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Research Article

Time Cost Trade-Off Problem Using Intuitionistic Fuzzy with Real Time Application in the Field of Construction

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Abstract: In order to assist in locating the critical path in an acyclic network, the Critical Path Method (CPM) employs intuitionistic triangular fuzzy number. Project crashing is a technique used to cut the project timeline fast by minimizing the duration of one or more critical task below their typical duration. The goal of crashing, often referred to as the time-cost trade-off, is to shorten the project's duration while lowering its cost. The intuitionistic fuzzy critical path technique and the time-cost trade-off issue were developed as a result of the uncertainty or ambiguity around the length of activities. The present investigation proposes an optimal method for handling the time-cost trade-off problem (TCTP). When the duration aspect is taken into account, the project's overall cost is effectively reduced. It is simpler to compute the time and monetary expenses of being on the critical path while using this approach. An existing application from the literature is selected and implemented to demonstrate the operation of this technology. Additionally, we suggested a ranking method for defuzzification called "Magnitude measure". Both the traditional approach and the path length ranking methodology are used for verification. MATLAB is used to give simulation results.

Keywords: critical path problem, triangular fuzzy number, intuitionistic triangular fuzzy number, acyclic network, crashing, optimization

MSC: MSC2020-90B10, 05C20, 90B35, 03B52

Abbreviation

FS	Fuzzy Set
IFS	Intuitionisitc Fuzzy Set
IFCP	Intuitionistic Fuzzy Critical Path
TFN	Triangular Fuzzy Number
ITFN	Intuitionistic Triangular Fuzzy Number
IFET	Intuitionistic Fuzzy Earliest Time
IL	Intuitionistic Fuzzy Length
IFTF	Intuitionistic Fuzzy Total Float
IFLT	Intuitionisitc Fuzzy Latest Time
IFPF	Intuitionistic Fuzzy Path Float

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1. Introduction

Successful project management techniques are hard to manipulate the projects for project managers and decision-makers in the contemporary world. It is vital to assess techniques so that managers can perform the projects and accomplish them within a specified time frame, cost estimate, and resource constraints. A project is considered as a set of interconnected tasks that can be performed in a precise situation in order to produce an impressive profit. The sophisticated project has several interrelated operations that rely on employees, machinery, and resources; it was unattainable for planners to put together and accomplish an optimum agenda. Yet, it was necessary to establish a new approach that would be more suitable and effective tactics owing to the intricacy of the few projects seen in the late 1950s. The Project Evaluation and Review Technique (PERT) and the Critical Path Method (CPM) are the two approaches that Operations Research has embraced. The US Naval Forces designed in the earlier in 1957, while James and Morgan was flourished [1]. The first CPM application was made in 1966 for the building of the old-World Trade Centre Twin Towers in New York City, a significant skyscraper. Both CPM and PERT are basically time-oriented techniques. The way that the time estimations were used differed most noticeably between CPM and PERT. Although they were deterministic in CPM, the value of time is assigned to be probabilistic in PERT. It was well renowned for being an important tool for programming and designing large-scale projects. The critical path approach can help the decision-maker to maintain and control over the project's budget, timeline, and quality of work. This approach is frequently used in several sectors to evaluate and boost a project's effectiveness [2].

To approach projects in the competing domain, decision-makers and administrators need effective project management solutions. Project managers must choose which strategies can complete projects and carry them out within a specific time frame [3]. In reality, securing the intended activity time was frequently challenging due to the data's unpredictability and the framework's inconsistency of levels. Because of this, Lofti Asker Zadeh developed the principles of fuzzy sets, which is important in this kind of decision-making environment [4]. In the open literature, a number of solutions to the fuzzy critical path (FCP) problem were disclosed. Chanas and Kamburowski proposed Fuzzy Program Evaluation and Review Technique (FPERT) was the first approach to identify an ideal path [5]. When project managers have deterministic data to locate a critical path, they can employ the Fuzzy Critical Path Method rather than FPERT, which assumes the time to find the critical path (FCPM). A technique to analyse the critical route in a project network using octagonal fuzzy numbers was provided by Stephen Dinakar and Rameshan [6]. A fresh approach to determining the critical path was put up by Balaganesan and Ganesan [7], in which the network diagram's ambiguous parameters take intuitionistic triangular fuzzy integers as opposed to exact numbers. To locate FCP, Jose Parvin Praveena et al. [8] suggested the innovative JOSE Algorithm, with 13 parameters and a ranking approach called the Euclidean ranking method. It was discovered that the critical path's dynamic encoding recursion may be expressed in terms of triskaidecagonal and fuzzy triskaidecagonal numbers. A novel analytical technique for locating essential pathways using a fuzzy project network was given by Shankar et al. The float time for each action in the project network was subjected to their novel defuzzification method for trapezoidal fuzzy numbers, which they applied, tabulated, and calculated the results for. They discovered the critical path by using table values [9]. In terms of intuitionistic triangular fuzzy numbers, they identified the critical path in an acyclic network and proposed the "Maximum edge distance" method [10]. As a result, there are many articles written about fuzzy critical path problems. [11–13]

The construction sector is becoming more challenging on a daily basis as new businesses enter the market and current businesses expand their employment prospects. Construction businesses strive to keep costs as low as possible in order to obtain an advantage over competitors. However, achieving this aim necessitates meticulous scheduling and planning of building projects. Critical tasks might be accelerated at an extra cost to reduce project length. Crashing a critical activity extends the project's length and raises the cost of building. The indirect cost is decreased as project time is shortened. When solving Time Cost Trade-off (TCT) problems, the lowest total of direct and indirect project costs is sought. [14, 15] were the first researchers to realise the TCT problem's significance more than 50 years ago, virtually with the emergence of project analytic methods. There have been created a number of heuristic algorithms that try to find the best answer to TCT issues. The heuristic approach developed by [16], which was the first significant effort to solve the TCT issue, was further modified by [17], Siemens and Gooding, and others. The methods give the lowest possible project cost for a given stipulated completion time. Further, for the computation of TCT with linear cost curves, [18, 19] introduced heuristic techniques. The mentioned algorithms offer the best answer for continuous crashing functions, but they cannot ensure convergence to the global optimum for non-linear or discrete crashing solutions. Siadat et al. proposed a methodology to found the critical path of the project network was established using Intervalued Intuitionistic Fuzzy after taking into account the risks associated with each activity [20]. They put forth an algorithm that was evaluated for both its rate of convergence and its ability to find the global optimum [21]. The algorithm converges to optimal or nearly optimal solutions noticeably quickly, according to test results [22]. The MAWA-TLBO algorithm was found to be efficient for the TCTP in the field of construction engineering and management [23]. They take into consideration a variety of project coefficients to make the proposed model realistic. Finally, there are numerous ways to apply uncertainty to the issue [24]. They used uncertain theory to analyse time resource-cost trade-off models for construction projects in uncertain environments [25]. The suggested VIKOR decision-making method provides the decision-maker with several options to access the infinite alternatives and is more suited than the current ones to cope with ambiguous and imprecise information. Additionally, since there is a larger difference between any two possible values, the outcome is more significant [26]. They define ITFM-numbers in terms of MCDM issues where the ITFM-numbers are used to convey the ratings of the alternatives. Several suggested operational rules make use of the t-norm and t-conorm. Next, a few operators for aggregation on ITFM-numbers are created. The alternative's similarity to the positive ideal solution is another factor used to determine its ranking order [27]. They provide a few TFM-number operating rules based on the t- and s-norms. Next, geometric and TFM-number arithmetic operators are suggested. Finally, in order to validate the presented decision-making techniques, we create an MCDM approach and apply it to an MCDM issue [28–30].

The goal of this research is to effectively cut project costs and duration. When critical activities are taken into account, crashing occurs. Using the suggested approach, each activity is crashed until the best possible outcome is achieved. We used a real-world example to illustrate the suggested methodology: constructing an eight-lane road from Salem to Chennai. We gathered the several cities that link Chennai and Salem and regarded them as nodes. The time it took to go to each city was now computed, and it was used as the edge length or time needed to do the task. This exercise is a representation of constructing an eight-way road. We have obtained data about the expenses associated with building a road. By employing the suggested approach, we were able to identify the critical path and shorten the time needed to finish the work. The aim of this study is to complete the project at a lower cost in a shorter span of time. Due to uncertainty to perform the real world problem we used intuitionistic triangular fuzzy number because we consider three criteria (if the work done fast, on time, delayed) (if it was too fast, on time, too delayed). So we have took intuitionistic triangular fuzzy number. In the literature, the critical path's time has simply been shortened; yet, when more external resources such as more labor, machinery, etc. were added, so that the cost increased during the crash. This research's innovation is in its ability to shorten the project's duration and expense.

The structure of this work is as follows: the preliminaries are in part 2, the methodology is in Section 3, the comparative research is highlighted in Section 4, and the conclusion is in Section 5.

2. Preliminaries

Definition 2.1 Fuzzy set [4]. The set which has limits that express a degree of membership function in the closed unit interval [0, 1] are known as fuzzy sets. Consider P as a non-empty set, then a fuzzy set X is therefore a set with ordered pairings in the form $X = \{(p, \alpha_A(p)) : p \in P\}$ where the membership function is written as $\alpha_x : P \to [0,1]$ and the degree of membership is denoted as $\alpha_x(p)$, for each element $p \in P$.

Definition 2.2 Intuitionisitc fuzzy set [31]. An intuitionistic fuzzy set X with the form $X = \{(p, \alpha_A(p), \beta_A(p)) : p \in P\}$ where the function $\alpha_x : P \to [0,1]$ is the degree of membership and $\beta_x : P \to [0,1]$ is the degree of non-membership of the element $p \in P$ to the set X and $p \in P$, $0 \le \alpha_A(p) + \beta_A(p) \le 1$.

Definition 2.3 Fuzzy number [4]. If the following three criteria are met, then the number is said to be fuzzy number, Piecewise continues, Convex and Normal.

Definition 2.4 Triangular fuzzy number [4]. A triplet $(p, q, r; 1)$, where $p < q < r$; $p, q, r \in \Re$ may be used to construct a triangular fuzzy number X. This is how the membership function $\alpha_x(p)$ is given as follows Figure 1:

$$\alpha_x(p) = \begin{cases} \frac{x-p}{q-p}, & p \leq x < q \\ 1, & x = q \\ \frac{r-x}{r-q}, & q < x \leq r \\ 0, & \text{otherwise} \end{cases}$$

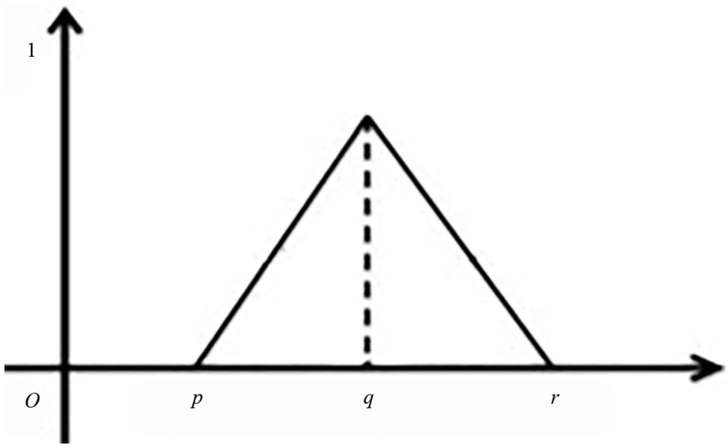


Figure 1. Triangular fuzzy number

Definition 2.5 A triplet $(p', q, r'; 1)$ can be used to define an intuitionistic triangular fuzzy number in figure 2, $p' < p < q < r < r'$; $p', q, r' \in \Re$ [31]. As stated in Definition 2.4, the membership function is similar. The following is the non-membership function $\beta_x(p)$;

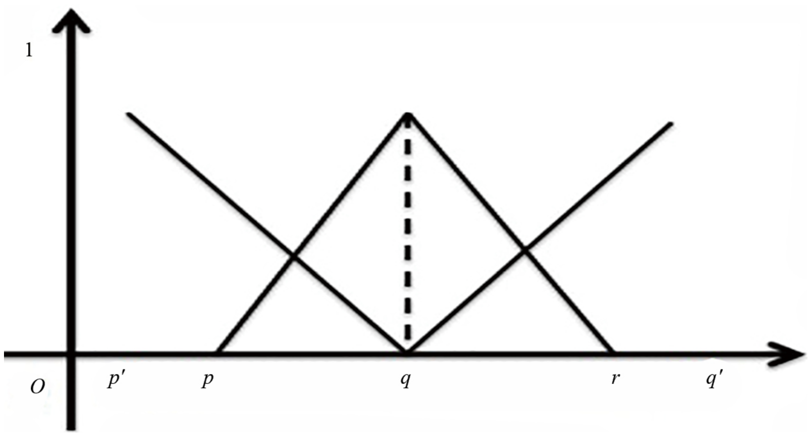


Figure 2. Intuitionisitc triangular fuzzy number

$$\beta_x(p) = \begin{cases} \dfrac{x - \bar{p}}{q - \bar{p}}, & \bar{p} \le x < q \\ 1, & x = q \\ \dfrac{\bar{r} - x}{\bar{r} - q}, & q < x \le \bar{r} \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.6 Operation on ITFN [31]. Let A = $(p_1, q_1, r_1)(p_1^{'}, q_1, r_1^{'})$ and B = $(p_2, q_2, r_2)(p_2^{'}, q_2, r_2^{'})$ be two ITFN, then

A $\oplus$ B = $(p_1 + p_2, q_1 + q_2, r_1 + r_2)(p_1^{'} + p_2^{'}, q_1 + q_2, r_1^{'} + r_2^{'})$.

A $\ominus$ B = $(p_1 - p_2, q_1 - q_2, r_1 - r_2)(p_1^{'} - p_2^{'}, q_1 - q_2, r_1^{'} - r_2^{'})$.

L_{max} = max $[(p_1, p_2), (q_1, q_2), (r_1, r_2)]$, max $[(p_1^{'}, p_2^{'}), (q_1, q_2), (r_1^{'}, r_2^{'})]$

L_{min} = min $[(p_1, p_2), (q_1, q_2), (r_1, r_2)]$, min $[(p_1^{'}, p_2^{'}), (q_1, q_2), (r_1^{'}, r_2^{'})]$

Definition 2.7 Acyclic network [32]. A graph with directed edges is referred to as a digraph. Consequently, a directed graph without a cycle is an acyclic digraph.

3. Methodology

3.1 *General technique in ITFN for intuitionistic fuzzy critical path*

The following is the procedure to calculate the IFCP:

Step 1 Build an acyclic network $G(V, E)$, where V is the vertices and E is the edges. Each arc length or edge weight in a practical situation relates to the cost, time, etc.

Step 2 Determine all feasible paths P_k, $k = 1$ to n from source 'i' to destination 'j' and the appropriate path lengths L_k, $k = 1$ to n. Let $L_k = (p_k, q_k, r_k)$.

Step 3 For each potential path $L_P = \sum_{ij \in p, p \in P_k} d_{ij}$, $k = 1$ to n, determine the duration of the path. The intuitionistic fuzzy critical path is recognised as the path with the highest path length.

3.2 *Proposed methodology to find intuitionistic fuzzy critical path*

The following are some definitions to calculate IFCP.

Intuitionistic Fuzzy Earliest Time: The earliest possible time at which an activity can begin is IFET. The symbol for it is IE_{ij}. It is determined by,

$$IE_{ij} = max_{p \in P_{1-k}} \{IL_p\} = max_{p \in P_{1-k}} \sum_{ij \in p} d_{ij} \tag{1}$$

Intuitionistic Fuzzy Latest Time: The IFLT is the latest time at which a task may be completed which is calculated as the difference between length of the path from 1 to n and the length of the path from k to n. The Symbol for it is IL_{ij}. It is formulated by,

$$IL_{ij} = max_{p \in P_{1-n}} \{IL_p\} - max_{p \in P_{k-n}} \{IL_p\}$$

$$IL_{ij} = max_{p \in P_{1-n}} \sum_{ij \in p} d_{ij} - max_{p \in P_{k-n}} \sum_{ij \in p} d_{ij} \tag{2}$$

Intuitionistic Fuzzy Total Float: The variation between IFLT and IFET is the amount of time that an activity can be delayed without having an impact on the project's new deadline which is denoted by ITF_{ij}. It is formulated as,

$$ITF_{ij} = IL_{ij} \ominus IE_{ij}$$

$$ITF_{ij} = max_{p \in P_{1-n}} \sum_{ij \in p} d_{ij} - max_{p \in P_{k-n}} \sum_{ij \in p} d_{ij} - max_{p \in P_{1-k}} \sum_{ij \in p} d_{ij} \tag{3}$$

Intuitionisitc Fuzzy Path Float: IFPF is the length of duration that an activity may be delayed after an early beginning without affecting the project's completion period which is calculated as the difference between the longest path (L_p^*) and length of the path (L_P). It is denoted as IPF_P. It is formulated as,

$$IPF_p = L_p^* - L_p = max_{p \in P_{1-n}} \sum_{ij \in p} d_{ij} - \sum_{ij \in p,\, p \in P_{1-n}} d_{ij} \tag{4}$$

Magnitude Measure: If $(p, q, r)\,(p', q, r')$ be the intuitionistic triangular fuzzy number then magnitude measure (Mag) is defined as,

$$Mag = \frac{p + 7q + r}{12}, \frac{2p' + 5q + 2r'}{12} \tag{5}$$

Theorem The IFTF for a network in an Intuitionistic fuzzy environment is,

$$ITF_{ij} = min_{p \in P_{1-n}} \{IPF_p | (i, j) \in p\} \tag{6}$$

Proof. Let us consider the R.H.S,

$$\Rightarrow min_{p \in P_{1-n}} \{IPF_p | (i, j) \in p\}$$

using equation (4),

$$\Rightarrow min_{p \in P_{1-n}} \left\{ max_{p \in P_{1-n}} \sum_{ij \in p} d_{ij} - \sum_{ij \in p,\, p \in P_{1-n}} d_{ij} \right\}$$

$$\Rightarrow min_{p \in P_{1-n}} \left\{ max_{p \in P_{1-n}} \sum_{ij \in p} d_{ij} + \left(- \sum_{ij \in p,\, p \in P_{1-n}} d_{ij} \right) \right\}$$

$$\Rightarrow min_{p \in P_{1-n}} \left\{ max_{p \in P_{1-n}} \sum_{ij \in p} d_{ij} \right\} + min_{p \in P_{1-n}} \left\{ - \sum_{ij \in p,\, p \in P_{1-n}} d_{ij} \right\}$$

$$\Rightarrow \left\{ max_{p\in P_{1-n}} \sum_{ij\in p} d_{ij} \right\} - max_{p\in P_{1-n}} \left\{ \sum_{ij\in p,\, p\in P_{1-n}} d_{ij} \right\}$$

$$\Rightarrow \left\{ max_{p\in P_{1-n}} \sum_{ij\in p} d_{ij} \right\} - max_{p\in P_{1-k}} \left\{ \sum_{ij\in p,\, p\in P_{1-k}} d_{ij} \right\} - max_{p\in P_{k-n}} \left\{ \sum_{ij\in p,\, p\in P_{k-n}} d_{ij} \right\}$$

$$ITF_{ij} = max_{p\in P_{1-n}} \sum_{ij\in p} d_{ij} - max_{p\in P_{k-n}} \sum_{ij\in p} d_{ij} - max_{p\in P_{1-k}} \sum_{ij\in p} d_{ij}$$

$$\Rightarrow ITF_{ij} \forall i,\, j\in p,\, p\in P.$$

$$\Rightarrow L.H.S$$

Hence Proved.

Algorithm

Step 1 Construct the network. Find all the possible paths and path length by $L_p = \sum_{ij\in p} d_{ij}$

Step 2 Find the path L_p^*, which has the maximum path length. $L_p^* = max_{p\in P} \sum_{ij\in p} d_{ij}$

Step 3 Find the IPF_p using equation (4) for all the paths in the network.

Step 4 Using IFPF, find the using equation (3) ITF_{ij} for all the activity.

Step 5 Utilizing the magnitude measure formula (5), defuzzify the value of IFTF. The critical activity and accompanying critical path are determined by the defuzzified values that reach zero.

3.3 *Proposed method to find time cost trade off problem*

The model of proposed methodology is given in Figure 3.

The Procedure to calculate time cost trade off problem.

Step 1 Identify the critical activity and the critical path using the above algorithm 3.2.

Step 2 By using below equation, determine the total critical path duration and total project cost (P_c). Direct Cost (D_c) is sum up of normal cost of the activities involved in the project.

$$P_c = D_c + (I_c * d_{ij});\; D_c = \sum_{i=1}^{n} (N_c)_i$$

Step 3 Calculate the crash time $(\Delta T)$and the crash cost (ΔC) to find the Cost Slope (C_s) using below equation.

$$C_s = \frac{\Delta T}{\Delta C};\ \text{where } \Delta T = N_T - C_T;\ \Delta C = C_c - N_c$$

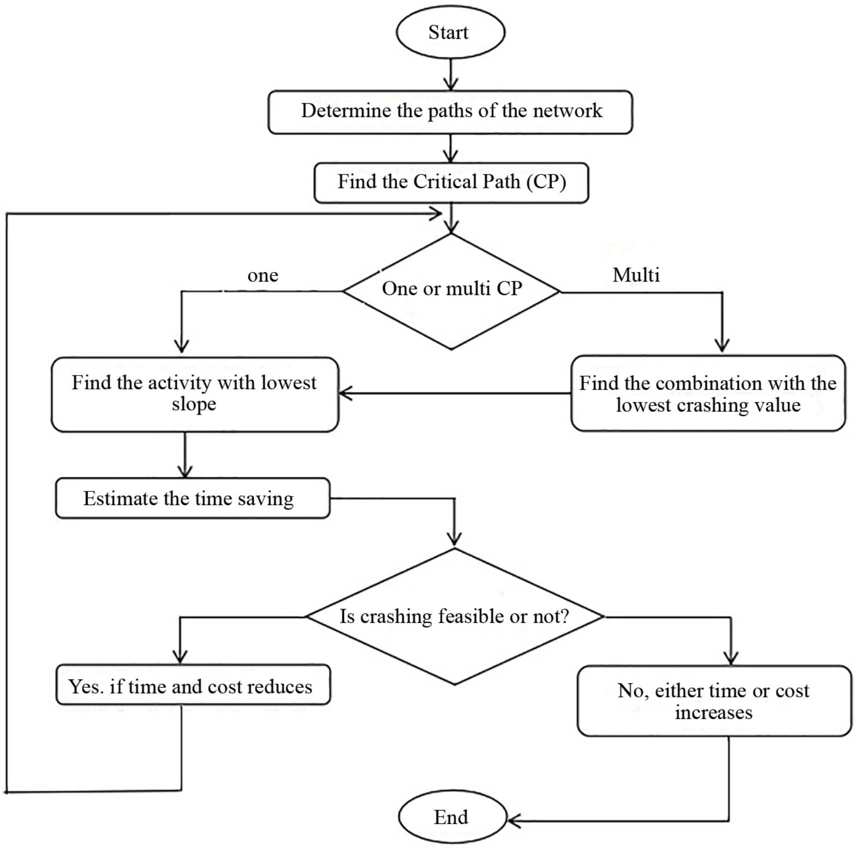
Step 4 Consider the critical path, identify the least cost slope among the critical activities and crash that activity by reducing the time.

$$d_{ij} = N_c - \Delta T$$

Step 5 Estimate the new project's cost using the formula.

$$P_c = D_c + C_c \text{ of } C_A + (I_c * d_{ij})$$

Step 6 Similarly, all the activities in the critical path are crashed. Obtain the new critical path and repeat the process until the P_c increases.



Note: Crashing of non-critical operations serves no purpose and has no effect on how long a project will take to complete. So, crashing is done only for the critical path

Figure 3. Flowchart

Table 1. Network description

Node	Description
1	Salem
2	Vellore
3	Polur
4	Thiruvanamalai
5	Cheyyar
6	Villupuram
7	Chennai

Figure 4. Geographical map

Numerical Illustration: Construct an eight-way road from Salem to Chennai in Figure 4, using critical path with minimum cost.

The description and node for the 4 is given in table 1. The Indirect Cost is (100, 100, 100) (100, 100, 100).

In Figure 4, the construction of roadway from Salem to Chennai was taken. In this construction, from one node to another node is considered as activity. For all the activity, the uncertainty occurs in time and cost which is denoted in the form of intuitionistic triangular fuzzy number. That is, the duration of node1 to node 2 is (25, 35, 55) (20, 35, 60).

In membership, the average duration to complete the activity is 35 days, minimum duration to complete the activity is 25 days and maximum duration to complete the activity is 55 days. In non-membership, the average duration to complete the activity is 35 days, least minimum duration to complete the activity is 20 days and utmost maximum duration to complete the activity is 60 days.

If we add any other extra resources like man power, machines etc the duration will be reduced which is denoted as crash time. The cost which takes to complete the activity is called normal cost, the cost corresponding to the crash time is called crash cost. For all the activities, we have considered the crash time, normal cost and crash cost which is given in 2. Using the proposed method Section 3.3, the crashing was done to reduce the time and cost of the construction of the roadway from Salem to Chennai.

Table 2. Duration and cost for the node

Activity	Duration	Crash time	Crash cost	Normal cost
1-2	(25, 35, 55) (20, 35, 60)	(13, 14, 15) (12, 14, 16)	(760, 800, 900) (750, 800, 1,000)	(400, 500, 600) (300, 500, 700)
1-3	(28, 44, 58) (22, 44, 65)	(13, 15, 17) (10, 15, 20)	(600, 700, 900) (500, 700, 950)	(410, 420, 430) (400, 420, 450)
1-4	(21, 30, 50) (15, 30, 55)	(8, 10, 12) (5, 10, 15)	(800, 900, 950) (700, 900, 1,000)	(600, 650, 680) (500, 650, 690)
2-5	(31, 45, 52) (28, 45, 58)	(7, 10, 13) (5, 10, 18)	(600, 700, 800) (500, 700, 900)	(200, 300, 400) (100, 300, 450)
3-5	(30, 47, 50) (29, 47, 60)	(10, 15, 20) (5, 15, 25)	(800, 850, 950) (700, 850, 1,000)	(400, 550, 600) (350, 550, 650)
3-6	(24, 37, 47) (20, 37, 55)	(8, 10, 12) (5, 10, 15)	(900, 950, 1,000) (800, 950, 1,050)	(700, 750, 780) (650, 750, 790)
4-6	(25, 43, 55) (20, 43, 60)	(15, 16, 17) (12, 16, 18)	(600, 700, 800) (500, 700, 900)	(300, 400, 450) (200, 400, 480)
5-7	(35, 52, 65) (30, 52, 70)	(18, 20, 22) (10, 20, 24)	(700, 800, 900) (600, 800, 950)	(400, 500, 550) (350, 500, 580)
6-7	(27, 37, 50) (25, 37, 55)	(7, 10, 12) (5, 10, 15)	(1,000, 1,100, 1,200) (900, 1,100, 1,500)	(700, 800, 850) (500, 800, 880)

Solution: Network Diagram was constructed in Figure 5 for the above construction of roadway.

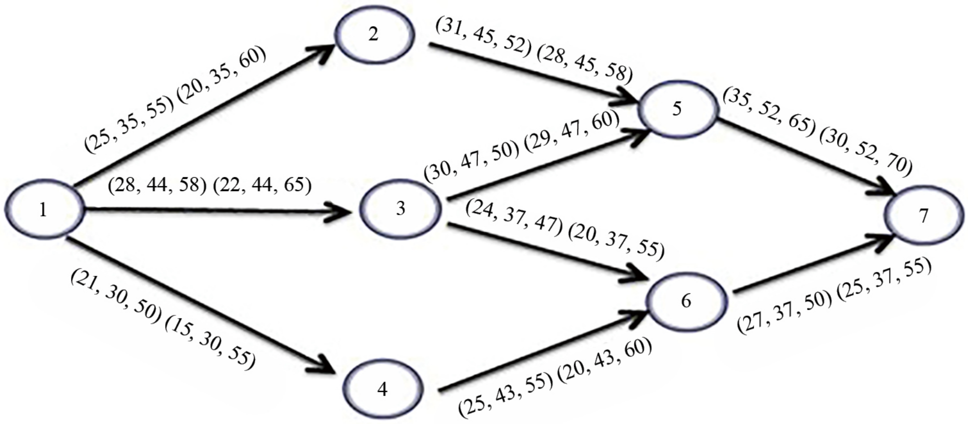


Figure 5. Network diagram

In Table 3, the possible path and path length was calculated using algorithm in step 2. The maximum value in Path Length is taken as L_P^*. Using equation (4), IPF_p is calculated.

Table 3. Result of IFPF

Path	Path length	IFPF
P_1: 1-2-5-7	(91, 132, 172) (78, 132, 188)	(-79, 11, 82) (-107, 11, 117)
P_2: 1-3-5-7	(93, 143, 173) (81, 143, 195) L_P^*	(-80, 0, 80) (-114, 0, 114)
P_3: 1-3-6-7	(79, 118, 155) (67, 118, 175)	(-62, 25, 94) (-94, 25, 128)
P_4: 1-4-6-7	(73, 110, 155) (60, 110, 170)	(-62, 33, 100) (-89, 33, 135)

From Table 4, conclude that the critical path using algorithm step 5. The activity which has zero IFTF is identified as critical path. That is, the activity 1-3, 3-5, 5-7 and the path 1-3-5-7 that is Salem-Polur-Cheyyar-Chennai and the path length that is total duration is (93, 143, 173) (81, 143, 195).

Table 4. Critical path

Activity	IFTF	Defuzzified Value eqn(5)
1-2	(-79, 11, 82) (-107, 11, 117)	(6.667, 6.25)
1-3	(-80, 0, 80) (-114, 0, 114)	(0, 0)
1-4	(-62, 33, 100) (-89, 33, 135)	(23.667, 26.333)
2-5	(-79, 11, 82) (-107, 11, 117)	(4.667, 7)
3-5	(-80, 0, 80) (-114, 0, 114)	(0, 0)
3-6	(-62, 25, 94) (-94, 25, 128)	(19, 59)
4-6	(-62, 33, 100) (-89, 33, 135)	(23.667, 26.333)
5-7	(-80, 0, 80) (-114, 0, 114)	(0, 0)
6-7	(-62, 25, 94) (-94, 25, 128)	(19, 59)

In step 3, given in proposed method Section 3.3, Cost slope, ΔC and ΔT was calculated and given in Table 5.

Table 5. Cost slope

Activity	ΔC	ΔT	Cost slope
1-2	(600, 300, 500) (50, 300, 700)	(10, 21, 42) (4, 21, 48)	(3.8095, 14.2857, 50) (1.0417, 14.2857, 175)
1-3	(170, 280, 490) (50, 280, 550)	(11, 29, 45) (2, 29, 55)	(3.7778, 9.6552, 275) (0.9091, 9.6552, 275)
1-4	(120, 250, 350) (10, 250, 500)	(9, 20, 42) (0, 20, 50)	(2.8571, 12.5, 38.8889) (0.2, 12.5, 10)
2-5	(200, 400, 600) (50, 400, 800)	(18, 35, 45) (10, 25, 53)	(4.444, 11.4286, 33.333) (0.9434, 11.4286, 80)
3-5	(200, 300, 550) (50, 300, 650)	(10, 32, 40) (4, 32, 55)	(5, 9.3750, 55) (10.9091, 9.3750, 162.5)
3-6	(120, 200, 300) (10, 200, 400)	(12, 27, 39) (5, 27, 50)	(3.0769, 7.4074, 25) (0.2, 7.4074, 80)
4-6	(150, 300, 500) (20, 300, 600)	(8, 27, 40) (2, 27, 48)	(3.75, 11.1111, 62.5) (0.4167, 11.1111, 350)
5-7	(150, 300, 500) (20, 300, 600)	(13, 32, 47) (6, 32, 60)	(3.1915, 9.375, 38.4615) (0.333, 9.3750, 100)
6-7	(150, 300, 500) (150, 300, 500)	(15, 27, 43) (10, 27, 50)	(3.4884, 11.1111, 33.333) (0.4, 11.1111, 100)

Crashing 1: $P_c = D_c + C_c$ of $C_A + (I_c * d_{ij})$

P_c = (6760, 7500, 8400) (5950, 7500, 9250) + (150, 300, 500) (200, 300, 600) + (100, 100, 100) (100, 100, 100) $*$ (46, 111, 160) (21, 111, 189)

$$P_c = (11510, 18900, 24900)(8250, 18900, 28750).$$

Continue the process till the cost increases. The result is given in Table 6.

Table 6. Crashing

Iteration	Crash	Crash duration	Crash cost
1	5-7 $\rightarrow$ (13, 32, 47) (6, 32, 60)	(46, 111, 160) (21, 111, 189)	(11,510, 18,900, 24,900) (8,250, 18,900, 28,750)
2	1-3 $\rightarrow$ (11, 29, 45) (2, 29, 55)	(1, 82, 149) (0, 82, 187)	(7,030, 15,980, 23,790) (6,000, 15,980, 28,500)
3	3-5 $\rightarrow$ (10, 32, 40) (4, 32, 55)	(0, 50, 139) (0, 50, 183)	(6,960, 12,800, 22,850) (6,000, 12,800, 28,200)

In step 5 and step 6, given in proposed method 3.3 crashing was done and tabulated in Table 6.

Therefore, the normal duration for constructing the eight way road from Salem to Chennai is (93, 143, 173) (81, 143, 195) and crashed to (0, 50, 139) (0, 50, 183) and the total direct cost (16,210, 22,100, 26,200) (8,070, 22,100, 29,350) which is crashed to (6,960, 12,800, 22,850) (6,000, 12,800, 28,200).

Simulation: The proposed algorithm is written in MATLAB, and a copy of the attachment is given in Figures 6-9. In Figure 6, the IFCP is shown, Figure 7 crashing was done, Figure 8 shows network and Figure 9 has crashing cost of the project.

MATLAB Code:

```
clear all close all a=[25 35 55 20 35 60]; Normal time
b=[28 44 58 22 44 65];
c=[21 30 50 15 30 55];
d=[31 45 52 28 45 58];
e=[30 47 50 29 47 60];
f=[24 37 47 20 37 55];
g=[25 43 55 20 43 60];
h=[35 52 65 30 52 70];
i=[27 37 50 25 37 55];
ca=[13 14 15 12 14 16];
cb=[13 15 17 10 15 20];
cc=[8 10 12 5 10 15];
cd=[7 10 13 5 10 18];
ce=[10 15 20 5 15 25];
cf=[8 10 12 5 10 15];
cg=[15 16 17 12 16 18];
ch=[18 20 22 10 20 24];
ci=[7 10 12 5 10 15];
cca=[760 800 900 750 800 1000];
ccb=[600 700 900 500 700 950];
ccc=[800 900 950 700 900 1000];
ccd=[600 700 800 500 700 900];
cce=[800 850 950 700 850 1000];
ccf=[900 950 1000 800 950 1050];
ccg=[600 700 800 500 700 900];
cch=[700 800 900 600 800 950];
cci=[1000 1100 1200 900 1100 1500];

nca=[400 500 600 300 500 700];
ncb=[410 420 430 400 420 450];
ncc=[600 650 680 500 650 690];
ncd=[200 300 400 100 300 450];
nce=[400 550 600 350 550 650];
ncf=[700 750 780 650 750 790];
ncg=[300 400 450 200 400 480];
nch=[400 500 550 350 500 580];
nci=[700 800 850 500 800 880];

p=a+d+h; q=b+e+h; r=b+f+i; s=c+g+i;
if(p>q & p>r & p>s
fprintf(' The Critical path = path 1');
```

```
fprintf(' The Critical path length = d', p);
disp(p)
elseif(q>r & q>s)
fprintf(' The Critical path = path 2');
fprintf(' The Critical path length = d', q);
disp(q)
elseif(r>s)
fprintf(' The Critical path = path 3');
fprintf(' The Critical path length = d', r);
else
fprintf(' The Critical path = path 4');
fprintf(' The Critical path length = d', s);
end

ta=[a(1,1)-ca(1,3) a(1,2)-ca(1,2) a(1,3)-ca(1,1) a(1,4)-ca(1,6) a(1,5)-ca(1,5) a(1,6)-ca(1,4)];
tb=[b(1,1)-cb(1,3) b(1,2)-cb(1,2) b(1,3)-cb(1,1) b(1,4)-cb(1,6) b(1,5)-cb(1,5) b(1,6)-cb(1,4)];
tc=[c(1,1)-cc(1,3) c(1,2)-cc(1,2) c(1,3)-cc(1,1) c(1,4)-cc(1,6) c(1,5)-cc(1,5) c(1,6)-cc(1,4)];
td=[d(1,1)-cd(1,3) d(1,2)-cd(1,2) d(1,3)-cd(1,1) d(1,4)-cd(1,6) d(1,5)-cd(1,5) d(1,6)-cd(1,4)];
te=[e(1,1)-ce(1,3) e(1,2)-ce(1,2) e(1,3)-ce(1,1) e(1,4)-ce(1,6) e(1,5)-ce(1,5) e(1,6)-ce(1,4)];
tf=[f(1,1)-cf(1,3) f(1,2)-cf(1,2) f(1,3)-cf(1,1) f(1,4)-cf(1,6) f(1,5)-cf(1,5) f(1,6)-cf(1,4)];
tg=[g(1,1)-cg(1,3) g(1,2)-cg(1,2) g(1,3)-cg(1,1) g(1,4)-cg(1,6) g(1,5)-cg(1,5) g(1,6)-cg(1,4)];
th=[h(1,1)-ch(1,3) h(1,2)-ch(1,2) h(1,3)-ch(1,1) h(1,4)-ch(1,6) h(1,5)-ch(1,5) h(1,6)-ch(1,4)];
ti=[i(1,1)-ci(1,3) i(1,2)-ci(1,2) i(1,3)-ci(1,1) i(1,4)-ci(1,6) i(1,5)-ci(1,5) i(1,6)-ci(1,4)];

coa=[cca(1,1)-nca(1,3) cca(1,2)-nca(1,2) cca(1,3)-nca(1,1) cca(1,4)-nca(1,6) cca(1,5)-nca(1,5) cca(1,6)-nca(1,4)];
cob=[ccb(1,1)-ncb(1,3) ccb(1,2)-ncb(1,2) ccb(1,3)-ncb(1,1) ccb(1,4)-ncb(1,6) ccb(1,5)-ncb(1,5) ccb(1,6)-ncb(1,4)];
coc=[ccc(1,1)-ncc(1,3) ccc(1,2)-ncc(1,2) ccc(1,3)-ncc(1,1) ccc(1,4)-ncc(1,6) ccc(1,5)-ncc(1,5) ccc(1,6)-ncc(1,4)];
cod=[ccd(1,1)-ncd(1,3) ccd(1,2)-ncd(1,2) ccd(1,3)-ncd(1,1) ccd(1,4)-ncd(1,6) ccd(1,5)-ncd(1,5) ccd(1,6)-ncd(1,4)];
coe=[cce(1,1)-nce(1,3) cce(1,2)-nce(1,2) cce(1,3)-nce(1,1) cce(1,4)-nce(1,6) cce(1,5)-nce(1,5) cce(1,6)-nce(1,4)];
cof=[ccf(1,1)-ncf(1,3) ccf(1,2)-ncf(1,2) ccf(1,3)-ncf(1,1) ccf(1,4)-ncf(1,6) ccf(1,5)-ncf(1,5) ccf(1,6)-ncf(1,4)];
cog=[ccg(1,1)-ncg(1,3) ccg(1,2)-ncg(1,2) ccg(1,3)-ncg(1,1) ccg(1,4)-ncg(1,6) ccg(1,5)-ncg(1,5) ccg(1,6)-ncg(1,4)];
coh=[cch(1,1)-nch(1,3) cch(1,2)-nch(1,2) cch(1,3)-nch(1,1) cch(1,4)-nch(1,6) cch(1,5)-nch(1,5) cch(1,6)-nch(1,4)];
coi=[cci(1,1)-nci(1,3) cci(1,2)-nci(1,2) cci(1,3)-nci(1,1) cci(1,4)-nci(1,6) cci(1,5)-nci(1,5) cci(1,6)-nci(1,4)];

csa=coa./ta; csb=[3.778 9.6552 275 0.9091 9.6552 275]; csc=coc./tc; csd=cod./td; cse=[5 9.3750 55 10.9091 9.3750 162.5];
csf=cof./tf; csg=cog./tg;
csh=[3.1915 9.375 38.4615 0.33 9.375 100]; csi=coi./ti;
IC=[100 100 100 100 100 100]; DC=nca+ncb+ncc+ncd+nce+ncf+ncg+nch+nci;
ncsb=sum(csb); ncse=sum(cse); ncsh=sum(csh);
A=[p; q; r; s];
B=max(A);
PC=DC+(IC.*B)
TempB=0;
TempE=0;
TempH=0;
ITL=1;
```

```
fprintf(' p: d ', p);fprintf(' q: d ', q);
fprintf(' r: d ', r); fprintf(' s: d ', s);
if(p>q && p>r && p>s)
fprintf(' Ranking for (P)=1');
elseif(q>r && q>s)
fprintf(' Ranking for (Q)=1');
elseif(r>s)
fprintf('Ranking for (R)=1');
else
fprintf(' Ranking for (S)=1');
end

A=[p; q; r; s];
ITM1=A;
A1=max(A, [], 1);
TC0= DC+(IC*A1);
fprintf(' Total Estimated Cost 0: d', TC0);
TC=TC0;
while (TC0>=TC)

TC0=TC;

A1=max(A, [], 1);

if(p>q && p>r && p>s)
A(1,:) = [];
elseif(q>r && q>s)
A(2,:) = [];
elseif(r>s)
A(3,:) = [];
elseI
A(4,:) = [];
end

FirstItreation

A2=max(A, [], 1);

Crash Critical Duration Start

if (A1-A2 =0)

I1=A1-A2;

fprintf('Crash Time Iteration : d', ITL);

fprintf(' Exepected Crash Time: d', I1);
```

Finding Minimum Cost Slope

```
if(p>q && p>r && p>s)

if (cs1<cs4)
MinCS=cs1;
CDTemp=cd1-I1;
DC1=CDTemp*cs1;
else
MinCS=cs4;
CDTemp=cd4-I1;
DC1=CDTemp*cs4;
end

elseif(q>r && q>s)
if (cs2<cs4 && cs2<cs5)
MinCS=cs2;
CDTemp=cd2-I1;
DC1=CDTemp*cs2;
elseif (cs4<cs5)
MinCS=cs4;
CDTemp=cd4-I1;
DC1=CDTemp*cs4;
else
MinCS=cs5;
CDTemp=cd5-I1;
DC1=CDTemp*cs5;
end
elseif(r>s)
if (cs2<cs6 && cs2<cs7)
MinCS=cs2;
CDTemp=cd2-I1;
DC1=CDTemp*cs2;
elseif (cs6<cs7)
MinCS=cs6;
CDTemp=cd6-I1;
DC1=CDTemp*cs6;
else
MinCS=cs7;
CDTemp=cd7-I1;
DC1=CDTemp*cs7;
end
else
if(cs3<cs7)
MinCS=cs3;
CDTemp=cd3-I1;
DC1=CDTemp*cs3;
else
```

```
MinCS=cs7;
CDTemp=cd7-I1;
DC1=CDTemp*cs7;
end
end
else

if (A1==p && A1==q)

if (cs1<cs4 && cs1<cs2 && cs1<cs5)
MinCS=cs1;
CDTemp=cd1-I1;
DC1=CDTemp*cs2;
elseif (cs2<cs4 && cs2<cs5)

MinCS=cs2;
CDTemp=cd2-I1;
DC1=CDTemp*cs2;
elseif (cs4<cs5)
MinCS=cs4;
CDTemp=cd4-I1;
DC1=CDTemp*cs4;
else
MinCS=cs5;
CDTemp=cd5-I1;
DC1=CDTemp*cs5;
end

TempMat=[cd1;cd2;cd4;cd5];
CDTemp=min(TempMat, [], 1);
elseif (A1==p && A1==r)

TempMat=[cd1;cd2;cd4;cd6;cd7];

CDTemp=min(TempMat, [], 1);

elseif (A1==p && A1==s)

TempMat=[cd1;cd3;cd4;cd7];

CDTemp=min(TempMat, [], 1);
elseif (A1==q && A1==r)
TempMat=[cd2;cd4;cd5;cd6;cd7];
CDTemp=min(TempMat, [], 1);
elseif (A1==q && A1==s) TempMat=[cd2;cd3;cd4;cd5;cd7];

CDTemp=min(TempMat, [], 1);
elseif (A1==s && A1==r)
```

```
TempMat=[cd2;cd3;cd6;cd7];
CDTemp=min(TempMat, [], 1);
end
end
TC=DC+(MinCS*CDTemp)+(A2*IC);
fprintf(' Total Estimated Cost d', ITL);
fprintf(' : d', TC );
```

Figure 6 shows the critical path numerical result. Figure 7 shows the numerical result of crashing. Figure 8 shows the simulation result of network diagram. Figure 9 shows the graph result of time cost trade off problem.

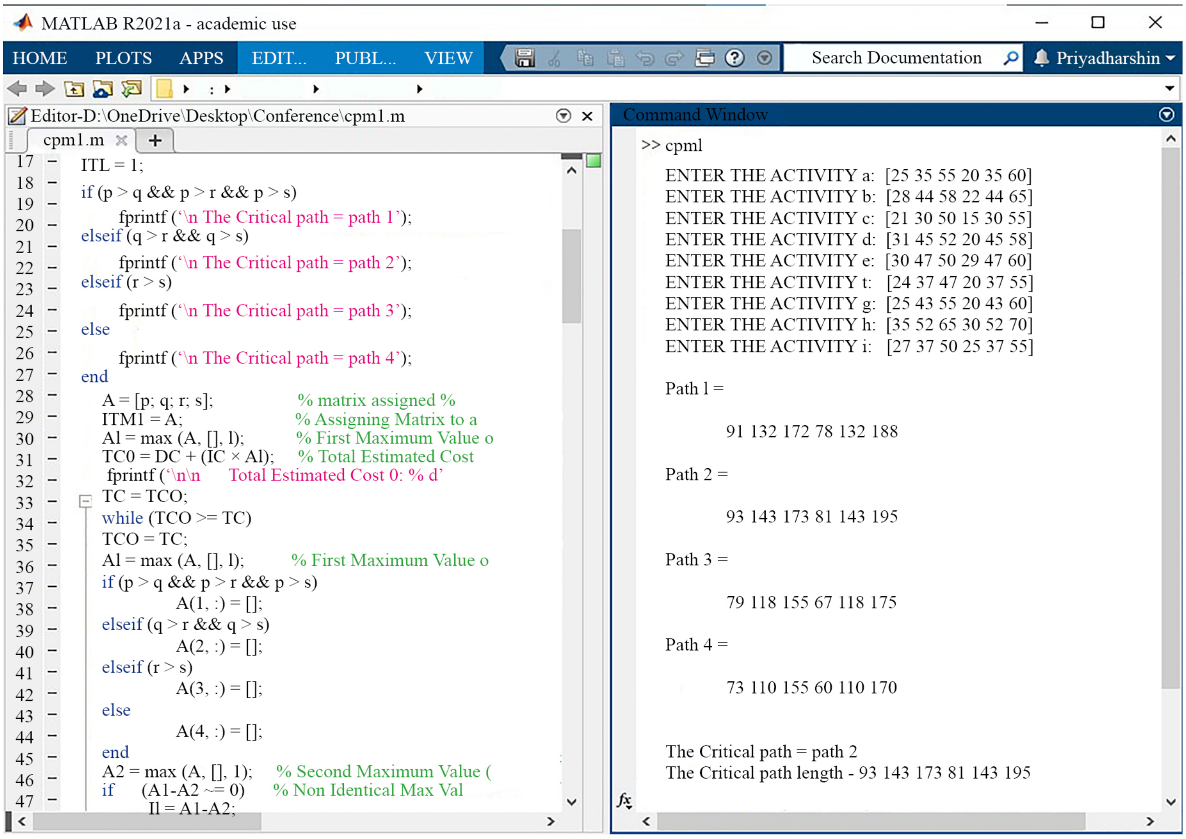


Figure 6. Critical path

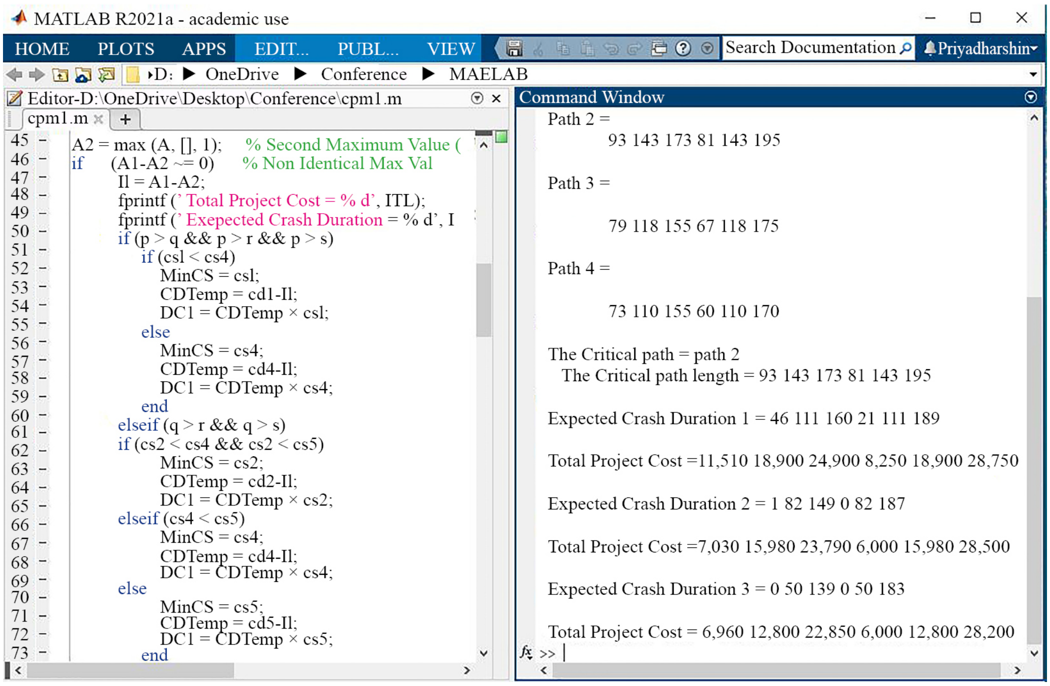


Figure 7. Crashing

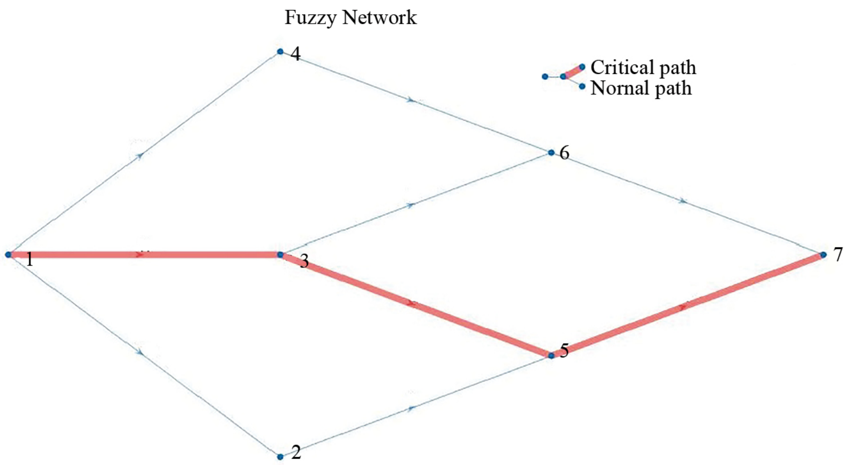


Figure 8. Network with critical path

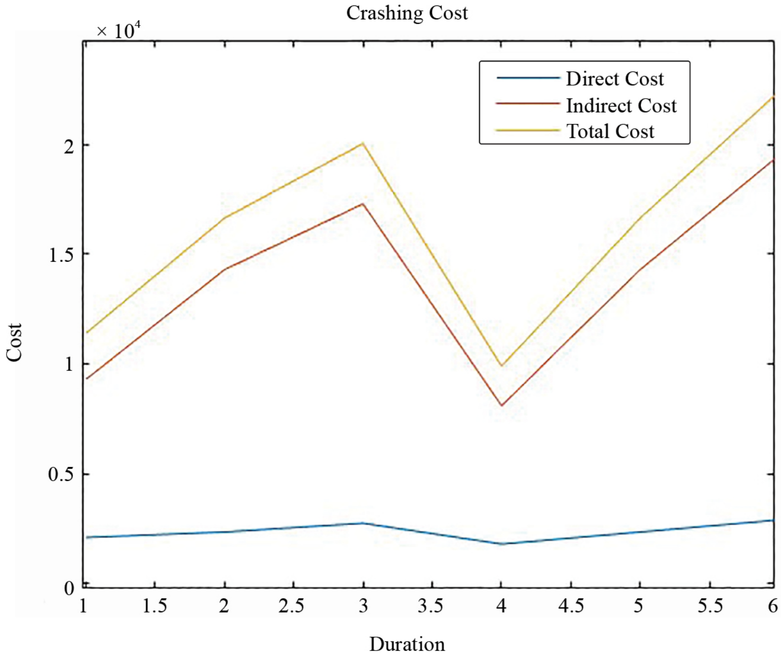


Figure 9. Cost trade graph

4. Results and discussions

Comparision:
In table 7 and 8 the comparision results are displayed.

Table 7. Comparision result for duration

Article	Total Duration	Crashed Duration
Abbasnia [33]	(40, 60, 65) (30, 60, 70)	(42, 55, 67) (38, 55, 70)
Biswasa [34]	(120, 140, 150) (110, 140, 160)	(80, 110, 115) (70, 110, 120)
Liu [35]	(99, 107, 125) (95, 107, 128)	(94, 114, 123) (93, 114, 121)
Weimin [36]	(25, 30, 32) (22, 30, 34)	(22, 24, 26) (20, 24, 27)

Table 8. Comparision result for cost

Article	Normal Cost	Crashed Cost
Abbasnia [33]	(170,000, 173,500, 174,800) (169,000, 173,500, 175,000)	(143,000, 154,500, 155,500) (1,429,000, 154,500, 155,500)
Biswasa [34]	(48,000, 48,900, 49,000) (47,500, 48,900, 50,000)	(45,500, 45,900, 46,000) (45,000, 45,900, 46,500)
Liu [35]	(11,500, 1,200, 12,300) (11,000, 1,200, 12,500)	(9,500, 1,000, 1,200) (9,400, 1,000, 12,000)
Weimin [36]	(18,402, 18,425, 18,450) (18,400, 18,425, 18,456)	(18,238, 18,247, 18,266) (18,232, 18,247, 18,271)

Verification: Using traditional forward and backward pass the verification was done.

Therefore, the IFCP is identified as P_2: 1-3-5-7 that is Salem-Polur-Cheyyar-Chennai using Table 9. The critical path found using the existing approach and the suggested method followed the same path. Calculate IFEST, IFEFT, IFLFT, IFLST, and IFTF to determine the IFCP for the existing approach, but IPFp and IFTF values for the suggested method. In order to reduce the amount of time required to compute the IFCP, the suggested method delivers the best result. The best solution was provided by the strategy suggested for the Time Cost Trade Off Problem that was introduced in an intuitionistic fuzzy environment. This means that 60 percentage of the project's total cost and time are optimised. Simulation was done in MATLAB. The proposed algorithm is written in MATLAB, and a copy of the attachment is given in Figures 6-9. The advantage of this research is to optimise both time and cost of the project. Using this approach the project can complete with short span of time. This strategy has very few drawbacks or restrictions.

Table 9. Traditional forward and backward pass calculation

Activity	IFEST	IFEFT	IFLFT	IFTF
1-2	(0, 0, 0) (0, 0, 0)	(25, 35, 55) (20, 35, 60)	(27, 46, 56) (23, 46, 67)	(2, 11, 1) (3, 11, 7)
1-3	(0, 0, 0) (0, 0, 0)	(28, 44, 58) (22, 44, 65)	(28, 44, 58) (22, 44, 65)	(0, 0, 0) (0, 0, 0)
1-4	(0, 0, 0) (0, 0, 0)	(21, 30, 50) (15, 30, 55)	(41, 63, 58) (36, 63, 80)	(20, 33, 8) (21, 33, 25)
2-5	(25, 35, 55) (20, 35, 60)	(56, 80, 107) (48, 80, 118)	(58, 91, 108) (51, 91, 125)	(2, 11, 1) (3, 11, 7)
3-5	(28, 44, 58) (22, 44, 65)	(58, 91, 108) (51, 91, 125)	(58, 91, 108) (51, 91, 125)	(0, 0, 0) (0, 0, 0)
3-6	(28, 44, 58) (22, 44, 65)	(52, 81, 105) (42, 81, 120)	(66, 106, 123) (56, 106, 140)	(14, 25, 18) (14, 25, 20)
4-6	(21, 30, 50) (15, 30, 55)	(46, 73, 105) (35, 73, 110)	(66, 106, 123) (56, 106, 140)	(20, 33, 18) (21, 33, 30)
5-7	(58, 91, 108) (51, 91, 125)	(93, 143, 163) (81, 143, 195)	(93, 143, 163) (81, 143, 195)	(0, 0, 0) (0, 0, 0)
6-7	(46, 73, 105) (35, 73, 115)	(73, 110, 155) (60, 110, 170)	(93, 143, 163) (81, 143, 195)	(20, 30, 58) (20, 30, 25)

5. Conclusion

To plan and manage complex projects, decision-makers can use intuitionistic fuzzy critical path and cost information. Unlike crisp models, the decision maker may model their project and include language variables for activity time, such as "maybe", "in between", "almost", and other expressions. An innovative method for handling network crashes in intuitionistic fuzzy triangular numbers is presented in this study. Hence, the average time required to build an eight-way road from Salem to Chennai is (93, 143, 173) (81, 143, 195) and is crashed to (0, 50, 139) (0, 50, 183). The total direct cost is (16,210, 22,100, 26,200) (8,070, 22,100, 29,350) also crashed to (6,960, 12,800, 22,850) (6,000, 12,800, 28,200). The benefit of this research is that the project's time and expense may be decreased effectively by utilizing the suggested method. The MATLAB simulation result, which also displays the same outcome, is what makes this study significant. As such, the methodology described in this study offers a means of determining the ideal cost and time in an intuitionistic framework. Future studies will be based on problems with several criteria. In addition to the crash duration and expense, other factors include probability ratio, performance, etc., We also attempt to use trapezoidal fuzzy numbers and hestitant fuzzy numbers in place of intuitionistic triangular fuzzy numbers. In Future, we can imply the proposed algorithm for the stake holders in construction industry.

Conflict of interest

The authors declare no competing financial interest.

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