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Research Article

Bright Optical Solitons for the Concatenation Model with Power-Law Nonlinearity: Laplace-Adomian Decomposition

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Abstract: This paper is a numerical study of the bright optical solitons that emerge from the concatenation model which is considered with power-law nonlinearity. The Laplace-Adomian decomposition scheme is the integration methodology adopted in the paper. The surface plots are exhibited for a range of values of the power-law parameter.

Keywords**:** solitons, concatenation model, power-law, Adomian polynomials

MSC: 78A60

1. Introduction

The concatenation model is one of the most innovative and very well-thought models that has been ever reported in the field of optical solitons. This is a combination of three widely recognized equations, namely the nonlinear Schrödinger's equation (NLSE), the Lakshmanan-Porsezian-Daniel (LPD) model, and the Sasa-Satsuma equation (SSE). The NLSE portion is considered with power-law of nonlinearity. Such a model with power-law has been well studied and several forms of preliminary results have been reported [1]. These include retrieval of optical solitons by the method of undetermined coefficients, enhanced Kudryashov's approach and the improved extended tanh-function algorithm and finally locating the conservation laws. Now, it is time to examine this model from a numerical standpoint. The current paper explores the model with the usage of the Laplace-Adomian decomposition scheme. Several values of the power-law parameter yielded the surface and contour plots of the bright solitons and they are presented in the rest of the paper after conducting the Laplace-Adomian decomposition scheme. The subsequent sections of this paper provide an in-depth examination of the methodology, following a brief introduction to the concatenation model and its associated concepts.

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2. Model of concatenation with power-law nonlinearity

The concatenation model with power-law nonlinearity has recently been studied in [2-4] and is given in its dimensional form by:

$$iq_t + aq_{xx} + b|q|^{2n} q + c_1\left[\sigma_1 q_{xxxx} + \sigma_2 \left(q_x\right)^2 q^* + \sigma_3 \left|q_x\right|^2 q + \sigma_4 |q|^{2n} q_{xx} + \sigma_5 q^2 q_{xx}^* + \sigma_6 |q|^{2n+2} q\right]$$

$$+ic_2\left[\sigma_7 q_{xxx} + \sigma_8 |q|^{2n} q_x + \sigma_9 q^2 q_x^*\right] = 0,\ i = \sqrt{-1}. \tag{1}$$

In the present research, a complex-valued function $q(x, t)$ is employed to describe the wave profile. t and x correspond, respectively, to temporal and spatial coordinates. The parameter n is derived from the power law of the self-phase modulation (SPM). Specifically, $0 < n < 2$ is necessary to prevent wave collapse [5]. In particular, b represents the coefficient of SPM and a represents the coefficient of chromatic dispersion (CD). Subsequently, the coefficient of c_1 is derived from the Lakshmanan-Porsezian-Daniel (LPD) model, while c_2 is derived from the Sasa-Satsuma equation (SSE).

The main motivation for this research arises from exhaustive theoretical and experimental inquiries extending over five decades, which have demonstrated the substantial influence of solitons in the practical realm of nonlinear wave mechanics. This study presents an algorithm for obtaining optical solitons that are capable of existing within the optical system as described by the model (1). This paper makes a valuable contribution to the science of nonlinear optics by investigating the characteristics and dynamics of bright solitons inside a model that incorporates various nonlinear processes. This find-ing improves our comprehension of intricate optical systems. Previous studies have examined the nonlinear Schrödinger equation with various potentials, which can be found in [6-10]. Additionally, there are more publications that address optical difficulties by utilizing fractional derivatives, which may be consulted in [11-15].

Bright solitons for the model (1) will be provided for the first time utilizing the Adomian decomposition procedure in conjunction with the well-known Laplace transform. The occurrence of solitons can also be guaranteed by several types of constraint requirements that arise inherently from the system's structure. In subsequent sections, specific details are enumerated and displayed.

3. The governing model's bright soliton

The bright soliton solution to (1), which was recently investigated employing the method of indeterminate coefficients in [2], is provided by

$$q(x, t) = A \operatorname{sech}^{\frac{1}{n}}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta_0)}, \tag{2}$$

where the bright soliton's velocity v is computed as

$$v = -2\kappa\left(a + 4c_1\sigma_1\kappa^2\right). \tag{3}$$

The frequency of the soliton κ is connected with some system coefficients as per

$$\kappa = \frac{c_2\sigma_8}{2c_1\sigma_4}. \tag{4}$$

The angular velocity ω is also calculated from the system coefficients, as

$$\omega = \frac{2\kappa^2\left[(a+12c_1\sigma_1\kappa^2)\sigma_2^2 - (a+4c_1\sigma_1\kappa^2)(\sigma_3+\sigma_5)^2\right]}{(\sigma_2+\sigma_3+\sigma_5)^2}. \tag{5}$$

The width of the soliton as a function of n and some coefficients of the model is obtained as follows

$$B = n\kappa\sqrt{\frac{3\sigma_2-\sigma_3-\sigma_5}{\sigma_2+\sigma_3+\sigma_5}}, \tag{6}$$

where the restriction must be imposed:

$$(3\sigma_2-\sigma_3-\sigma_5)(\sigma_2+\sigma_3+\sigma_5) > 0. \tag{7}$$

In finality, the amplitude of the soliton can be obtained by

$$A = \sqrt[2n]{\frac{\kappa^2(3\sigma_2-\sigma_3-\sigma_5)[\sigma_2+\sigma_3+\sigma_5(n+1)]}{(\sigma_2+\sigma_3+\sigma_5)\sigma_6}}, \tag{8}$$

resulting in the ensuing restriction:

$$\kappa^2\sigma_6(3\sigma_2-\sigma_3-\sigma_5)[\sigma_2+\sigma_3+\sigma_5(n+1)](\sigma_2+\sigma_3+\sigma_5) > 0. \tag{9}$$

4. Materials and methods

In this part, we will provide a concise exposition of the widely used Adomian decomposition method and its enhanced version achieved through the integration of the approach with the Laplace transform [16-17]. The proposed methodology will be employed to acquire bright solitons for the novel concatenation model with power-law nonlinearity (1).

In general, using operators we can write Eq. (1) as

$$D_t q(x, t) + Lq(x, t) + Nq(x, t) = 0 \tag{10}$$

subject to an initial condition

$$q(x, 0) = f(x). \tag{11}$$

In the context of the operational Eq. (10), the operators involved act on a complex-valued function q as:

$$D_t q = iq_t, \tag{12}$$

$$Lq(x, t) = aq_{xx} + c_1\sigma_1 q_{xxxx} + ic_2\sigma_7 q_{xxx}, \tag{13}$$

$$Nq(x,t) = b\,|q|^{2n} q + c_1\sigma_2 (q_x)^2 q^* + c_1\sigma_3 |q_x|^{2n} q + c_1\sigma_4 |q|^{2n} q_{xx} + c_1\sigma_5 q^2 q_{xx}^*$$

$$+c_1\sigma_6 |q|^{2n+2} q + ic_2(\sigma_8 |q|^{2n} q_x + \sigma_9 q^2 q_x^*). \tag{14}$$

It is clear that the operator N is nonlinear. Consequently, according to the Adomian decomposition approach, it may be decomposed into a series:

$$Nq(x,t) = \sum_{k=0}^{\infty} M_k(q_0, \ldots, q_k), \tag{15}$$

where each of the M_n is an Adomian polynomial [18]. Also, by the Adomian decomposition method we have

$$q(x,t) = \sum_{k=0}^{\infty} q_k(x,t). \tag{16}$$

To conveniently represent the nonlinear operator denoted by (15), we may write it as

$$Nq(x,t) = (N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8) q(x,t), \tag{17}$$

where

$$N_1 q = b\,|q|^{2n} q, \quad N_2 q = c_1\sigma_2 (q_x)^2 q^*, \quad N_3 q = c_1\sigma_3 |q_x|^2 q, \quad N_4 q = c_1\sigma_4 |q|^{2n} q_{xx},$$

$$N_5 q = c_1\sigma_5 q^2 q_{xx}^*, \quad N_6 q = c_1\sigma_6 |q|^{2n+2} q, \quad N_7 q = ic_2\sigma_8 |q|^{2n} q_x, \quad N_8 q = ic_2\sigma_9 q^2 q_x^*, \tag{18}$$

and all nonlinear components $N_1, \ldots, N_8$ can be decomposed into infinite series of Adomian polynomials given by:

$$N_j q = \sum_{k=0}^{\infty} M_k^j(q_0, q_1, \ldots, q_k), \; j = 1, 2, \ldots, 8. \tag{19}$$

M_k^j represents the Adomian polynomials for each $j = 1, 2, \ldots, 8$ in Eq. (19), which can be calculated using the formulas established in [19], i.e.

$$M_k^j(q_0, q_1, \ldots, q_n) = \begin{cases} N_j(q_0), & k = 0 \\ \dfrac{1}{k}\displaystyle\sum_{i=0}^{k-1} (i+1) q_{i+1} \dfrac{\partial}{\partial q_0} M_{n-1-i}^j, & k = 1, 2, 3, \ldots \end{cases} \tag{20}$$

In this context, the symbol $\mathscr{L}$ will be used to represent the Laplace transform, while $\mathscr{L}^{-1}$ will represent its inverse operator. Next, we apply the Laplace transform $\mathscr{L}$ to both sides of the operational Eq. (10) to obtain

$$\mathscr{L}\{D_t q(x,t) + Lq(x,t) + Nq(x,t)\} = 0. \tag{21}$$

By utilizing the initial condition, which is obtained from the initial profiles of the solitons f, we acquire

$$\mathscr{L}\{q(x,t)\} = \frac{1}{s} f(x) - \frac{1}{s}\Big(\mathscr{L}\{Lq(x,t)\} + \mathscr{L}\{Nq(x,t)\}\Big). \tag{22}$$

By substituting the Eqs. (15), (16) and (19) into Eq. (22), we get

$$\mathscr{L}\Big\{\sum_{k=0}^{\infty} q_k(x,t)\Big\} = \frac{1}{s} f(x) - \frac{1}{s}\Big(\mathscr{L}\Big\{L\Big(\sum_{k=0}^{\infty} q_k(x,t)\Big)\Big\} + \mathscr{L}\Big\{\sum_{j=1}^{8}\sum_{k=0}^{\infty} M_k^j(q_0, \ldots, q_k)\Big\}\Big). \tag{23}$$

By equating both sides of Eq. (23), we can calculate the Laplace transform of each individual component of the solution, that is

$$\mathscr{L}\{q_0(x,t)\} = \frac{1}{s} f(x) \tag{24}$$

and for every $m \geq 1$, the recursive relations are given by

$$\mathscr{L}\{q_m(x,t)\} = -\frac{1}{s}\Big(\mathscr{L}\{Lq_{m-1}(x,t)\} + \mathscr{L}\Big\{\sum_{j=1}^{8}\sum_{m=0}^{\infty} M_{m-1}^j(q_0, \ldots, q_{m-1})\Big\}\Big). \tag{25}$$

In order to calculate Adomian polynomials, we will focus on the nonlinear operators N_j acting on the function q described in Eq. (18). By applying the formula (20), for example, for $n = 1$, we may get the following results:

$$M_0^1 = bq_0^2 q_0^*,$$

$$M_1^1 = b(q_1^* q_0^2 + 2q_0^* q_1 q_0),$$

$$M_2^1 = b(q_2^* q_0^2 + q_0^* q_1^2 + 2q_0^* q_2 q_0 + 2q_1^* q_1 q_0),$$

$$M_3^1 = b(q_3^* q_0^2 + q_1^* q_1^2 + 2q_2^* q_1 q_0 + 2q_1^* q_2 q_0 + 2q_0^* q_3 q_0 + 2q_0^* q_1 q_2),$$

$$M_4^1 = b(q_4^* q_0^2 + q_2^* q_1^2 + q_0^* q_2^2 + 2q_3^* q_1 q_0 + 2q_2^* q_2 q_0 + 2q_1^* q_3 q_0 + 2q_0^* q_4 q_0 + 2q_1^* q_1 q_2 + 2q_0^* q_1 q_3),$$

$$\vdots$$

$$M_0^2 = c_1 \sigma_2 q_{0x}^2 q_0^*,$$

$$M_1^2 = c_1 \sigma_2 (q_1^* q_{0x}^2 + 2q_0^* q_{1x} q_{0x}),$$

$$M_2^2 = c_1\sigma_2(q_2^* q_{0x}^2 + q_0^* q_{1x}^2 + 2q_0^* q_{2x} q_{0x} + 2q_1^* q_{1x} q_{0x}),$$

$$M_3^2 = c_1\sigma_2(q_3^* q_{0x}^2 + q_1^* q_{1x}^2 + 2q_2^* q_{1x} q_{0x} + 2q_1^* q_{2x} q_{0x} + 2q_0^* q_{3x} q_{0x} + 2q_0^* q_{1x} q_{2x}),$$

$$M_4^2 = c_1\sigma_2(q_4^* q_{0x}^2 + q_2^* q_{1x}^2 + q_0^* q_{2x}^2 + 2q_3^* q_{1x} q_{0x} + 2q_2^* q_{2x} q_{0x} + 2q_1^* q_{3x} q_{0x} + 2q_0^* q_{4x} q_{0x} + 2q_1^* q_{1x} q_{2x}$$
$$+ 2q_0^* q_{1x} q_{3x}),$$

$$\vdots$$

$$M_0^3 = c_1\sigma_3 q_0 q_{0x}^* q_{0x},$$

$$M_1^3 = c_1\sigma_3(q_0 q_{0x}^* q_{1x} + q_0 q_{1x}^* q_{0x} + q_1 q_{0x}^* q_{0x}),$$

$$M_2^3 = c_1\sigma_3(q_0 q_{0x}^* q_{2x} + q_0 q_{1x}^* q_{1x} + q_0 q_{2x}^* q_{0x} + q_1 q_{0x}^* q_{1x} + q_1 q_{1x}^* q_{0x} + q_2 q_{0x}^* q_{0x}),$$

$$M_3^3 = c_1\sigma_3(q_0 q_{0x}^* q_{3x} + q_0 q_{1x}^* q_{2x} + q_0 q_{2x}^* q_{1x} + q_0 q_{3x}^* q_{0x} + q_1 q_{0x}^* q_{2x} + q_1 q_{1x}^* q_{1x} + q_1 q_{2x}^* q_{0x} + q_2 q_{0x}^* q_{1x}$$
$$+ q_2 q_{1x}^* q_{0x} + q_3 q_{0x}^* q_{0x}),$$

$$M_4^3 = c_1\sigma_3(q_0 q_{0x}^* q_{4x} + q_0 q_{1x}^* q_{3x} + q_0 q_{2x}^* q_{2x} + q_0 q_{3x}^* q_{1x} + q_0 q_{4x}^* q_{0x} + q_1 q_{0x}^* q_{3x} + q_1 q_{1x}^* q_{2x}$$
$$+ q_1 q_{2x}^* q_{1x} + q_1 q_{3x}^* q_{0x} + q_2 q_{0x}^* q_{2x} + q_2 q_{1x}^* q_{1x} + q_2 q_{2x}^* q_{0x} + q_3 q_{0x}^* q_{1x} + q_3 q_{1x}^* q_{0x} + q_4 q_{0x}^* q_{0x}),$$

$$\vdots$$

$$M_0^4 = c_1\sigma_4 q_0^* q_0 q_{0xx},$$

$$M_1^4 = c_1\sigma_4(q_1^* q_0 q_{0xx} + q_0^* q_1 q_{0xx} + q_0^* q_0 q_{1xx}),$$

$$M_2^4 = c_1\sigma_4(q_2^* q_0 q_{0xx} + q_1^* q_1 q_{0xx} + q_0^* q_2 q_{0xx} + q_1^* q_0 q_{1xx} + q_0^* q_1 q_{1xx} + q_0^* q_0 q_{2xx}),$$

$$M_3^4 = c_1\sigma_4(q_3^* q_0 q_{0xx} + q_2^* q_1 q_{0xx} + q_1^* q_2 q_{0xx} + q_0^* q_3 q_{0xx} + q_2^* q_0 q_{1xx} + q_1^* q_1 q_{1xx} + q_0^* q_2 q_{1xx} + q_1^* q_0 q_{2xx}$$
$$+ q_0^* q_1 q_{2xx} + q_0^* q_0 q_{3xx}),$$

$$M_4^4 = c_1\sigma_4(q_4^* q_0 q_{0xx} + q_3^* q_1 q_{0xx} + q_2^* q_2 q_{0xx} + q_1^* q_3 q_{0xx} + q_0^* q_4 q_{0xx} + q_3^* q_0 q_{1xx} + q_2^* q_1 q_{1xx} + q_1^* q_2 q_{1xx}$$
$$+ q_0^* q_3 q_{1xx} + q_2^* q_0 q_{2xx} + q_1^* q_1 q_{2xx} + q_0^* q_2 q_{2xx} + q_1^* q_0 q_{3xx} + q_0^* q_1 q_{3xx} + q_0^* q_0 q_{4xx}),$$

$$M_0^5 = c_1\sigma_5 q_0^2 q_{0xx}^*,$$

$$M_1^5 = c_1\sigma_5 (q_0^2 q_{1xx}^* + 2q_0 q_1 q_{0xx}^*),$$

$$M_2^5 = c_1\sigma_5 (q_1^2 q_{0xx}^* + 2q_0 q_2 q_{0xx}^* + 2q_0 q_1 q_{1xx}^* + q_0^2 q_{2xx}^*),$$

$$M_3^5 = c_1\sigma_5 (q_1^2 q_{1xx}^* + 2q_1 q_2 q_{0xx}^* + 2q_0 q_3 q_{0xx}^* + 2q_0 q_2 q_{1xx}^* + 2q_0 q_1 q_{2xx}^* + q_0^2 q_{3xx}^*),$$

$$M_4^5 = c_1\sigma_5 (q_1^2 q_{2xx}^* + 2q_1 q_2 q_{1xx}^* + q_2^2 q_{0xx}^* + 2q_1 q_3 q_{0xx}^* + 2q_0 q_4 q_{0xx}^* + 2q_0 q_3 q_{1xx}^* + 2q_0 q_2 q_{2xx}^*$$

$$+2q_0 q_1 q_{3xx}^* + q_0^2 q_{4xx}^*),$$

$$\vdots$$

$$M_0^6 = c_1\sigma_6 q_0^{*2} q_0^3,$$

$$M_1^6 = c_1\sigma_6 (2q_0^* q_1^* q_0^3 + 3q_0^{*2} q_1 q_0^2),$$

$$M_2^6 = c_1\sigma_6 (q_1^{*2} q_0^3 + 2q_0^* q_2^* q_0^3 + 6q_0^* q_1^* q_1 q_0^2 + 3q_0^{*2} q_2 q_0^2 + 3q_0^{*2} q_1^2 q_0),$$

$$M_3^6 = c_1\sigma_6 (2q_1^* q_2^* q_0^3 + 2q_0^* q_3^* q_0^3 + 3q_1^{*2} q_1 q_0^2 + 6q_0^* q_2^* q_1 q_0^2 + 6q_0^* q_1^* q_2 q_0^2 + 3q_0^{*2} q_3 q_0^2 + 6q_0^* q_1^* q_1^2 q_0$$

$$+6q_0^{*2} q_1 q_2 q_0 + q_0^{*2} q_1^3),$$

$$M_4^6 = c_1\sigma_6 (q_2^{*2} q_0^3 + 2q_1^* q_3^* q_0^3 + 2q_0^* q_4^* q_0^3 + 6q_1^* q_2^* q_1 q_0^2 + 6q_0^* q_3^* q_1 q_0^2 + 3q_1^{*2} q_2 q_0^2 + 6q_0^* q_2^* q_2 q_0^2$$

$$+6q_0^* q_1^* q_3 q_0^2 + 3q_0^{*2} q_4 q_0^2 + 3q_1^{*2} q_1^2 q_0 + 6q_0^* q_2^* q_1^2 q_0 + 3q_0^{*2} q_2^2 q_0 + 12q_0^* q_1^* q_1 q_2 q_0 + 6q_0^{*2} q_1 q_3 q_0$$

$$+2q_0^* q_1^* q_1^3 + 3q_0^{*2} q_1^2 q_2),$$

$$\vdots$$

$$M_0^7 = ic_2\sigma_8 q_0 q_0^* q_{0x},$$

$$M_1^7 = ic_2\sigma_8 (q_1 q_0^* q_{0x} + q_0 q_1^* q_{0x} + q_0 q_0^* q_{1x}),$$

$$M_2^7 = ic_2\sigma_8 (q_2 q_0^* q_{0x} + q_1 q_1^* q_{0x} + q_0 q_2^* q_{0x} + q_1 q_0^* q_{1x} + q_0 q_1^* q_{1x} + q_0 q_0^* q_{2x}),$$

$$M_3^7 = ic_2\sigma_8(q_3q_0^*q_{0x} + q_2q_1^*q_{0x} + q_1q_2^*q_{0x} + q_0q_3^*q_{0x} + q_2q_0^*q_{1x} + q_1q_1^*q_{1x} + q_0q_2^*q_{1x} + q_1q_0^*q_{2x}$$

$$+q_0q_1^*q_{2x} + q_0q_0^*q_{3x}),$$

$$M_4^7 = ic_2\sigma_8(q_4q_0^*q_{0x} + q_3q_1^*q_{0x} + q_2q_2^*q_{0x} + q_1q_3^*q_{0x} + q_0q_4^*q_{0x} + q_3q_0^*q_{1x} + q_2q_1^*q_{1x} + q_1q_2^*q_{1x}$$

$$+q_0q_3^*q_{1x} + q_2q_0^*q_{2x} + q_1q_1^*q_{2x} + q_0q_2^*q_{2x} + q_1q_0^*q_{3x} + q_0q_1^*q_{3x} + q_0q_0^*q_{4x}),$$

$$\vdots$$

$$M_0^8 = ic_2\sigma_9 q_{0x}^* q_0^2,$$

$$M_1^8 = ic_2\sigma_9(q_{1x}^*q_0^2 + 2q_{0x}^*q_1q_0),$$

$$M_2^8 = ic_2\sigma_9(q_{2x}^*q_0^2 + 2q_{1x}^*q_1q_0 + 2q_{0x}^*q_2q_0 + q_{0x}^*q_1^2),$$

$$M_3^8 = ic_2\sigma_9(q_{3x}^*q_0^2 + 2q_{2x}^*q_1q_0 + 2q_{1x}^*q_2q_0 + 2q_{0x}^*q_3q_0 + q_{1x}^*q_1^2 + 2q_{0x}^*q_1q_2),$$

$$M_4^8 = ic_2\sigma_9(q_{4x}^*q_0^2 + 2q_{3x}^*q_1q_0 + 2q_{2x}^*q_2q_0 + 2q_{1x}^*q_3q_0 + 2q_{0x}^*q_4q_0 + q_{2x}^*q_1^2 + 2q_{1x}^*q_1q_2 + 2q_{0x}^*q_1q_3),$$

and so on for other Adomian polynomials.

Eventually, when contemplating the inverse Laplace transform $\mathscr{L}^{-1}$, the components q_0, q_1, q_2, and so forth, are subsequently ascertained through an iterative procedure, which is given as:

$$\begin{cases} q_0(x, t) = f(x), \\ q_1(x, t) = -\mathscr{L}^{-1}\Big(\frac{1}{s}\mathscr{L}\{Rq_0(x, t)\} + \frac{1}{s}\Big[\mathscr{L}\Big\{\sum_{j=1}^{8} P_0^j(q_0)\Big\}\Big]\Big), \\ q_2(x, t) = -\mathscr{L}^{-1}\Big(\frac{1}{s}\mathscr{L}\{Rq_1(x, t)\} + \frac{1}{s}\Big[\mathscr{L}\Big\{\sum_{j=1}^{8} P_1^j(q_0, q_1)\Big\}\Big]\Big), \\ \vdots \\ q_m(x, t) = -\mathscr{L}^{-1}\Big(\frac{1}{s}\mathscr{L}\{Rq_{m-1}(x, t)\} + \frac{1}{s}\Big[\mathscr{L}\Big\{\sum_{j=1}^{8} P_{m-1}^j(q_0, \ldots, q_{m-1})\Big\}\Big]\Big), \ m \geq 1. \end{cases} \tag{26}$$

where q_0 is referred to as the zero-th component, which is taken as the initial condition in this method.

The solution functions q in the Laplace-Adomian decomposition method are derived as

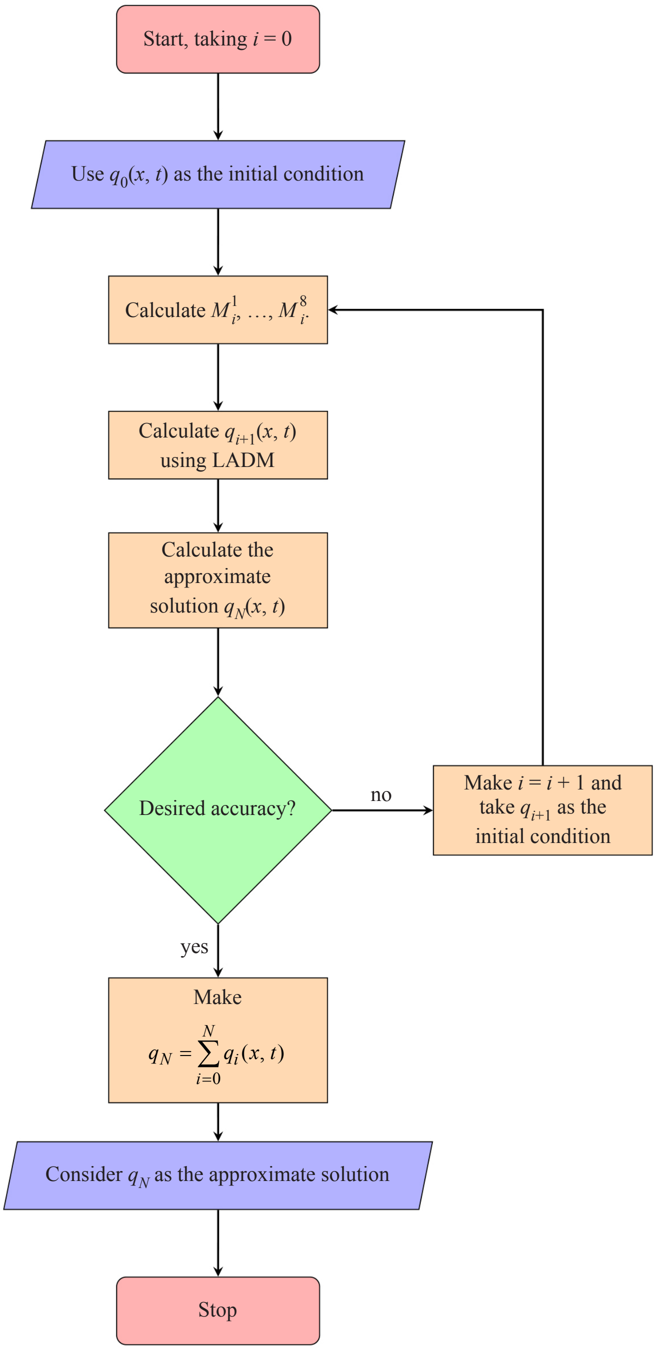
$$q(x, t) = \sum_{i=0}^{\infty} q_i(x, t). \tag{27}$$

The following is the approximate N-step solution obtained:

$$q_N = \sum_{i=0}^{N} q_i(x, t). \tag{28}$$

The series solution (28) can be used for numerical purposes. For further detail about the convergence of the proposed method, we refer [20].

The below flowchart presents a concise visual representation of the algorithm derived from the combined use of the Adomian approach and the Laplace transform.



5. Results

Using the method we discussed in the last part, we will simulate how the bright solitons in the concatenation model behave when the parameter n has different values.

5.1 *Bright soliton simulation for n = 1/3*

The coefficients of the concatenation model (1) as presented in Table 1 are to be taken into consideration for the simulations.

Table 1. Parameters for bright soliton simulation and with $N = 13$ steps

Cases	a	b	c_1	c_2	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8	σ_9	|Max Error|
Case 1	2.2	0.5	9.0	0.6	2.3	6.1	0.1	3.1	0.3	8.9	2.2	0.3	6.2	2.4×10^{-7}
Case 2	0.8	2.2	3.4	2.8	0.3	4.8	1.1	6.1	0.8	2.0	8.2	1.1	5.5	8.2×10^{-7}

Consider, for the two cases presented in Table 1, the condition at time $t = 0$ provided by

$$q(x, 0) = f(x) = A\,\mathrm{sech}^3[B(x)]e^{i(-\kappa x + \theta_0)}. \tag{29}$$

Figures 1 and 2 illustrate the results obtained from simulations performed on the scenarios enumerated in Table 1.

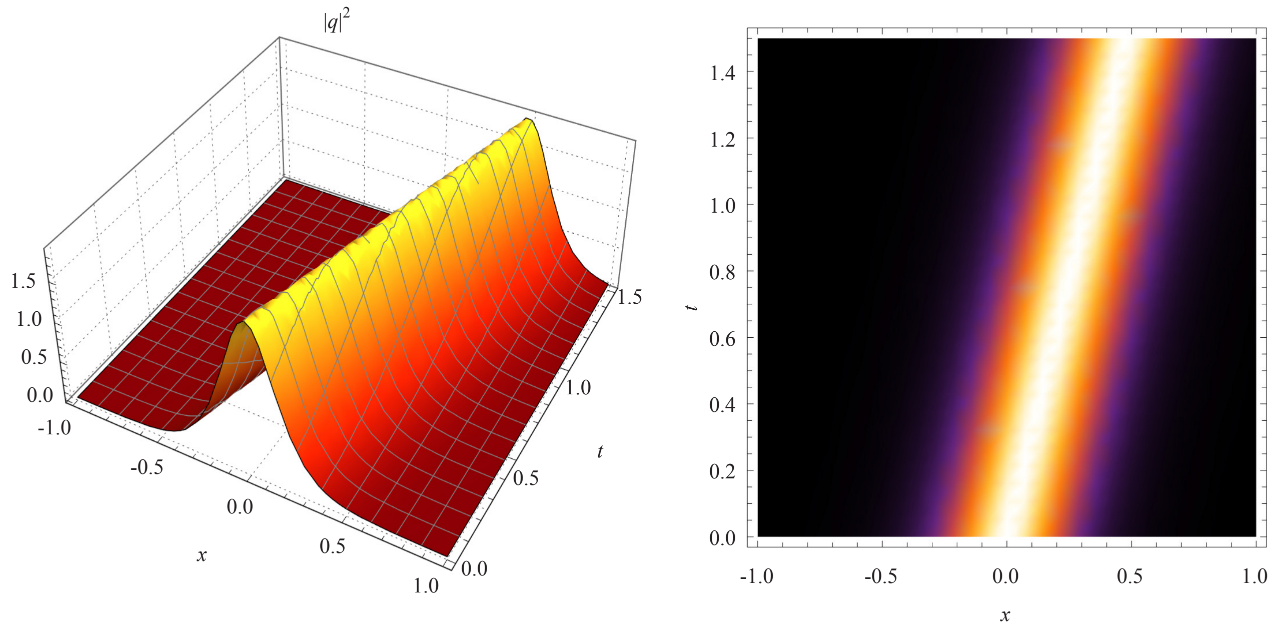


Figure 1. (left) 3D optical bright soliton using LADM for the parameters given in Case 1. (right) 2D density graphs represent bright soliton solutions of Eq. (1)

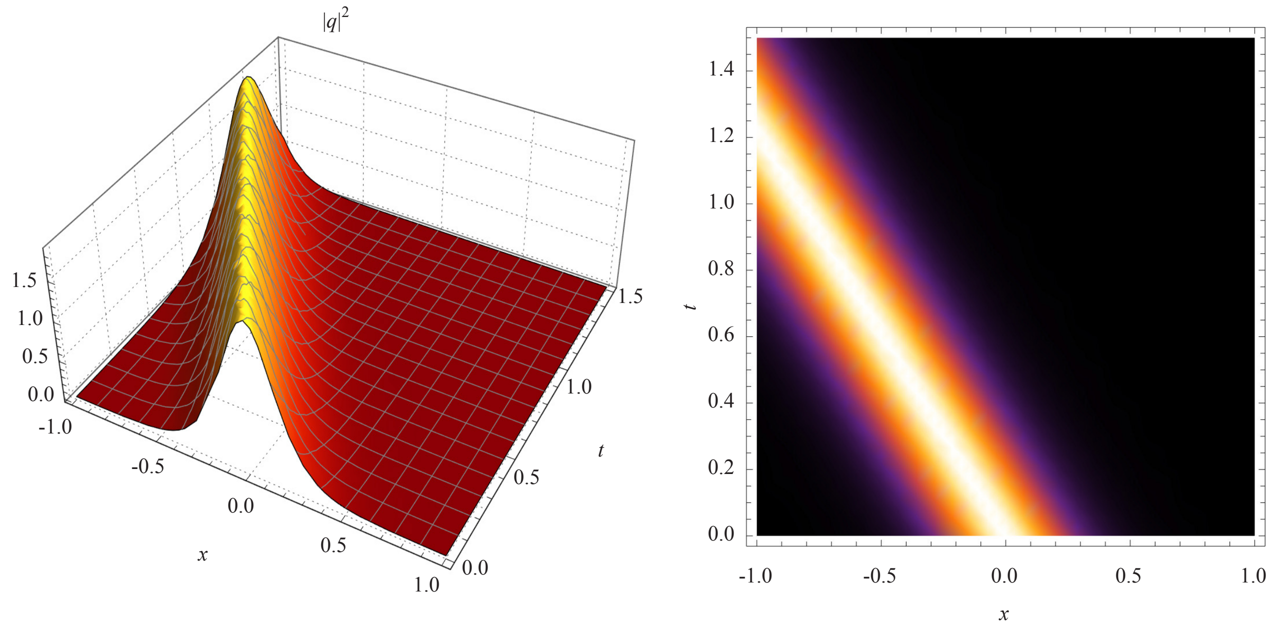


Figure 2. (left) 3D optical bright soliton using LADM for the parameters given in Case 2. (right) 2D density graphs represent bright soliton solutions of Eq. (1)

5.2 *Bright soliton simulation for n = 1/2*

The coefficients of the concatenation model (1) as presented in Table 2 are to be taken into consideration for the simulations.

Table 2. Parameters for bright soliton simulation and with $N = 13$ steps

Cases	a	b	c_1	c_2	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8	σ_9	|Max Error|
Case 3	3.2	0.2	0.4	4.2	5.3	4.9	8.4	0.3	0.2	0.1	2.2	1.5	1.2	7.2×10^{-8}
Case 4	2.5	1.0	7.7	3.2	5.7	6.8	6.2	4.4	0.4	1.8	0.6	2.0	2.4	5.5×10^{-7}

Consider, for the two cases presented in Table 2, the condition at time $t = 0$ provided by

$$q(x, 0) = f(x) = A\,\mathrm{sech}^2[B(x)]e^{i(-\kappa x+\theta_0)}. \tag{30}$$

Figures 3 and 4 illustrate the results obtained from simulations performed on the scenarios enumerated in Table 2.

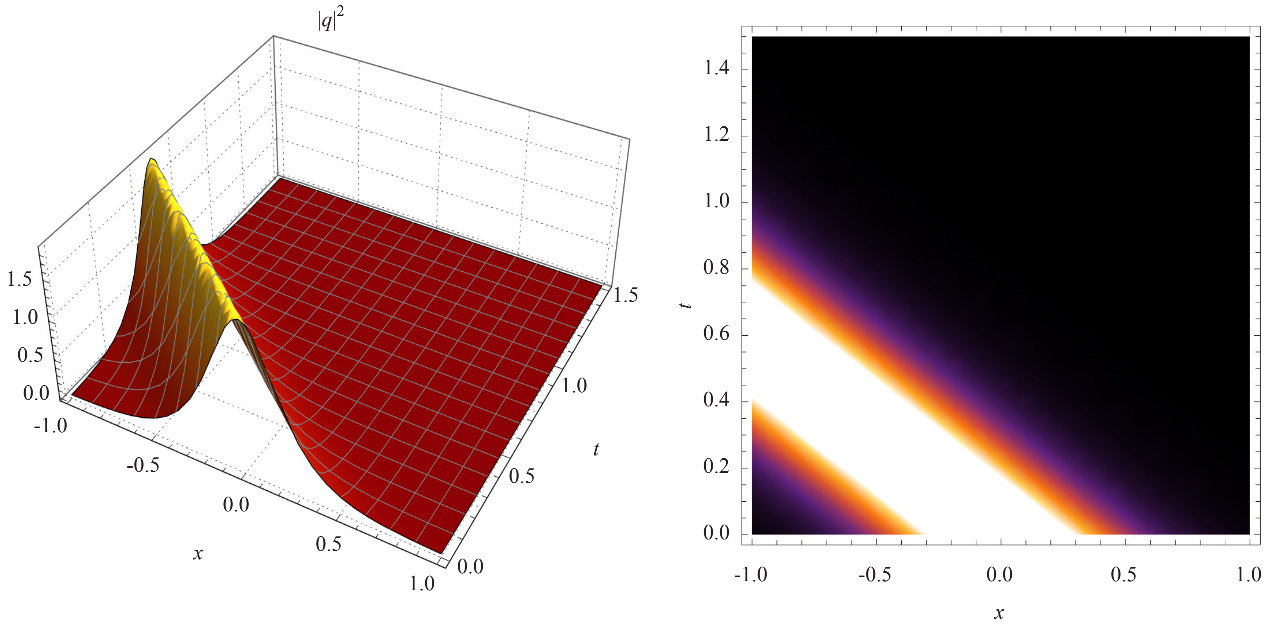


Figure 3. (left) 3D optical bright soliton using LADM for the parameters given in Case 3. (right) 2D density graphs represent bright soliton solutions of Eq. (1)

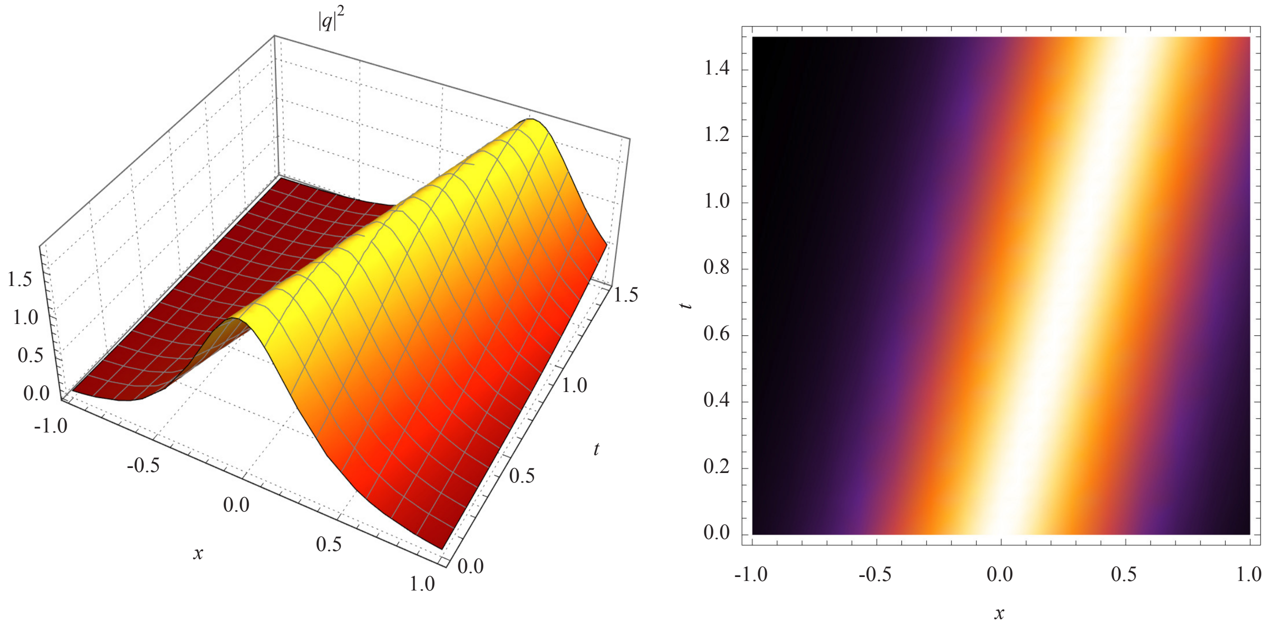


Figure 4. (left) 3D optical bright soliton using LADM for the parameters given in Case 4. (right) 2D density graphs represent bright soliton solutions of Eq. (1)

5.3 *Bright soliton simulation for n = 3/2*

The coefficients of the concatenation model (1) as presented in Table 3 are to be taken into consideration for the simulations.

Table 3. Parameters for bright soliton simulation and with $N = 13$ steps

Cases	a	b	c_1	c_2	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8	σ_9	|Max Error|
Case 3	9.2	2.3	7.7	1.1	0.4	7.0	0.4	5.5	0.3	4.4	3.6	0.5	1.9	5.1×10^{-7}
Case 4	0.4	1.8	9.0	6.2	3.2	4.4	1.9	4.7	1.1	0.1	0.9	2.9	6.6	6.7×10^{-8}

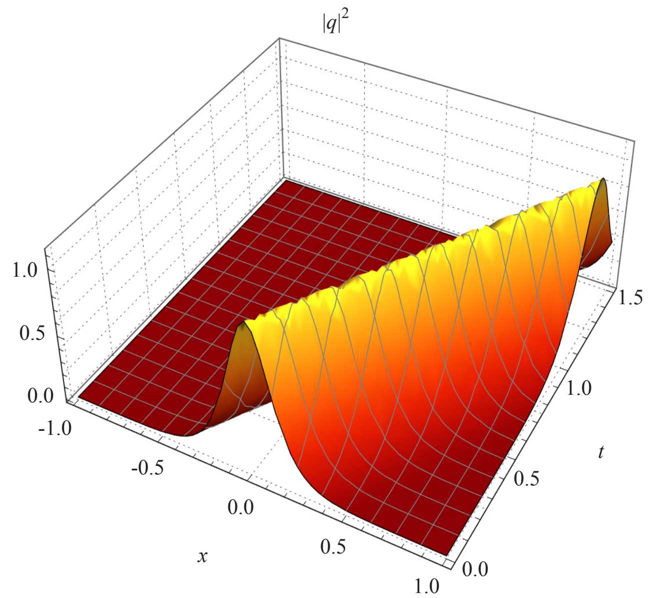
Consider, for the two cases presented in Table 3 , the condition at time $t = 0$ provided by

$$q(x, 0) = f(x) = A \operatorname{sech}^{\frac{2}{3}}[B(x)] e^{i(-\kappa x + \theta_0)}. \tag{31}$$

Figures 5 and 6 illustrate the results obtained from simulations performed on the scenarios enumerated in Table 3.

In this part, we illustrate the graphical representations of some solutions to Eq. (1). Figures 1-6 depict the solutions derived in the present study. In order to achieve this objective, we choose specific values for the parameters that have been acquired.

Based on the aforementioned data, it is evident that the retrieved solutions exhibit the presence of brigh soliton solutions as described in Eq. (1). Furthermore, these plots effectively depict the characteristics of the solutions, providing readers with a comprehensive understanding of how the solutions behave across various values of n, in particular in relation to different values of SPM.



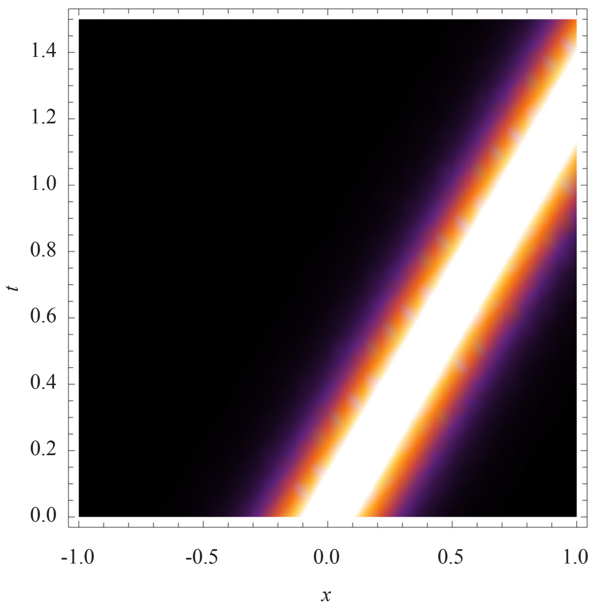


Figure 5. (left) 3D optical bright soliton using LADM for the parameters given in Case 5. (right) 2D density graphs represent bright soliton solutions of Eq. (1)

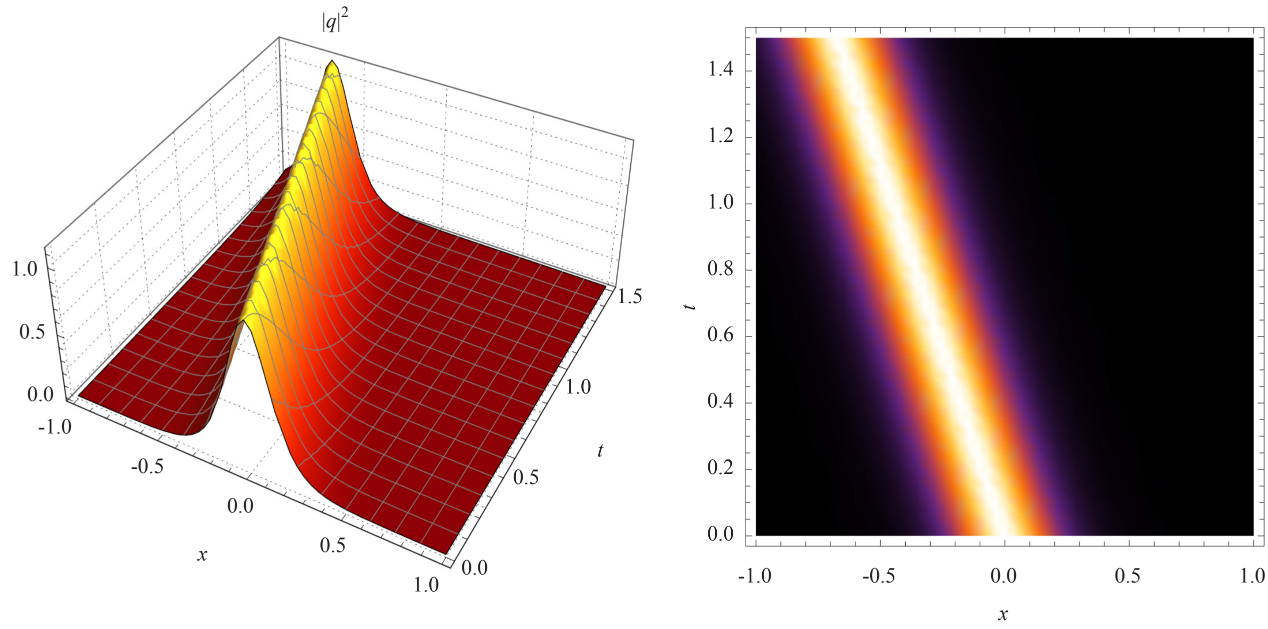


Figure 6. (left) 3D optical bright soliton using LADM for the parameters given in Case 6. (right) 2D density graphs represent bright soliton solutions of Eq. (1)

6. Conclusions

The concatenation model via the via the Adomian decomposition method combined with the Laplace transform was addressed in the present study. The bright solitons solutions were examined for some specific values of the power-law nonlinearity parameter n. These typical values of n are chosen to tacitly avoid the self-focusing singularity issue. It must be noted that the numerical scheme for the dark solitons is not considered in this paper. This is because the dark solitons for power-law nonlinearity is possible only when the power-law nonlinearity parameter n collapses to unity. This means dark solitons are only possible when power-law nonlinearity reduces to Kerr law as established during 2023 [1].

One notable benefit of this approach is in its direct applicability to a wide range of differential equations, encompassing both linear and nonlinear forms, as well as homogeneous and inhomogeneous variations. Furthermore, this method can effectively handle equations with either constant or variable coefficients. An further noteworthy aspect is that the Adomian decomposition method combined with the Laplace transform is capable of addressing non-linear problems without the need for linearization, discretization, or perturbed parameters.

The future of this paper holds strong in this work. Later, the other form of the concatenation model will be taken up for its numerical scheme, namely the dispersive concatenation model. The results of these research activities are currently awaited and would be made visible once recovered.

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Conflict of interest

The authors declare no competing financial interest.

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