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Research Article

Control Policy, Manageable Arrival Rates, and Reverse Balking in Mutually Dependent Queueing Model

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Abstract: A single server has a limited amount of storage and processing power dependent queueing model with examination of a control strategy, manageable rates of arrival and balking in reverse is examined in this paper. Using this model, a steady-state solution and system characteristics may be found. To aid comprehension, numerical findings are provided, as well as a pertinent conclusion.

***Keywords*:** single server, finite capacity, queueing system, mutually dependent, primary arrival and service processes, reverse balking, N-policy

MSC: 60K25, 68M20, 90B22

1. Introduction

Today's environment of advancement and liberalization has made operating a firm difficult. Customers now use greater discretion. The frequency of brand change has increased. Customers become increasingly frustrated with a certain company more frequently as a result of their increased expectations. In the business world, customer dissatisfaction has become a major issue. For different service systems dealing with impatient customers, queueing theory provides a number of models. There are several studies that discuss this idea of balking According to these models, the size of the system and the length of the delay affect balking. More balking occurs as the system size increases. However, in the case of investment, the presence of more clients at a specific company attracts clients who are interested in investing. Thus, in industries like customer investment, the chance of balking will be lower when the system size is greater and vice versa, which is known as reverse balking.

The N-policy for queueing has been investigated by various researchers. This strategy is quite helpful in maintaining systemic stability. The N-policy implies that the server will remain idle until there are N consumers in the queue. The service begins with the arrival of the nth customer, and the active time continues until the system is vacant.

The designing of a manufacturing structure is formulated as an N-policy single server bulk input queueing system with a transferable and non-dependable server. The most suitable working method is achieved in the technique of total costs involved and the complexity in a model's outcome is presented in [1]. [2] Focuses on the action of leaving the

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system with stoppage and assembling period with N-policy and to examine a state that happens in a pump industrial production. Using N-policy an algorithmic program is proposed in [3] to adjudge the most desirable control strategy. N-policy for single server general queue has been changed slightly in [4-5]. [6] investigated a two-stage single server finite capacity queuing system with N-policy for comprehensive group service with and without blocking. N-policy and a single server queue with various operational vacations have been studied in [7] Moreover [8] have considered a loss and detain queueing model under the limitation of N-policy.

Manageable arrival rates in queueing model have been discussed in [9-11] and [12-13] have expanded their research to include adjustable arrival rates in retrial queue. Busy duration perusal of correlated queues with associated incoming and outgoing procedure has been evaluated in [14]. The stability conditions and the system characteristics of mutually dependent queueing model dealt upon, as in [15]. The basic analytical rules required to investigate the theory of probability in queues have been described in [16]. The relationship between a capacitated queueing system and dissatisfied customers and a machine mend problem with replacements are analysed. Dependence between arrival stage possibilities and performance measures for queueing systems are also obtained in [17]. The steady state is derived by considering that the requirement for service does not glut the service process in [18]. Further, [19-21] the notion of invert balking and invert reneging into the single server queuing system has been developed and incorporated. A short queue problem, which involves multiple beam satellite structure has been explored in [22]. A formulation is derived in [23] for the proportionate loss in custom generated by loss of confidence. [24] have discussed refinements to various server asymptotic and staffing rule for a Markov process queueing model with abandoned customers. [25] have studied the synchronised abandonment M/M/1 queue.

Arrival and service are often seen as separate operations. However, in many cases, the arrival and service procedures must be considered mutually dependent. Interdependent queuing model is a queueing model where arrivals and services are linked. Mutually dependent queueing models with manageable incoming assess have been widely discussed in the literature. At present, a tendency has been diverted and moved to explore more practical effecting estimate of the system as contrasted to general theoretical perspective that incorporate only just a bit of application. Here, some pertinent researches are in follow up.

The single server manageable queueing model with finite capacity, reverse balking, and control policy are developed in this paper. Postulates are outlined in Section 2. The equations for the steady state are deduced in section 3. As a result of part 4, the model's attributes are obtained. Analytical findings are shown.

2. Model summary

Consider a one-server queueing system with limited space whose client arrive in accordance with the Poisson movement λ_1 and λ_2 and the distribution of service times is exponential with rate μ. Upon arrival $[X_1(t)]$ and outgoing technique $[X_2(t)]$ of the system are in mutual relationship and two events where the mean period between the events is recognized, yet the required period of the event is arbitrary with the intersection of two events:

$$P(X_1 = x_1, X_2 = x_2; t) = e^{-(\lambda_i + \mu - \in)t} \sum_{j=0}^{\min(x_1, x_2)} \frac{(\in t)^j [(\lambda_i - \in)t]^{x_1 - j} [(\mu - \in)t]^{(x_2 - j)}}{j!\,(x_1 - j)!(x_2 - j)!}$$

Where $x_1, x_2 = 0, 1, 2, \ldots, 0 < \lambda_i, \mu; 0 \le \in < \min(\lambda_i, \mu), i = 1, 2.$

$$n = 0, 1, 2, \ldots\ N-1, N, N+1\ \ldots\ g-1, g, g+1, \ldots, G-1, G, G+1, \ldots, K$$

with parameters $\lambda_1, \lambda_2, \mu$ and $\in$ as average entry rates, average departure rates, and average dependence rates (covariance between the entry and departure process).

The system capacity is assumed to be limited, let's say to K. The queue order policy is first-come, first-served. Customers may not balk with probability $u(= 1 - v)$ when the system is empty, but they do so with probability v. Let

$R = \frac{1}{K-1}$, Customers balk both probabilistically $(1 - nR)$ and non probabilistically nR when the system is not empty. It is termed as reverse balking.

The incoming and outgoing processes of the system are assumed to be associated and two variables follow Poisson process in this article, which examines one server, limited capacity interdependent queueing model with controlled arrival rates. Here, the arrival rate is defined as, λ_1- a higher arrival rate and λ_2- a moderate arrival rate. At some defined number G ($G > N$), the arrival rate drops from 1 to 2, and it remains at that rate, till the queue becomes bigger than some other required number g ($g \geq 0$ & $g < G$). This is true until the queue size drops below G. After g, the arrival rate returns to λ_1 and the same procedure happen in continuous.

The postulates of the model are

(i) There is a possibility that no arrivals with reverse balking will occur for any given length of time h and no service will be provided, if the system observes an higher arrival rate is $1 - [(\lambda_1 - \in)u + (\mu - \in)]h + o(h)$.

(ii) There is a possibility that one arrival with reverse balking will occur for any given length of time h and no service will be provided, if the system observes an higher arrival rate is $(\lambda_1 - \in)uh + o(h)$.

(iii) There is a possibility that no arrival with reverse balking will occur for any given length of time h and no service will be provided, if the system observes moderate arrival rate $1 - [nR(\lambda_2 - \in)u + (\mu - \in)]h + o(h)$ and higher arrival rate $1 - [nR(\lambda_1 - \in)u + (\mu - \in)h + o(h)]$.

(iv) The possibility of no incoming customers with balking in reverse and one service happens in a short time h, when the system arrivals are high or moderate is $(\mu - \in)h + o(h)$.

(v) The probability that one arrival with reverse balking and one service completion will occur within a brief period of time h when the system is operating at either a higher or a moderate arrival rate, is $\in h + o(h)$.

3. Determination of steady state

Constant-state notation includes the following symbols:

$P_{0n}(0)$ = If the system is undergoing a greater rate of advents and the server is vacant, there is a good possibility of having n consumers in the queue.

$P_{1n}(0)$ = When the system has a higher rate of arrivals and the server is overwhelmed, there is a likelihood that n clients are in the line.

$P_{1n}(1)$ = While the system's advent rate is dropping and the server is occupied, the chance of n consumers in the queue.

We may see that $P_{0n}(0)$ exist for $1 \leq n \leq N - 1$, $P_{1n}(0)$ exist for $0 \leq n \leq G - 1$ and $P_{1n}(1)$ exist for $n \geq g + 1$.

The equations for the steady state have this dependent structure and are hence

$$-(\lambda_1 - \in)P_{0n}(0) + (\lambda_1 - \in)u\, P_{0n-1}(0) = 0 \qquad n = 1, 2, 3 .... N-1 \tag{1}$$

$$-(\lambda_1 - \in)uP_{00}(0) + (\mu - \in)P_{1,1}(0) = 0 \tag{2}$$

$$-\left[R(\lambda_1 - \in) + (\mu - \in)\right]P_{11}(0) + (\mu - \in)P_{12}(0) = 0 \tag{3}$$

$$-\left[nR(\lambda_1 - \in) + (\mu - \in)\right]P_{1n}(0) + (\mu - \in)\, P_{1n+1}(0)$$

$$+\left(n-1\right)R(\lambda_1 - \in)P_{1n-1}(0) = 0 \qquad 2 \leq n \leq N-1 \tag{4}$$

$$-\left[nR(\lambda_1-\in)+(\mu-\in)\right]P_{1N}(0)+\left(n-1\right)R(\lambda_1-\in)P_{0\ N-1}(0)$$

$$+(\mu-\in)\ P_{1N+1}(0)+\left(n-1\right)R(\lambda_1-\in)P_{1\ N-1}(0)=0 \tag{5}$$

$$-\left[nR(\lambda_1-\in)+(\mu-\in)\right]P_{1n}(0)+(\mu-\in)P_{1n+1}(0)$$

$$\left(n-1\right)R(\lambda_1-\in)P_{1n-1}(0)\ =0, \qquad n=N+1, N+2, N+3\ \dots\ g-1 \tag{6}$$

$$-\left[gR(\lambda_1-\in)+(\mu-\in)\right]P_{1g}(0)+(\mu-\in)P_{1\ g+1}(0)$$

$$+(\mu-\in)P_{1\ g+1}(1)+\left(g-1\right)R(\lambda_1-\in)P_{1\ g-1}(0)=0 \tag{7}$$

$$-\left[nR(\lambda_1-\in)+(\mu-\in)\right]P_{1n}(1)+(\mu-\in)\ P_{1n+1}(1)$$

$$+\left(n-1\right)R(\lambda_1-\in)P_{1n-1}(1)=0, \qquad n=g+1, g+2,\ \dots,\ G-2 \tag{8}$$

$$-\left[\left(G-1\right)R(\lambda_1-\in)+(\mu-\in)\right]P_{1\ G-1}(1)$$

$$+\left(G-2\right)R\ (\lambda_1-\in)P_{1\ G-2}(1)=0 \tag{9}$$

$$-\left[\left(nR\right)(\lambda_2-\in)+(\mu-\in)\right]P_{1n}(1)+(\mu-\in)\ P_{1n+1}(1)$$

$$+\left(n-1\right)R\ \ (\lambda_2-\in)P_{1n-1}(1)=0, \qquad g+2\le n\le G-1, \tag{10}$$

$$-\left[GR(\lambda_2-\in)+(\mu-\in)\right]P_{1\ G}(1)+(\mu-\in)P_{1\ G+1}(1)$$

$$+\left(G-1\right)R(\lambda_1-\in)P_{1\ G-1}(0)+\left(G-1\right)R(\lambda_2-\in)P_{1\ G-1}(1)=0 \tag{11}$$

From equation (10)

$$G+1\le n\le K-1 \tag{12}$$

$$\left(n-1\right)R(\lambda_2-\in)P_{1K-1}(1)-(\mu-\in)P_{1K}(1)=0 \tag{13}$$

From equation (1) we get,

$$P_{0n}(0)=uP_{00}(0), \qquad 1\le n\le N-1 \tag{14}$$

Write

$$s_0 = \frac{\lambda_1 - \in}{\mu - \in}, s_1 = \frac{\lambda_2 - \in}{\mu - \in}$$

From equation (2) to (4) we get,

$$P_{1n}(0) = uR^{n-1}(n-1)!{s_0}^n P_{00}(0), \qquad n = 2, \ \ldots, \ N-1 \tag{15}$$

From equation (5) we get,

$$P_{1N}(1) = uR^{N-1}(N-1)!{s_0}^N P_{00}(0) \tag{16}$$

From equations (6) to (9) we get

$$P_n(1) = \begin{Bmatrix} uR^{n-1}(n-1)!{s_0}^n P_{0,0}(0) \ -\Big[\left(Rs_0\right)^{n-g-1}(n-1)P_{n-g-1} + \\ \left(Rs_0\right)^{n-g-2}(n-1)p_{n-g-2} + \ldots + \left(Rs_0\right)^{n\text{-}G+1}(n-1)P_{n-G+1}\Big] DP_{0,0}(0) \\ n = N+1 \ldots\ldots g, g+1, g+2, \ \ldots, \ G-1 \end{Bmatrix} \tag{17}$$

From equations (10) to (13) we get

$$P_n(1) = \Big[\left(Rs_1\right)^{n-g-1}(n-1)P_{n-g-1} + \left(Rs_1\right)^{n\text{-}g\text{-}2}(n-1) \ P_{n-g-2} + \ldots$$

$$+ \left(Rs_1\right)^{n\text{-}G}(n-1) \ P_{n-G} \Big] DP_0(0), \qquad n = g+1, g+2, \ \ldots, \ K-1, K \tag{18}$$

where

$$D = \frac{uR^{G-1}(G-1)!\left(s_0\right)^G}{\frac{(G-1)!}{r!}\left(Rs_0\right)^{G-g-1} + \frac{(G-1)!}{(g+1)!}\left(Rs_0\right)^{G-g-2} + \ldots + 1}$$

The normalising condition $P_{0,0}(0)$ may be used to determine the likelihood that the system is empty $P(0) + P(1) = 1$.

4. Model detailed information

Probability $P(0)$ of system's higher arrival rate

$$P(0) = P_0(0) + \sum_{n=1}^{g} P_n(0) + \sum_{n=g+1}^{G-1} P_n(0), \qquad 0 \le n \le G-1 \tag{19}$$

Probability $P(1)$ of system's moderate arrival rate

$$P(1) = \sum_{n=g+1}^{K} P_n(1), \qquad n = g+1, g+2, \ldots, G-2, G-1, \ldots, K \tag{20}$$

The number of anticipated clients in the structure is determined by

$$L_{so} = \sum_{n=0}^{g} nP_n(0) + \sum_{n=g+1}^{G-1} nP_n(0) \text{ and } L_{s1} = \sum_{n=g+1}^{G-1} nP_n(1) + \sum_{n=G}^{K} nP_n(1) \tag{21}$$

From (17) and (18), we get

$$L_s = \sum_{n=1}^{G-1} n u R^{n-1} (n-1)! {s_0}^n P_0(0)$$

$$-\left[\sum_{n=g+1}^{G-1} n\left[(Rs_0)^{n-g-1}(n-1)P_{n-g-1} + (Rs_0)^{n-g-2}(n-1)P_{n-g-2} + \ldots + (Rs_0)^{n-G+1}(n-1)P_{n-G+1} \right] \right.$$

$$\left. + \sum_{n=g+1}^{N} \left[(Rs_1)^{n-g-1}(n-1)P_{n-g-1} + (Rs_1)^{n-g-2}(n-1)P_{n-g-2} + \ldots + (Rs_1)^{n-G}(n-1)P_{n-G} \right] \right] DP_0(0) \tag{22}$$

Waiting time for clients may be estimated using Little's formula.

$$W_s = \frac{L_s}{\bar{\lambda}} \tag{23}$$

Where $\bar{\lambda} = \lambda_1 P(0) + \lambda_2 P(1)$.

5. Numerical analysis

In the following two Tables, the mean value of the customers in the queue and system, the waiting time of the customers in the queue and system are calculated by varying higher rate of arrivals, moderate rate of arrivals, service rate, mean dependence rate, v, u and keeping the other parameters fixed.

Table 1. Variation in $P(0)$ and $P(1)$ with respect to $\lambda_1, \lambda_2, \mu, \in, v, u$

S.No	g	G	K	N	λ_1	λ_2	μ	$\in$	v	u	$P_{0,0}(0)$	$P(0)$	$P(1)$
1	4	9	12	4	4	3	7	0	0	1	0.6294	0.9972	0.0028
2	4	9	12	4	5	4	7	0	0	1	0.6291	0.9978	0.0022
3	4	9	12	4	5	3	7	0.7	0	1	0.6392	0.9981	0.0019
4	4	9	12	4	5	3	7	1	0	1	0.7126	0.9999	0.0001
5	4	9	12	4	5	3	6	0	0	1	0.5635	0.9958	0.0042
6	4	9	12	4	5	3	6	0.7	0	1	0.5638	0.9965	0.0035
7	4	9	12	4	5	3	5	0	0	1	0.4633	0.9585	0.0415
8	4	9	12	4	5	3	5	0.7	0.9	0.1	0.8368	0.9945	0.0055
9	4	9	12	4	5	3	5	0	1	0	1.0000	1.0000	0.0000
10	4	9	12	4	5	3	5	0	0.6	0.4	0.6321	0.9725	0.0275
11	4	9	12	4	5	3	5	0	0.7	0.3	0.7411	0.9670	0.0330
12	4	9	12	4	5	3	5	0	0.8	0.2	0.6321	0.9725	0.0275

Table 2. Variation in Lq, Wq, L_S and W_S with respect to $\lambda_1, \lambda_2, \mu, \in, v, u$

S.No	g	G	K	N	λ_1	λ_2	μ	$\in$	v	u	Lq	Wq	L_S	W_S
1	4	9	12	4	4	3	7	0	0	1	0.5776	0.1445	0.6887	0.2557
2	4	9	12	4	5	4	7	0	0	1	0.5802	0.1452	0.6915	0.2562
3	4	9	12	4	5	3	7	0.7	0	1	0.4754	0.1186	0.5854	0.2298
4	4	9	12	4	5	3	7	1	0	1	0.4426	0.1172	0.5565	0.2291
5	4	9	12	4	5	3	6	0	0	1	0.7067	0.1786	0.8282	0.2898
6	4	9	12	4	5	3	6	0.7	0	1	0.7005	0.1617	0.8157	0.2729
7	4	9	12	4	5	3	5	0	0	1	0.6123	0.2662	0.7241	0.3774
8	4	9	12	4	5	3	5	0.7	0.9	0.1	0.4215	0.1057	0.5335	0.2169
9	4	9	12	4	5	3	5	0	1	0	0.0000	0.0000	0.0000	0.0000
10	4	9	12	4	5	3	5	0	0.6	0.4	0.7674	0.1761	0.8785	0.2873
11	4	9	12	4	5	3	5	0	0.7	0.3	0.8785	0.2873	0.9896	0.3984
12	4	9	12	4	5	3	5	0	0.8	0.2	0.8874	0.2761	0.9985	0.3873

6. Conclusions

This model encompasses the prior models as specific cases. For instance, when $\in = 0$ then the concept reduces to a single server with limited capability and manageable arrival rates, N-policy, reverse balking and without interdependence. When λ_1 goes to λ_2, this model simplified into single server finite capacity interdependence model with control policy and reverse balking. When λ_1 tends to λ_2 and $\in = 0$, this model turn down to queuing system with a solitary client and restricted storage with reverse balking model and N policy.

The above Tables 1 and 2 shows numerical results for finite capacity, one server manageable advent rates, and reverse balking in a mutually dependent queueing model. By increasing advent rate, Lq, Wq, L_S and W_S increase as happens in real life congestion situations. The average queue length and waiting duration reduces, when the number of servers and the rate of service raises. All other factors stay unchanged, Lq, Wq, L_S and W_S drop as the mean dependency rate rises. Observation reveals, the predicted size of the system is zero when v is 1. Increase in impatience rate and remaining factors keeps Ls at its maximum when v balking rate is zero. Lq, Wq, Ls, and Ws keep increasing when the advent rate rises, but the other variables remain the same. Lq, Wq, L_S and W_S drop when the outgoing number goes up while the supporting variables remain persistent. The reverse balking and N policy concepts are integrated into an M/M/1/K interdependent queueing system with manageable arrival rates in this work. Some significant performance metrics are determined from the model's steady-state study. The model's sensitivity analysis is also carried out. This model is useful for investment firms dealing with impatient clients. The multiserver situation of the concept can be researched in the future.

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Conflict of interest

The authors declare no competing financial interest.

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