Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

Research Article

Bifurcation Analysis and Chaotic Behavior of the Concatenation Model with Power-Law Nonlinearity

Lu Tang[1], **Anjan Biswas**[2,3,4,5], **Yakup Yildirim**[6,7*], **Asim Asiri**[3]

[1]School of Mathematics and Physics, Chengdu University of Technology, Chengdu, China
[2]Department of Mathematics and Physics, Grambling State University, Grambling, LA, USA
[3]Mathematical Modeling and Applied Computation Research Group, Center of Modern Mathematical Sciences and their Applications, Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia
[4]Department of Applied Sciences, Cross-Border Faculty of Humanities, Economics and Engineering, University of Galati, Galati, Romania
[5]Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa, South Africa
[6]Department of Computer Engineering, Biruni University, Istanbul, Turkey
[7]Department of Mathematics, Near East University, Nicosia, Cyprus
E-mail: yyildirim@biruni.edu.tr

Received: 1 September 2023; **Revised:** 18 October 2023; **Accepted:** 2 November 2023

Abstract: This paper presents a comprehensive analysis of the concatenation model with power-law nonlinearity. The research encompasses multiple key aspects, providing a detailed exploration of the model's behavior and implications within the context of nonlinear dynamics and optics. The study commences with an in-depth bifurcation analysis, aiming to unravel the intricate dynamics and transitions within the system. This analysis not only uncovers the system's behavior under varying conditions but also sheds light on its stability and the emergence of bifurcation phenomena. Our research delves into the retrieval of soliton solutions within the model. The exploration of solitons is of paramount significance, offering insights into localized, self-sustaining waveforms that often play a crucial role in nonlinear systems. These soliton solutions are identified, characterized, and their relevance to the model is established. The paper addresses the complex dynamics of the system in the presence of perturbation terms. By incorporating perturbations into the analysis, we elucidate how external influences impact the system's behavior and lead to chaotic phenomena. This analysis helps uncover the system's sensitivity to external factors and provides a deeper understanding of chaotic behavior.

Keywords: bifurcation analysis, concatenation model, power-law nonlinearity, chaotic behavior, perturbation effects

MSC: 78A60

1. Introduction

The concatenation model has appeared in the field of nonlinear optics for about a decade [1, 2]. After its inception, a deluge of results is visible from a wide variety of journals across the board. The coverage is wide and varied. Many results that are mathematically intense as well as results that are applicable to fiber-optic communication systems have

DOI: https://doi.org/10.37256/cm.4420233606

emerged. A few of the topics that have been touched base upon for the model are conservation laws, Painleve analysis, magneto-optic solitons, quiescent solitons for nonlinear chromatic dispersion, numerical analysis, bifurcation analysis, birefringent fibers, just to enlist a few [3-8]. While the previous round of bifurcation analysis of the model is with Kerr law of self-phase modulation (SPM), the current paper is a generalized version of the previous counterpart [6]. The bifurcation analysis of the concatenation model with power-law of SPM is the focus of this paper [4, 5]. In addition to this analysis, the chaotic dynamics of the model will also be addressed. Subsequently, the soliton solutions to the model will be derived using this analysis. The details are exhibited in the rest of the paper after a quick and succinct introduction to the model.

Our study is significant in the field of nonlinear optics as it builds on a decade of research on the concatenation model. This model has generated a wealth of results in various journals, spanning topics like conservation laws, solitons, and fiber-optic systems. While previous work primarily focused on the Kerr law of SPM, our study presents a more generalized analysis, specifically examining the concatenation model with the power-law of SPM. Additionally, we explore the model's chaotic dynamics and derive soliton solutions. This research contributes to the field by offering new insights and potential applications in nonlinear optics.

Our study enhances understanding of the concatenation model with power-law SPM but has limitations. It focuses on a specific model aspect, involves simplifications and theoretical emphasis. We assume practical relevance to fiber-optic systems. Researchers should consider these limitations when applying our findings to real-world situations.

1.1 *Governing model*

In the current paper, we will devote ourselves to considering following model [3-8]:

$$iq_t + aq_{xx} + b|q|^{2n} q + \alpha_1 \left[\delta_1 q_{xxxx} + \delta_2 (q_x)^2 q^* + \delta_3 |q_x|^2 q + \delta_4 |q|^{2n} q_{xx} + \delta_5 q^2 q_{xx}^* + \delta_6 |q|^{2n+2} q \right]$$

$$+ i\alpha_2 \left[\delta_7 q_{xxx} + \delta_8 |q|^{2n} q_x + \delta_9 q^2 q_x^* \right] = 0. \tag{1}$$

The concatenation model given by (1) is true to its name. The model is a concatenation of version of the familiar nonlinear Schrödinger's equation (NLSE) [3-8], Sasa-Satsuma equation (SSE) [3-8] and the Lakshmanan-Porsezian-Daniel (LPD) model [3-8]. Thus in (1), the first three terms stem from the NLSE with power-law nonlinearity with n being the power-law nonlinearity parameter and the coefficients of α_1 and α_2 being from LPD model and SSE respectively. For $\alpha_1 = 0$, (1) reduces to the familiar SSE, while for $\alpha_2 = 0$, equation (1) reduces to LPD model. But for $\alpha_1 = \alpha_2 = 0$, (1) collapses to the familiar NLSE with power-law nonlinearity.

The physical motivation for considering the model presented in Eq. (1) is to create a versatile and adaptable framework that unifies elements from different equations, offering a valuable tool for studying various aspects of nonlinear optics and the influence of different parameters on system behavior.

2. Phase portraits and optical solitons

In order to obtain bifurcation phase portraits, optical soliton solutions and chaotic behaviors for the model (1), we firstly postulate assumption

$$q(x,t) = \Phi(x,t)e^{i\Psi(x,t)} = \Phi(\xi)e^{i\Psi(x,t)},\ \xi = x - V_0 t,\ \Psi(x,t) = -\mu x + \lambda t + \kappa_0, \tag{2}$$

where $\Phi(x, t)$ stands for the amplitude component of the wave form, while the coefficient λ denotes the wave number. Also, the coefficients μ and κ_0 represent the frequency and phase constant, respectively. V_0 stands for the velocity, while x denotes the normalized propagation and t represents the time. Substituting (2) into (1) and separating it into real and imaginary parts, one has

Real part

$$(\alpha_1\delta_1\mu^4 - \alpha_2\delta_7\mu^3 - a\mu^2 - \lambda)\Phi - [\alpha_1(\delta_2 - \delta_3 + \delta_5)\mu^2 + \alpha_2\delta_9\mu]\Phi^3$$

$$+(b + \alpha_2\delta_8\mu - \alpha_1\delta_4\mu^2)\Phi^{2n+1} + \alpha_1\delta_6\Phi^{2n+3} + (a - 6\alpha_1\delta_1\mu^2 + 3\alpha_2\delta_7\mu)\Phi''$$

$$+\alpha_1\delta_1\Phi^{(4)} + \alpha_1\delta_4\Phi^{2n}\Phi'' + \alpha_1\delta_5\Phi^2\Phi'' + \alpha_1(\delta_2 + \delta_3)\Phi(\Phi')^2 = 0. \tag{3}$$

Imaginary part

$$(-2a\mu - 3\alpha_2\delta_7\mu^2 + 4\alpha_1\delta_1\mu^3 - V_0)\Phi' + (\alpha_2\delta_8 - 2\alpha_1\delta_4\mu)\Phi^{2n}\Phi' + [\alpha_2\delta_9 - 2\alpha_1\mu(\delta_2 - \delta_5)]\Phi^2\Phi'$$

$$+(\alpha_2\delta_7 - 4\mu\alpha_1\delta_1)\Phi''' = 0. \tag{4}$$

Certain restrictions are provided by Equation (3) when the coefficients of its linearly independent functions are set to zero, as shown below

$$\alpha_1\delta_1 = 0, \tag{5}$$

$$\alpha_1\delta_4 = 0, \tag{6}$$

$$\alpha_1\delta_5 = 0, \tag{7}$$

$$\alpha_1\left(\delta_2 + \delta_3\right) = 0, \tag{8}$$

$$-6\alpha_1\delta_1\mu^2 + 3\alpha_2\delta_7\mu + a = 0, \tag{9}$$

$$\alpha_1\delta_6 = 0, \tag{10}$$

$$-\alpha_1\delta_4\mu^2 + \alpha_2\delta_8\mu + b = 0, \tag{11}$$

$$\alpha_1\delta_2\mu^2 - \alpha_1\delta_3\mu^2 + \alpha_1\delta_5\mu^2 + \alpha_2\delta_9\mu = 0, \tag{12}$$

and

$$\delta_1\alpha_1\mu^4 - \alpha_2\delta_7\mu^3 - a\mu^2 - \lambda = 0. \tag{13}$$

The parameter constraints are yielded by equations (5) through (13) as follows:

$$a = -3\alpha_2\delta_7\mu, \tag{14}$$

$$b = -\alpha_2\delta_8\mu, \tag{15}$$

$$\lambda = 2\alpha_2\delta_7\mu^3, \tag{16}$$

$$\delta_1 = 0, \tag{17}$$

$$\delta_2 = -\frac{\alpha_2\delta_9}{2\mu\alpha_1}, \tag{18}$$

$$\delta_3 = \frac{\alpha_2\delta_9}{2\mu\alpha_1}, \tag{19}$$

$$\delta_4 = 0. \tag{20}$$

$$\delta_5 = 0, \tag{21}$$

and

$$\delta_6 = 0. \tag{22}$$

The use of equations (14)-(22) results in equation (3) vanishing, and the soliton profile is provided by the integration of equation (4). From the imaginary part (4), we know the soliton speed

$$V_0 = -2\mu(a + 4\alpha_1\delta_1\mu^2), \tag{23}$$

whenever

$$\alpha_2\delta_8 = 2\alpha_1\delta_4\mu, \tag{24}$$

$$\alpha_2\delta_9 = 2\alpha_1\mu(\delta_2 - \delta_5), \tag{25}$$

and

$$\alpha_2\delta_7 = 4\mu\alpha_1\delta_1. \tag{26}$$

In reference [6], our research focused on the implications of Kerr law nonlinearity, while in the present study, we have shifted our attention to power law nonlinearity. Thus, set $n = 2$, one has

$$(-2a\mu - 3\alpha_2\delta_7\mu^2 + 4\alpha_1\delta_1\mu^3 + V_0)\Phi' + (\alpha_2\delta_8 - 2\alpha_1\delta_4\mu)\Phi^4\Phi' + [\alpha_2\delta_9 - 2\alpha_1\mu(\delta_2 - \delta_5)]\Phi^2\Phi'$$

$$+(\alpha_2\delta_7 - 4\mu\alpha_1\delta_1)\Phi''' = 0. \tag{27}$$

Integrating (27) once, we get

$$(-2a\mu - 3\alpha_2\delta_7\mu^2 + 4\alpha_1\delta_1\mu^3 + V_0)\Phi + \frac{\alpha_2\delta_8 - 2\alpha_1\delta_4\mu}{5}\Phi^5 + \frac{\alpha_2\delta_9 - 2\alpha_1\mu(\delta_2 - \delta_5)}{3}\Phi^3$$

$$+(\alpha_2\delta_7 - 4\mu\alpha_1\delta_1)\Phi'' = 0. \tag{28}$$

Therefore, the equation (28) can be rewritten

$$\Phi'' - \Xi_1\Phi(\Phi^4 + \Xi_2\Phi^2 + \Xi_3) = 0, \tag{29}$$

where $\Xi_1 = \frac{-10a\mu - 15\alpha_2\delta_7\mu^2 + 20\alpha_1\delta_1\mu^3 + 5V_0}{\alpha_2\delta_8 - 2\alpha_1\delta_4\mu}$, $\Xi_2 = \frac{5\alpha_2\delta_9 - 10\alpha_1\mu(\delta_2 - \delta_5)}{3\alpha_2\delta_8 - 6\alpha_1\delta_4\mu}$, $\Xi_1 = -\frac{\alpha_2\delta_8 - 2\alpha_1\delta_4\mu}{5\alpha_2\delta_7 - 20\mu\alpha_1\delta_1}$.

In order to obtain the bifurcation phase portraits and chaotic behaviors of the Eq. (1) in detail, we firstly suppose that $\Phi' = p$, therefore we can obtain the plane dynamical system

$$\begin{cases} \dfrac{d\Phi}{d\xi} = p, \\ \dfrac{dp}{d\xi} = \Xi_1\Phi(\Phi^4 + \Xi_2\Phi^2 + \Xi_3), \end{cases} \tag{30}$$

with the Hamiltonian system

$$H(\Phi, p) = \frac{1}{2}p^2 - \frac{\Xi_1}{6}\Phi^6 - \frac{\Xi_1\Xi_2}{4}\Phi^4 - \frac{\Xi_1\Xi_3}{2}\Phi^2 = h. \tag{31}$$

To analysis the dynamical behaviors of system (30) in detail, we firstly assume that $G(\Phi) = \Xi_1\Phi(\Phi^4 + \Xi_2\Phi^2 + \Xi_3)$ and $\Delta = \Xi_2^2 - 4\Xi_3$. Thus, it is easy to notice that all equilibrium points of system (30) are located on the Φ-axis. Moreover, Φ_e (e = 1, 2, 3 ...) stands for the real root of the function $G(\Phi)$. Suppose that $M_e(\Phi_e, 0)$ denotes the equilibrium point for system (30).

Next, we set

$$J(\Phi_e, p) = \begin{vmatrix} 0 & 1 \\ G'(\Phi_e) & 0 \end{vmatrix} = -G'(\Phi_e). \tag{32}$$

As a consequence, according to the theory of planar-dynamical systems [9-14], we can derive the following conclusions.

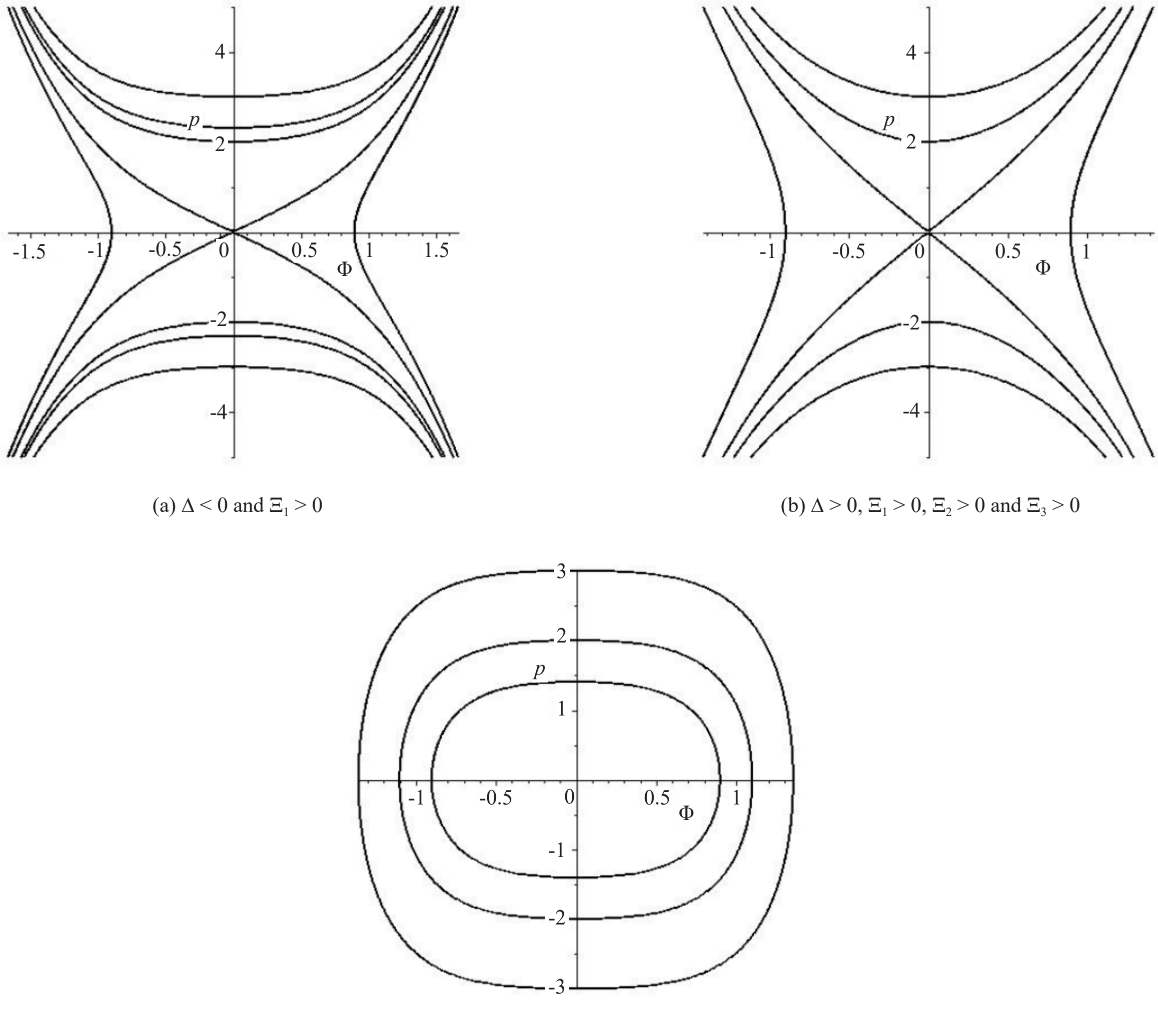
(1) If $\Delta < 0$ and $\Xi_1 > 0$ or $\Delta > 0$, $\Xi_3 > 0$, $\Xi_2 > 0$ and $\Xi_1 > 0$, it is easy to find that system (30) owns only one equilibrium point at $M_0(0, 0)$. Therefore, we deduce that $J(0, 0) < 0$. As is shown in Figure 1a and Figure 1b, we obtain that $E_0(0, 0)$ represents the saddle point.

(2) If $\Delta = 0$, $\Xi_1 > 0$, $\Xi_2 < 0$ and $\Xi_3 > 0$, we observe that system (30) owns only one equilibrium point at $M_0(0, 0)$. We derive that $J(0, 0) > 0$. As is shown in Figure 1c, we get that $E_0(0, 0)$ represents the center point.

(3) If $\Delta > 0$, $\Xi_1 < 0$, $\Xi_2 > 0$ and $\Xi_3 > 0$, it is easy to notice that system (30) owns three equilibrium points, which include $M_0(0, 0)$, $M_{1\pm}\left(\pm\sqrt{-\frac{\Xi_2}{2}}, 0\right)$. Next, when $J(0, 0) > 0$, $M_0(0, 0)$ stands for the center point. When $J\left(\pm\sqrt{-\frac{\Xi_2}{2}}, 0\right)$ = 0 and Poincaré index is equal to zero, as is vivid shown in Figure 2a, we obtain that $M_{1\pm}\left(\pm\sqrt{-\frac{\Xi_2}{2}}, 0\right)$ denote cusp points.

(4) If $\Delta = 0$, $\Xi_1 < 0$, $\Xi_2 < 0$ and $\Xi_3 > 0$, we notice that system (30) owns three equilibrium points, which include

$M_0(0, 0)$, $M_{2\pm}\left(\pm\sqrt{-\frac{\Xi_2}{2}},0\right)$. Next, when $J(0, 0) > 0$, $M_0(0, 0)$ represents the center point. When $J\left(\pm\sqrt{-\frac{\Xi_2}{2}},0\right) = 0$ and Poincaré index is equal to zero, as is shown in Figure 2b, we obtain that $M_{2\pm}\left(\pm\sqrt{-\frac{\Xi_2}{2}},0\right)$ denote cusp points.



(a) $\Delta < 0$ and $\Xi_1 > 0$

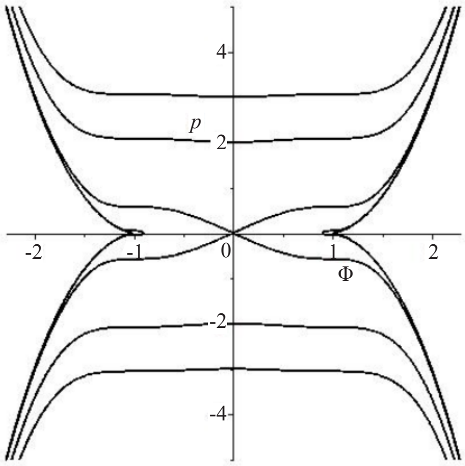
(b) $\Delta > 0$, $\Xi_1 > 0$, $\Xi_2 > 0$ and $\Xi_3 > 0$

(c) $\Delta > 0$, $\Xi_1 < 0$, $\Xi_2 > 0$ and $\Xi_3 > 0$

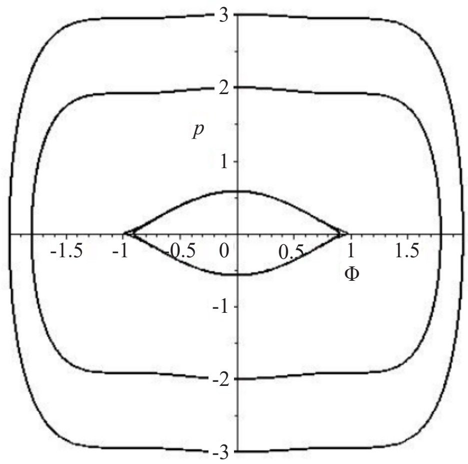
Figure 1. The bifurcation phase portraits of system (30)

(5) If $\Xi_1 < 0$ and $\Xi_3 < 0$, we notice that system (30) owns three equilibrium points, which include $M_0(0, 0)$, $M_{3\pm}\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)$. Next, when $J(0, 0) < 0$, $M_0(0, 0)$ represents the saddle point. When $J\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right) > 0$, $M_{3\pm}\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)$ stand for the center points (see Figure 2c). When $\Xi_1 > 0$ and $\Xi_3 < 0$, we derive that system

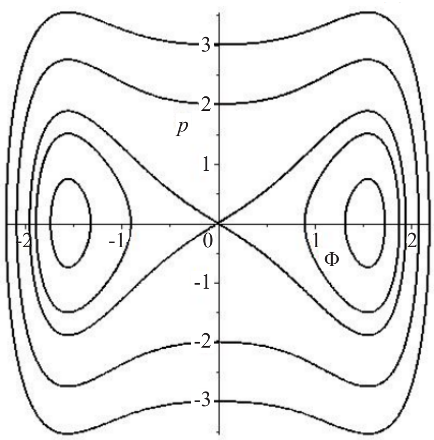
(30) owns three equilibrium points, which include $M_0(0, 0)$, $M_{4\pm}\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}}, 0\right)$. Next, when $J(0, 0) > 0$, $M_0(0, 0)$ represents the center point. When $J\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}}, 0\right) < 0$, $M_{4\pm}\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}}, 0\right)$ stand for the saddle points (see Figure 3a).



(a) $\Delta = 0$, $\Xi_1 > 0$, $\Xi_2 < 0$ and $\Xi_3 > 0$



(b) $\Delta = 0$, $\Xi_1 < 0$, $\Xi_2 < 0$ and $\Xi_3 > 0$



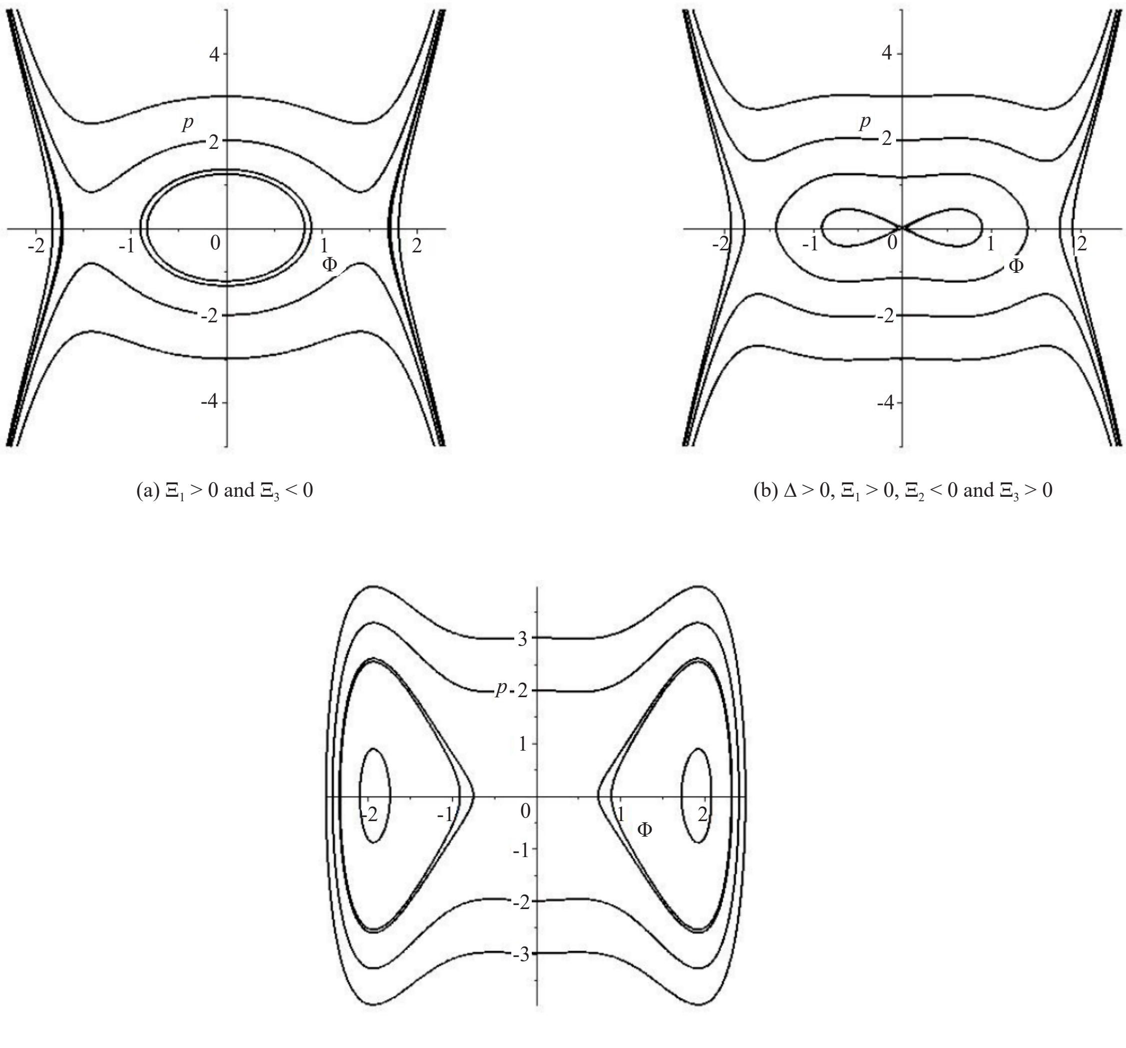
(c) $\Xi_1 < 0$ and $\Xi_3 < 0$

Figure 2. The bifurcation phase portraits of system (30)

(6) If $\Delta > 0$, $\Xi_1 < 0$, $\Xi_2 < 0$ and $\Xi_3 > 0$, we notice that system (30) owns five equilibrium points, which include $M_0(0, 0)$, $M_{5\pm}\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}}, 0\right)$ and $M_{6\pm}\left(\pm\sqrt{\frac{-\Xi_2-\sqrt{\Xi_2^2-4\Xi_3}}{2}}, 0\right)$. Next, when $J(0, 0) > 0$, $M_0(0, 0)$ represents the center point (see Figure 3b). When $J\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}}, 0\right) < 0$, $M_{5\pm}\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}}, 0\right)$ stand for the center

points. When $J\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)<0$, $M_{6\pm}\left(\pm\sqrt{\frac{-\Xi_2-\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)$ stand for the center points (see Figure 3c).

(7) If $\Delta>0$, $\Xi_1>0$, $\Xi_2<0$ and $\Xi_3>0$, we notice that system (30) owns five equilibrium points, which include $M_0(0, 0)$, $M_{7\pm}\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)$ and $M_{8\pm}\left(\pm\sqrt{\frac{-\Xi_2-\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)$. Next, when $J(0, 0)>0$, $M_0(0, 0)$ represents the saddle point. When $J\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)<0$, $M_{7\pm}\left(\pm\sqrt{\frac{-\Xi_2+\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)$ stand for the saddle points. When $J\left(\pm\sqrt{\frac{-\Xi_2-\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)<0$, $M_{8\pm}\left(\pm\sqrt{\frac{-\Xi_2-\sqrt{\Xi_2^2-4\Xi_3}}{2}},0\right)$ stand for the center points.



(a) $\Xi_1>0$ and $\Xi_3<0$

(b) $\Delta>0$, $\Xi_1>0$, $\Xi_2<0$ and $\Xi_3>0$

(c) $\Delta>0$, $\Xi_1<0$, $\Xi_2<0$ and $\Xi_3>0$

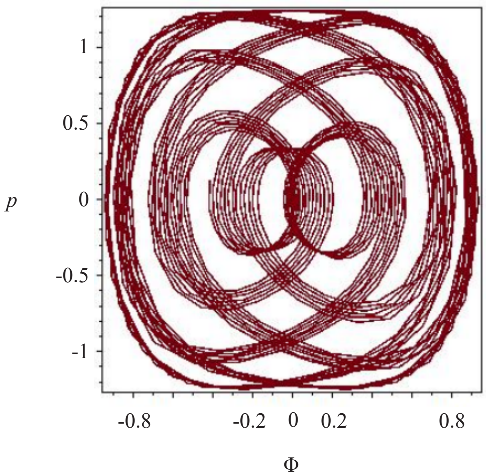
Figure 3. The bifurcation phase portraits of system (30)

3. Chaotic behavior with perturbation terms

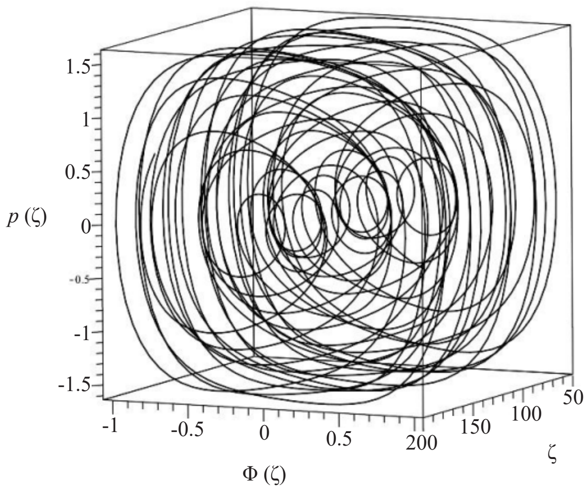
In this section, we study the two-dimensional planar dynamical system (30) with perturbation term

$$\begin{cases} \dfrac{d\Phi}{d\xi} = p, \\ \dfrac{dp}{d\xi} = \Xi_1 \Phi(\Phi^4 + \Xi_2 \Phi^2 + \Xi_3) + \Xi_0 \cos(\omega\xi), \end{cases} \tag{33}$$

where Ξ_0 represents the amplitude of the system (33), and ω stands for the frequency of (33). In Figure 4, 2D and 3D phase portraits are illustrated for $\Delta > 0$, $\Xi_1 < 0$, $\Xi_2 > 0$ and $\Xi_3 > 0$, $\Xi_0 = 0.6$ $\omega = 1$. In Figure 5, 2D and 3D phase portraits are displayed for $\Xi_1 < 0$, $\Xi_3 < 0$, $\Xi_0 = 0.6$ $\omega = 1$. In addition, in Figure 6, 2D and 3D phase portraits are visualized for $\Delta > 0$, $\Xi_1 < 0$, $\Xi_2 < 0$ and $\Xi_3 > 0$, $\Xi_0 = 0.6$ $\omega = 1$.



(a) 2D phase portrait with perturbation term

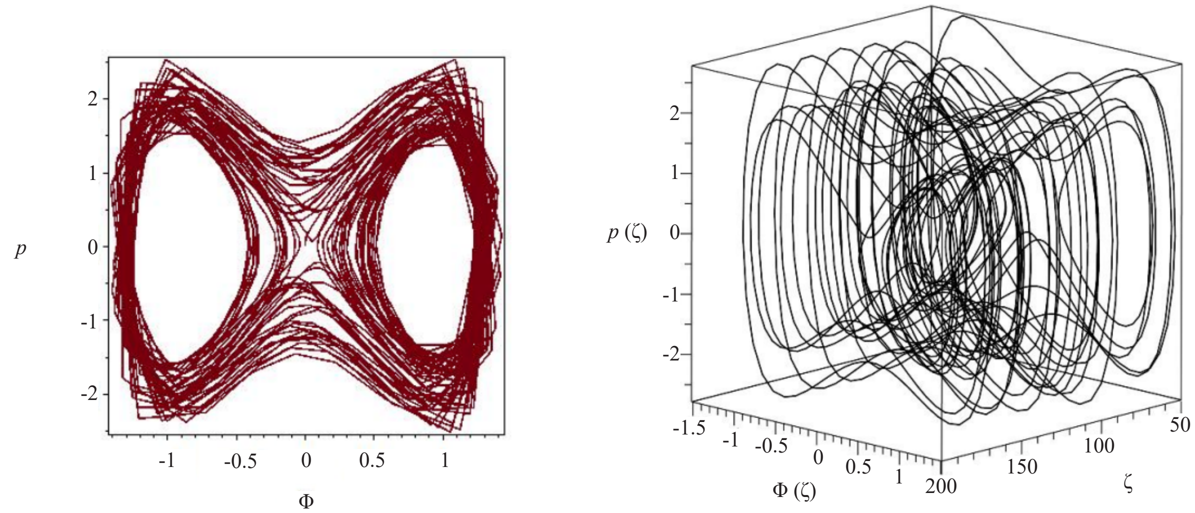


(b) 3D phase portrait with perturbation term

Figure 4. The portraits of system (33) for $\Delta > 0$, $\Xi_1 < 0$, $\Xi_2 > 0$ and $\Xi_3 > 0$

4. Conclusions

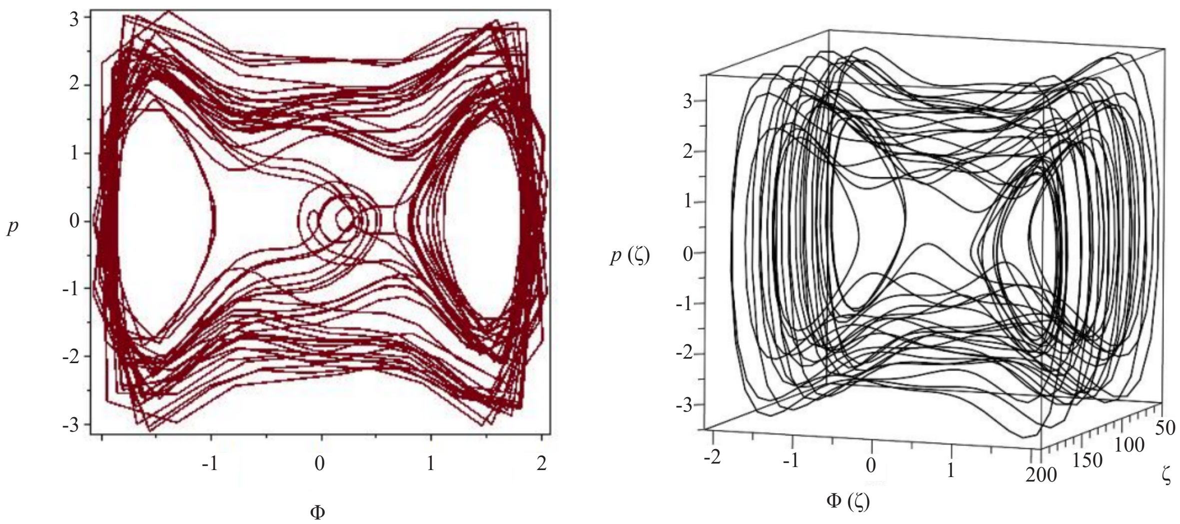
This paper is a comprehensive study of the bifurcation analysis of the concatenation model that is with power-law of nonlinearity. The results are based on a continued effort from the previous model [6]. The power-law nonlinearity parameter played a crucial role in the analysis of this paper. The phase portraits of the model and the soliton solutions are recovered. With the perturbation terms turned on, the chaotic behavior is studied and analyzed. The numerical schemes have exhibited the technical details of the analysis. Figure 1, 2, and 3 show how the system's behavior changes with varying parameters, helping us understand critical points, stability, and transitions in system dynamics, while Figure 4, 5, and 6 illustrate the effects of perturbation on the system, offering insights into phase space, attractors, and sensitivity to external factors. These figures are significant in our study as they visually represent the dynamic behavior of the systems under consideration. They help researchers understand the effects of bifurcation and perturbation, providing insights that can be valuable for both theoretical exploration and practical applications in nonlinear dynamics and optics.



(a) 2D phase portrait with perturbation term

(b) 3D phase portrait with perturbation term

Figure 5. The portraits of system (33) for $\Xi_1 < 0$ and $\Xi_3 < 0$



(a) 2D phase portrait with perturbation term

(b) 3D phase portrait with perturbation term

Figure 6. The portraits of system (33) for $\Delta > 0$, $\Xi_1 < 0$, $\Xi_2 < 0$ and $\Xi_3 > 0$

Our study's significance lies in its contribution to the evolving field of nonlinear optics by offering a comprehensive examination of the concatenation model with a novel perspective on power-law SPM, exploration of chaotic dynamics, and the derivation of soliton solutions. These findings extend the current knowledge and have the potential to impact various applications within the research landscape of nonlinear optics [15-27].

The results of this work provides a strong footing to further future analysis of the project. Later this bifurcation analysis will be studied for the concatenation model with differential group display and the extension of this study to dispersive concatenation model is also on the horizon. The results of such research activities will be disseminated with time after aligning them with the results of the preexisting works [28-38].

Conflict of interest

The authors claim there is no conflict of interest in this work.

References

[1] Ankiewicz A, Akhmediev N. Higher-order integrable evolution equation and its soliton solutions. *Physics Letters*. 2014; 378(4): 358-361. Available from: doi: 10.1016/j.physleta.2013.11.031.

[2] Ankiewicz A, Wang Y, Wabnitz S, Akhmediev N. Extended nonlinear Schrödinger equation with higher-order odd and even terms and its rogue wave solutions. *Physical Review E*. 2014; 89(1). Available from: doi: 10.1103/physreve.89.012907.

[3] Moraru L. Optical solitons and conservation laws for the concatenation model with spatio-temporal dispersion (internet traffic regulation). *Journal of the European Optical Society: Rapid Publications*. 2023; 19(2): 35. Available from: doi: 10.1051/jeos/2023031.

[4] Arnous AH, Biswas A, Kara AH, Yıldırım Y, Moraru L, Iticescu C, et al. Optical solitons and conservation laws for the concatenation model: Power-law nonlinearity. *Ain Shams Engineering Journal*. 2023; 102381. Available from: doi: 10.1016/j.asej.2023.102381.

[5] Biswas A, Vega-Guzmán J, Yıldırım Y, Moshokoa SP, Aphane M, Alghamdi AS. Optical solitons for the concatenation model with power-law nonlinearity: undetermined coefficients. *Ukrainian Journal of Physical Optics*. 2023; 24(3): 185-192. Available from: doi: 10.3116/16091833/24/3/185/2023.

[6] Tang L, Biswas A, Yıldırım Y, Alghamdi AA. Bifurcation analysis and optical solitons for the concatenation model. *Physics Letters*. 2023; 480: 128943. Available from: doi: 10.1016/j.physleta.2023.128943.

[7] Triki H, Sun Y, Zhou Q, Biswas A, Yıldırım Y, Alshehri HM. Dark solitary pulses and moving fronts in an optical medium with the higher-order dispersive and nonlinear effects. *Chaos Solitons & Fractals*. 2022; 164: 112622. Available from: doi: 10.1016/j.chaos.2022.112622.

[8] Yıldırım Y, Biswas A, Moraru L, Alghamdi AA. Quiescent optical solitons for the concatenation model with nonlinear chromatic dispersion. *Mathematics*. 2023; 11(7): 1709. Available from: doi: 10.3390/math11071709.

[9] Li JB, Dai HH. *On the Study of Singular Nonlinear Traveling Wave Equations: Dynamical System Approach*. Beijing: Science Press; 2007.

[10] Li JB. *Singular Nonlinear Traveling Wave Equations: Bifurcation and Exact Solutions*. Beijing: Science Press; 2013.

[11] Tang L. Bifurcation analysis and multiple solitons in birefringent fibers with coupled Schrödinger-Hirota equation. *Chaos, Solitons & Fractals*. 2022; 161(161): 112383. Available from: doi: 10.1016/j.chaos.2022.112383.

[12] Tang L. Bifurcations and optical solitons for the coupled nonlinear Schrödinger equation in optical fiber Bragg gratings. *Journal of Optics*. 2022; 52. Available from: doi: 10.1007/s12596-022-00963-4.

[13] Tang L. Phase portraits and multiple optical solitons perturbation in optical fibers with the nonlinear Fokas-Lenells equation. *Journal of Optics*. 2023. Available from: doi: 10.1007/s12596-023-01097-x.

[14] Wang MY. Optical solitons with perturbed complex Ginzburg-Landau equation in kerr and cubic-quintic-septic nonlinearity. *Results in Physics*. 2022; 33: 105077. Available from: doi: 10.1016/j.rinp.2021.105077.

[15] Zhou Q. Influence of parameters of optical fibers on optical soliton interactions. *Chinese Physics Letters*. 2022; 39(1): 010501. Available from: doi: 10.1088/0256-307x/39/1/010501.

[16] Younis M, Iftikhar M, Rehman HU. Exact solutions to the nonlinear schrödinger and eckhaus equations by modified simple equation method. *Journal of Advanced Physics*. 2014; 3(1): 77-79. Available from: doi: 10.1166/jap.2014.1104.

[17] Ur Rehman H, Mustafa I, Asjad MI, Habib A, Munir Q. New soliton solutions for the space-time fractional modified third order Korteweg-de Vries equation. *Journal of Ocean Engineering and Science*. 2022. Available from: doi: 10.1016/j.joes.2022.05.032.

[18] Eldidamony HA, Ahmed HM, Zaghrout AS, Ali YS, Arnous AH. Optical solitons with Kudryashov's quintuple power law nonlinearity having nonlinear chromatic dispersion using modified extended direct algebraic method. *Optik*. 2022; 262: 169235. Available from: doi: 10.1016/j.ijleo.2022.169235.

[19] Ullah N, Asjad MI, Ur Rehman H, Akgül A. Construction of optical solitons of Radhakrishnan-KunduLakshmanan equation in birefringent fibers. *Nonlinear Engineering*. 2022; 11(11): 80-91. Available from: doi: 10.1515/nleng-2022-0010.

[20] Asjad MI, Ullah N, Rehman HU. Construction of optical solitons of magneto-optic waveguides with anti-cubic law nonlinearity. *Optical and Quantum Electronics*. 2021; 53(11). Available from: doi: 10.1007/s11082-021-03288-x.

[21] Tariq KU, Younis M, Rezazadeh H, Rizvi STR, Osman MS. Optical solitons with quadratic-cubic nonlinearity and fractional temporal evolution. *Modern Physics Letters B*. 2018; 32(26): 1850317. Available from: doi: 10.1142/s0217984918503177.

[22] Kumar D, Park C, Tamanna N, Paul GC, Osman MS. Dynamics of two-mode Sawada-Kotera equation: Mathematical and graphical analysis of its dual-wave solutions. *Results in Physics*. 2020; 19: 103581. Available from: doi: 10.1016/j.rinp.2020.103581.

[23] Ismael HF, Bulut H, Park C, Osman MS. M-lump, N-soliton solutions, and the collision phenomena for the (2 + 1)-dimensional Date-Jimbo-Kashiwara-Miwa equation. *Results in Physics*. 2020; 19: 103329. Available from: doi: 10.1016/j.rinp.2020.103329.

[24] Ali KK, Abd El Salam MA, Mohamed EMH, Samet B, Kumar S, Osman MS. Numerical solution for generalized nonlinear fractional integro-differential equations with linear functional arguments using Chebyshev series. *Advances in Difference Equations*. 2020; 2020(1). Available from: doi: 10.1186/s13662-020-02951-z.

[25] Kumar S, Niwas M, Osman MS, Abdou MA. Abundant different types of exact soliton solution to the (4 + 1)-dimensional Fokas and (2 + 1)-dimensional breaking soliton equations. *Communications in Theoretical Physics*. 2021; 73(10): 105007. Available from: doi: 10.1088/1572-9494/ac11ee.

[26] Rahman RU, Qousini M, Alshehri A, Eldin SM, El-Rashidy K, Osman MS. Evaluation of the performance of fractional evolution equations based on fractional operators and sensitivity assessment. *Results in Physics*. 2023; 49(49): 106537. Available from: doi: 10.1016/j.rinp.2023.106537.

[27] Md. Arif C, Mamun Miah M, Iqbal M, Alshehri HM, Dumitru Băleanu, Osman MS. Advanced exact solutions to the nano-ionic currents equation through MTs and the soliton equation containing the RLC transmission line. *The European Physical Journal Plus*. 2023; 138(6). Available from: doi: 10.1140/epjp/s13360-023-04105-y.

[28] Wang S. Novel soliton solutions of CNLSEs with Hirota bilinear method. *Journal of Optics*. 2023; 52(3): 1602-1607. Available from: doi: 10.1007/s12596-022-01065-x.

[29] Han T, Zhao L, Li C, Zhao L. Bifurcations, stationary optical solitons and exact solutions for complex Ginzburg-Landau equation with nonlinear chromatic dispersion in non-Kerr law media. *Journal of Optics*. 2022; 52(2): 831-844. Available from: doi: 10.1007/s12596-022-01041-5.

[30] Nandy S, Lakshminarayanan V. Adomian decomposition of scalar and coupled nonlinear Schrödinger equations and dark and bright solitary wave solutions. *Journal of Optics*. 2015; 44(4). Available from: doi: 10.1007/s12596-015-0270-9.

[31] Chen W, Shen M, Kong Q, Wang Q. The interaction of dark solitons with competing nonlocal cubic nonlinearities. *Journal of Optics*. 2015; 44. Available from: doi: 10.1007/s12596-015-0255-8.

[32] Xu SL, Petrović N, Belić MR. Two-dimensional dark solitons in diffusive nonlocal nonlinear media. *Journal of Optics*. 2015; 44(2): 172-177. Available from: doi: 10.1007/s12596-015-0243-z.

[33] Ravi KD, Prabhakara RB. Influences of third-order dispersion on linear birefringent optical soliton transmission systems. *Journal of Optics*. 2011; 40(3): 132-142. Available from: doi: 10.1007/s12596-011-0045-x.

[34] Singh M, Sharma AK, Kaler RS. Investigations on optical timing jitter in dispersion managed higher order soliton system. *Journal of Optics*. 2011; 40. Available from: doi: 10.1007/s12596-010-0021-x.

[35] Vijay J. Formation and propagation-dynamics of primary and secondary soliton-like pulses in bulk nonlinear media. *Journal of Optics*. 2008; 37(1): 1-8. Available from: doi: 10.1007/bf03354831.

[36] Hasegawa A. Application of optical solitons for information transfer in fibers - A tutorial review. *Journal of Optics*. 2004; 33(3): 145-156. Available from: doi: 10.1007/bf03354760.

[37] Mahalingam AA, Uthayakumar PA. Dispersion and nonlinearity managed multisoliton propagation in an erbium doped inhomogeneous fiber with gain/loss. *Journal of Optics*. 2013; 42(3): 182-188. Available from: doi: 10.1007/s12596-012-0105-x.

[38] Jawad AJM, Abu-AlShaeer MJ. Highly dispersive optical solitons with cubic law and cubic-quinticseptic law nonlinearities by two methods. *Al-Rafidain Journal of Engineering Sciences*. 2023; 1(1): 1-8.