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Research Article

Application of Network Reconstruction Algorithm to Compute Maximum Flow for Water Supply Network: A Case Study of the City of Bulawayo, Zimbabwe

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Abstract: Determining the maximum flow value in Water Supply Networks (WSN) is a common problem that is being faced by many cities during and after designing of WSN. In this article, the network reconstruction (NR) algorithm is applied to compute the maximum flow value for the city's WSN based on the actual data. The city of Bulawayo has been selected for the following reasons, availability of research data, the city is facing severe water shedding and several studies have focused only on other issues such as alternative water sources, leakage detection and demand forecasting. The goal of this study is to determine the maximum flow value from the sources to the reservoirs. The results have revealed that the computed maximum flow value is within the estimated range of 110,000 cubic meters to 190,000 cubic meters. Dam sensitivity analysis were considered to determine the sources to give maintenance priority before the rain season. Several recommendations were suggested to improve the water supply situation.

Keywords: water supply network, maximum flow, case study, network reconstruction algorithm

MSC: 90B10, 90B90, 65C85, 68W99

1. Introduction

City Bulawayo is the second largest city in Zimbabwe with a population of approximately 1.2 million. It was founded in the 1840s. City of Bulawayo has been struggling to secure sustainable and long term water resources for over one hundred years, due to factors such as climate effects, geographical location, and economic decline in Baker [1]. Annual rainfall of 590 mm has been recorded for the city of Bulawayoin in Mkandla et al. [2]. Baker [1] pointed out that the city of Bulawayo recorded an increase in water demand over the past three decades. Currently there are six dams, most of them built between 1958 and 1992 that are supplying water to the city, thus Upper Ncema, Lower Ncema, Mtshabezi, Umzingwane, Inyankuni, and Insiza dam as shown in Figure 1. On average the supplying dams are about 50 Km from the city of Bulawayo. To manage the water supply problem, the city is facing severe water shedding thus 48-hour and

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72-hour shedding in any given week. Water rationing has resulted in the outbreak of diseases such as cholera, diarrhea and others. In this research, the actual maximum flow that the city receives is computed so that recommendations can made to the city to improve the situation.

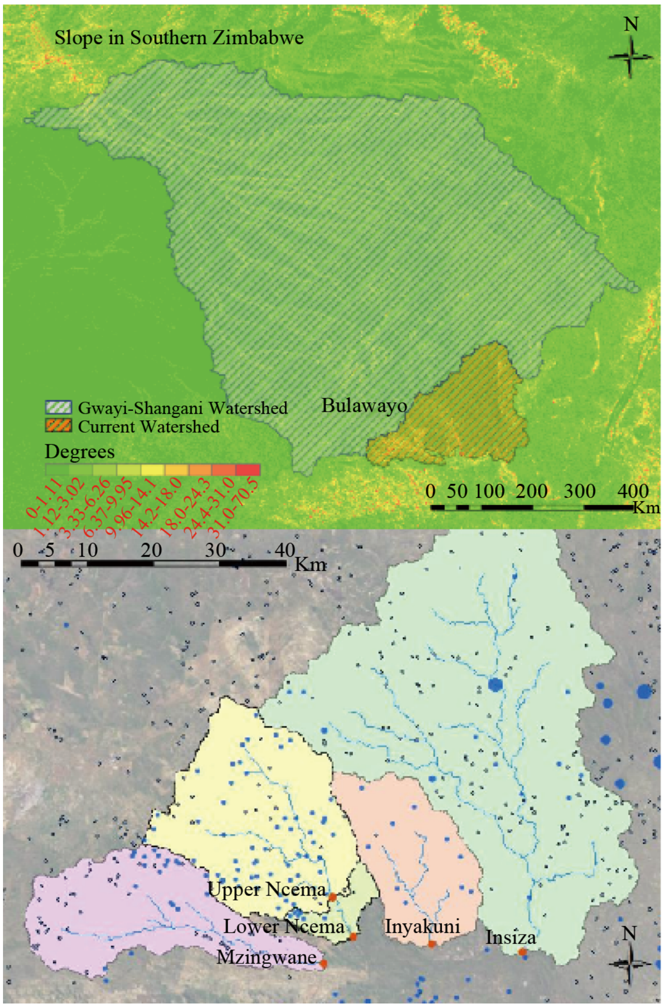

Figure 1. Supply dams in the city of Bulawayo

1.1 *Motivation*

Severe water rationing that causes residents, hospitals, and schools to go without water for more than 24 hours has motivated this research to determine the actual maximum flow value that passes through the city of Bulawayo's WSN. Disease outbreak due to water shortages has caused many people to lose their lives has motivated this research. Application of the newly developed maximum flow algorithm to water supply problems to save humanity has also motivated this research.

2. Literature review

Water supply networks (WSNs) consist of nodes such as dams, tanks, boreholes, reservoirs and pipe junctions interlinked using pipes. The objective of the WSN is to transport water from the source (Dams, tanks, boreholes) to destination (Reservoirs) at the required water pressure and flow rate. In today's world, maximum flow problem (MFP) has many applications such as maximizing the flow of water from dams, boreholes, to treatment sites, etc then to the residents, used to solve regular sewer bursting problems, maximizing flow of oil through a system of pipes, and optimizing traffic between two or more cities. Creaco et al. [3] applied the minimum transport driven algorithm for partitioning of water distribution networks (WDNs) into district metered areas (DMAs). The algorithm performed better than other algorithms in the literature. Kyi et al. [4] applied Ford-fulkerson algorithm to determine the maximum flow in water supply network of Pyigytagon Township, Myanmar and the goal of application was to determine the maximum flow from the source to sink. Rodríguez et al. [5] developed a statistical model to determine the maximum flow for the irrigation distribution network. The results revealed that the model is sometimes underestimated due to demand peak times. Munapo et al. [6] determined the maximum flow using the network reconstruction method, the algorithm identified the outmost path and remove it in such a way that the original network maximum flow value is not affected. The algorithm is simple and computes exact solutions. Dumora et al. [7] presented a data- oriented algorithm for estimation of flow rates in water distribution network from sensors. The algorithm was implemented on Bordeaux Metropolis water distribution network. The sensors helped to detect underestimation and overestimation of the algorithm. Liu et al. [8] considered water distribution networks with accidents such as pipe burst. The optimization cost obtained when considering accident situation were higher that when considering a normal situation. Loubser et al. [9] presented a linear regression model for evaluating water distribution potential maximum peak flow rate that can be supplied given that the total pipeline length is known. Kanneganti et al. [10] used machine learning to estimate sewage flow rate in Jefferson County, Kentucky. The model managed to estimate the sewage flow rate with 91.7% accuracy. Chagwiza et al. [11] optimized water distribution cost using the modified soccer league competition (MSCL) algorithm with stubborn payers. The results revealed that the proposed method requires less computational time compared to the soccer league competition (SLC) algorithm. Karadirek [12] analyzed customer water meter accuracy under various water pressures and flow rates that might exist in the water distribution networks. The results revealed that water pressure has no effect on higher flow rates and causes customer water meter inaccuracies in lower flow rates. Jain and Khare [13] optimized WDNs using the ParameterLess Rao Algorithm through pressure driven analysis. The aim was to minimize network cost and the algorithm reduced cost by 1.7% for the New York Tunnel network. Akram et al. [14] solved the maximum flow problem with linguistic capacities and flows using the Pythagorean fuzzy maximum flow algorithm. The algorithm was evaluated on two case studies thus, Pyigyitagon Township network and 14-Bus electricity network. Tawanda et al. [15] presented an exact intelligent node labelling maximum flow algorithm that determines maximum flow in any given WSN in two steps. Tawanda et al. [16] proposed a matrix based extreme min-cut max-flow algorithm to calculate maximum flow value without making used of augmenting paths. Zhou et al. [17] developed a deep learning framework to help identify burst locations in water distribution networks, thus reducing water pollution and loss. Ngancha et al. [18] modelled and simulated timer switching control for local city water pumping system, this research helps to save cost. Burgan [19] predicted daily stream flows using Artificial Neural Networks (ANN) and multiple linear regression methods in Kocasu river, Turkey. Water demand for the city of Iraq was predicted in Zabaidi et al. [20] using different models. These models were based on treated water. This kind of prediction is very complex given population growth and global warming.

3. Methodology

Network reconstruction algorithm recently developed in Munapo et al. [21] is applied to compute the maximum flow value. The algorithm is simple in that it computes the maximum flow rate by reducing the original network at each iteration without changing the maximum flow rate value that can be computed in the original water distribution network. The algorithm reduces the original network by identifying the outmost path in the network and removes some links in such a way that the original maximum flow rate remains the same. Another advantage of the algorithm is that convergence is guaranteed in finite number of steps. In general the algorithm is simple to understand, reduces complexity as iterations increases, it does not make use of augmenting paths as it computes exact solutions. The steps of Network reconstruction algorithm are presented in Algorithm 1 below.

Algorithm 1 Network Reconstruction Algorithm.

Input Directed flow network.

Output Maximum flow value.

1. Reduce the network measurements to their simplest values.

2. Identify an outmost route in the network. Remove this identified route in such a way that the reduced new network has the same maximal flow as the original network.

3. Using the reduced new network, return to Step 2 until the network is small and simple and that the maximal flow is easy and obvious to determine.

The flow chart describing the steps of Algorithm 1 is shown above. The flow chart Figure 2 also shows that the proposed algorithm is very simple to implement.

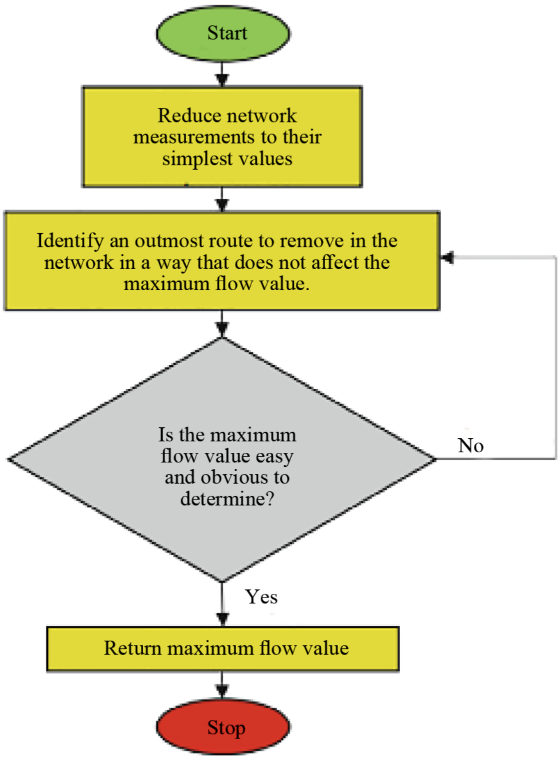

Figure 2. Flow chart for the network reconstruction algorithm

Given pipe sizes and water velocity in respective pipes in the network the following equations are used to calculate the flow rate in each pipe.

$$Q = A \times V \tag{1}$$

where Q is the flow rate per unit time, A is the pipe area, and V is the water velocity. Pipe area is given by the following equation.

$$A = \frac{\pi d^2}{4} \tag{2}$$

where d is the pipe diameter.

4. Case study
4.1 *City of Bulawayo water supply network*

The WSN for the city of Bulawayo, southern of Zimbabwe is a directed WSN that has five source nodes and three demand reservoirs. The WSN has 17 pipes with diameter which ranges between (0.15 to 0.84 meters), velocity ranges between (1308 to 17,677 meters per hour) and flow rate ranges between (166 to 5,000 cubic meters per hour). A dummy source and sink nodes were introduced in the network to integrate the sources and sink nodes respectively. Table 1 below gives the names of the nodes in the city of Bulawayo WSN.

Table 1. Names of the nodes in the WSN

Node	Node name
A	dummy source node
B	zinwa tanks
C	insiza dam
D	inyankuni dam
E	umzingwane dam
F	umzingwane dam
G	flowserves reservoir
H	pipe junction 1
I	pipe junction 2
J	ncema treatment works
K	flowserves line boaster pump
L	sulzer line boaster pump
M	cowdray park boaster pump
N	umzingwane dam
O	criterion raw reservoir
P	mzinyathi irrigation
Q	pipe junction 3
R	criterion treament works
S	tuli reservoir
T	criterion clear reservoir
U	esigodini tanks
V	magwegwe reservoir
W	dummy sink node

Pipe diameter and velocity data were collected from the WSN and then used to calculate the flow rate in each pipe in the network using the following equation (Equation (1)). Table 2 below shows the calculated flow rates for each pipe in the city of Bulawayo WSN.

Table 2. Calculated pipe flow rates

Pipe No.	Pipe	Diameter (m)	Velocity (m/h)	Flow rate (m^3/h)
1	(A, B)	0.6	1,326	375
2	(A, C)	0.9	3,799	2,417.62
3	(A, D)	0.6	3,412	965
4	(A, E)	0.84	2,630	1,458.3
5	(A, F)	0.6	6,452	1,875
6	(B, M)	0.6	1,326	375
7	(C, G)	0.9	3,799	2,417.62
8	(D, H)	0.6	3,412	965
9	(E, I)	0.84	2,630	1,458.3
10	(F, J)	0.6	6,629	1,875
11	(G, K)	0.84	7,147	3,962.08
12	(H, L)	0.825	7,167	3,833
13	(I, P)	0.15	11,783	208.3
14	(J, Q)	0.75	4,242	1,875
15	(M, V)	0.6	1,308	370
16	(K, O)	0.84	7,147	3,962.08
17	(L, O)	0.75	8,673	3,833
18	(P, W)	0.15	11,783	208.3
19	(Q, S)	0.75	3,865	1,708
20	(Q, U)	0.15	9,390	166
21	(V, W)	0.6	1,308	370
22	(S, W)	0.75	3,865	1,708
23	(U, W)	0.15	9,390	166
24	(O, R)	0.9	7,856	5,000
25	(R, T)	0.6	17,677	5,000
26	(T, W)	0.6	17,677	5,000

Bulawayo water supply network is given in Figure 3. Node A and node W are the source and destination nodes, respectively.

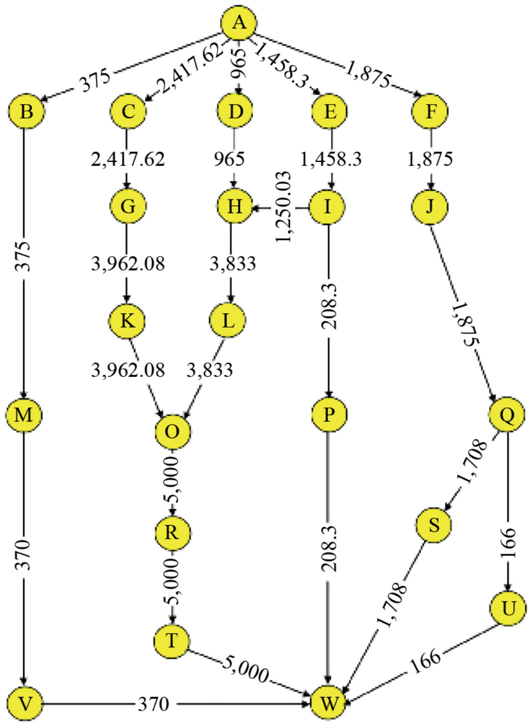

Figure 3. Water supply network for city of Bulawayo

4.2 *Solution using network reconstruction algorithm*

Iteration 1 Removing path $A \to B \to M \to V \to W$ with the flow rate bottleneck capacity of 370 m^3/h. Flow rate of 370 m^3/h is added to the path $A \to C \to G \to K \to O \to R \to T \to W$ without changing the maximum flow rate in the WSN. Figure 4 shows the reduced WSN.

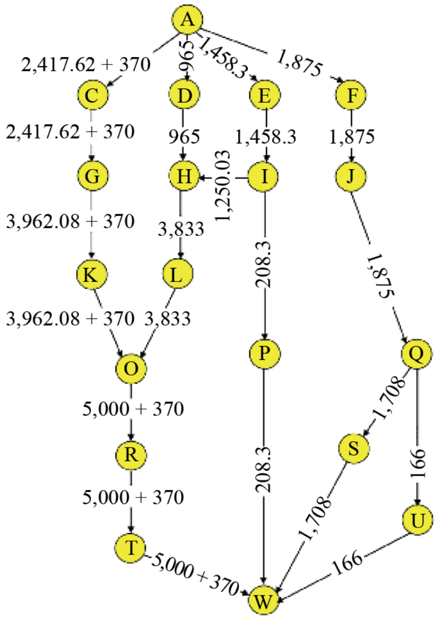

Figure 4. Reduced WSN after iteration 1

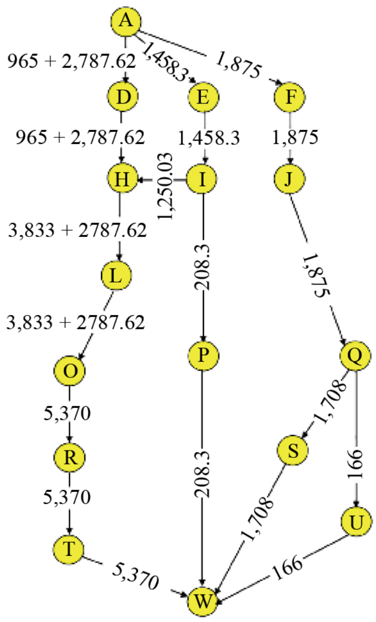

Figure 5. Reduced WSN after iteration 2

Iteration 2 Removing path $A \to C \to G \to K \to O$ with the flow rate bottleneck capacity of 2,787.62 m^3/h. Adding this bottleneck capacity to the path $A \to D \to H \to L \to O$ without changing the maximum flow rate in the WSN. Figure 5 shows the reduced WSN.

Iteration 3 Removing path $Q \to U \to W$ with the flow rate bottleneck capacity of 166 m^3/h. Adding this bottleneck capacity to the path $Q \to S \to W$ without changing the maximum flow rate in the WSN. Figure 6 shows the reduced WSN.

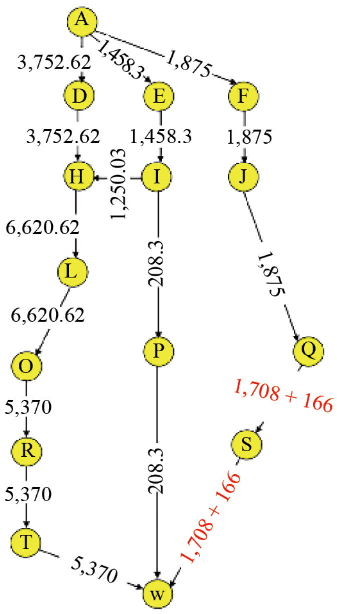

Figure 6. Reduced WSN after iteration 3

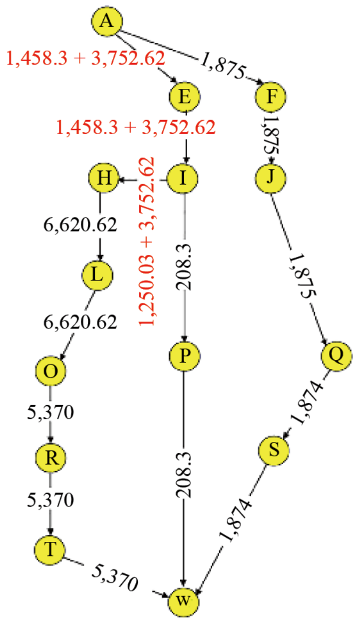

Figure 7. Reduced WSN after iteration 4

Iteration 4 Removing path $A \to D \to H$ with the flow rate bottleneck capacity of 3,752.62 m^3/h. Adding this bottleneck capacity to the path $A \to E \to I \to H$ without changing the maximum flow rate in the WSN. Figure 7 shows the reduced WSN.

Iteration 5 Removing path $A \to F \to J \to Q \to S \to W$ with the flow rate bottleneck capacity of 1,874 m^3/h. Adding this bottleneck capacity to the path $A \to E \to I \to P \to W$ without changing the maximum flow rate in the WSN. Figure 8 shows the reduced WSN.

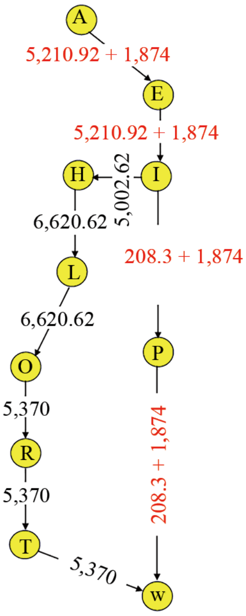

Figure 8. Reduced WSN after iteration 5

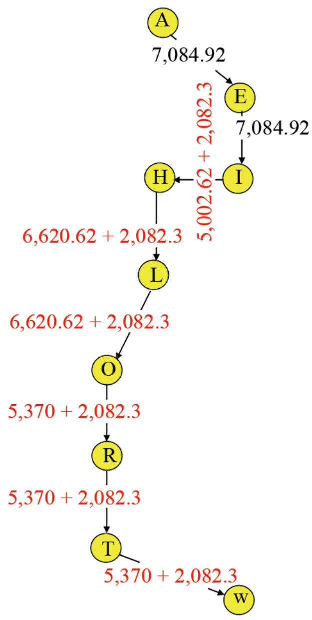

Figure 9. Reduced WSN after iteration 6

Iteration 6 Removing path $I \to P \to W$ with the flow rate bottleneck capacity of 2,082.3 m^3/h. Adding this bottleneck capacity to the path $I \to H \to L \to O \to R \to T \to W$ without changing the maximum flow rate in the WSN. Figure 9 shows the reduced WSN.

Water supply network has been reduced in such a way that no further reductions are possible and as a result the maximum flow rate value is now obviously the bottleneck of the linearly reduced WSN given in Figure 10.

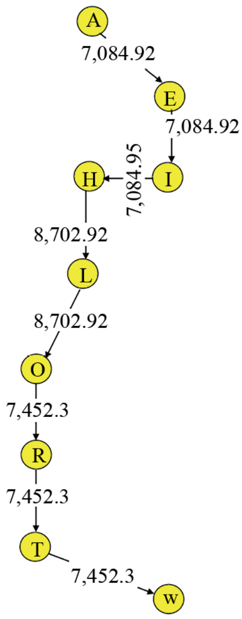

Figure 10. Simplified WSN

The Maximum flow value is obviously 7,084.92 m^3/h which gives 170,038.08 m^3/day and this value is in the estimated range of (110,000 to 190,000 m^3/day) in Mkandla et al. [2].

4.3 *Dam sensitivity analysis*

We also considered five scenarios, thus a supplying source is assumed to be not supplying due to burst pipes that move water from the dams to the network or pump failure. The NR algorithm was applied to determine the maximum flow in 5 scenarios, thus (1) Node B (ZINWA Tanks), (2) Node C (Insiza dam), (3) Node D (Inyankuni dam), (4) Node E (Umzingwane dam) and (5) Node F (Lower Ncema dam). Maximum flow values and flow reduction values due to these scenarios are shown in Table 3 below.

Table 3. Maximum flow given different scenario

Scenario	Dam	f^* (m^3/h)	f^* (m^3/day)	Loss (m^3/h)	Loss %
1	Zinwa tanks (B)	6,714.92	161,158.08	370	5.22%
2	Insiza dam (C)	4,667.3	112,015.2	2,417.62	34.12%
3	Inyankuni dam (D)	6,119.92	146,878.08	965	13.62%
4	Umzingwane dam (E)	5,626.62	135,038.88	1,458.3	20.58%
5	Lower Ncema dam (F)	5,210.92	1,250,620.8	1,874	26.45%

5. Recommendations

Pipes and pumps should be replaced or maintained giving priority in the following order of the dams, Insiza dam, Lower Ncema dam, Umzingwane dam, Inyankuni dam, and lastly ZINWA tanks to reduce 48-72 hour water shedding. This priority list is supported by the results in Table 3. The city of Bulawayo should have a pipe replacement schedule to reduce water distribution losses. Mutsvangwa [21] pointed out that 12,000 pipe burst are recorded in the city of Bulawayo per month. Water distribution losses is also caused by leakages due to old pipes, technology to detect leakages should be put in place so as to reduce unaccounted water. Population growth will also see water demand rising, as a result new water sources should be considered and connected to the current water distribution network. Currently Gwayi-shangani dam that was proposed 100 years ago is under construction. The city of Bulawayo is also recommended to apply water demand management thus, formulating water reuse strategies in Taigbenu and Ncube [22], water resource management, and technology supported billing systems, among others. Due to high temperatures in the region that causes evaporation loss. Bulawayo is also recommended to consider alternative water sources such as underground water pumping.

6. Conclusion

In this paper, network reconstruction algorithm was applied to determine the maximum flow values in water supply network for the city of Bulawayo. The algorithm computed the maximum flow value in WSN by reducing the size of the network at each iteration. We have also considered different supply scenarios and computed the respective maximum flow values, this helps the authorities to know supply sources to give priority in terms of maintenance and dredging. Maximum flow values in WSN helps the authorities to know if there is need to adjust network design parameters such pump size and pipe size so as to meet water demand requirement at any given time. Several recommendations such as dams and pipes to give priority, to consider technology to detect leakages in Zhou et al. [17] and to consider new water sources due to population growth were given to improve the water supply situation in the city of Bulawayo.

Data availability statement

All relevant data are contained in this Funding: The APC was funded by the North West University, South Africa.

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Conflict of interest

Authors declare there is no conflict of interest at any point with reference to research findings.

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