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Research Article

Numerical Solutions of Fuzzy Differential Equations by Harmonic Mean and Cubic Mean of Modified Euler's Method

Balaji R[1], Antline Nisha B[2], Saradha M[3], R. Udhayakumar[4*]

[1]Department of Mathematics, Panimalar Engineering College, Chennai-600123, Tamilnadu, India
[2]Department of Mathematics, St. Joseph's Institute of Technology, Chennai-600119, Tamilnadu, India
[3]Departmentof Mathematics, School of Applied Sciences, REVA University, Bangalore-560064, India
[4]Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore-632014, Tamilnadu, India
E-mail: udhayaram.v@gmail.com

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Abstract: We aimed to solve first-order differential equations using two novel techniques: the harmonic mean and the cubic mean of Euler's modified approach for fuzzy primary value in this research proposal. We present a new formulation of Euler's classic approach based on Zadeh's extension concept to address this dependency issue in a fuzzy situation. In the literature, numerical approaches for solving differential equations with fuzzy main values often disregard this issue. With a few examples, we show how our approach outperforms more traditional fuzzy approaches based on Euler's method.

Keywords: modified Euler's method, harmonic mean method, cubic mean method, differential equation, fuzzy initial value, fuzzy solutions

MSC: 34A07

1. Introduction

Differential equations play an important role in modern life, just as they do in science and engineering. This proposed research solves the ordinary differential equation (ODE) problems [1]. The tangent line method is another name for the Euler's method, and it is a simple way to solve first order differential equation (FODE) problems [2-5]. Leonhard Euler [6] discovered a method for rapid training, simple implementation, and low computational cost. However, the accuracy factor leads the investigator to use a new intricate technique to replace the Euler method. Ultimately, the investigators of this study hope to improve the accuracy factor [7] and determine the exact solution by implementing the various mean values. Thus, changes to the current Euler method result in the Modified Euler's method [8-9]. The modification method is used to determine the average gradients. The proposed method is intended to be an improvement on Euler's method, incorporating the harmonic mean and cubic mean.

In recent years, the uncertainty in fuzzy differential equations has become increasingly important in fuzzy analysis. The terms "fuzzy differential equation" [10], "fuzzy differential and integral equations" [11], "fuzzy differential inclusion" [12], and "fuzzy differential inclusion" [13] are used interchangeably because the differential equations with

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fuzzy initial values [14] or fuzzy boundary values are calculated alongside the functions for fuzzy numbers (Seikkala [15], Abbasbandy [16] and Osmo [17]).

The Zadeh's extension principle is most commonly used in fuzzy set theory and fuzzy set operations. While fuzzy logic and fuzzy control systems are not directly used for solving differential equations, they can be applied to differential equations in some cases. Fuzzy control systems, which use fuzzy logic and reasoning to design controllers for systems described by differential equations, can be used to design controllers for systems described by differential equations. These systems may be characterized by uncertainty, imprecise inputs, or complex relationships that are difficult to accurately model using traditional methods.

In such cases, the fuzzy control system can use fuzzy sets and rules to map input variables (which may be related to the state of the system described by differential equations) to output actions, thereby creating a control strategy that accounts for the problem's imprecision and uncertainty. While the Zadeh's extension principle does not directly solve differential equations, it does serve as the foundation for fuzzy logic and control, which can be applied to a wide range of engineering and scientific problems, including differential equations. In cases where precise mathematical models may be difficult to establish, the key is to use fuzzy logic to manage uncertainty and approximate solutions.

In this paper, we discuss the following topics: Section 2 determines some basic terms for triangular fuzzy numbers and fuzzy derivatives [17], as well as the results. In Section 3, we propose two modified Euler's methods for solving the first order fuzzy differential equation with a fuzzy initial value condition. Section 4 illustrates and solves a numerical example using the proposed method. Furthermore, the results of the proposed methods' approximation solutions are compared to the other Euler methods with different step sizes of h.

2. Preliminaries

In this section we will discuss the completion of Harmonic mean and Cubic mean of Modified Euler's [18-19] approach towards to resolve the first order differential equations accompany with the fuzzy initial value problem.

2.1 *Triangular fuzzy number [19]*

A triangular fuzzy number $\tilde{u}$ can be defined by a triplet (a, b, c), the membership function is defined as follows.

$$\tilde{u}(x) = \begin{cases} 0, & if\ x<a \\ \dfrac{x-a}{b-a}, & if\ a \le x < b \\ \dfrac{c-x}{c-b}, & if\ b \le x \le c \\ 0 & if\ x>c \end{cases}.$$

The α-level of the fuzzy number $\tilde{u}$ is $\tilde{u}_\alpha = [a + (b - c)\alpha, c - (c - b)\alpha]$ for any $\alpha \in [0, 1]$.

2.2 *Modified Euler's method using harmonic mean*

Consider a, b and c are the fundamentals of Harmonic sequence. The Harmonic mean is calculated as $c = \frac{2ab}{a+b}$. Then, the anticipated Harmonic mean of the two center points are definite by

$$\left\{ \frac{2x_0x_1}{x_0 + x_1}, \frac{2y_0y_1}{y_0 + y_1} \right\}. \tag{1}$$

The equation also can be expressed as

$$\left\{\frac{2x_0(x_0+h)}{x_0+(x_0+h)}, \frac{2y_0(y_0+hf(x_0,y_0))}{y_0+(y_0+hf(x_0,y_0))}\right\}. \tag{2}$$

Using an equation in the neighborhood of a point, such $P(x_0, y_0)$, and the gradient up the equation, we may derive a new equation written as,

$$y = y_0 + hf\left(\frac{2x_0(x_0+h)}{x_0+(x_0+h)}, \frac{2y_0(y_0+hf(x_0,y_0))}{y_0+(y_0+hf(x_0,y_0))}\right). \tag{3}$$

By using the modified and stable slope function at the predictable central points of (x_0, y_0) and (x_1, y_1) to estimated y_{n+1} the Euler method will be more stable. The above-mentioned equation is termed as Harmonic Mean of Euler's modified method [19]. It can be defined as,

$$y_{n+1} = y_n + hf\left(\frac{2x_n(x_n+h)}{x_n+(x_n+h)}, \frac{2y_n(y_n+hf(x_n,y_n))}{y_n+(y_n+hf(x_n,y_n))}\right). \tag{4}$$

2.3 *Cubic mean of modified Euler's method*

Consider a, b and c are the fundamental elements of Geometric Progression, in that case the Geometric mean of cubic average point is defined as $c = \sqrt[n]{\frac{a^n+b^n}{2}}$. As a result, the reckoning middling of Geometric cubic mean of the two midpoints are expressed as,

$$\left\{\left(\frac{x_0^3+x_1^3}{2}\right)^{1/3}, \left(\frac{y_0^3+y_1^3}{2}\right)^{1/3}\right\}. \tag{5}$$

The equation can be defined as,

$$\left\{\left(\frac{x_0^3+(x_0+h)^3}{2}\right)^{1/3}, \left(\frac{y_0^3+(y_0+hf(x_0,y_0))^3}{2}\right)^{1/3}\right\}. \tag{6}$$

If an equation passes to the course of a point, like $P(x_0, y_0)$, in the midst of the slope throughout the equation, a new equation can be generated and it is given as,

$$y = y_0 + hf\left(\left(\frac{x_0^3+(x_0+h)^3}{2}\right)^{1/3}, \left(\frac{y_0^3+(y_0+hf(x_0,y_0))^3}{2}\right)^{1/3}\right). \tag{7}$$

Modified Euler method provide new sure and precise outcomes. The above equation is called as Cubic Mean of modified Euler's method. It can be written as

$$y_{n+1} = y_n + hf\left(\left(\frac{x_n^3 + (x_n + h)^3}{2}\right)^{1/3}, \left(\frac{y_n^3 + (y_n + hf(x_n, y_n))^3}{2}\right)^{1/3}\right). \tag{8}$$

3. Fuzzy Initial Value Problem (FIVP)

3.1 *Technique for precisely solving FIVP*

Take a look at the FIVP [20]

$$x'(t) = \begin{cases} f(t, x) \\ x(t_0) = \left(\underline{x}_0, x_0^{\bullet}, \overline{x}_0\right) \end{cases}. \tag{9}$$

Here a fuzzy starting condition is in term of triangular fuzzy numbers. Normal differential equations may be recast as an Eigen value problem. By using Taylor's method [9, 12, 16] to find their accurate solutions. The solution to FIVP is obtained using fuzzy beginning conditions.

3.2 *Proposed methods to FIVP*

In this case, we examine the fuzzy initial value issue as

$$x'(t) = \begin{cases} f(t, x) \\ x(t_0) = \left(\underline{x}_0, x_0^{\bullet}, \overline{x}_0\right) \end{cases}. \tag{10}$$

The triangular fuzzy starting condition may be expressed using an r-cut technique as

$$\left[\left(x^{\bullet} - \underline{x}_0\right)r + \underline{x}_0,\ \overline{x}_0 + \left(x^{\bullet} - \overline{x}_0\right)r\right],\ 0 \le r \le 1.$$

The current modified Euler's method has been designed to solve the FIVP. At this time, all the new upper and lower bound possible permutation are evaluated by implementing the Euler's modified method [21]. The grid points are calculated as t_n and the results are shown below.

$$\underline{x}_{n+1}^{(1)}\left(t_{n+1} : r\right) = \underline{x}\left(t_n : r\right) + F\left[x\left(t_n : r\right)\right],$$

$$\overline{x}_{n+1}^{(1)}\left(t_{n+1} : r\right) = \overline{x}\left(t_n : r\right) + G\left[x\left(t_n : r\right)\right],$$

$$\underline{x}_{n+1}^{(2)}\left(t_{n+1} : r\right) = \underline{x}\left(t_n : r\right) + G\left[x\left(t_n : r\right)\right],$$

$$\overline{x}_{n+1}^{(2)}\left(t_{n+1} : r\right) = \overline{x}\left(t_n : r\right) + F\left[x\left(t_n : r\right)\right]. \tag{11}$$

Then are intended for inferior and superior value of the independent variable x, from that we adopt the values from minimum to maximum. So that it produces the better exact solutions.

$$\underline{x}_{n+1} = \min\left\{\underline{x}(t_n : r) + F\left[x(t_n : r)\right],\ \underline{x}(t_n : r) + G\left[x(t_n : r)\right]\right\},$$

$$\overline{x}_{n+1} = \max\left\{\overline{x}(t_n : r) + G\left[x(t_n : r)\right],\ \overline{x}(t_n : r) + F\left[x(t_n : r)\right]\right\}. \tag{12}$$

3.2.1 *Harmonic mean of modified Euler's method*

The anticipated Harmonic mean of Modified Euler's method is definite described by

$$\underline{x}_{n+1} = \underline{x}(t_n : r) + F\left[x(t_n : r)\right],$$

$$\overline{x}_{n+1} = \overline{x}(t_n : r) + G\left[x(t_n : r)\right]. \tag{13}$$

Where

$$F = hf\left(\frac{2(t_n : r)((t_n : r) + h)}{(t_n : r) + ((t_n : r) + h)},\ \frac{2(\underline{x}_n : r)((\underline{x}_n : r) + hf((t_n : r), (\underline{x}_n : r)))}{(\underline{x}_n : r) + ((\underline{x}_n : r) + hf((t_n : r), (\underline{x}_n : r)))}\right),$$

$$G = hf\left(\frac{2(t_n : r)((t_n : r) + h)}{(t_n : r) + ((t_n : r) + h)},\ \frac{2(\overline{x}_n : r)((\overline{x}_n : r) + hf((t_n : r), (\overline{x}_n : r)))}{(\overline{x}_n : r) + ((\overline{x}_n : r) + hf((t_n : r), (\overline{x}_n : r)))}\right).$$

3.2.2 *Cubic mean of modified Euler's method*

The proposed Cubic mean of Modified Euler's method is defined by

$$\underline{x}_{n+1} = \underline{x}(t_n : r) + F\left[x(t_n : r)\right],$$

$$\overline{x}_{n+1} = \overline{x}(t_n : r) + G\left[x(t_n : r)\right] \tag{14}$$

where

$$F = hf\left(\left(\frac{(t_n : r)^3 + ((t_n : r) + h)^3}{2}\right)^{1/3},\ \left(\frac{(\underline{x}_n : r)^3 + ((\underline{x}_n : r) + hf((t_n : r), (x_n : r)))^3}{2}\right)^{1/3}\right),$$

$$G = hf\left(\left(\frac{(t_n:r)^3+((t_n:r)+h)^3}{2}\right)^{1/3}, \left(\frac{(\bar{x}_n:r)^3+((\bar{x}_n:r)+hf((t_n:r),(\bar{x}_n:r)))^3}{2}\right)^{1/3}\right).$$

4. Resultand discussionby numerical example

Consider the differential equation with fuzzy initial value problem,

$$x' = -t^3 x,\ x(0) = (0.8+0.2r,\ 1.25-0.25r) \text{ where } 0 \le r \le 1. \tag{15}$$

Solution:
The exact solution is given by

$$\underline{x}(t:r) = \underline{x}(t:r)e^{-\frac{t^4}{4}} \text{ and } \bar{x}(t:r) = \bar{x}(t:r)e^{-\frac{t^4}{4}}, \tag{16}$$

then the solutions at $t = 1$,

$$x(1:r) = [(0.8+0.2r)e^{-0.25},\ (1.25-0.25r)e^{-0.25}],\ 0 \le r \le 1. \tag{17}$$

The precise and fairly accurate solution is acquired by Euler's method. The harmonic mean of modified Euler method and also cubic mean of modified Euler method by means of h = 0.1 is given below:

Table 1. Numerical Solutions of Fuzzy Differential Equation with h = 0.1

r	Exact Solution		Harmonic Mean Solution		Cubic Mean Solution		Euler Solution	
	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy
0.0	0.6230406265	0.9735009788	0.6230167317	0.9734636433	0.6230665947	0.9735415542	0.6257143798	0.9776787184
0.1	0.6386166421	0.9540309593	0.6385921500	0.9539943705	0.6386432595	0.9540707231	0.6413572393	0.9581251441
0.2	0.6541926578	0.9345609397	0.6541675683	0.9345250976	0.6542199244	0.9345998920	0.6570000988	0.9385715697
0.3	0.6697686734	0.9150909201	0.6697429866	0.9150558247	0.6697965893	0.9151290609	0.6726429583	0.9190179953
0.4	0.6853446891	0.8956209005	0.6853184049	0.8955865519	0.6853732541	0.8956582298	0.6882858178	0.8994644210
0.5	0.7009207048	0.8761508810	0.7008938232	0.8761172790	0.7009499190	0.8761873988	0.7039286773	0.8799108466
0.6	0.7164967204	0.8566808614	0.7164692415	0.8566480061	0.7165265839	0.8567165677	0.7195715368	0.8603572722
0.7	0.7320727361	0.8372108418	0.7320446598	0.8371787333	0.7321032487	0.8372457366	0.7352143963	0.8408036979
0.8	0.7476487517	0.8177408222	0.7476200781	0.8177094604	0.7476799136	0.8177749055	0.7508572558	0.8212501235
0.9	0.7632247674	0.7982708026	0.7631954964	0.7982401875	0.7632565785	0.7983040744	0.7665001153	0.8016965491
1.0	0.7788007831	0.7788007831	0.7787709147	0.7787709147	0.7788332433	0.7788332433	0.7821429747	0.7821429747

Table 2. Error Analysis between Exact and Proposed Methods h = 0.1

r	Harmonic Mean Solution		Cubic Mean Solution		Euler Solution	
	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy
0.0	0.0000238947	0.0000373355	0.0000259682	0.0000405753	0.0026737533	0.0041777396
0.1	0.0000244921	0.0000365888	0.0000266174	0.0000397638	0.0027405972	0.0040941848
0.2	0.0000250895	0.0000358421	0.0000272666	0.0000389523	0.0028074410	0.0040106300
0.3	0.0000256868	0.0000350954	0.0000279158	0.0000381408	0.0028742848	0.0039270752
0.4	0.0000262842	0.0000343487	0.0000285650	0.0000373293	0.0029411287	0.0038435204
0.5	0.0000268816	0.0000336020	0.0000292142	0.0000365178	0.0030079725	0.0037599656
0.6	0.0000274789	0.0000328553	0.0000298635	0.0000357063	0.0030748163	0.0036764108
0.7	0.0000280763	0.0000321085	0.0000305127	0.0000348948	0.0031416602	0.0035928560
0.8	0.0000286737	0.0000313618	0.0000311619	0.0000340833	0.0032085040	0.0035093013
0.9	0.0000292710	0.0000306151	0.0000318111	0.0000332718	0.0032753478	0.0034257465
1.0	0.0000298684	0.0000298684	0.0000324603	0.0000324603	0.0033421917	0.0033421917

The Fuzzy differential equation problem is solved by known method. That is Euler's method, the approximate solution is displayed in the above Table 1. Also, we solved our proposed two methods like Harmonic Mean and Cubic Mean, the approximate solution is displayed in the above Table 1. Table 2 shows Error Analysis between Exact and Proposed Methods when h = 0.1. Here the overall analysis of the solution obtained by our proposed methods gives nearly accurate solution other than the Euler's method. So, it can be concluded that our recommended procedure is better than the Euler's method.

The Exact solution and the approximate solutions are obtained by the Euler's method, Harmonic mean of modified Euler method and also cube mean of modified Euler method with h = 0.01. Further, to obtain the approximate solutions using proposed methods with step size h = 0.001 is given below.

The Fuzzy differential equation problem is solved by a known method. That is Euler's method, the approximate solution is displayed in the above Table 3. Also, we solved our proposed two methods like Harmonic Mean and Cubic Mean, the approximate solution is displayed in the above Table 3. Table 4 shows Error Analysis between the Exact and Proposed Methods when h = 0.001. Here the overall analysis of the solution obtained by our proposed methods gives nearly more accurate solution other than the Euler's method. So, it concludes that our recommended procedure is more better than Euler's method.

Table 3. Numerical Solutions of Fuzzy Differential Equation with h = 0.001

r	Exact Solution		Harmonic Mean Solution		Cubic Mean Solution		Euler Solution	
	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy
0.0	0.6230406265	0.9735009788	0.6230403876	0.9735006056	0.6230408861	0.9735013845	0.6257143798	0.9776787184
0.1	0.6386166421	0.9540309593	0.6386163973	0.9540305935	0.6386169082	0.9540313568	0.6413572393	0.9581251441
0.2	0.6541926578	0.9345609397	0.6541924070	0.9345605814	0.6541929304	0.9345613291	0.6570000988	0.9385715697
0.3	0.6697686734	0.9150909201	0.6697684167	0.9150905693	0.6697689525	0.9150913014	0.6726429583	0.9190179953
0.4	0.6853446891	0.8956209005	0.6853444264	0.8956205572	0.6853449747	0.8956212737	0.6882858178	0.8994644210
0.5	0.7009207048	0.8761508810	0.7009204361	0.8761505451	0.7009209968	0.8761512460	0.7039286773	0.8799108466
0.6	0.7164967204	0.8566808614	0.7164964458	0.8566805330	0.7164970190	0.8566812183	0.7195715368	0.8603572722
0.7	0.7320727361	0.8372108418	0.7320724554	0.8372105209	0.7320730411	0.8372111907	0.7352143963	0.8408036979
0.8	0.7476487517	0.8177408222	0.7476484651	0.8177405087	0.7476490633	0.8177411630	0.7508572558	0.8212501235
0.9	0.7632247674	0.7982708026	0.7632244748	0.7982704966	0.7632250854	0.7982711353	0.7665001153	0.8016965491
1.0	0.7788007831	0.7788007831	0.7788004845	0.7788004845	0.7788011076	0.7788011076	0.7821429747	0.7821429747

Table 4. Error Analysis between Exact and Proposed Methods h = 0.001

r	Harmonic Mean Solution		Cubic Mean Solution		Euler Solution	
	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy	Lower Fuzzy	Upper Fuzzy
0.0	0.0000002388	0.0000003732	0.0000002596	0.0000004056	0.0026737533	0.0041777396
0.1	0.0000002448	0.0000003657	0.0000002661	0.0000003975	0.0027405972	0.0040941848
0.2	0.0000002508	0.0000003583	0.0000002726	0.0000003894	0.0028074410	0.0040106300
0.3	0.0000002568	0.0000003508	0.0000002791	0.0000003813	0.0028742848	0.0039270752
0.4	0.0000002627	0.0000003433	0.0000002856	0.0000003732	0.0029411287	0.0038435204
0.5	0.0000002687	0.0000003359	0.0000002921	0.0000003651	0.0030079725	0.0037599656
0.6	0.0000002747	0.0000003284	0.0000002986	0.0000003570	0.0030748163	0.0036764108
0.7	0.0000002806	0.0000003209	0.0000003050	0.0000003489	0.0031416602	0.0035928560
0.8	0.0000002866	0.0000003135	0.0000003115	0.0000003407	0.0032085040	0.0035093013
0.9	0.0000002926	0.0000003060	0.0000003180	0.0000003326	0.0032753478	0.0034257465
1.0	0.0000002986	0.0000002986	0.0000003245	0.0000003245	0.0033421917	0.0033421917

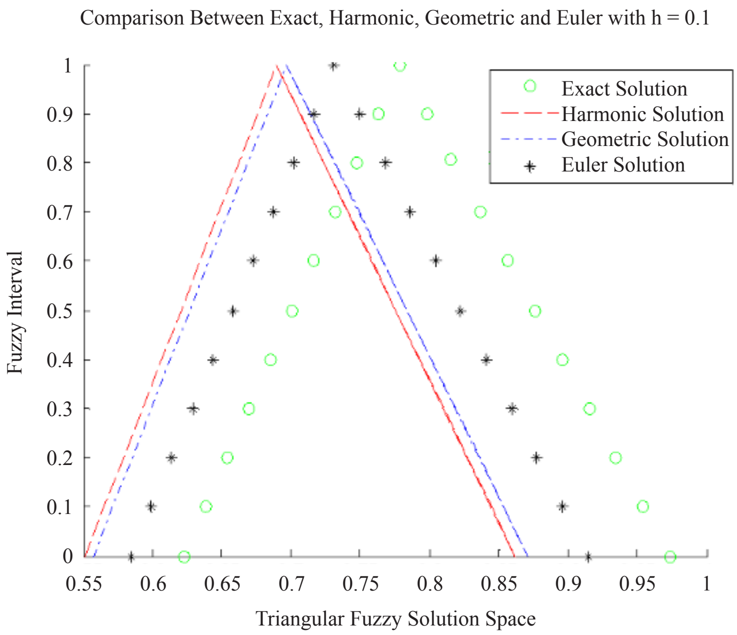

Figure 1. Graphical Representation of Fuzzy Solutions with h = 0.1

Figure 1 shows that, the value of Exact solution are marked at Green Colour, Harmonic Solution are marked at Red Colour, Geometric Solution are marked at Blue Colour and Euler solution marked at Black Colour when h = 0.1. Every solution Fuzzy triangular number format clearly our proposed solution graph more accurate.

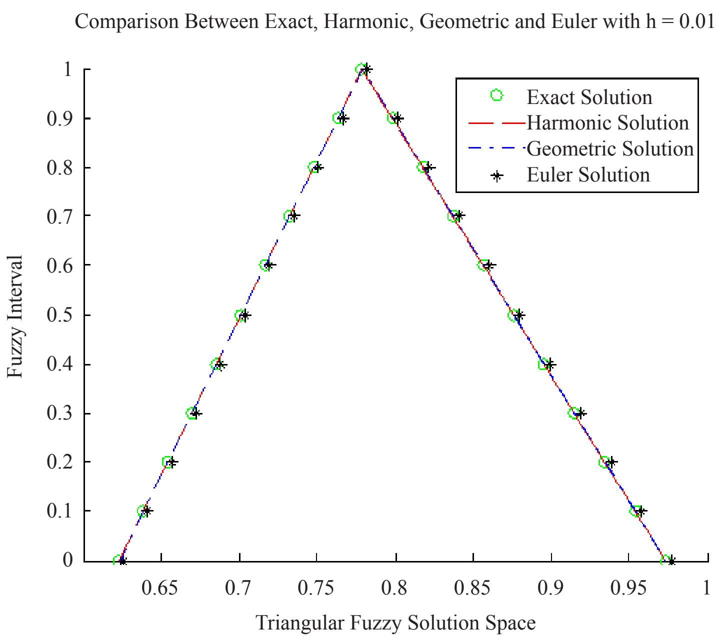

Figure 2. Graphical Representation of Fuzzy Solutions with h = 0.01

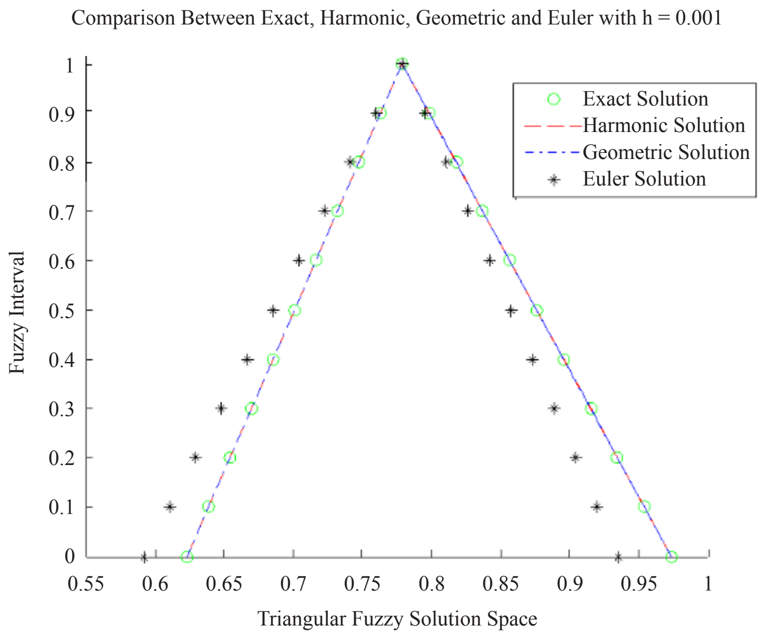

Figure 3. Graphical Representation of Fuzzy Solutions with h = 0.001

For the different values of step size h = 0.01, 0.001, this technique provides the very Exact solution for fuzzy differential equations at shown in the Figure 2 and Figure 3 and the numerical calculations for this paper are performed using C-programming language with matplotlib.

5. Conclusion

In comparison to the prior approach, the new strategy we introduce as part of our research will result in an accurate solution. Various upper and lower bound values (h = 0.1, 0.01, 0.001, 0.0001) will be used to assess our method. Other than the Euler's method, our modified Euler's method provides the most comprehensive and accurate solution. When compared to our proposed approach, h = 0.1 is the optimum value. Euler's approach is more precise when h = 0.01.

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Conflict of interest

The authors declare no competing financial interest.

References

[1] Dubois D, Prade H. Towards fuzzy differential calculus part 3: Differentiation. *Fuzzy Sets and Systems*. 1982; 8(3): 225-233. Available from: https://doi.org/10.1016/s0165-0114(82)80001-8.
[2] Allahviranloo T, Ahmady N, Ahmady E. Numerical solution of fuzzy differential equations by predictor-corrector

method. *Information Sciences*. 2007; 177(7): 1633-1647. Available from: https://doi.org/10.1016/j.ins.2006.09.015.
[3] Allahviranloo T, Abbasbandy S, Ahmady N, Ahmady E. Improved predictor-corrector method for solving fuzzy initial value problems. *Information Sciences*. 2009; 179(7): 945-955. Available from: https://doi.org/10.1016/j.ins.2008.11.030.
[4] Allahviranloo T, Ahmady N, Ahmady E. Erratum to "Numerical solution of fuzzy differential equations by predictor-corrector method". *Information Sciences*. 2008; 178(6): 1780-1782. Available from: https://doi.org/10.1016/j.ins.2007.11.009.
[5] Allahviranloo T, Ahmady E, Ahmady N. Nth-order fuzzy linear differential equations. *Information Sciences*. 2008; 178(5): 1309-1324. Available from: https://doi.org/10.1016/j.ins.2007.10.013.
[6] Friedman M, Ma M, Kandel A. Numerical solutions of fuzzy differential and integral equations. *Fuzzy Sets and Systems*. 1999; 106(1): 35-48. Available from: https://doi.org/10.1016/s0165-0114(98)00355-8.
[7] Ma M, Friedman M, Kandel A. Numerical solutions of fuzzy differential equations. *Fuzzy Sets and Systems*. 1999; 105(1): 133-138. Available from: https://doi.org/10.1016/s0165-0114(97)00233-9.
[8] Tapaswini S, Chakraverty S. A new approach to fuzzy initial value problem by improved Euler method. *Fuzzy Information and Engineering*. 2012; 4(3): 293-312. Available from: https://doi.org/10.1007/s12543-012-0117-x.
[9] Usha B, Duraisamy C. Another approach to solution of fuzzy differential equations by Modified Euler's method. *2010 International Conference on Communication and Computational Intelligence (INCOCCI)*. Erode, India: IEEE; 2010. p.52-56.
[10] Osmo K. The cauchy problem for fuzzy differential equations. *Fuzzy Sets and Systems*. 1990; 35(3): 389-396. Available from: https://doi.org/10.1016/0165-0114(90)90010-4.
[11] Javad S. Numerical solution of fuzzy differential equations. *Applied Mathematical Sciences*. 2007; 1(45): 2231-2246.
[12] Bede B. Note on "Numerical solutions of fuzzy differential equations by predictor-corrector method". *Information Sciences*. 2008; 178(7): 1917-1922. Available from: https://doi.org/10.1016/j.ins.2007.11.016.
[13] Gerald CF, Wheatley PO. *Applied Numerical Analysis*. 7th ed. Amazon. Boston, Mass. Munich: Pearson College Div; 2003. Available from: https://www.amazon.com/Applied-Numerical-Analysis-Curtis-Gerald/dp/0321133048.
[14] Mohankumar S, Venkatesh A, Manikandan R. Step-stress and truncated acceptance sampling plan model for the analysis of vasopressin. *International Journal of Pure and Applied Mathematics*. 2017; 117(6): 107-114.
[15] Seikkala S. On the fuzzy initial value problem. *Fuzzy Sets and Systems*. 1987; 24(3): 319-330. Available from: https://doi.org/10.1016/0165-0114(87)90030-3.
[16] Abbasbandy S, Viranloo TA. Numerical solutions of fuzzy differential equations by Taylor method. *Computational Methods in Applied Mathematics*. 2002; 2(2): 113-124. Available from: https://doi.org/10.2478/cmam-2002-0006.
[17] Osmo K. Fuzzy differential equations. *Fuzzy Sets and Systems*. 1987; 24(3): 301-317. Available from: https://doi.org/10.1016/0165-0114(87)90029-7.
[18] Zulzamri Salleh. Ordinary differential equations by Euler's technique, scilab programming. *Mathematical Modelsand Methods in Modern Science.* 2012; 20(4): 264-269.
[19] Kaufmann A, Gupta MM. *Introduction to Fuzzy Arithmetic: Theory and Applications, With a foreword by Lotfi A. Zadeh, Van Nostrand Reinhold Electrical/Computer Science and Engineering Series*. Van Nostrand Reinhold Co., New York; 1985.
[20] Chandio MS, Memon AG. Improving the efficiency of Heun's method. *Sindh University Research Journal (Science Series)*. 2010; 42(2): 85-88.
[21] Fadugba S, Ogunrinde B, Okunlola T. Euler's method for solving initial value problems in ordinary differential equation. *The Pacific Journal of Science Technology*. 2012; 13(2): 152-158.