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Research Article

Implicit Quiescent Optical Solitons for the Dispersive Concatenation Model with Nonlinear Chromatic Dispersion by Lie Symmetry

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Abstract: The primary purpose of this paper is to investigate and recover implicit quiescent optical solitons in the context of the dispersive concatenation model in nonlinear optics. Specifically, the study focuses on a model that incorporates nonlinear chromatic dispersion and includes Kerr and power-law self-phase modulation effects. The objective is to identify and characterize these soliton solutions within this complex optical system. To achieve this purpose, we employ the Lie symmetry analysis method. Lie symmetry analysis is a powerful mathematical technique commonly used in physics and engineering to identify symmetries and invariance properties of differential equations. In this context, it is used to uncover the underlying symmetries of the nonlinear optical model, which in turn aids in the recovery of the quiescent optical solitons. This method involves mathematical derivations and calculations to determine the solutions. The outcomes of the current paper include the successful recovery of implicit quiescent optical solitons for the dispersive concatenation model with nonlinear chromatic dispersion, Kerr, and power-law self-phase modulation. The study provides mathematical expressions and constraints on the model's parameters that yield upper and lower bounds for these solutions. Essentially, this paper presents a set of mathematical descriptions for the optical solitons that can exist within the described optical system. The present paper contributes to the field of nonlinear optics by exploring the behavior of optical solitons in a model that combines multiple nonlinear effects. This extends our understanding of complex optical systems.

***Keywords*:** solitons, quiescent, dispersive, Lie transform

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1. Introduction

One of the most fascinating and intriguing equations in nonlinear optics that have been proposed during 2014 is

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the concatenation model that describes the propagation of solitons through optical fibers [1, 2]. This model is a linkage of the familiar nonlinear Schrödinger's equation (NLSE), Lakshmanan-Porsezian-Daniel (LPD) model and the Sasa-Satsuma equation (SSE). This model has been extensively studied and a plethora of results, from various angles, have been reported. They range from the Painleve analysis, application of the method of undetermined coefficients to recover a full spectrum of solitons, conservation laws, bifurcation analysis, magneto-optic solitons, gap solitons, stochasticity with white noise, quiescent solitons with nonlinear chromatic dispersion (CD), application of the Laplace-Adomian decomposition scheme. Later, this model was also addressed with spatio-temporal dispersion in addition to the linear CD and finally the study of the model was conducted with differential group delay.

Thereafter, during 2015, another form of the concatenation model was proposed by the same group in Australia which is a dispersive form of the concatenation model [3-5]. This model was consequently referred to as the dispersive concatenation model. This is a combination of the Schrödinger-Hirota equation (SHE), the LPD and the fifth-order NLSE. Some preliminary results have started pouring in starting with the retrieval of optical solitons [6, 7]. The current paper will retrieve the implicit quiescent optical solitons for the model with nonlinear CD and having Kerr law as well as power-law of self-phase modulation (SPM) by Lie symmetry. It must be noted that implicit quiescent solitons have been recovered for the concatenation model with Kerr law, power-law and in absence of SPM [8-10]. This paper for the first time is diving into the retrieval of the implicit quiescent optical solitons for the dispersive concatenation with both forms of SPM by Lie symmetry.

This methodology was also successfully implemented, in the past, for many models such as the complex Ginzburg-Landau equation and others [11]. For both forms of SPM taken into account, the method fails to study the concatenation model with generalized temporal evolution. It is only the linear temporal evolution that yields the results for the two forms of SPM. The details of the derivation of quiescent optical solitons, for the two forms of SPM, are exhibited in the next two subsections, after a quick introduction to the respective models.

This paper holds significance in several aspects. It contributes to the field of nonlinear optics by exploring the behavior of optical solitons in a model that combines multiple nonlinear effects. This extends our understanding of complex optical systems. The use of Lie symmetry analysis showcases advanced mathematical techniques for solving intricate physical problems, which can be valuable for researchers in various scientific disciplines. Understanding the existence and constraints of optical solitons in such systems is crucial for practical applications in optical communication, signal processing, and the design of optical devices. The upper and lower bounds derived from the parameter constraints provide theoretical insights into the range of possible soliton solutions, aiding in the interpretation and prediction of optical behaviors. As a result, this paper employs mathematical analysis and the Lie symmetry method to recover and describe quiescent optical solitons in a complex nonlinear optical system. The findings offer valuable contributions to both theoretical optics and potential practical applications in the field.

While the present paper explores a fascinating and complex topic within nonlinear optics, it is essential to acknowledge its limitations to provide a comprehensive understanding of the scope and applicability of the research. The following are some of the limitations of the current paper. The methodology used fails to study the concatenation model with generalized temporal evolution and is limited to linear temporal evolution. This constraint may affect the model's applicability to scenarios where nonlinear temporal evolution is prevalent.

1.1 *Kerr law*

The dimensionless form of the dispersive concatenation model with linear temporal evolution, nonlinear CD and Kerr law of SPM is structured as [6, 8-11]:

$$iq_t + a\left(|q|^n q\right)_{xx} + b|q|^2 q - i\delta_1\left[\sigma_1 q_{xxx} + \sigma_2 |q|^2 q_x\right]$$

$$+\delta_2\left[\sigma_3 q_{xxxx} + \sigma_4 |q|^2 q_{xx} + \sigma_5 |q|^4 q + \sigma_6 |q_x|^2 q + \sigma_7 (q_x)^2 q^* + \sigma_8 q_{xx}^* q^2\right]$$

$$-i\delta_3\left[\sigma_9 q_{xxxxx} + \sigma_{10} |q|^2 q_{xxx} + \sigma_{11} |q|^4 q_x + \sigma_{12} q q_x q_{xx}^* + \sigma_{13} q^* q_x q_{xx} + \sigma_{14} q q_x^* q_{xx} + \sigma_{15} (q_x)^2 q_x^*\right] = 0. \quad (1)$$

Here, in (1), the dependent variable $q(x, t)$ is a complex valued function that represents the wave variable with x and t respectively representing the spatial and temporal co-ordinates. The coefficient of a is the nonlinear CD while the coefficient of b is the Kerr law of SPM. Also, the coefficient of the linear temporal evolution, namely the first term, is $i = \sqrt{-1}$. The coefficient of δ_1 is the SHE operator that extends the NLSE to SHE by Lie transform. The coefficient of δ_2 is from LPD equation while the coefficient of δ_3 is from the fifth-order NLSE. Thus, equation (1) stands as the dispersive concatenation model for Kerr law with nonlinear CD.

In comparing the current paper with the relevant existing literature, it's important to highlight the key distinctions and contributions of the present work. This paper recovers implicit quiescent optical solitons in the dispersive concatenation model. Also, we consider nonlinear CD and two forms of SPM: Kerr law and power law. Moreover, we use Lie symmetry analysis to obtain solutions in terms of Gauss' hypergeometric functions. Lastly, we derive parameter constraints that define upper and lower bounds for the solutions. Reference [6] is closely related as it also deals with the dispersive concatenation model. Reference [8] explores quiescent optical solitons within the concatenation model but focuses on cases with nonlinear chromatic dispersion. The current paper extends this work by considering the added complexities of both nonlinear chromatic dispersion and two forms of SPM (Kerr law and power-law) within the same model. Reference [9] investigates solitons within the concatenation model but excludes self-phase modulation effects. The current paper expands upon this by introducing SPM (Kerr law and power-law) into the model, thus exploring soliton behavior under more realistic conditions. Reference [10] focuses on solitons within the concatenation model with nonlinear CD and power-law SPM. The current paper broadens the scope by considering both Kerr law and power-law SPM, providing a more comprehensive analysis of SPM effects on soliton dynamics. As a result, the current paper extends the existing literature by addressing the dispersive concatenation model with nonlinear CD and two forms of SPM (Kerr law and power-law) simultaneously. It provides a more comprehensive understanding of the behavior of implicit quiescent optical solitons under these complex conditions, making it a valuable contribution to the field of nonlinear optics.

This model will be analyzed with Lie symmetry to search for the quiescent solitons that it support. The dynamics of implicit quiescent optical solitons within the context of the dispersive concatenation model in nonlinear optics is characterized by several key aspects. Quiescent optical solitons are localized, self-sustaining waveforms that propagate through optical fibers without changing their shape. They maintain their stability and coherence over long distances, making them valuable for signal transmission in optical communication systems. The presence of nonlinear CD in the model introduces dispersion effects that can influence the dynamics of the solitons. Depending on the specific parameters, nonlinear CD can lead to various behaviors, including dispersion-managed solitons and soliton compression or broadening. Two forms of SPM, namely Kerr law and power-law, are considered in the model. SPM effects arise due to the intensity-dependent refractive index of the optical medium. These effects can induce changes in the soliton's shape and velocity, impacting its dynamics. The solutions for the quiescent optical solitons are expressed in terms of Gauss' hypergeometric functions. These solutions provide a mathematical description of the soliton's profile and behavior. The hypergeometric functions incorporate the nonlinear and dispersive effects, allowing for the analysis of how these effects influence the soliton's dynamics. The hypergeometric functions also yield parameter constraints that define the upper and lower bounds of the solutions. These constraints provide insights into the range of possible soliton behaviors within the model. Parameter values within these bounds lead to implicit quiescent soliton propagation, while values beyond may result in different optical phenomena. Generalized temporal evolutions result in solutions in terms of quadratures. This suggests that, under certain conditions, the dynamics of the solitons become more complex, requiring quadrature-based representations rather than closed-form solutions. The findings in the present paper may have practical applications in optical communication and signal processing, where solitons play a crucial role. The results can be further aligned with existing works to enhance the understanding of optical solitons in a broader context. Therefore, the dynamics of implicit quiescent optical solitons within the dispersive concatenation model is influenced by nonlinear CD, SPM effects, and the mathematical representations provided by hypergeometric functions. Understanding these dynamics is essential for the design and optimization of optical communication systems and signal processing techniques. Additionally, this study highlights the importance of parameter constraints and the potential complexity of soliton dynamics under certain conditions.

With nonlinear CD, (1) cannot support mobile solitons and hence the starting hypothesis for the soliton solutions would naturally be

$$q(x,t)=\phi(x)\mathrm{e}^{i\lambda t}, \tag{2}$$

where λ would be the wave number of the quiescent soliton. Substituting (2) into (1) and decomposing into real and imaginary parts would yield

$$an(n+1)\phi^{n}(x)\{\phi'(x)\}^{2}+a(n+1)\phi^{n+1}(x)\phi''(x)+b\phi^{4}(x)+\delta_2\sigma_3\phi^{(iv)}(x)\phi(x)$$

$$+\delta_2(\sigma_4+\sigma_8)\phi^{3}(x)\phi''(x)+\delta_2(\sigma_6+\sigma_7)\phi^{2}(x)\{\phi'(x)\}^{2}+\delta_2\sigma_5\phi^{6}(x)-\lambda\phi^{2}(x)=0, \tag{3}$$

and

$$\delta_3\sigma_9\phi^{(v)}(x)\phi(x)+\delta_3\sigma_{10}\phi'''(x)\phi^{3}(x)+\delta_1\sigma_1\phi'''(x)\phi(x)+\delta_3\sigma_{11}\phi^{5}(x)\phi'(x)+\delta_1\sigma_2\phi^{3}(x)\phi'(x)$$

$$+\delta_3\sigma_{15}\phi(x)\{\phi'(x)\}^{3}+\delta_3(\sigma_{12}+\sigma_{13}+\sigma_{14})\phi^{2}(x)\phi'(x)\phi''(x)=0, \tag{4}$$

respectively. For integrability, one must make the following choices:

$$\sigma_1=\sigma_2=\sigma_3=\sigma_5=\sigma_9=\sigma_{10}=\sigma_{11}=\sigma_{15}=0, \tag{5}$$

$$\sigma_4+\sigma_8=0, \tag{6}$$

$$\sigma_6+\sigma_7=0, \tag{7}$$

$$\sigma_{12}+\sigma_{13}+\sigma_{14}=0. \tag{8}$$

With such choice of parameters, the governing model (1) condenses to

$$iq_t+a\left(|q|^{n}q\right)_{xx}+b|q|^{2}q$$

$$-\delta_2\left[\sigma_8|q|^{2}q_{xx}+\sigma_7|q_x|^{2}q-\sigma_7(q_x)^{2}q^{*}-\sigma_8q_{xx}^{*}q^{2}\right]$$

$$+i\delta_3\left[(\sigma_{13}+\sigma_{14})qq_xq_{xx}^{*}-\sigma_{13}q^{*}q_xq_{xx}-\sigma_{14}qq_x^{*}q_{xx}\right]=0. \tag{9}$$

Consequently, the real part equation (3) would reduce to:

$$a(n+1)\phi^{n}(x)\left[n\{\phi'(x)\}^{2}+\phi(x)\phi''(x)\right]+\phi^{2}(x)\left\{b\phi^{2}(x)-\lambda\right\}=0. \tag{10}$$

The above equation admits a single Lie point symmetry, namely $\dfrac{\partial}{\partial x}$.

With this symmetry equation (10) integrates to

$$x = \pm \frac{\phi^{\frac{n}{2}}}{n\lambda}\sqrt{\frac{2a(n+1)(n+2)\left[\{(n+4)\lambda\} - b(n+2)\phi^2\right]}{n+4}}\,{}_2F_1\left(1, \frac{n+2}{4}, \frac{n+4}{4}; \frac{b(n+2)\phi^2}{(n+4)\lambda}\right), \tag{11}$$

where the Gauss' hypergeometric function is:

$$F(\alpha, \beta; \gamma; z) = \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{(\gamma)_n} \frac{z^n}{n!}, \tag{12}$$

and the Pochhammer symbol is:

$$(p)_n = \begin{cases} 1 & n = 0, \\ p(p+1)\cdots(p+n-1) & n > 0. \end{cases} \tag{13}$$

The convergence criterion for the series is:

$$|z| < 1, \tag{14}$$

which for (11) it means

$$-\left[\frac{\lambda(n+4)}{b(n+2)}\right]^{\frac{1}{2}} < \phi(x) < \left[\frac{\lambda(n+4)}{b(n+2)}\right]^{\frac{1}{2}}. \tag{15}$$

Additionally the coefficient of the hypergeometric function in (11) compels:

$$a\left[\{(n+4)\lambda\} - b(n+2)\phi^2\right] > 0. \tag{16}$$

1.2 *Power law*

For power-law, the dispersive concatenation model given by (1) generalizes to

$$iq_t + a\left(|q|^n q\right)_{xx} + b|q|^{2m} q - i\delta_1\left[\sigma_1 q_{xxx} + \sigma_2 |q|^{2m} q_x\right]$$

$$+\delta_2\left[\sigma_3 q_{xxxx} + \sigma_4 |q|^2 q_{xx} + \sigma_5 |q|^4 q + \sigma_6 |q_x|^2 q + \sigma_7 (q_x)^2 q^* + \sigma_8 q_{xx}^* q^2\right]$$

$$-i\delta_3\left[\sigma_9 q_{xxxxx} + \sigma_{10}|q|^2 q_{xxx} + \sigma_{11}|q|^4 q_x + \sigma_{12} q q_x q_{xx}^* + \sigma_{13} q^* q_x q_{xx} + \sigma_{14} q q_x^* q_{xx} + \sigma_{15} (q_x)^2 q_x^*\right] = 0, \tag{17}$$

where m is the power law nonlinearity parameter. Inserting the same solution hypothesis as in (2) into (17) yields the real and imaginary parts as

$$an(n+1)\phi^n(x)\{\phi'(x)\}^2 + a(n+1)\phi^{n+1}(x)\phi''(x) + b\phi^{2m+2}(x) + \delta_2\sigma_3\phi^{(iv)}(x)\phi(x)$$

$$+\delta_2\left(\sigma_4+\sigma_8\right)\phi^3(x)\phi''(x)+\delta_2\left(\sigma_6+\sigma_7\right)\phi^2(x)\{\phi'(x)\}^2+\delta_2\sigma_5\phi^6(x)-\lambda\phi^2(x)=0, \tag{18}$$

and (4). For integrability, the same parameter constraints (5)-(8) emerge. Therefore equation (17) reduces to

$$iq_t+a\left(|q|^n q\right)_{xx}+b|q|^{2m}q$$

$$-\delta_2\left[\sigma_8|q|^2 q_{xx}+\sigma_7|q_x|^2 q-\sigma_7\left(q_x\right)^2 q^*-\sigma_8 q_{xx}^* q^2\right]$$

$$+i\delta_3\left[\left(\sigma_{13}+\sigma_{14}\right)qq_x q_{xx}^*-\sigma_{13}q^* q_x q_{xx}-\sigma_{14}qq_x^* q_{xx}\right]=0. \tag{19}$$

Consequently, the real part equation (18) would reduce to:

$$a(n+1)\phi^n(x)\left[n\{\phi'(x)\}^2+\phi(x)\phi''(x)\right]+\phi^2(x)\left\{b\phi^{2m}(x)-\lambda\right\}=0. \tag{20}$$

The above equation again admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$.
With this symmetry equation (20) integrates in terms of Gauss' hypergeometric function to

$$x=\pm\frac{\phi^{\frac{n}{2}}}{n\lambda}\sqrt{\frac{2a(n+1)(n+2)\left[\{(n+2m+2)\lambda\}-b(n+2)\phi^{2m}\right]}{n+2m+2}}\,{}_2F_1\left(1,\frac{1}{2}+\frac{n}{4m};1+\frac{n}{4m};\frac{b(n+2)\phi^{2m}}{(n+2m+2)\lambda}\right). \tag{21}$$

The convergence criteria in this case is: which for (11) it means

$$-\left[\frac{\lambda(n+2m+2)}{b(n+2)}\right]^{\frac{1}{2m}}<\phi(x)<\left[\frac{\lambda(n+2m+2)}{b(n+2)}\right]^{\frac{1}{2m}}. \tag{22}$$

Additionally, the coefficient of the hypergeometric function in (21) compels:

$$a\left[\{(n+2m+2)\lambda\}-b(n+2)\phi^{2m}\right]>0. \tag{23}$$

2. Conclusions

This paper studied the evolution of implicit quiescent optical solitons for the dispersive concatenation model having nonlinear CD with linear temporal evolution by Lie symmetry analysis. In our analysis, we identified the Lie point symmetry $\frac{\partial}{\partial x}$, which corresponds to a translational symmetry along the x-axis. It's important to note that the existence of additional Lie point symmetries depends on the specific form and complexity of the differential equation describing the physical model. Some equations may have multiple symmetries, while others may have only one or none at all. Two forms of SPM are studied and they are with Kerr law and power-law. The solutions are in terms of Gauss' hypergeometric functions for both forms of SPM. The parameter constraints that evolve from the hypergeometric functions also lead to the upper and lower bounds of the solutions. The results are thus very interesting. It must be noted that unlike the previous works, the generalized temporal evolutions yielded solutions that are in terms of quadratures and are consequently not listed. These results are indeed extremely promising and will be applied to a variety of other models from optics. They are currently awaited and will be disseminated with time after aligning them with the pre-existing works [12-30].

This paper contributes to the fundamental understanding of quiescent optical solitons within the dispersive concatenation model, which is an important topic in nonlinear optics. Investigating the behavior of solitons is crucial because they have significant applications in optical communication, signal processing, and information transmission. This paper explores a complex optical model that combines nonlinear CD with linear temporal evolution, along with two forms of SPM-Kerr law and power-law. Understanding how these elements interact is essential for modeling and predicting the behavior of optical signals in real-world optical systems. The use of Lie symmetry analysis and the expression of solutions in terms of Gauss' hypergeometric functions showcase advanced mathematical techniques for solving intricate physical problems. These methods can be valuable for researchers working in various scientific disciplines. This paper highlights the limitations of generalized temporal evolutions in yielding solutions in terms of quadratures. This provides insights into the conditions under which explicit solutions can be obtained, which is relevant for researchers working on similar models.

The results obtained in the present paper may have practical implications in optical communication systems. Understanding the behavior of optical solitons with nonlinear CD and SPM effects is crucial for designing efficient optical communication networks. Optical solitons are essential components in signal processing applications. The findings in this paper may aid in the development of advanced signal processing techniques that utilize solitons. The current paper mentions that the results will be aligned with pre-existing works. This suggests that the research has the potential to contribute to the alignment and integration of various optical models, leading to a more comprehensive understanding of optical phenomena. This paper opens avenues for further research in the field of nonlinear optics. Researchers can build upon the findings to explore related models, validate theoretical predictions through experiments, and develop practical applications in optical technology.

This paper is significant in the field of nonlinear optics because it recovers implicit quiescent optical solitons within a complex model that includes nonlinear chromatic dispersion and two forms of SPM. It employs advanced mathematical techniques, like Lie symmetry analysis, and identifies parameter constraints that yield upper and lower bounds for the solutions. Our findings have the potential to impact nonlinear optics in practical and theoretical ways. They can be applied in designing efficient optical communication and signal processing systems. The research extends our understanding of soliton behavior in complex optical models and can contribute to the development of more accurate theoretical frameworks. Aligning the findings with existing works can further enhance our comprehension of optical solitons. Future work may include experimental validation of the recovered soliton solutions in real optical systems, exploring additional nonlinear effects, and investigating applications in optical devices. Integration with other models and conducting comparative studies can provide insights into soliton behavior in diverse optical scenarios.

Conflict of interest

The authors declare that there is no conflict of interest.

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