TC_P}{\partial \xi^2} & \frac{\partial^2 TC_P}{\partial \xi \partial T_2} & \frac{\partial^2 TC_P}{\partial \xi \partial T_3} \\ \frac{\partial^2 TC_P}{\partial T_2 \partial \xi} & \frac{\partial^2 TC_P}{\partial {T_2}^2} & \frac{\partial^2 TC_P}{\partial T_2 \partial T_3} \\ \frac{\partial^2 TC_P}{\partial T_3 \partial \xi} & \frac{\partial^2 TC_P}{\partial T_3 \partial T_2} & \frac{\partial^2 TC_P}{\partial {T_3}^2} \end{bmatrix}$$

Step 5 Find all possible principal minors of matrix G (i.e. G_{11}, G_{22} and G_{33}), where G_{11}, and G_{22}, and G_{33} denote the first, second and third principal minors respectively.

Step 6 If all the principal minors of matrix G are positive (i.e. $G_{11} > 0$, $G_{22} > 0$ and $G_{33} > 0$), then total cost function is convex.

Theorem 1 The total cost function of production inventory system $TC_P(T_2, T_3, \xi)$ is a pseudo convex function and attained minimum value at (T_2^*, T_3^*, ξ^*) for the discrete values of preservation investment (ξ), if $W \geq 0$ and $WZ \geq \Psi^2$ where the value of W, Z and Ψ are given in Appendix A.

Proof. Differentiating Eq. (19) w. r. to T_2 and T_3, we get

$$\frac{\partial TC_P}{\partial T_2}=\frac{(1-e^{-rNT})}{T(1-e^{-rT})}\left\{\begin{array}{c} -ap^{-b}L_s(1-B)e^{-rT_2}+ap^{-b}BC_s\left(\frac{e^{-rT_3}-e^{-rT_2}}{r}\right) \\ \frac{ap^{-b}h_c}{\theta}\left(\theta e^{\theta T_2}\left(\frac{e^{-(\theta+r)T_1}-e^{-(\theta+r)T_2}}{\theta+r}\right)\right) \end{array}\right\} \tag{20}$$

$$\frac{\partial TC_P}{\partial T_3}=\frac{(1-e^{-rNT})}{T(1-e^{-rT})}\left\{\begin{array}{c} ap^{-b}L_s(1-B)e^{-rT_3}-KK_ce^{-rT_3}-C_se^{-rT_3} \\ \left((K-ap^{-b})(T-T_3)+ap^{-b}B(T_2-T_3)\right) \end{array}\right\} \tag{21}$$

Solving the following equations $\frac{\partial TC_P}{\partial T_2}=0$ and $\frac{\partial TC_P}{\partial T_3}=0$, we can find the value T_2 and T_3 respectively.

Again differentiating equation (20) and (21) partially w. r. to T_2 and T_3 respectively, we get

$$\frac{\partial^2 TC_P}{\partial {T_2}^2}=\frac{(1-e^{-rNT})}{T(1-e^{-rT})}\left\{\begin{array}{c} arp^{-b}L_s(1-B)e^{-rT_2}+ap^{-b}BC_se^{-rT_2}+\frac{arp^{-b}h_ce^{-rT_2}}{\theta} \\ \frac{ap^{-b}h_c}{\theta}\left(e^{\theta T_2}\left((\theta-r)e^{-(\theta+r)T_2}+\frac{(e^{-(\theta+r)T_1}-e^{-(\theta+r)T_2})\theta^2}{\theta+r}\right)\right) \end{array}\right\}$$

$$\frac{\partial^2 TC_P}{\partial {T_3}^2}=\frac{(1-e^{-rNT})}{T(1-e^{-rT})}\left\{\begin{array}{c} -arp^{-b}L_s(1-B)e^{-rT_3}-KrK_ce^{-rT_3}+C_se^{-rT_3} \\ \left(ap^{-b}B+(K-ap^{-b})\left(1+r(T-T_3)\right)+arp^{-b}B(T_2-T_3)\right) \end{array}\right\}$$

And

$$\frac{\partial^2 TC_P}{\partial T_2 \partial T_3}=\frac{\partial^2 TC_P}{\partial T_3 \partial T_2}=\left\{-\frac{ap^{-b}BC_se^{-rT_3}(1-e^{-rNT})}{(1-e^{-rT})T}\right\}$$

Now

$$G_{11} = \left|\frac{\partial^2 TC_P}{\partial T_2^{\ 2}}\right| = \frac{(1-e^{-rNT})}{T(1-e^{-rT})}\left\{\begin{array}{c} arp^{-b}L_s(1-B)e^{-rT_2} + ap^{-b}BC_s e^{-rT_2} + \dfrac{arp^{-b}h_c e^{-rT_2}}{\theta} \\ \dfrac{ap^{-b}h_c}{\theta}\left(e^{\theta T_2}\left((\theta-r)e^{-(\theta+r)T_2} + \dfrac{\left(e^{-(\theta+r)T_1} - e^{-(\theta+r)T_2}\right)\theta^2}{\theta+r}\right)\right)\end{array}\right\}$$

$$G_{11} = W > 0$$

$$G_{22} = \begin{vmatrix} \dfrac{\partial^2 TC_P}{\partial T_2^{\ 2}} & \dfrac{\partial^2 TC_P}{\partial T_2 \partial T_3} \\ \dfrac{\partial^2 TC_P}{\partial T_3 \partial T_2} & \dfrac{\partial^2 TC_P}{\partial T_3^{\ 2}} \end{vmatrix} = \left(\frac{\partial^2 TC_P}{\partial T_2^{\ 2}}\right)\left(\frac{\partial^2 TC_P}{\partial T_3^{\ 2}}\right) - \left(\frac{\partial^2 TC_P}{\partial T_2 \partial T_3}\right)^2 = WZ - \Psi^2$$

$G_{22} > 0$ as $WZ - \Psi^2 > 0$.

Since all the principal minors of Hessian matrix G are positive therefore the Hessian matrix G is positive definite. Hence the total cost function TC_P of production inventory system is convex in nature.

6. Numerical illustration of the model

To illustrate the proposed inventory model, we consider a numerical example with the help of following parametric values.

Example K = 40 unit/month, p = \$ 400, h_c = 100 \$/unit/month, B = 0.7 unit/month, C_s = 140 \$/unit/month, a = 50, b = 0.1, R_w = 150 \$/unit/month, S = 200 \$/setup, r = 0.08, T_1 = 2 months, T = 6 months, K_c = 120 \$/unit/month, ρ = 0.6, I_c = 60 \$/month, δ = 0.8, N = 5, L_s = 100 \$/unit/month, θ_o = 0.5, M = 4,

Solution Total Cost TC = \$ 6,816.62, Preservation Investment ξ = 13.0748,

Time T_2 = 4.29751 months, Time T_3 = 5.58549 months.

6.1 *Graphical representation of the convexity*

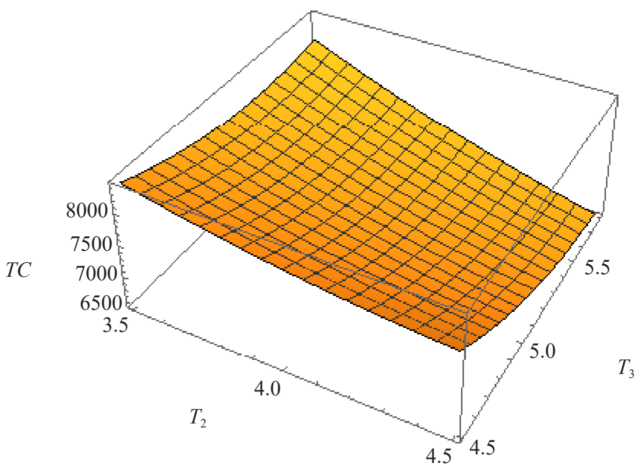

Figure 3. Convexity of total cost (TC) w. r. to T_2 and T_3

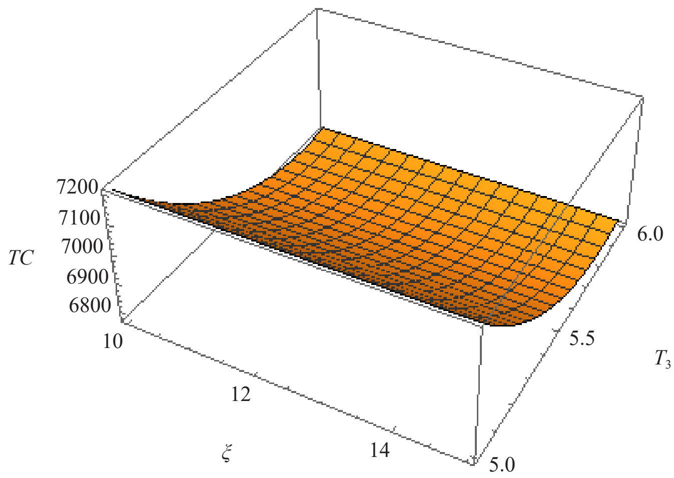

Figure 4. Convexity of total cost (TC) w. r. to T_3 and ξ

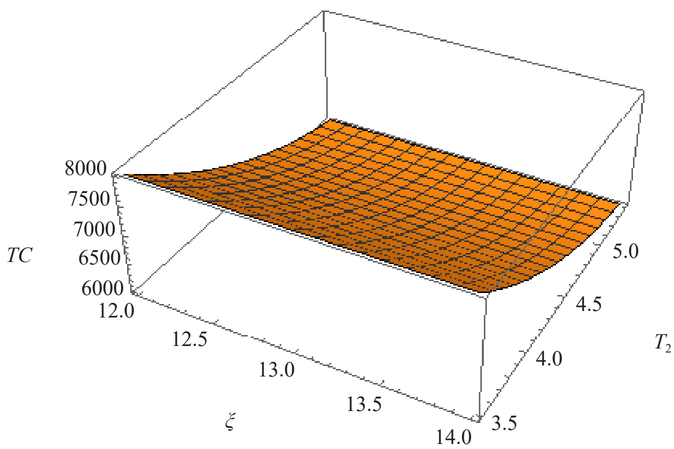

Figure 5. Convexity of total cost (TC) w. r. to T_2 and ξ

Figure 3, 4 and 5 represent the Convexity of the total cost function with respect to decision variables.

6.2 *Comparison study for special cases*

Table 2. Changes in total cost for special cases

Special cases	Preservation investment	Time T_2	Time T_3	Total cost (TC)
With preservation investment	13.0748	4.29751	5.58549	6,816.62
Without preservation investment	-	3.35508	5.29323	9,927.73

From the above Table 2, it is cleared that by implementation of the preservation technology provides a substantial decrease in the overall cost of imperfect production inventory system. This highlights the cost-saving impact of adopting preservation methods in inventory management. This cost reduction is attributed to minimized spoilage, enhanced inventory management, and streamlined production processes. The findings emphasize the financial and operational advantages of adopting preservation techniques in modern business contexts. The effect of change in total cost *TC* is shown in Figure 6.

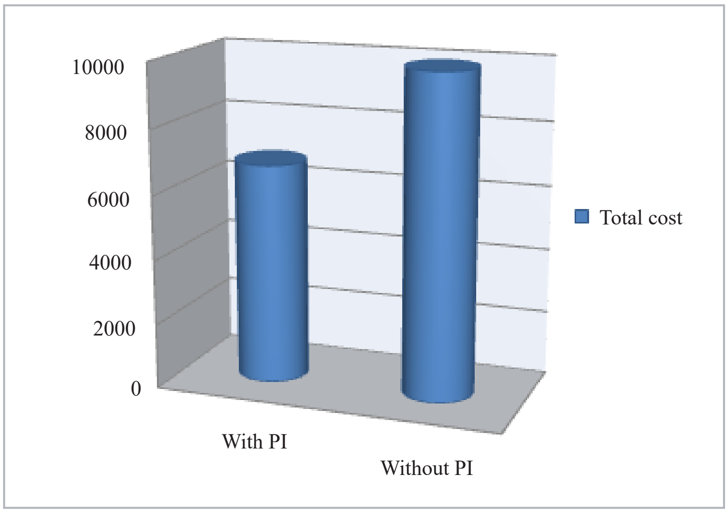

Figure 6. Pictorial representation of total cost with or without PI

7. Sensitivity analysis

Performing the sensitivity analysis by varying the parameters from -20% to +20%, for different values of the parameters K, r, p, b, K_c, B, a, I_c, h_c, θ_o, ρ, C_s, T, staking one parameter at a time, in turn remaining parameters are treated as a constant. The results of which are presented from in Table 3.

Table 3. Sensitivity analysis for various parameters

Parameters	Changes in %	Preservation investment	Time T_2	Time T_3	Total cost (TC)
$K = 40$	-20	13.8738	4.26109	5.12339	5,737.55
	-10	13.3995	4.28361	5.34098	6,303.19
	+10	11.9362	4.36620	5.76831	7,020.62
	+20	11.7190	4.56614	5.91479	7,896.47
$p = 400$	-20	13.5968	4.29596	5.53845	6,850.69
	-10	13.3856	4.29793	5.56341	6,831.37
	+10	13.0693	4.30079	5.60555	6,798.97
	+20	12.9198	4.30941	5.62443	6,787.74

$r = 0.08$	-20	8.85062	3.81700	5.55814	8,420.00
	-10	10.8544	4.11147	5.57235	7,322.22
	+10	15.7900	4.30567	5.59785	6,578.46
	+20	17.5552	4.35113	5.65278	5,929.44
$K_c = 120$	-20	13.1422	4.24728	5.34120	6,114.13
	-10	13.1031	4.26913	5.36788	6,512.52
	+10	13.0479	4.35980	5.73347	7,090.05
	+20	13.0144	4.42251	5.87648	7,344.29
$I_c = 120$	-20	13.0748	4.29751	5.58549	6,463.95
	-10	13.0748	4.29751	5.58549	6,640.29
	+10	13.0748	4.29751	5.58549	6,992.96
	+20	13.0748	4.29751	5.58549	7,169.29
$h_c = 100$	-20	12.5738	4.56979	5.75412	6,461.21
	-10	12.8919	4.43008	5.68098	6,636.13
	+10	13.2645	4.19047	5.51553	6,965.04
	+20	13.6049	4.09906	5.44778	7,097.91
$a = 50$	-20	12.0163	4.22874	5.94987	6,594.73
	-10	12.7889	4.27766	5.78098	6,692.89
	+10	13.5319	4.31164	5.36016	6,978.97
	+20	13.7507	4.34503	5.10400	7,162.63
$b = 0.1$	-20	14.2982	3.75281	5.24509	7,723.98
	-10	13.1217	4.28808	5.44731	6,917.61
	+10	12.9827	4.29036	5.69942	6,740.97
	+20	12.8208	4.29569	5.80094	6,676.42
$C_s = 140$	-20	13.6181	4.22471	5.74399	6,710.21
	-10	13.2109	4.25474	5.65136	6,776.52
	+10	12.5814	4.36962	5.54148	6,865.40
	+20	12.3435	4.50485	5.51028	6,902.35
$B = 0.7$	-20	13.7514	4.27917	5.69977	6,811.59
	-10	13.3522	4.29229	5.64396	6,814.40
	+10	12.8923	4.29887	5.53780	6,822.15
	+20	12.5315	4.31842	5.53096	6,825.56

$S = 200$	-20	13.0748	4.29751	5.58549	6,799.12
	-10	13.0748	4.29751	5.58549	6,807.87
	+10	13.0748	4.29751	5.58549	6,825.37
	+20	13.0748	4.29751	5.58549	6,834.13
$\theta_o = 0.5$	-20	13.9395	4.29527	5.59520	6,819.67
	-10	13.6206	4.29748	5.58634	6,818.97
	+10	12.8742	4.30084	5.58308	6,814.78
	+20	12.7361	4.30644	5.58099	6,805.20
$T = 6$	-20	7.55203	3.50723	4.91169	10,599.8
	-10	10.2802	3.84647	5.23298	89,584.2
	+10	13.5974	4.43127	5.90581	6,370.94
	+20	14.9454	4.72172	6.20873	5,637.89
$\rho = 0.6$	-20	13.0748	4.29751	5.58549	6,810.30
	-10	13.0748	4.29751	5.58549	6,813.46
	+10	13.0748	4.29751	5.58549	6,819.78
	+20	13.0748	4.29751	5.58549	6,822.95

7.1 *Graphical representation of sensitivity analysis*

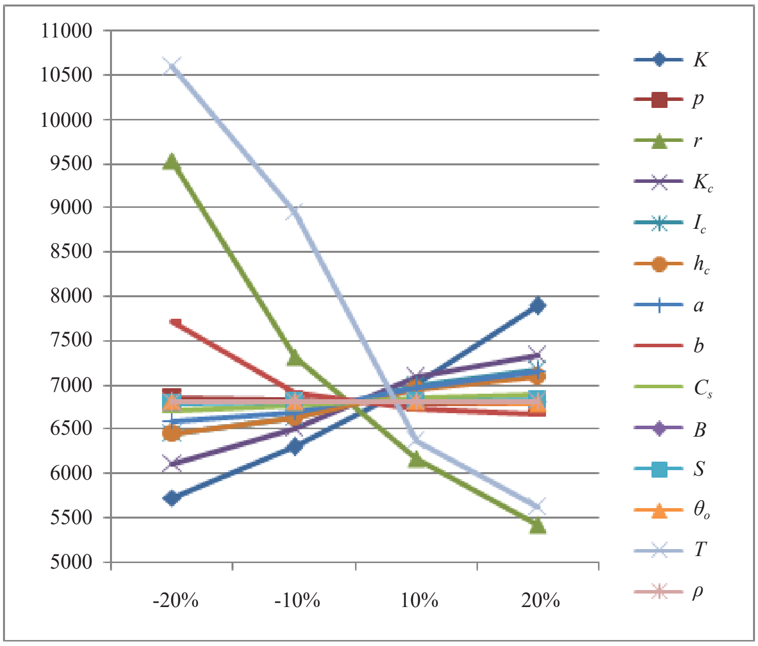

Figure 7. Change in total cost function w. r. to various parameters

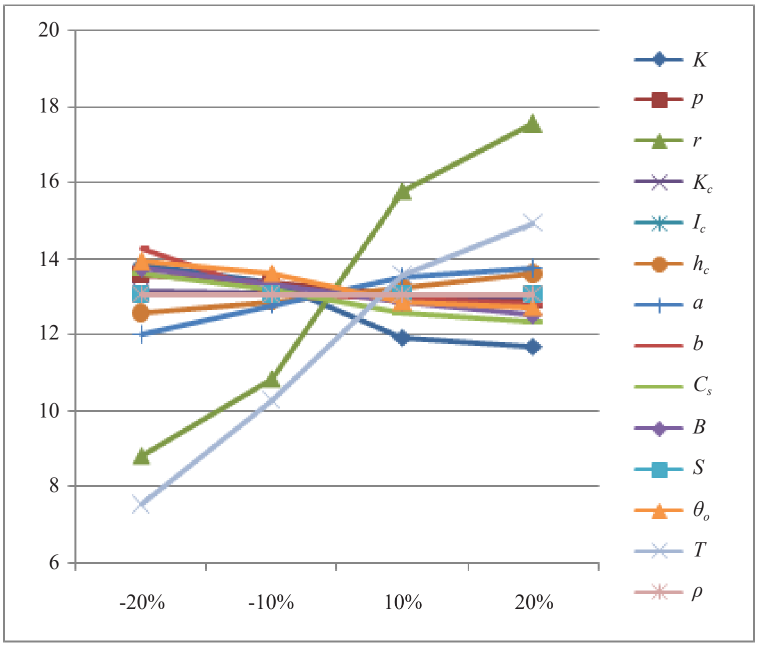

Figure 8. Change in preservation investment w. r. to various parameters

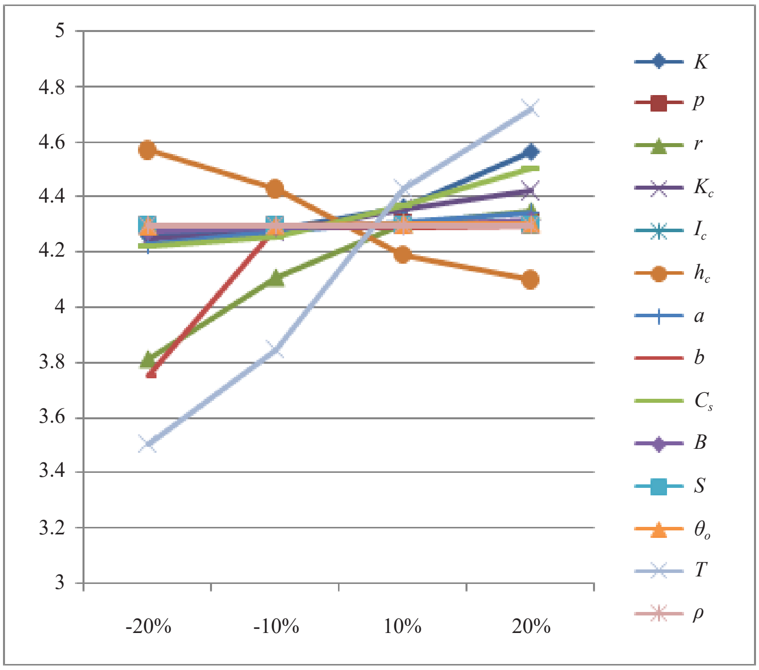

Figure 9. Change in time T_2 w. r. to various parameters

8. Observations and managerial insights

To assess the impact of variations in various key parameters related to the production inventory system on total cost changes, the effects of these different parameters visually depicted from Figures 7, 8, 9 and 10.

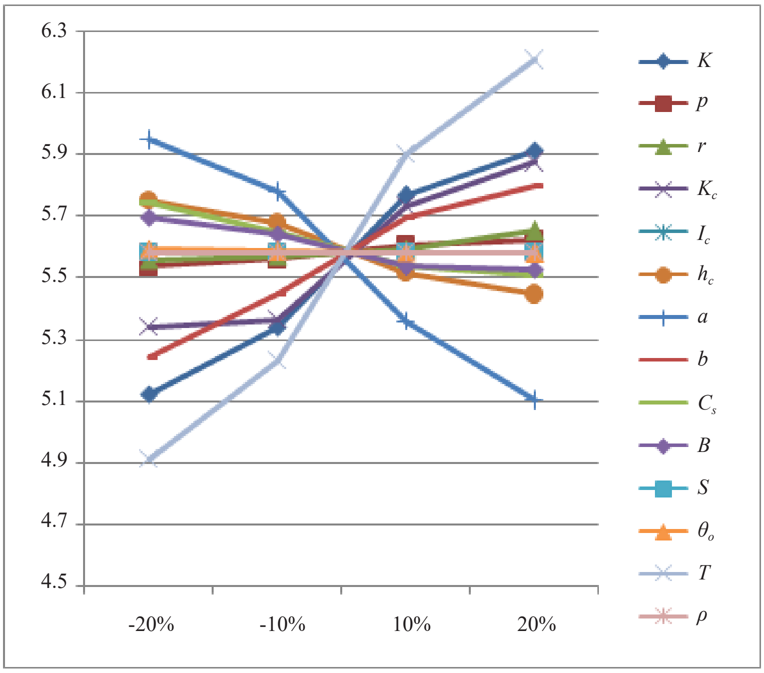

Figure 10. Change in time T_3 w. r. to various parameters

8.1 *Observations*

• As the parameter K is increased, the values of the time parameters T_2 and T_3 exhibit minimal sensitivity to changes in parameter K, whereas the total cost TC and preservation investment ξ demonstrate a high degree of sensitivity.

• It is seen that when we take different values of r, then by performing sensitivity analysis by changing the parameter by -20%, to +20%, then preservation investment ξ, time T_2 and T_3 are increases while total cost TC decreases.

• With the increment in selling price p, the total cost TC and the preservation investment ξ decreases while the values of time T_2 and T_3 are increases. A same type of effect has been seen for the demand parameter b.

• With the increment in holding cost parameter h_c, the total cost TC and the preservation investment ξ increases while the values of time T_2 and T_3 are decreases.

• With an increment in the value of demand parameter a, total cost TC, preservation investment ξ and time T_2 increases while the time T_3 decreases.

• With an increment in the initial deterioration rate θ_o, the total cost TC, preservation investment ξ and time T_3 decreases while the time T_2 moderately increases.

• High parameter values of K_c results in higher value of total cost while the values of parameters T_2 and T_3 are moderately increases and the value preservation investment ξ is moderately decreases.

• High parameter values of I_c results in higher value of total cost while the values of time T_2 and T_3 and preservation investment ξ are less sensitive with changes in I_c. A same type of effect has been seen for the fraction of imperfect items ρ.

• With the increment in backlogging parameter B, the total cost TC and time T_2 increases while the preservation investment ξ and time T_3 are decreases. A same type of effect has been seen for the shortage cost parameter C_s.

• High parameter values of cycle time T results in lower value of total cost while the time T_2 and T_3 and preservation investment ξ all are increases with increment in T.

8.2 *Managerial implication*

The managerial implications of this research paper are broad and touch upon various aspects of inventory management, like pricing strategies, risk management, and financial planning. Implementing the insights from this research could lead to more efficient and effective inventory management practices, ultimately contributing to improved overall business performance. Here are some managerial implications that can be derived from the key components of the research:

• The inclusion of preservation technology in the model suggests that managers should consider investing in technologies that can extend the shelf life of deteriorating items. This decision can impact the overall inventory holding costs and improve the efficiency of the supply chain.

• With selling price-dependent demand, the research may shed light on how pricing strategies can influence inventory levels. Managers could explore dynamic pricing models that take into account the impact of pricing on demand and use this information to optimize both pricing and inventory decisions.

• Understanding the effect of inflation on the time value of money is crucial for making accurate financial decisions. Managers can incorporate this knowledge into their cost-benefit analyses, helping them make more informed decisions regarding inventory investments, pricing, and financial planning.

• To implement the proposed inventory management strategies effectively, the research may highlight the importance of advanced information systems. Investing in technology that allows for real-time tracking, data analysis, and forecasting can enhance decision-making processes.

• Implementing the findings of the research may require a certain level of expertise. Managers may need to invest in reworkprocess for reducing waste and minimized total cost of inventory system.

9. Conclusion, limitations and scope for future research

9.1 *Conclusion*

In today's competitive market environment, uncertainty about future inflation can affect savings and investments. Due to which the rate of inflation may increase and due to increased inflation rate there may be shortage of goods. This concern may result in hoarding of goods or services by the consumers, which will increase the prices in future. Therefore, ignoring inflation when developing an inventory system can lead to misleading results. This paper develops a comprehensive approach for inventory model for managing deteriorating items with selling price dependent demand and impact of inflation on the time value of money within the imperfect production process. As selling price dependent demand is significantly influences the decision-making process, so this paper highlights the importance of such a demand in inventory models for deteriorating items. By taking the imperfect production process, the model appears more realistic, which helps in overcoming the challenges faced by real world businesses. Furthermore, analyzing the effect of inflation on the time value of money emphasizes the need for a nuanced understanding of financial considerations in inventory management.

9.2 *Limitations*

The effectiveness of any model is highly dependent on the quality and accuracy of the data used for calibration and validation. If the data used in the research is incomplete or inaccurate, it can affect the reliability of the model. As inventory management is influenced by external factors such as market dynamics, consumer behaviour, and economic conditions. If the research does not account for these dynamic elements, the model's predictions may not hold over time. Concepts of imperfect production process have been presented through the proposed research paper. In reality, production processes may be subject to disruptions, quality issues, and other imperfections that the model cannot fully account for. Additionally, this paper also presents its ideas on the impact of inflation on the time value of money, but it cannot fully capture the broader effects of inflation on other aspects of the supply chain, such as procurement costs, transportation costs and pricing strategies. It is essential to critically evaluate the assumptions and methods used in the research and consider the practical implications of the findings in real-world business scenarios. Additionally, acknowledging these limitations in the paper demonstrates transparency and helps guide future research in addressing these challenges.

9.3 *Scope for future research*

In the future, potential avenues for research include examining the influence of supply chain integration on the proposed inventory model. This investigation could delve into the ways collaboration and coordination among various entities within the supply chain impact inventory management, particularly in the context of handling deteriorating items under imperfect production processes. Additionally, there is an opportunity to explore the dynamics of pricing strategies, specifically in relation to selling price-dependent demand. An exploration of how dynamic pricing models can be seamlessly integrated into the inventory management framework to optimize both revenue and inventory levels would be beneficial. Furthermore, the integration of emerging technologies such as the Internet of Things (IoT), block-chain, or artificial intelligence into the proposed inventory model presents another avenue for investigation. This research could assess how these technologies enhance visibility, traceability, and decision-making within the inventory management process.

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Conflict of interest

There is no conflict of interest for this study.

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Appendix A

$$W = \frac{\partial^2 TC_P}{\partial {T_2}^2}$$

$$Z = \frac{\partial^2 TC_P}{\partial {T_3}^2}$$

and

$$\Psi = \frac{\partial^2 TC_P}{\partial T_2 \partial T_3} = \frac{\partial^2 TC_P}{\partial T_3 \partial T_2}$$