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Research Article

Optical Solitons for the Dispersive Concatenation Model: Undetermined Coefficients

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Abstract: This paper recovers optical solitons to the dispersive concatenation model that is studied with Kerr law of self-phase modulation. The method of undetermined coefficients is the adopted integration algorithm, enabling this retrieval possible. A full spectrum of optical solitons is recovered. The parameter constraints for the existence of the solitons, that naturally emerge during the course of their derivation, are also presented. The practical applications of this research include advancements in optical communication, nonlinear optics, and optical signal processing, as well as the potential for optimizing optical soliton-based technologies. In our current work, we have achieved the following novel findings: optical soliton recovery, integration algorithm innovation, and parameter constraints.

***Keywords*:** concatenation, solitons, dispersion, parameter constraints

MSC: 78A60

1. Introduction

The concatenation model that was first proposed by Ankiewicz et al. during 2014 is a conjunction of the familiar nonlinear Schrödinger's equation (NLSE), the Lakshmanan-Porsezian-Daniel (LPD) model and the Sasa-Satsuma equation (SSE) [1, 2]. This was extensively studied from several spheres of the photonics field. A few of the features that have been addressed are the retrieval of soliton solutions, locating conservation laws, Painleve analysis, quiescent optical solitons for the nonlinear chromatic dispersion (CD), bifurcation analysis, magneto-optics, numerical simulation using the Laplace-Adomian decomposition scheme and others. Subsequently this model was studied in the context of birefringent fibers and its soliton solutions were recovered there as well.

Incidentally, later during 2014 and during the subsequent year, another version of the concatenation model was

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made visible [3-5]. This is by conjoining the Schrödinger-Hirota equation (SHE), LPD model and the fifth-order NLSE which means that the model includes fifth-order dispersion. Hence this model was conveniently being referred to as the dispersive concatenation model [6-8]. The current paper addresses it and the task is to locate its soliton solution. The integration algorithm adopted in this paper is the method of undetermined coefficients. A full spectrum of optical soliton solutions is revealed together with their existence criteria that is yielded from the solvability criteria as detailed in this paper.

Stepping back in time, it must be noted that additional integration methodologies were implemented earlier to recover the soliton solutions to this dispersive concatenation model. Those schemes involved enhanced Kudryashov's method as well as the Weierstrass' type Riccati equation expansion scheme, projective Riccati equation approach, the generalized sine-Gordon equation method and several others [6-8]. The simplicity of the method of undetermined coefficients makes this algorithm stand above the rest. The details of the current scheme are exhibited in the rest of the paper after a quick ride through and revisiting to the model.

This paper relies on the method of undetermined coefficients for its integration procedure. This method of integration is a simpler approach than the Kudryashov's approach or the classic methods such as Inverse Scattering Transform or the Hirota's bilinear approach that requires a thorough knowledge and to be well-versed with Hirota Calculus. The other advantage of this approach is that the method of undetermined coefficients would prove that dark solitons can exist for power-law provided the power law condenses to Kerr law nonlinearity. This has been successfully demonstrated for the regular concatenation model [9]. Inverse Scattering Transform, Kudryashov's approach as well as Hirota's bilinear approach fail to prove this important conclusive fact that has also been experimentally proven.

The following novel findings have been achieved in our current work. The research successfully recovers a full spectrum of optical solitons within the dispersive concatenation model, which is studied in conjunction with the Kerr law of self-phase modulation. This achievement is a significant contribution to the field [10-18]. The utilization of the method of undetermined coefficients as the integration algorithm is a novel approach that enabled the retrieval of optical solitons. This methodology represents a new and effective way to address the problem. The study uncovers parameter constraints for the existence of solitons. These constraints naturally emerged during the soliton derivation process, and their identification is a novel aspect of the research. These novel findings advance our understanding of optical solitons in the context of the dispersive concatenation model, offering potential applications in optical communication, nonlinear optics, optical signal processing, and the optimization of soliton-based technologies [19-25].

1.1 *Governing model*

The dimensionless of the system to be studied herein is [6-8]:

$$
\begin{aligned}
& iq_t + aq_{xx} + b|q|^2 q - i\delta_1\left[\sigma_1 q_{xxx} + \sigma_2 |q|^2 q_x\right] \\
& +\delta_2\left[\sigma_3 q_{xxxx} + \sigma_4 |q|^2 q_{xx} + \sigma_5 |q|^4 q + \sigma_6 |q_x|^2 q + \sigma_7 q_x^2 q^* + \sigma_8 q_{xx}^* q^2\right] \\
& -i\delta_3\left[\sigma_9 q_{xxxxx} + \sigma_{10} |q|^2 q_{xxx} + \sigma_{11} |q|^4 q_x + \sigma_{12} q q_x q_{xx}^* + \sigma_{13} q^* q_x q_{xx} + \sigma_{14} q q_x^* q_{xx} + \sigma_{15} q_x^2 q_x^*\right] = 0.
\end{aligned} \tag{1}
$$

Equation (1) is the dispersive concatenation model that is with Kerr law of self-phase modulation (SPM). The dependent variable is $q(x, t)$ that represents the soliton profile. The independent variables x and t are from the spatial and temporal coordinates respectively. In equation (1), $i = \sqrt{-1}$ while a and b are the coefficients of CD and SPM respectively. The coefficient of δ_1 contains the remaining terms of the SHE that is recoverable from the standard NLSE via Lie transform. Next, the coefficients of δ_2 and δ_3 are from LPD and SHE respectively.

This expression given by (1) was first proposed as the concatenation model during 2014 by Ankeiciwz et al. [1, 2]. Later during 2015, this model was extended with a dispersive component added to it and is being referred to as the dispersive concatenation model [3-5]. The physical application of the proposed model is to study the dynamics of solitons that propagate through single-mode fibers across trans-oceanic and trans-continental distances [26-34].

In order to integrate the dispersive concatenation model (1), the following hypothesis is applied:

$$q(x,t) = P(x,t)e^{i(-\kappa x+\omega t+\Theta)}. \tag{2}$$

On the proposed hypothesis the term $P(x, t)$ represents the wave form, which is unique for each type of soliton, κ denotes the soliton frequency, ω represents the wave number, and Θ portrays a phase constant. Substituting these hypotheses into the system (1) leads to the real part identity,

$$\begin{aligned}
&\left[\delta_2\left(\sigma_6+\sigma_7\right)+\kappa\delta_3\left\{(2\sigma_{12}-2\left(\sigma_{13}+\sigma_{14}\right)-\sigma_{15}\right\}\right]P\left(\frac{\partial P}{\partial x}\right)^2 \\
&+\left(a-3\kappa\delta_1\sigma_1-6\kappa^2\delta_2\sigma_3+10\kappa^3\delta_3\sigma_9\right)\frac{\partial^2 P}{\partial x^2}+\left(\sigma_3\delta_2-5\kappa\sigma_9\delta_3\right)\frac{\partial^4 P}{\partial x^4} \\
&+\left[\delta_2\left(\sigma_4+\sigma_8\right)-\kappa\delta_3\left(3\sigma_{10}+\sigma_{12}+\sigma_{13}-\sigma_{14}\right)\right]P^2\frac{\partial^2 P}{\partial x^2}+\left(\sigma_5\delta_2-\kappa\sigma_{11}\delta_3\right)P^5 \\
&+\left[b-\kappa\sigma_2\delta_1-\delta_2\left(\sigma_4-\sigma_6+\sigma_7+\sigma_8\right)\kappa^2+\delta_3\left(\sigma_{10}+\sigma_{12}+\sigma_{13}-\sigma_{14}-\sigma_{15}\right)\kappa^3\right]P^3 \\
&+\left(\sigma_1\delta_1\kappa^3+\sigma_3\delta_2\kappa^4-\sigma_9\delta_3\kappa^5-\omega-a\kappa^2\right)P=0,
\end{aligned} \tag{3}$$

while the resulting imaginary counterpart is given by,

$$\begin{aligned}
&\frac{\partial P}{\partial t}+\left[-2a\kappa+3\sigma_1\delta_1\kappa^2+\left(4\sigma_3\delta_2-5\kappa\sigma_9\delta_3\right)\kappa^3\right]\frac{\partial P}{\partial x}-\left[\sigma_1\delta_1+2\kappa\left(2\sigma_3\delta_2-5\kappa\sigma_9\delta_3\right)\right]\frac{\partial^3 P}{\partial x^3}-\sigma_9\delta_3\frac{\partial^5 P}{\partial x^5} \\
&-\left[\sigma_2\delta_1+2\kappa\delta_2\left(\sigma_4+\sigma_7-\sigma_8\right)+\kappa^2\delta_3\left(-3\sigma_{10}+\sigma_{12}-3\sigma_{13}+\sigma_{14}+\sigma_{15}\right)\right]P^2\frac{\partial P}{\partial x} \\
&-\sigma_{10}\delta_3P^2\frac{\partial^3 P}{\partial x^3}-\sigma_{11}\delta_3P^4\frac{\partial P}{\partial x}-\sigma_{15}\delta_3\left(\frac{\partial P}{\partial x}\right)^3-\delta_3\left(\sigma_{12}+\sigma_{13}+\sigma_{14}\right)P\left(\frac{\partial P}{\partial x}\right)\left(\frac{\partial^2 P}{\partial x^2}\right)=0.
\end{aligned} \tag{4}$$

In fact, one can compute the soliton speed from (4), which in this case gives,

$$v = 2\kappa(a-\kappa\sigma_1\delta_1) \tag{5}$$

as long as the conditions

$$\sigma_9 = \sigma_{10} = \sigma_{11} = \sigma_{15} = 0, \tag{6}$$

$$\sigma_1\delta_1 + 4\kappa\sigma_3\delta_2 = 0, \tag{7}$$

$$\sigma_{12} + \sigma_{13} + \sigma_{14} = 0, \tag{8}$$

$$\sigma_2\delta_1 + 2\kappa\delta_2(\sigma_4 + \sigma_7 - \sigma_8) = 4\sigma_{13}\kappa^2\delta_3 \tag{9}$$

are secured. In view of the above conditions, the real part equation (3) reduces to

$$\sigma_3\delta_2\frac{\partial^4 P}{\partial x^4} + \left[a - 3\kappa\left(\sigma_1\delta_1 + 2\kappa\sigma_3\delta_2\right)\right]\frac{\partial^2 P}{\partial x^2} + \left[\delta_2\left(\sigma_6 + \sigma_7\right) + 4\kappa\sigma_{12}\delta_3\right]P\left(\frac{\partial P}{\partial x}\right)^2$$

$$+\left[\delta_2\left(\sigma_4 + \sigma_8\right) + 2\kappa\sigma_{14}\delta_3\right]P^2\frac{\partial^2 P}{\partial x^2} + \left(\sigma_1\delta_1\kappa^3 + \sigma_3\delta_2\kappa^4 - \omega - a\kappa^2\right)P$$

$$+\left[b - \sigma_2\kappa\delta_1 - \delta_2\left(\sigma_4 - \sigma_6 + \sigma_7 + \sigma_8\right)\kappa^2 - 2\sigma_{14}\delta_3\kappa^3\right]P^3 + \sigma_5\delta_2 P^5 = 0. \tag{10}$$

The last equation will be essential to recover four different types of optical soliton solutions from the dispersive concatenation model (1).

2. Soliton solutions

We now proceed to construct optical soliton solutions for the dispersive concatenation model. Four different solitons are explored throughout the next four subsections by studying the integrability of the real part equation (10) according to the corresponding waveform $P(x, t)$.

2.1 *Bright solitons*

The first type of waveform to be studied is the bright soliton, whose waveform has typically a structure of the form,

$$P(x,t) = A\operatorname{sech}^p\tau,\ \ \tau = B(x - vt), \tag{11}$$

where A is the soliton amplitude with their width being B. The unknown exponent p will fall out by the aid of balancing principle. Substituting (11) into the real part (10) transforms the last into

$$\left[\left(\sigma_1\delta_1\kappa^3 + \sigma_3\delta_2\kappa^4 - a\kappa^2 - \omega\right) + p^4\delta_2\sigma_3 B^4 + p^2 Z_1 B^2\right]\operatorname{sech}^p\tau$$

$$+\left[p^2\left(Z_2 + Z_3\right)B^2 + Z_4\right]A^2\operatorname{sech}^{3p}\tau + \delta_2\sigma_5 A^4\operatorname{sech}^{5p}\tau$$

$$-p(p+1)\left[Z_1 + 2\left\{2 + p\left(2 + p\right)\right\}\delta_2\sigma_3 B^2\right]B^2\operatorname{sech}^{p+2}\tau$$

$$+p(p+1)(p+2)(p+3)\delta_2\sigma_3 B^4\operatorname{sech}^{p+4}\tau$$

$$-p\left[pZ_2 + \left(1 + p\right)Z_3\right]A^2B^2\operatorname{sech}^{3p+2}\tau = 0. \tag{12}$$

On the above we have labeled the coefficients of the real part equation (10) as

$$Z_1 = a - 3\kappa\left(\delta_1\sigma_1 + 2\kappa\delta_2\sigma_3\right), \tag{13}$$

$$Z_2 = \delta_2\left(\sigma_6 + \sigma_7\right) + 4\kappa\delta_3\sigma_{12}, \tag{14}$$

$$Z_3 = \delta_2\left(\sigma_4 + \sigma_8\right) + 2\kappa\delta_3\sigma_{14}, \tag{15}$$

$$Z_4 = b - \kappa\delta_1\sigma_2 - \kappa^2\delta_2\left(\sigma_4 - \sigma_6 + \sigma_7 + \sigma_8\right) - 2\kappa^3\delta_3\sigma_{14} \tag{16}$$

for the sake of simplicity. The balancing principle allows to compare the exponents $3p$ and $p + 2$. It follows that $p = 1$. Upon substituting the resulting value of the parameter p on the identity (12), and setting to zero the coefficients of the functions $\operatorname{sech}^j\tau$ for $j = 1, 3, 5$ lead to the wave number

$$\omega = \kappa^3\delta_1\sigma_1 + \left(\kappa^4 + B^4\right)\delta_2\sigma_3 + Z_1B^2 - a\kappa^2, \tag{17}$$

and the soliton amplitude as

$$A = \sqrt{\frac{2\left(Z_1 + 10\delta_2\sigma_3B^2\right)B^2}{Z_4 + \left(Z_2 + Z_3\right)B^2}} \tag{18}$$

subject to

$$\left(Z_1 + 10\delta_2\sigma_3B^2\right)\left\{Z_4 + \left(Z_2 + Z_3\right)B^2\right\} > 0. \tag{19}$$

Both ω and A are expressed in terms of the soliton width B, which for bright soliton solution resulted to be

$$B = \left\{\frac{\left[\left(Z_2 + Z_3\right)\left(Z_2 + 2Z_3\right) - 40\delta_2^2\sigma_3\sigma_5\right]Z_1 - 2\delta_2\sigma_3\left(7Z_2 + 2Z_3\right)Z_4}{4\delta_3\sigma_3\left[\left(Z_2 - 4Z_3\right)\left(Z_2 + Z_3\right) + 100\delta_2^2\sigma_3\sigma_5\right]}\right.$$

$$\left.\pm\frac{\sqrt{\left[Z_1\left(Z_2 + Z_3\right) - 10\delta_2\sigma_3Z_4\right]^2\left[\left(Z_2 + 2Z_3\right)^2 - 96\delta_2^2\sigma_3\sigma_5\right]}}{4\delta_3\sigma_3\left[\left(Z_2 - 4Z_3\right)\left(Z_2 + Z_3\right) + 100\delta_2^2\sigma_3\sigma_5\right]}\right\}^{\frac{1}{2}}, \tag{20}$$

along with corresponding constraint. Therefore, the bright 1-soliton solution for the dispersive concatenation system (1) is given by

$$q(x, t) = A\operatorname{sech}[B(x - vt)]e^{i(-\kappa x + \omega t + \Theta)}, \tag{21}$$

where the amplitude obtained in (18) and the wave number (17) are expressed in terms of the the soliton width depicted on (20) along with corresponding solvability conditions. The speed of the soliton was obtained earlier on (5). It is imperative to point out that the bright soliton solution is valid if in addition the constraints (6)-(9) are satisfied.

Figures 1 and 2 provide a visual representation of numerical simulations for bright and dark optical soliton solutions. In the context of these plots, the parameter values utilized are as follows: $a = 1$, $b = 1$, $\kappa = 1$, $\delta_1 = 1$, $\delta_2 = 1$, $\delta_3 = 1$, $\sigma_1 = 1$, $\sigma_2 = 1$, $\sigma_3 = 1$, $\sigma_4 = 1$, $\sigma_5 = 1$, $\sigma_6 = 1$, $\sigma_7 = 1$, $\sigma_8 = 1$, $\sigma_{12} = 1$, and $\sigma_{14} = 1$.

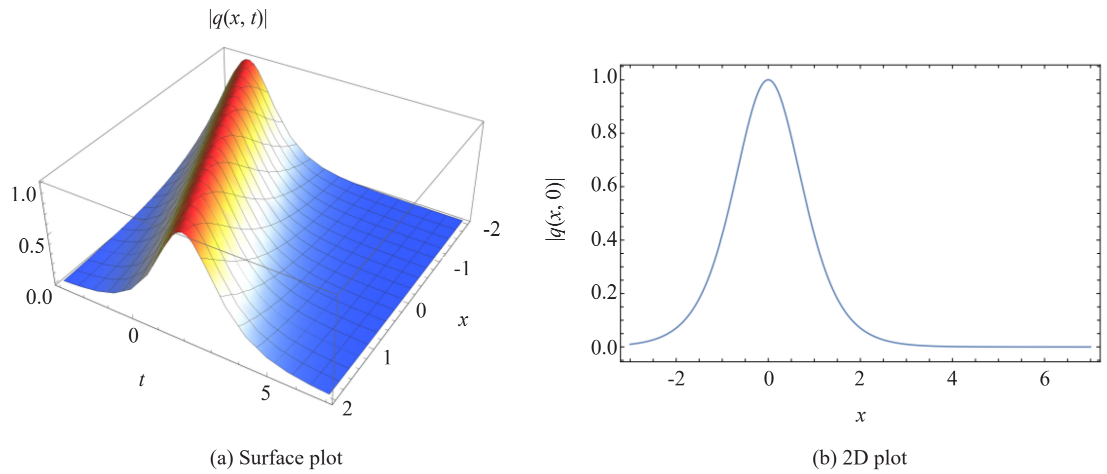


Figure 1. Graphical depiction of a bright soliton solution (21)

2.2 *Dark solitons*

The second case to be considered is the dark soliton, whose assumption for the wave form $P(x, t)$ is

$$P(x,t) = A\tanh^{p}\tau,\ \ \tau = B(x - vt). \tag{22}$$

Here, v represents the soliton speed, while A and B are free parameters. The parameter p will be calculated according to the balancing principle. Upon substituting (22) into (10), it would lead to the simplified expression

$$\begin{aligned}
&\left[\left(\sigma_1\delta_1\kappa^3+\sigma_3\delta_2\kappa^4-a\kappa^2-\omega\right)-2p^2\left\{Z_1-\left(5+3p^2\right)\delta_2\sigma_3B^2\right\}B^2\right]\tanh^{p}\tau\\
&+\left[Z_4-2p^2\left(Z_2+Z_3\right)B^2\right]A^2\tanh^{3p}\tau+\delta_2\sigma_5A^4\tanh^{5p}\tau\\
&+p(p+1)\left[Z_1-4\left\{2+p\left(2+p\right)\right\}\delta_2\sigma_3B^2\right]B^2\tanh^{p+2}\tau\\
&+p(p+1)(p+2)(p+3)\delta_2\sigma_3B^4\tanh^{p+4}\tau\\
&+p\left\{pZ_2+\left(1+p\right)Z_3\right\}A^2B^2\tanh^{3p+2}\tau\\
&-p(1-p)\left[Z_1-4\left\{2-p\left(2-p\right)\right\}\delta_2\sigma_3B^2\right]B^2\tanh^{p-2}\tau\\
&-p(1-p)(2-p)(3-p)\delta_2\sigma_3B^4\tanh^{p-4}\tau\\
&+p\left[pZ_2-\left(1-p\right)Z_3\right]A^2B^2\tanh^{3p-2}\tau=0.
\end{aligned}\tag{23}$$

Notice we have adopted the same notation (13)-(16) as we did for bright soliton. In this case the balance between nonlinearity and dispersion leads also to $p = 1$. Next, by substituting the resulting value of p into (23), and setting to zero the coefficients of the functions $\tanh^j\tau$ for $j = 1, 3, 5$ we obtain, in terms of the free parameter B, the wave number

$$\omega = \frac{\left[\kappa^3\delta_1\sigma_1 - a\kappa^2 - 2Z_1B^2 + \left(\kappa^4 + 16B^4\right)\delta_2\sigma_3\right]Z_4 - 2\left(Z_2 + Z_3\right)B^2 - 2Z_2\left(Z_1 - 20\delta_2\sigma_3B^2\right)B^4}{Z_4 - 2\left(Z_2 + Z_3\right)B^2}, \tag{24}$$

and the free parameter

$$A = \sqrt{-\frac{2\left(Z_1 - 20\delta_2\sigma_3B^2\right)B^2}{Z_4 - 2\left(Z_2 + Z_3\right)B^2}} \tag{25}$$

restricted to

$$\left(Z_1 - 20\delta_2\sigma_3B^2\right)\left\{Z_4 - 2\left(Z_2 + Z_3\right)B^2\right\} < 0. \tag{26}$$

Here, the resulting parameter B is given by

$$B = \left\{\frac{2\delta_2\sigma_3\left(7Z_2 + 2Z_3\right)Z_4 - \left[\left(Z_2 + Z_3\right)\left(Z_2 + 2Z_3\right) - 40\delta_2^2\sigma_3\sigma_5\right]Z_1}{8\delta_2\sigma_3\left[\left(Z_2 - 4Z_3\right)\left(Z_2 + Z_3\right) + 100\delta_2^2\sigma_3\sigma_5\right]}\right.$$
$$\left.\pm\frac{\sqrt{\left[Z_1\left(Z_2 + Z_3\right) - 10\delta_2\sigma_3Z_4\right]^2\left[\left(Z_2 + 2Z_3\right)^2 - 96\delta_2^2\sigma_3\sigma_5\right]}}{8\delta_2\sigma_3\left[\left(Z_2 - 4Z_3\right)\left(Z_2 + Z_3\right) + 100\delta_2^2\sigma_3\sigma_5\right]}\right\}^{\frac{1}{2}}, \tag{27}$$

where corresponding constraints are assumed to be valid. Thus, the single dark soliton solution for the concatenation model (1) is

$$q(x,t) = A\tanh[B(x - vt)]e^{i(-\kappa x + \omega t + \Theta)}, \tag{28}$$

where the free parameters A and B are given in (25) and (27) respectively, while the wave number, as with the free parameter A, is expressed in terms of B on (24). The speed was retrieved on (5). For dark soliton solution, as for bright soliton, the solvability conditions (6)-(9) along with (26) and resulting constraints from (27) must be satisfied in order for the soliton to evolve on time.

2.3 *Singular solitons (type-I)*

For the first type of singular soliton solution, the waveform assumption to be considered is

$$P(x,t) = A\operatorname{csch}^p\tau, \ \ \tau = B(x - vt), \tag{29}$$

where A and B are set as free parameters. This assumption, when inserted into (10) would give:

$$\left[\left(\sigma_1\delta_1\kappa^3 + \sigma_3\delta_2\kappa^4 - a\kappa^2 - \omega\right) + p^4\delta_2\sigma_3B^4 + p^2Z_1B^2\right]\operatorname{csch}^p\tau$$

$$+\left[p^2\left(Z_2+Z_3\right)B^2+Z_4\right]A^2\operatorname{csch}^{3p}[\tau]+\delta_2\sigma_5 A^4\operatorname{csch}^{5p}\tau$$

$$+p(p+1)\left[Z_1+2\left\{2+p\left(2+p\right)\right\}\delta_2\sigma_3 B^2\right]B^2\operatorname{csch}^{p+2}\tau$$

$$+p(p+1)(p+2)(p+3)\delta_2\sigma_3 B^4\operatorname{csch}^{p+4}\tau$$

$$+p\left[pZ_2+\left(1+p\right)Z_3\right]A^2B^2\operatorname{csch}^{3p+2}\tau=0, \tag{30}$$

where the notation (6)-(9) has been adopted for simplicity. Notice the balancing principle leads again to $p = 1$. Thus, upon substituting such value into (30), and setting to zero the coefficients of the resulting linearly independent functions $\operatorname{csch}^j\tau$ for $j = 1, 3, 5$ the wave number (17) is retrieved. However, the resulting the parameter A is

$$A=\sqrt{-\frac{2\left(Z_1+10\delta_2\sigma_3 B^2\right)B^2}{Z_4+\left(Z_2+Z_3\right)B^2}}, \tag{31}$$

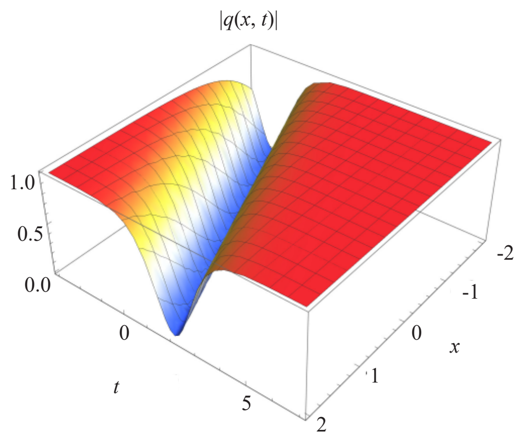
which is valid as long as

$$\left(Z_1+10\delta_2\sigma_3 B^2\right)\left\{Z_4+\left(Z_2+Z_3\right)B^2\right\}<0. \tag{32}$$

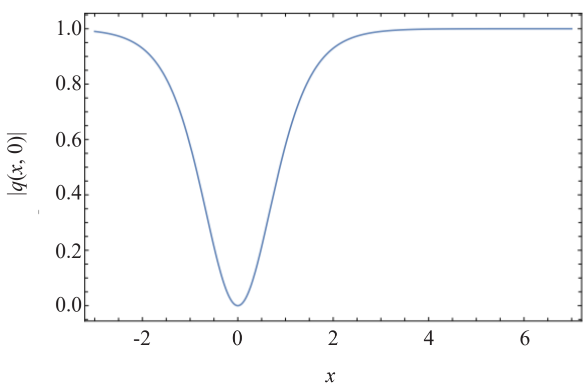
Here, the resulting parameter B is the same as the case for bright soliton (20) along with corresponding constraints. Therefore, the type-I singular soliton solution for the considered concatenation system resulted to be

$$q(x,t)=A\operatorname{csch}[B(x-vt)]e^{i(-\kappa x+\omega t+\Theta)}, \tag{33}$$

where the parameter A is given in (31), while the wave number and the parameter B are the same as in bright 1-soliton given on (17) and (20) respectively, along with their corresponding solvability constraints.



(a) Surface plot



(b) 2D plot

Figure 2. A depiction of a dark soliton solution (28)

2.4 *Singular solitons (type-II)*

In this case, the assumption for the waveform portion, $P(x, t)$ is

$$P(x,t) = A\coth^{p} \tau,\ \ \tau = B(x - vt). \tag{34}$$

Here too, A and B are free parameters. Upon substituting into (10), it would lead to the relation

$$\begin{aligned}
&\left[\left(\sigma_1\delta_1\kappa^3 + \sigma_3\delta_2\kappa^4 - a\kappa^2 - \omega\right) - 2p^2\left\{Z_1 - \left(5 + 3p^2\right)\delta_2\sigma_3 B^2\right\} B^2\right]\coth^{p} \tau \\
&+\left[Z_4 - 2p^2\left(Z_2 + Z_3\right)B^2\right]A^2 \coth^{3p} \tau + \delta_2\sigma_5 A^4 \coth^{5p} \tau \\
&+p(p+1)\left[Z_1 - 4\left\{2 + p\left(2 + p\right)\right\}\delta_2\sigma_3 B^2\right]B^2 \coth^{p+2} \tau \\
&+p(p+1)(p+2)(p+3)\delta_2\sigma_3 B^4 \coth^{p+4} \tau \\
&+p\left\{pZ_2 + \left(1 + p\right)Z_3\right\}A^2 B^2 \coth^{3p+2} \tau \\
&-p(1-p)\left[Z_1 - 4\left\{2 - p\left(2 - p\right)\right\}\delta_2\sigma_3 B^2\right]B^2 \coth^{p-2} \tau \\
&-p(1-p)(2-p)(3-p)\delta_2\sigma_3 B^4 \coth^{p-4} \tau \\
&+p\left[pZ_2 - \left(1 - p\right)Z_3\right]A^2 B^2 \coth^{3p-2} \tau = 0.
\end{aligned} \tag{35}$$

The delicate balance between nonlinearity and dispersion, as well as the coefficients of the stand alone elements $\coth^{p-4}\tau$ and $\coth^{p-2}\tau$ results into to the same value of p as in the previous three cases $p = 1$. As usual, substituting the resulting value of p into the equation (10) leads to exactly the same results as in the above case for dark soliton (24)-(27) with corresponding solvability conditions. Finally, the type-II of the soliton solutions for the system (1) is

$$q(x,t) = A\coth[B(x - vt)]e^{i(-\kappa x + \omega t + \Theta)}, \tag{36}$$

where the parameters along with the corresponding solvability constraints resulted to be the same as for dark solitons.

3. Conclusions

The current paper is a revistation of the dispersive concatenation model that retrieves a full spectrum of optical solitons using the undetermined coefficients approach. The parameter constraints naturally emerge from the integration scheme that are essential restrictions and must hold for the soliton solutions to exist. These preliminary results that are recovered serves us with a lot of promise. The same scheme will be implemented later to obtain the soliton solutions when the model will be studied in birefringent fibers and eventually with Dense Wavelength Division Multiplexing (DWDM) topology. The quasimonochromatic dynamics of the solitons will also be recovered by the aid of soliton perturbation theory. The stochastic aspect of the solitons with the inclusion of white noise will be later studied. These topics and much more are in the pipeline and the results of such research activities will be available after connecting them with the pre-existing ones [11, 12].

Conflict of interest

The authors claim there is no conflict of interest.

References

[1] Ankiewicz A, Nail A. Higher-order integrable evolution equation and its soliton solutions. *Physics Letters*. 2014; 378(4): 358-361. Available from: doi: 10.1016/j.physleta.2013.11.031.

[2] Ankiewicz A, Wang Y, Wabnitz S, Nail A. Extended nonlinear Schrödinger equation with higher-order odd and even terms and its rogue wave solutions. *Physical Review E*. 2014; 89(1). Available from: doi: 10.1103/physreve.89.012907.

[3] Amdad C, Kedziora DJ, Ankiewicz A, Nail A. Soliton solutions of an integrable nonlinear Schrödinger equation with quintic terms. *Physical Review E*. 2014; 90(3). Available from: doi: 10.1103/physreve.90.032922.

[4] Amdad C, Kedziora DJ, Ankiewicz A, Nail A. Breather solutions of the integrable quintic nonlinear Schrödinger equation and their interactions. *Physical Review E*. 2015; 91(2). Available from: doi: 10.1103/physreve.91.022919.

[5] Amdad C, Kedziora DJ, Ankiewicz A, Nail A. Breather-to-soliton conversions described by the quintic equation of the nonlinear Schrödinger hierarchy. *Physical Review E*. 2015; 91(3). Available from: doi: 10.1103/physreve.91.032928.

[6] Arnous AH, Mirzazadeh M, Biswas A, Yildirim Y, Triki H, Asiri A. A wide spectrum of optical solitons for the dispersive concatenation model. *Journal of Optics*. 2023. Available from: doi: 10.1007/s12596-023-01383-8.

[7] Triki H, Azzouzi F, Biswas A, Moshokoa SP, Belic M. Bright optical solitons with Kerr law nonlinearity and fifth order dispersion. *Optik*. 2017; 128: 172-177. Available from: doi: 10.1016/j.ijleo.2016.10.026.

[8] Zayed EME, Gepreel KA, El-Horbaty M, Biswas A, Yildirim Y, Triki H, et al. Optical solitons for the Dispersive Concatenation Model. *Contemporary Mathematics.* 2023; 4(3): 593. Available from: doi: 10.37256/cm.4320233321.

[9] Biswas A, Vega-Guzmán J, Yildirim Y, Moshokoa SP, Aphane M, Alghamdi AS. Optical solitons for the concatenation model with power-law nonlinearity: undetermined coefficients. *Ukrainian Journal of Physical Optics*. 2023; 24(3): 185-192. Available from: doi: 10.3116/16091833/24/3/185/2023.

[10] Zhou Q, Xu M, Sun Y, Zhong Y, Mirzazadeh M. Generation and transformation of dark solitons, anti-dark solitons and dark double-hump solitons. *Nonlinear Dynamics*. 2022; 110(2): 1747-1752. Available from: doi: 10.1007/s11071-022-07673-3.

[11] Zhou Q. Influence of parameters of optical fibers on optical soliton interactions. *Chinese Physics Letters*. 2022; 39(1): 010501. Available from: doi: 10.1088/0256-307x/39/1/010501.

[12] Wang S. Novel soliton solutions of CNLSEs with Hirota bilinear method. *Journal of Optics*. 2023; 52(3): 1602-1607. Available from: doi: 10.1007/s12596-022-01065-x.

[13] Han T, Zhao L, Li C, Zhao L. Bifurcations, stationary optical solitons and exact solutions for complex Ginzburg-Landau equation with nonlinear chromatic dispersion in non-Kerr law media. *Journal of Optics*. 2022; 52(2): 831-844. Available from: doi: 10.1007/s12596-022-01041-5.

[14] Zhou Q, Xu M, Sun Y, Zhong Y, Mirzazadeh M. Generation and transformation of dark solitons, anti-dark solitons and dark double-hump solitons. *Nonlinear Dynamics*. 2022; 110(2): 1747-1752. Available from: doi: 10.1007/s11071-022-07673-3.

[15] Chen W, Shen M, Kong Q, Wang Q. The interaction of dark solitons with competing nonlocal cubic nonlinearities. *Journal of Optics*. 2015. Available from: doi: 10.1007/s12596-015-0255-8.

[16] Xu SL, Petrović N, Belić MR. Two-dimensional dark solitons in diffusive nonlocal nonlinear media. *Journal of Optics*. 2015; 44(2): 172-177. Available from: doi: 10.1007/s12596-015-0243-z.

[17] Dowluru RK, Bhima PR. Influences of third-order dispersion on linear birefringent optical soliton transmission systems. *Journal of Optics*. 2011; 40(3): 132-142. Available from: doi: 10.1007/s12596-011-0045-x.

[18] Singh M, Sharma AK, Kaler RS. Investigations on optical timing jitter in dispersion managed higher order soliton system. *Journal of Optics*. 2011. Available from: doi: 10.1007/s12596-010-0021-x.

[19] Janyani V. Formation and propagation-dynamics of primary and secondary soliton-like pulses in bulk nonlinear

media. *Journal of Optics*. 2008; 37(1): 1-8. Available from: doi: 10.1007/bf03354831.
[20] Hasegawa A. Application of optical solitons for information transfer in fibers - A tutorial review. *Journal of Optics*. 2004; 33(3): 145-156. Available from: doi: 10.1007/bf03354760.
[21] Mahalingam A, Uthayakumar A, Anandhi P. Dispersion and nonlinearity managed multisoliton propagation in an erbium doped inhomogeneous fiber with gain/loss. *Journal of Optics*. 2013; 42(3): 182-188. Available from: doi: 10.1007/s12596-012-0105-x.
[22] Jawad AJM, Abu-AlShaeer MJ. Highly dispersive optical solitons with cubic law and cubic-quinticseptic law nonlinearities by two methods. *Al-Rafidain Journal of Engineering Sciences*. 2023; 1(1): 1-8.
[23] Raza N, Sheela Rani B, Chahlaoui Y, Ali Shah N. A variety of new rogue wave patterns for three coupled nonlinear Maccari's models in complex form. *Nonlinear Dynamics*. 2023; 111(19): 18419-18437. Available from: doi: 10.1007/s11071-023-08839-3.
[24] Rafiq M, Raza N, Jhangeer A. Nonlinear dynamics of the generalized unstable nonlinear Schrödinger equation: a graphical perspective. *Optical and Quantum Electronics*. 2023; 55(7). Available from: doi: 10.1007/s11082-023-04904-8.
[25] Rafiq M, Jannat N, Rafiq M. Sensitivity analysis and analytical study of the three-component coupled NLS-type equations in fiber optics. *Optical and Quantum Electronics*. 2023; 55(7). Available from: doi: 10.1007/s11082-023-04908-4.
[26] Rafiq MH, Raza N, Jhangeer A. Dynamic study of bifurcation, chaotic behavior and multi-soliton profiles for the system of shallow water wave equations with their stability. *Chaos, Solitons & Fractals*. 2023; 171: 113436. Available from: doi: 10.1016/j.chaos.2023.113436.
[27] Rafiq MH, Jhangeer A, Raza N. The analysis of solitonic, supernonlinear, periodic, quasiperiodic, bifurcation and chaotic patterns of perturbed Gerdjikov-Ivanov model with full nonlinearity. *Communications in Nonlinear Science and Numerical Simulation*. 2023; 116: 106818. Available from: doi: 10.1016/j.cnsns.2022.106818.
[28] Alquran M, Jaradat I. Identifying combination of dark-bright binary-soliton and binary-periodic waves for a new two-mode model derived from the (2 + 1)-dimensional nizhnik-novikov-veselov equation. *Mathematics*. 2023; 11(4): 861. Available from: doi: 10.3390/math11040861.
[29] Ali M, Alquran M, BaniKhalid A. Symmetric and asymmetric binary-solitons to the generalized two-mode KdV equation: Novel findings for arbitrary nonlinearity and dispersion parameters. *Results in Physics*. 2023; 45: 106250. Available from: doi: 10.1016/j.rinp.2023.106250.
[30] Alquran M, Ali M, Gharaibeh F, Qureshi S. Novel investigations of dual-wave solutions to the Kadomtsev-Petviashvili model involving second-order temporal and spatial-temporal dispersion terms. *Partial Differential Equations in Applied Mathematics*. 2023; 8: 100543. Available from: doi: 10.1016/j.padiff.2023.100543.
[31] Alquran M. Classification of single-wave and bi-wave motion through fourth-order equations generated from the Ito model. *Physica Scripta*. 2023; 98(8): 085207. Available from: doi: 10.1088/1402-4896/ace1af.
[32] Vijayalekshmi S, Mahalingam A, Uthayakumar A, Mani Rajan MS. Oscillating soliton propagation in SPNLS equation with symmetric potentials. *Optik*. 2020; 221: 165143. Available from: doi: 10.1016/j.ijleo.2020.165143.
[33] Vijayalekshmi S, Mahalingam A, Uthayakumar A, Mani Rajan MS. Multi-soliton propagation in generalized inhomogeneous NLS equation with symmetric potentials. *Optik*. 2019; 181: 948-955. Available from: doi: 10.1016/j.ijleo.2018.12.186.
[34] Rajan MSM. Transition from bird to butterfly shaped nonautonomous soliton and soliton switching in erbium doped resonant fiber. *Physica Scripta*. 2020; 95(10): 105203. Available from: doi: 10.1088/1402-4896/abb2df.