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Cooperative Games with Agreements Self-Implemented

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Abstract: In classical cooperative game theory, it is often assumed that agreements are implemented by a third party, this paper provides an analytical framework for cooperative games with agreements self-implemented in three scenarios: (1) the possible opportunistic behaviors in the distribution process are ignored; (2) coalitions centralize all the payoffs their members get in the game to inhibit the possible opportunistic behaviors in the distribution process; (3) coalitions distribute their cooperative payoffs before the game begins to inhibit the possible opportunistic behaviors in the distribution process. In each scenario, this paper examines the formation of the coalitions and the distribution process of the cooperative payoff of a coalition, defines and provides the existence proof of the coalition equilibrium of a cooperative game with agreements self-implemented, defines and provides the existence proof of the equilibrium in the bargaining game of a coalition on the distribution of its cooperative payoff, when its members cooperate in the game or not.

***Keywords*:** cooperative game, self-implemented agreement, coalition equilibrium, escape-payoff deriving from deviation, common payoff; bargaining game

MSC: 91A12

1. Introduction

Since von Neumann and Morgenstern [1] founded modern game theory, cooperative game theory has achieved tremendous success, just as non-cooperative game theory. However, researchers in classical cooperative game theory did not pay much attention to the agreement implementation, the agreements are often assumed to be implemented by a third party cost free. While in fact, if there is no third party who implements the agreements by force, or the supervision of a third party is invalid, many agreements are self-implemented rather than implemented by a third party. Many problems in social or economic cooperative games are related to the implementation of the agreements.

While assuming that the agreement is implemented by a third party, literature of cooperative games usually assumes that there is only one coalition composed of all the game players except the "dummies", and ignores the possibility that these dummies excluded from the unique coalition may also form one or more coalitions if there are synergies between them. Of course, such an assumption is also contrary to the reality: in a society there may be many competing families, enterprises, or political parties at the same time, not just only one.

In a cooperative game, players will form different coalitions and compete to maximize the cooperative payoffs of their coalitions. The Nash equilibrium of a cooperative game is the Nash equilibrium of (non-cooperative) game between

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the coalitions. The literature on cooperative games only focused on the coalition formation and the cooperative payoff distribution in a coalition, especially on the latter.

Konishi and Ray [2] introduced the equilibrium definition of coalition formation, which is extended by Hyndman and Ray [3], and also by Gomes and Jehiel [4] and Gomes [5]. Aumann [6] defined the equilibrium process of coalition formation (EPCF) in repeated cooperative game, which is regarded as a process of coalition formation with the property that at every history, every active coalition, faced with a given set of potential partners, makes a profitable and maximal move. Aumann studied an information-asymmetric dynamic repeated game, and didn't define the equilibrium of coalition formation in an information-symmetric, one-off cooperative game.

Many literatures focus on cooperative payoff distribution in a coalition, which is called the "solution" of a cooperative game. By excluding some obviously unreasonable distribution schemes of the cooperative payoff, some researchers proposed some sets of feasible distribution schemes, in which the equilibrium distribution scheme exists, such as the stable set, the core, and the bargaining set (Aumann and Maschler [7], Gillies [8], Shapley [9], von Neumann and Morgenstern [1]). Harsanyi [10], Aumann and Myerson [11] expanded these concepts to introduce the farsightedness. Chwe [12], Ray and Vohra [13], Diamantoudi and Xue [14] further developed the farsightedness notion. However, these literatures did not end up with the cooperative payoff distribution equilibrium in a coalition, as they only provide a set of feasible solutions containing the distribution equilibrium.

Many people tried to obtain a single-point cooperative payoff distribution equilibrium in a coalition, say, the solution to the cooperative game, the main achievements include the Shapley value (Shapley [15]) and the Nucleolus (Schmeidler [16]), etc. However, these models are usually based on some "collectivism", that is to say, in their models, in the distribution process the coalition members are assumed to pursue some collective goal, or adhere to some rule of collectivist behavior. In the non-cooperative game between the coalitions, the members of a coalition do pursue some collective goal, that is, to maximize the cooperative payoff of the coalition. However, the distribution process between the coalition members is a non-cooperative game and there is no synergy between them, rational coalition members will not replace their own welfare goals with a collective one. Nash [17] recognized that the cooperative payoff distribution in a coalition is a non-cooperative game among its members, pointed out that the cooperative payoff distribution equilibrium in a coalition is the result of the bargaining among the members. Under several necessary axiomatic assumptions, Nash showed that the unique bargaining process exists when the Nash axioms are all satisfied, which is so called Nash bargaining solution. Unfortunately, because some Nash axiom is unreasonable, Nash bargaining solution is obviously not the equilibrium of the bargaining game between the coalition members.

In non-fully cooperative game theory, researchers introduced the participation levels of the players. For the first time Aubin [18, 19] introduced the cooperative game with fuzzy coalitions, as the extensions of Auman's research, Butnariu [20], Tsurumi, Tanino and Inuiguchi [21, 22] investigated the Shapley value, Branzei, Dimitrov and Tijs [23] considered the equalizer solution and Sakawa and Nishizaki [24], Molina and Tejiada [25] proposed the lexicographical solution in a cooperative game with fuzzy coalitions.

However, if information is symmetric between the players and they either participate in the cooperation or give up the cooperation with some coalitions, what is the coalition equilibrium in a cooperative game? How a coalition distribute its cooperative payoff?

Chen [26] defined the coalition equilibrium of an information-symmetric cooperative game with agreements implemented by a third party, and inherited Nash's ideas, regarded the cooperative payoff distribution scheme is determined by the equilibrium of the bargaining game among the coalition members. Chen introduced the concept of common payoff of a member set and investigated the equilibrium of the bargaining game on the distribution of the common payoff of the member set. Applying the distribution rule of the common payoffs, Chen proposed the Nash equilibrium of the bargaining game on the distribution of the cooperative payoff of a coalition. Chen [26] examines only cooperative games with agreements implemented by a third party, however, the basic methodology proposed in his paper can still be well applied when agreements are self-implemented: the coalition equilibrium is still the equilibrium of the coalition-choosing game between the players who pursue the maximization of their cooperative payoff distributions. Of course, due to the lack of a third party to implement the agreements, in a cooperative game with agreements self-implemented, each member of a coalition must be trusted by others and won't take opportunistic behavior; when agreements are self-implemented, if the common payoff of a member set of a coalition is well defined, the distribution equilibrium of the common payoff will be determined, thus we can get the equilibrium of the bargaining game on the distribution of the cooperative payoff of a coalition.

On the basis of Chen [26], this paper examines cooperative games with agreements self-implemented, investigates the coalition formation, or, the achievement of the coalition equilibrium in a cooperative game with agreements self-implemented, and the equilibrium of the bargaining game on the distribution of the cooperative payoff of a coalition, when the coalition members are cooperative or non-cooperative in the bargaining game.

A self-implemented cooperation agreement of a coalition means that all the coalition members would take the actions they promised in the cooperation agreement automatically without the supervision of a third party and its punishment when necessary, that is, there would be no opportunism behavior against the cooperation agreement, both in the implementing of the cooperation agreement and in the distribution process.

In Section 2, for the convenience of discussion, the possible opportunistic behaviors of the coalition members in the distribution process are ignored; and it is assumed that the coalitions centralize all the payoffs that their members get in the game in Section 3, and that the coalitions distribute their cooperative payoffs before the game begins in Section 4, to inhibit the opportunistic behaviors of the coalition members in the distribution process.

2. Self-implemented agreements: ignoring the possible opportunistic behaviors in the distribution process

In this section, we'll investigate the coalition formation and the distribution of the cooperative payoff of a coalition in a cooperative game with agreements self-implemented, ignoring the possible opportunistic behaviors of the players in the distribution process.

2.1 *Coalition situations*

First, we define an n-player cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ as follows:

(1) the set of players is $N = \{1, 2, \cdots, n\}$;

(2) the strategy set of any player i is $S_i = \{s_{i1}, s_{i2}, \cdots, s_{im_i}\}$, $i = 1, 2, \cdots, n$;

(3) the payoff function of any player i is u_i, $i = 1, 2, \cdots, n$. When each player i decides his strategy $s_i \in S_i (i = 1, 2, \cdots, n)$, we get a strategic situation in the game, $s = (s_1, s_2, \cdots, s_n) \in \prod_{i \in N} S_i$. As for strategic situation s , the payoff function of player i can be expressed as follows:

$$u_i(s) = u_i(s_1, s_2, \cdots, s_n), \; i = 1, 2, \cdots, n.$$

Suppose that the agreements are self-implemented in the cooperative game, and that the players can freely choose the coalitions they want to join, the players can form up to n non-empty coalitions. Given the arrangement of coalitions, $C_1, C_2, \cdots, C_n$, through free assemble, some coalitions may be empty, but we still define it a coalition (with no member). When each player selects a coalition-choosing strategy, a coalition situation is formed.

In cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, coalition situation $c = (c_1, c_2, \cdots, c_n)$ is defined as the strategic situation when any player i selects strategy c_i in the coalition-choosing game, that is, he chooses to join coalition C_{c_i}.

When all the players select their coalition-choosing strategy independently, the n coalitions they form in different coalition situations may be the same, with the only difference being the order of arrangement. Chen [26] set a special rule on coalition-choosing shown as follows, to ensure that the n coalitions formed in different coalition situations are not duplicated (regardless of the order of coalition arrangement). Of course, this rule does not limit the autonomy of any player in choosing his coalition.

Rule on coalition-choosing (Chen [26]). In the coalition-choosing game of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$, the players are assumed to select their coalition-choosing strategy according to the following rule:

$$c_1 = 1;$$

$$c_2 = 1, 2;$$

$$c_3 = 1, 2, 3 (\text{ if } c_2 \neq 2, c_3 \neq 2);$$

$$\cdots\cdots$$

$$c_i = 1, 2, 3, \cdots, i \ (\text{if } c_j \neq j, c_i \neq j, j < i);$$

$$\cdots\cdots$$

$$c_n = 1, 2, 3, \cdots, n (\text{ if } c_j \neq j, c_n \neq j, j < n).$$

Chen [26] defined the escape-payoff of player $i (i \in C_j)$ in coalition situation $c = (c_1, c_2, \cdots, c_n)$ as the maximum distribution that he can get when he withdraws from coalition C_j he originally belongs to and joins another coalition. When he escapes from coalition C_j and joins some coalition C_k, due to the competition between the coalitions to attract players to join, the cooperative payoff distribution he can get is his marginal contribution to coalition C_k. Obviously, when player i withdraws from coalition C_j, he will join the coalition that can give him the maximum cooperative payoff distribution. Therefore, the escape-payoff of player i in coalition situation $c = (c_1, c_2, \cdots, c_n)$ is defined as follows:

$$w_i (C_j) = \max_{\substack{C_h = C_1, \cdots, C_n \\ C_h \neq C_j}} Mv_i (C_j, C_h),$$

where $w_i (C_j)$, $Mv_i (C_j, C_h)$ are player i's escape-payoff and his marginal contribution to coalition C_h when he escapes from coalition C_j and joins coalition C_h, respectively. Obviously, the escape target coalition chosen by player i at this time is:

$$C^T = \arg\max_{\substack{C_h = C_1, \cdots, C_n \\ C_h \neq C_j}} Mv_i (C_j, C_h).$$

However, if in a cooperative game the agreements are self-implemented rather than implemented by a third party, when player i withdraws from the coalition he belongs to and join another one, other coalitions will not be informed except his escape target coalition. Although his opportunistic behavior is not conducive to the coalition he originally belongs to, it is conducive to improving the cooperative payoff of his escape target coalition, and therefore improving his own cooperative payoff distribution. Next, we will examine the cooperative payoff distribution that player i can get from the escape target coalition when he withdraws from the coalition he originally belongs to and joins another coalition through deviation.

2.2 *Escape-payoff deriving from deviation*

If member i of some coalition carries out opportunistic behaviors after he promises to his coalition to play the equilibrium strategy s_i^* (his equilibrium strategy in the Nash equilibrium of the non-cooperative game among coalitions), member i must get more distribution (which is called his escape-payoff deriving from deviation from the coalition he joins after his opportunistic behavior) than the distribution he can get from his original coalition, otherwise, he has no motivation to betray his original coalition and deviate from his equilibrium strategy s_i^*.

Denote the coalition-choosing game of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ as $\Gamma_T(N, \{C_i\}, \{w_i\})$, assume that when agreements are implemented by a third party, the coalition equilibrium of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ (that is, the equilibrium situation of coalition-choosing game $\Gamma_T(N, \{C_i\}, \{w_i\})$ is $(c_1^*, c_2^*, \cdots, c_i^*, \cdots, c_n^*)$, and we get n coalitions, $C_1^*, C_2^*, \cdots, C_n^*$ (including empty coalitions) in the game. Without loss of generality, assume that in the coalition-choosing game the strategies of players $1, 2, \cdots, i, \cdots, m$ are the same, $c_1^* = c_2^* = \cdots = c_i^* = \cdots = c_m^*$, namely, they form some coalition C in the game. When agreements are implemented by a third party, each member of coalition C will take action to maximize the cooperative payoff of coalition C. Assume that the Nash equilibrium of the non-cooperative game among coalitions is $(s_1^*, s_2^*, \cdots, s_i^*, \cdots, s_n^*)$, and that in coalition C , the distribution vector of members $1, 2, \cdots, i, \cdots, m$ is $x = (x_1, x_2, \cdots, x_i, \cdots, x_m)$. Obviously, when agreements are self-implemented, member i chooses to betray coalition C only because his escape-payoff deriving from deviation is more than his distribution $\tilde{x}_i$ from coalition C.

Obviously, in above distribution vector $x = (x_1, x_2, \cdots, x_i, \cdots, x_m)$, the distribution of some member i should be no less than his escape-payoff, otherwise, he will not choose to join coalition C . However, if member i betrays coalition C and escapes through deviation (when agreements are self-implemented, his opportunistic behavior will not incur punishment), the strategy combination of coalition C may no longer be optimal. If member i chooses to betray coalition C, his escape-payoff deriving from deviation is the maximum of the marginal contributions when he joins other coalitions through deviation.

Player i's marginal contribution to some coalition C_k when he betrays the coalition he originally belongs to and joins coalition C_k is:

$$Mv_i(C_k) = V_{C_k \cup \{i\}} - V_{C_k},$$

where V_{C_k} is the cooperative payoff of coalition C_k in Nash equilibrium s^* of the non-cooperative game between coalitions; $V_{C_k \cup \{i\}}$ is the cooperative payoff of coalition C_k when i chooses to betray coalition C and join coalition C_k through deviation.

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, assume that when player i takes deviation action, except player i himself and members of coalition C_k, other players in the game have no idea, therefore, other players in the game would still keep their equilibrium strategies $s_{-C_k-i}^*$ unchanged. Assume that $s_{C_k}^\circ$, s_i° are the optimal strategic combination of the members of coalition C_k and the optimal strategy of player i when player i takes opportunistic action and withdraws from C, respectively,

$$\left(s_{C_k}^\circ,\ s_i^\circ\right) = \arg \max_{\left(s_{C_k},\ s_i\right)} \left[\sum_{\substack{j \in C_k \\ j \neq i}} u_j\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^*\right) + u_i\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^*\right) \right],$$

$$V_{C_k \cup \{i\}} = \sum_{\substack{j \in C_k \\ j \neq i}} u_j\left(s_{C_k}^\circ,\ s_i^\circ,\ s_{-C_k,\ -i}^*\right) + u_i\left(s_{C_k}^\circ,\ s_i^\circ,\ s_{-C_k,\ -i}^*\right).$$

Therefore, when player i takes opportunistic action and betrays coalition C and joins coalition C_k, his marginal contribution to coalition C_k is:

$$
\begin{aligned}
Mv_i(C_k) =& V_{C_k\cup\{i\}}(s^{\circ}_{C_k},\ s^{\circ}_i,\ s^{*}_{-C_k,\ -i}) - V_{C_k}(s^{*}_{C_k},\ s^{*}_{-C_k}) \\
=& \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^{\circ}_{C_k},\ s^{\circ}_i,\ s^{*}_{-C_k,\ -i}) + u_i(s^{\circ}_{C_k},\ s^{\circ}_i,\ s^{*}_{-C_k,\ -i}) - \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^{*}_{C_k},\ s^{*}_{-C_k}).
\end{aligned}
$$

The cooperative payoff that coalition C_k gets is higher than before after the joining of player i,

$$
V_{C_k\cup\{i\}}\left(s^{\circ}_{C_k},\ s^{\circ}_i,\ s^{*}_{-C_k,\ -i}\right) > V_{C_k}\left(s^{*}_{C_k},\ s^{*}_{-C_k}\right),
$$

and the maximum distribution that coalition C_k can offer to player i is his marginal contribution:

$$
Mv_i\left(C_k\right) = V_{C_k\cup\{i\}}\left(s^{\circ}_{C_k},\ s^{\circ}_i,\ s^{*}_{-C_k,\ -i}\right) - V_{C_k}\left(s^{*}_{C_k},\ s^{*}_{-C_k}\right).
$$

So in coalition situation c the escape-payoff deriving from deviation of player i is

$$
\begin{aligned}
w_i^{-c} =& \max Mv_i\left(C_k\right) \\
=& \max\left\{V_{c_i\cup(i)}\left(s^{\circ}_{c_k},\ s^{\circ}_i,\ s^{*}_{-c_k,\ -i}\right) - V_{c_k}\left(s^{*}_{c_k},\ s^{*}_{-c_k}\right)\right\} \\
=& \max\left\{\sum_{\substack{j\in c_i \\ j\neq i}} u_j\left(s^{\circ}_{C_k},\ s^{\circ}_i,\ s^{*}_{-C_k,\ -i}\right) + u_i\left(s^{\circ}_{C_k},\ s^{\circ}_i,\ s^{*}_{-C_k,\ -i}\right) - \sum_{\substack{j\in C_k \\ j\neq i}} u_j\left(s^{*}_{C_k},\ s^{*}_{-C_k}\right)\right\}, \\
& C_k \neq C,
\end{aligned}
$$

where $\left(s^{\circ}_{C_k},\ s^{\circ}_i\right) = \underset{(s_{C_k},\ s_i)}{\arg\max}\left[\sum_{\substack{j\in C_k \\ j\neq i}} u_j\left(s_{C_k},\ s_i,\ s^{*}_{-C_k,\ i}\right) + u_i\left(s_{C_k},\ s_i,\ s^{*}_{-C_k,\ -i}\right)\right]$.

The marginal loss of coalition C when player i takes opportunistic action and withdraws from C is

$$
\begin{aligned}
\tilde{M}o_i(C) =& V_C - V_{C\setminus\{i\}} \\
=& \sum_{\substack{j\in C \\ j\neq i}} u_j(s^{*}_C,\ s^{*}_{-C}) + u_i(s^{*}_C,\ s^{*}_{-C}) - \sum_{\substack{j\in C \\ j\neq i}} u_j(s^{\circ}_{C_k},\ s^{\circ}_i,\ s^{*}_{-C_k,\ -i}).
\end{aligned}
$$

Apparently, the self-implemented agreement of a coalition must make sure that each of its members has no motivation to betray the coalition, because the opportunistic action of a member usually means the loss of the coalition, and sometimes even serious decrease of the cooperative payoff. So, ignoring the opportunistic behaviors in the cooperative payoff distribution process, in distribution vector $\tilde{x} = (\widetilde{x}_1,\ \widetilde{x}_2,\ \cdots,\ \tilde{x}_i,\ \cdots,\ \widetilde{x}_m)$ of coalition C , distribution $\tilde{x}_i$ of member i must satisfy:

$$Mv_i(C) \geq \tilde{x}_i \geq w_i^{-C},\ i \in C,$$

where the equation holds if and only if there is one and only member in coalition C.

It's easy to prove that when the agreement in a coalition is self-implemented, ignoring the opportunistic behaviors in the cooperative payoff distribution process, to some member i the competitive distribution condition $\tilde{x}_i \geq w_i(i \in C)$ is satisfied at the same time.

Theorem 2.1 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition situation c of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, the escape-payoff of some member $i(i \in C)$ is no more than his escape-payoff deriving from deviation, that is,

$$w_i^{-C} \geq w_i(C),\ i \in C,$$

where the equation holds if and only if there is one and only member in coalition C.

Proof. Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition situation c of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, assume that the equilibrium strategy combination of all the players is $(s_1^*,\ s_2^*,\ \cdots,\ s_i^*,\ \cdots,\ s_n^*)$, if player $i(i \in C)$ escapes from his coalition and join another coalition C_k (not through deviation), the escape-payoff that member i can get is

$$\begin{aligned} w_i(C,\ C_k) &= Mv_i(C,\ C_k) \\ &= V_{C_k \cup \{i\}}(s_{C_k}^{**},\ s_i^{**},\ s_{-C_k,\ -i}^{**}) - V_{C_k}(s_{C_k}^{*},\ s_{-C_k}^{*}) \\ &= \sum_{\substack{j \in C_k \\ j \neq i}} u_j(s_{C_k}^{**},\ s_i^{**},\ s_{-C_k,\ -i}^{**}) + u_i(s_{C_k}^{**},\ s_i^{**},\ s_{-C_k,\ -i}^{**}) - \sum_{\substack{j \in C_k \\ j \neq i}} u_j(s_{C_k}^{*},\ s_{-C_k}^{*}), \\ C_k &\neq C, \end{aligned}$$

where $\left(s_{C_k}^{**},\ s_i^{**},\ s_{-C_k,\ -i}^{**}\right) = (s_1^{**},\ s_2^{**},\ \cdots,\ s_i^{**},\ \cdots,\ s_n^{**})$ is the equilibrium strategy combination of all the players after player i has withdrawn from coalition C and joins coalition C_k.

If player i doesn't give up his coalition-choosing strategy, but betrays coalition C after his joining, player i and the members of coalition C_k which he chooses to join would benefit from his deviation and be convenient to decide their strategic combination: other players in the game will play their original equilibrium strategies $s_{-C_k-i}^{*}$ because of asymmetrical information, so player i and the members of coalition C_k can choose their optimal strategic combination to maximize the cooperative payoff of coalition C_k:

$$\left(s_{C_k}^{\circ},\ s_i^{\circ}\right) = \underset{(s_{C_k},\ s_i)}{\arg\max} \left[\sum_{\substack{j \in C_k \\ j \neq i}} u_j\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^{*}\right) + u_i\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^{*}\right) \right].$$

Therefore,

$$\max_{(s_{C_k},\, s_i)} \{ \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s_{C_k},\ s_i,\ s^*_{-C_k,-i}) + u_i(s_{C_k},\ s_i,\ s^*_{-C_k,-i}) \}$$

$$= \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^\circ_{C_k},\ s^\circ_i,\ s^*_{-C_k,-i}) + u_i(s^\circ_{C_k},\ s^\circ_i,\ s^*_{-C_k,-i})$$

$$\geq \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^{**}_{C_k},\ s^{**}_i,\ s^{**}_{-C_k,-i}) + u_i(s^{**}_{C_k},\ s^{**}_i,\ s^{**}_{-C_k,-i}).$$

Player i's marginal contribution to coalition C_k is:

$$\begin{aligned} Mv_i(C_k) =& V_{C_k\cup\{i\}}(s^\circ_{C_k},\ s^\circ_i,\ s^*_{-C_k,\ -i}) - V_{C_k}(s^*_{C_k},\ s^*_{-C_k}) \\ =& \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^\circ_{C_k},\ s^\circ_i,\ s^*_{-C_k,\ -i}) + u_i(s^\circ_{C_k},\ s^\circ_i,\ s^*_{-C_k,\ -i}) - \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^*_{C_k},\ s^*_{-C_k}) \\ \geq& \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^{**}_{C_k},\ s^{**}_i,\ s^{**}_{-C_k,\ -i}) + u_i(s^{**}_{C_k},\ s^{**}_i,\ s^{**}_{-C_k,\ -i}) - \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^*_{C_k},\ s^*_{-C_k}). \end{aligned}$$

The escape-payoff deriving from deviation of player i satisfies:

$$\begin{aligned} w_i^{-C} =& \max Mv_i(C_k) \\ =& \max\{V_{C_k\cup\{i\}}(s^\circ_{C_k},\ s^\circ_i,\ s^*_{-C_k,\ -i}) - V_{C_k}(s^*_{C_k},\ s^*_{-C_k})\} \\ =& \max\{ \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^\circ_{C_k},\ s^\circ_i,\ s^*_{-C_k,\ -i}) + u_i(s^\circ_{C_k},\ s^\circ_i,\ s^*_{-C_k,\ -i}) - \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^*_{C_k},\ s^*_{-C_k})\} \\ \geq& \max\{ \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^{**}_{C_k},\ s^{**}_i,\ s^{**}_{-C_k,\ -i}) + u_i(s^{**}_{C_k},\ s^{**}_i,\ s^{**}_{-C_k,\ -i}) - \sum_{\substack{j\in C_k \\ j\neq i}} u_j(s^*_{C_k},\ s^*_{-C_k})\} \\ =& w_i(C). \end{aligned}$$

□

Although the cooperative payoff of a coalition is no less than the sum of the escape-payoffs of all its members, it may be less than the sum of the escape-payoffs deriving from deviation of all its members if members are able to betray the coalition through deviation behaviors after they join the coalition when there is no third party implementing agreements. That's to say, in a cooperative game, the agreements implemented by a third party may be not self-implemented.

Furthermore, the agreements of some coalitions perhaps are not self-implemented even though the escape-payoff deriving from deviation of each member is less than his marginal contribution to the coalition he belongs to. For example, in a tribal society in a state of anarchy, when three young men, $a,\ b$ and c cooperated, they were able to hunt a buffalo, but, if anyone was absent, the remaining two would be empty-handed. Assume that in this society, there was also a strong young man, d, who was not good at hunting, but good at stealing (if he knew where the buffalo was hidden, he could steal it). According to the practice of this thief, half of the stolen goods would be left to the one who provided information. In this cooperative game, if $a,\ b$ and c reached a self-implemented agreement, and formed some coalition C, the cooperative payoff of coalition C was a buffalo, and the marginal contribution of each member to the coalition was a buffalo too. If the agreement that these young men reached was implemented by a third party, the distribution of each member was $1/3$ buffalo, the escape-payoff of each member was 0, and the escape-payoff deriving from deviation was 1 buffalo when their agreement was self-implemented (When he betrayed his coalition and formed a new coalition with d, he could help d steal the buffalo, and his marginal contribution to the new coalition is 1 buffalo). In this case, apparently the marginal contribution of any coalition member was no less than his escape-payoff deriving from deviation, the agreement was still unable to be implemented, for the cooperative payoff of the coalition was less than the sum of the escape-payoffs deriving from deviation of all its members, the cooperative payoff of coalition C was not enough to meet the requirements for reservation distribution of all its members.

Inoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition C in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, only if the marginal contribution of each member and the marginal contribution of each member set meet the conditions below, the agreement of the coalition would be self-implemented (the equations hold if and only if there is one and only member in the coalition):

$$Mv_i(C) \geq w_i^{-C},\ i \in C;$$

$$Mv_{T_h}(C) \geq \sum\nolimits_{j \in T_h} w_j^{-C},\ i \in T_h \subseteq C.$$

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, the cooperative payoff distribution vector of any coalition C must satisfy the following conditions:

(1) $Mv_i(C) \geq \tilde{x}_i \geq w_i^{-C},\ i \in C$ (the equations hold if and only if there is one and only member in the coalition);

(2) $\sum_{i \in C} \tilde{x}_i = V_C$,

where condition (1) is referred to as the individual rationality condition (or competitive distribution condition), and condition (2) is referred to as the collective rationality condition.

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, the escape-payoff of player i is no more than his escape-payoff deriving from deviation,

$$w_i^{-C} \geq w_i,\ i \in C,$$

where the equation holds if and only if there is one and only member in the coalition.

Not only that, ignoring the opportunistic behaviors in the cooperative payoff distribution process, the sum of escape-payoffs of the members in any member set of coalition C is no more than the sum of their escape-payoffs deriving from deviation,

$$\sum_{j\in T_h} w_j^{-C} \geq \sum_{j\in T_h} w_j,\ i\in T_h\subseteq C,$$

where the equation holds if and only if there is one and only member in the coalition.

So, if the agreements are self-implemented, the distribution of each member of a coalition must be at least no less than his escape-payoff deriving from deviation, in order to prevent the members from deviating; and the distribution of each member set of the coalition must be at least no less than the sum of the escape-payoffs deriving from deviation of the members in the set, in order to prevent the members in the set from deviating. So, for coalition C, the individual rationality conditions (or competitive distribution conditions) when the agreement is self-implemented are as follows:

$$Mv_i(C) \geq \tilde{x}_i \geq w_i^{-C},\ i\in C;$$

$$Mv_{T_h}(C) \geq \sum_{j\in T_h} \tilde{x}_j \geq \sum_{j\in T_h} w_j^{-C},\ i\in T_h\subseteq C,$$

where the equations hold if and only if there is one and only member in the coalition.

When the agreements are self-implemented, if the cooperative payoff of some coalition is just equal to the sum of escape-payoffs deriving from deviation of all its members, to guarantee the implementation of its agreement, the distribution of each member is only determined by his escape-payoffs deriving from deviation:

$$\tilde{x}_i = w_i^{-C}(i\in C),\ \text{if}\ \sum_{j\in C} u_j(s^*) = \sum_{j\in C} w_j^{-C}.$$

However, this can only happen in a one-member coalition, otherwise, player i cannot join the coalition because his marginal contribution to the coalition is just equal to his reservation distribution, that is to say, player i has no synergy with the coalition.

According to the analysis above on the distribution condition of the self-implemented agreements, an agreement implemented by a third party is not necessarily self-implemented. Although an agreement implemented by a third party can guarantee that the distribution of each member is no less than his escape-payoff, this agreement can’t guarantee that his distribution is always no less than his escape-payoff deriving from deviation, or, it can’t guarantee that the sum of distributions of the members in a member set is always no less than the sum of their escape-payoffs deriving from deviation. Certainly, if the agreement implemented by a third party of a coalition can guarantee that the distribution of each member is at least no less than his escape-payoff deriving from deviation, and that the distribution of each member set of this coalition is at least no less than the sum of the escape-payoffs deriving from deviation of its members by accident, this agreement implemented by a third party is also self-implemented. But this is only a particular case, the more possible case is that when the agreement is implemented by a third party, the cooperative payoff distribution of some member $i(i\in C)$ is less than his escape-payoff deriving from deviation:

$$\tilde{x}_i < w_i^{-C};$$

or the sum of distributions of the members in some member set T_h is less than the sum of their escape-payoffs deriving from deviation:

$$\sum_{j\in T_h}\tilde{x}_j < \sum_{j\in T_h} w_j^{-C};$$

or the sum of the escape-payoffs deriving from deviation of all the members of the coalition is more than its cooperative payoff:

$$\sum_{j\in C} u_j(s^*) < \sum_{j\in C} w_j^{-C}.$$

2.3 *Coalition equilibrium*

When a third party implementing the agreements is absent, that is to say, the agreements are self-implemented in a cooperative game, in each coalition situation, the cooperative payoff distribution equilibrium of a coalition is quite different from the one in the same coalition situation of the game with agreements implemented by a third party, this will lead to a different coalition equilibrium and different distribution equilibria of the coalitions.

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, in some coalition C, if $\sum_{i\in C} u_i(s^*) < \sum_{i\in C} w_i^{-C}$, that is to say, if the sum of the escape-payoffs deriving from deviation of all the members is more than the coalition's cooperative payoff, there must be one or some members whose distributions don't satisfy the distribution condition:

$$Mv_i(C) \geq \tilde{x}_i \geq w_i^{-C}(i \in C).$$

Therefore, because the coalition members' escape-payoffs deriving from deviation are more than their distributions from the coalition, coalition C can't reach its self-implemented agreement.

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, assume that each player in the game aims to minimize his escape-payoff deriving from deviation, if coalition situation $c^* = (c_1^*, c_2^*, \cdots, c_n^*)$ is feasible, and that the coalition-choosing strategy of each player is his best response to the strategy combination of other players, the following coalition situation $c^* = (c_1^*, c_2^*, \cdots, c_n^*)$ is defined as the coalition equilibrium of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other), namely,

$$\forall i = 1, 2, \cdots, n,$$

$$c_i^* = \begin{cases} i, \quad \text{if for any } c_i \neq i,\ w_i^{-C_i}\left(i, c_{-i}^*\right) \leq w_i^{-C_{c_i}}\left(c_i, c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq w_i^{-C_{c_i}}, \ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum\limits_{j\in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\min w_i^{-C_{c_i}}\left(c_i, c_{-i}^*\right), \ \text{ if at least for a certain } c_i \neq i,\ w_i^{-C_i}\left(i, c_{-i}^*\right) > w_i^{-C_{c_i}}\left(c_i, c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > w_i^{-C_{c_i}}, \ \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum\limits_{j\in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}, \end{cases}$$

where c_i^* is player i's equilibrium coalition-choosing strategy; $w_i^{-C_i}(\cdot)$ is player i's escape-payoff deriving from deviation in the corresponding coalition situation, $\left(i,\ c_{-i}^*\right)$; $Mv_i\left(C_{c_i}\right)$, $w_i^{-c_{c_i}}$ are player i's marginal contribution to coalition C_{c_i}, and his escape-payoff deriving from deviation in the corresponding coalition situation $\left(c_i,\ c_{-i}^*\right)$, respectively; $Mv_{T_h}\left(C_{c_i}\right)$, $\sum_{j\in T_h} w_j^{-c_{c_i}}$ are member set T_h's marginal contribution to coalition C_{c_i} to which player i belongs, and the sum of the escape-payoffs deriving from deviation of the members in set T_h in the corresponding coalition situation $\left(c_i,\ c_{-i}^*\right)$, respectively.

That is, keeping coalition-choosing strategy combination c_{-i}^* of other players unchanged, player i has no motivation to unilaterally withdraw from his equilibrium coalition-choosing strategy c_i^*. Here, "$c_i^* = i$, if for any $c_i \neq i$, $w_i^{-C_i}\left(i,\ c_{-i}^*\right) \leq w_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right)$" represents that when player i gets the lowest escape-payoff deriving from deviation in each coalition, he will join C_i (that is, he will take part in the game solely) rather than join another coalition randomly.

Theorem 2.2 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, there exists the coalition equilibrium under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other):

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i, \ \text{if for any } c_i \neq i,\ w_i^{-C_i}\left(i,\ c_{-i}^*\right) \leq w_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq w_i^{-C_{c_i}},\ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum\limits_{j\in T_h} w_j^{-c_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\min w_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right),\ \text{if at least for a certain } c_i \neq i,\ w_i^{-C_i}\left(i,\ c_{-i}^*\right) > w_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > w_i^{-c_{c_i}},\ \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum\limits_{j\in T_h} w_j^{-c_{c_i}},\ i \in T_h \subseteq C_{c_i}, \end{cases}$$

and also there exists the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other):

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i, \ \text{if for any } \ c_i \neq i,\ \tilde{x}_i\left(i,\ c_{-i}^*\right) \geq \tilde{x}_i\left(c_i,\ c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq w_i^{-c_{c_i}},\ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum\limits_{j\in T_h} w_j^{-c_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\max \tilde{x}_i\left(c_i,\ c_{-i}^*\right),\ \text{ if at least for a certain } c_i \neq i,\ \tilde{x}_i\left(i,\ c_{-i}^*\right) < \tilde{x}_i\left(c_i,\ c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > w_i^{-C_{c_i}},\ \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum\limits_{j\in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}, \end{cases}$$

where $\widetilde{x}_i(\cdot)$ is the cooperative payoff distribution that player i gets in the corresponding coalition situation.

Proof. Herein, we only prove that, ignoring the opportunistic behaviors in the cooperative payoff distribution process, in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, there exists the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other):

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i, \text{ if for any } c_i \neq i,\ \tilde{x}_i\left(i, c_{-i}^*\right) \geq \tilde{x}_i\left(c_i, c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq w_i^{-C_{c_i}}, \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\max \tilde{x}_i\left(c_i,\ c_{-i}^*\right), \text{ if at least for a certain } c_i \neq i,\ \tilde{x}_i\left(i,\ c_{-i}^*\right) < \tilde{x}_i\left(c_i,\ c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > w_i^{-C_{c_i}}, \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum_{j \in T_h} w_j^{-c_{c_i}},\ i \in T_h \subseteq C_{c_i}. \end{cases}$$

If in coalition situation c, there is synergy between each player i and his coalition, player i and each member set he belongs to are trusted by his coalition, then coalition situation c is feasible; if in the coalition situation, there is no synergy between some player i and his coalition, or, he or some member set to which he belongs is not trusted by his coalition, coalition situation c is infeasible.

Assume that C is the set of the feasible situations. First and foremost, C must not be an empty set because at least the coalition situation in which each player chooses to create a coalition with only one member is feasible.

In cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$, with agreements self-implemented, when the opportunistic behaviors in the distribution process are ignored, all the players must make a consistent decision through negotiations on selecting coalition situation. Therefore, the probabilities with which all the players select the same coalition situation are the same. Assume that all the players select the feasible coalition situations, $c_1,\ c_2,\ \cdots,\ c_M$ (where M means that there is M feasible coalition situations in the feasible coalition situation set), with an identical probability vector $p = (p_1,\ p_1,\ \cdots,\ p_M)$, the decision-making problem of any player i is

$$\Psi(N,\ p) : \tilde{x}_i(p^*) = \max_{p \in P} \tilde{x}_i(p),\ \forall i \in N.$$

The above decision-making problem can be further expressed as follows:

$$\max_p \tilde{x}_i(p) = \max_p \sum_{I=1}^{M} p_I \tilde{x}_i(c_I).$$

$$s.t. \max_p \tilde{x}_{-i}(p) = \max_p \sum_{I=1}^{M} p_I \tilde{x}_{-i}(c_I);$$

$$(p_1,\ p_2,\ \cdots,\ p_M) |\, p_I(c_I) \geq 0,\ I = 1,\ 2,\ \cdots,\ M,\ p = (p_1,\ p_2,\ \cdots,\ p_M) \in P;$$

$$\sum_{I=1}^{M} p_I = 1.$$

If the optimal solution p^* to the above decision-making problem exists, p^* is the mixed-strategic coalition equilibrium in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented when the opportunistic behaviors in the distribution process are ignored.

By using the Kakutani fixed point theorem, it's easy to prove that the optimal solution to the above decision-making problem exists, however, for the saddle point does not exist, it may not be unique.

Similarly, we can prove that, ignoring the opportunistic behaviors in the cooperative payoff distribution process, when the agreements are self-implemented, there exists the coalition equilibrium in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other):

$$\forall i = 1, 2, \cdots, n,$$

$$c_i^* = \begin{cases} i, \text{ if for any } c_i \neq i, \ w_i^{-C_i}\left(i, c_{-i}^*\right) \leq w_i^{-C_{c_i}}\left(c_i, c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq w_i^{-C_{c_i}}, \ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum_{j \in T_h} w_j^{-C_{c_i}}, \ i \in T_h \subseteq C_{c_i}; \\ \arg\min w_i^{-C_{c_i}}\left(c_i, c_{-i}^*\right), \ \text{ if at least for a certain } c_i \neq i, \ w_i^{-C_i}\left(i, c_{-i}^*\right) > w_i^{-C_{c_i}}\left(c_i, c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > w_i^{-C_{c_i}}, \ \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum_{j \in T_h} w_j^{-C_{c_i}}, \ i \in T_h \subseteq C_{c_i}. \end{cases}$$

□

For any player i, the objective that he pursues when he selects his equilibrium coalition-choosing strategy is not his minimum escape-payoff deriving from deviation but the maximum distribution he can get from the coalition he joins. In the cooperative game with agreements self-implemented, in coalition C the competitive distribution condition means:

$$Mv_i(C) \geq \tilde{x}_i \geq w_i^{-C}, \ i \in C,$$

the equations hold if and only if there is one and only member in coalition C.

In the following, we'll prove that, ignoring the opportunistic behaviors in the cooperative payoff distribution process, in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, if the cooperative payoff distribution scheme of each coalition satisfies the competitive distribution condition mentioned above, the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other) is equivalent to the one under the criterion of minimum escape-payoff deriving from deviation (when members in a coalition trust each other).

Theorem 2.3 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, if the competitive distribution condition is satisfied in all coalitions,

$Mv_i(C) \geq \tilde{x}_i \geq w_i^{-C},\ i \in C$ (the equations hold if and only if there is one and only member in coalition C), the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other) of the game is equivalent to the one under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other) of the game. That is,

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i,\ \text{ if for any } c_i \neq i,\ w_i^{-C_i}\left(i,\ c_{-i}^*\right) \leq w_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq w_i^{-C_{c_i}},\ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\min w_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right),\ \text{ if at least for a certain } c_i \neq i,\ w_i^{-C_i}\left(i,\ c_{-i}^*\right) > w_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > w_i^{-C_{c_i}},\ \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}. \end{cases}$$

$$\Leftrightarrow c_i^* = \begin{cases} i,\ \text{ if for any } c_i \neq i,\ \tilde{x}_i\left(i,\ c_{-i}^*\right) \geq \tilde{x}_i\left(c_i,\ c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq w_i^{-C_{c_i}},\ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\max \tilde{x}_i\left(c_i,\ c_{-i}^*\right),\ \text{ if at least for a certain } c_i \neq i,\ \tilde{x}_i\left(i,\ c_{-i}^*\right) < \tilde{x}_i\left(c_i,\ c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > w_i^{-C_{c_i}},\ \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum_{j \in T_h} w_j^{-c_{c_i}},\ i \in T_h \subseteq C_{c_i}. \end{cases}$$

Proof. Ignoring the opportunistic behaviors in the cooperative payoff distribution process, when competitive distribution condition,

$$Mv_i(C) \geq \tilde{x}_i \geq w_i^{-C}, i \in C,$$

is satisfied in each coalition, obviously,

$$c_i^* = i,\ \text{if for any } c_i \neq i,\ \tilde{x}_i(i,\ c_{-i}^*) \geq \tilde{x}_i(c_i,\ c_{-i}^*),\ \text{or, } Mv_i(C_{c_i}) \leq w_i^{-C_{c_i}},\ \text{or, } Mv_{T_h}(C_{c_i}) \leq \textstyle\sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}.$$

$$\Leftrightarrow c_i^* = i,\ \text{if for any} c_i \neq i,\ w_i^{-C_i}(i,\ c_{-i}^*) \leq w_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \text{or, } Mv_i(C_{c_i}) \leq w_i^{-C_{c_i}},\ \text{or,}$$

$$Mv_{T_h}(C_{c_i}) \leq \textstyle\sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}.$$

To prove Theorem 2.3, what we need to do is to prove that when the agreements are self-implemented,

$$c_i^* = \arg\min w_i^{-C_{c_i}}(c_i,\, c_{-i}^*) \Leftrightarrow c_i^* = \arg\max \tilde{x}_i(c_i,\, c_{-i}^*).$$

That is, when some player i minimizes his escape-payoff deriving from deviation in the coalition equilibrium under the criterion of minimum escape-payoff deriving from deviation, he can get the highest cooperative payoff distribution. Meanwhile, when player i maximizes his distribution in the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other), he can get the lowest escape-payoff deriving from deviation at the same time.

(1) If $c_i^* = \arg\max \tilde{x}_i\left(c_i,\, c_{-i}^*\right)$, according to the definition, coalition situation $c^* = \left(c_i^*,\, c_{-i}^*\right)$ is the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other). In coalition equilibrium $\left(c_i^*,\, c_{-i}^*\right)$, the distribution $\tilde{x}_i(c_i^*,\, c_{-i}^*)$ of player i is assumed to meet the competitive distribution condition:

$$Mv_i(C_{c_i^*})(c_i^*,\, c_{-i}^* \geq \tilde{x}_i(c_i^*,\, c_{-i}^*) \geq w_i^{-C_{c_i^*}}(c_i^*,\, c_{-i}^*).$$

Assume that player i select another coalition-choosing strategy $c_i,\ c_i \neq c_i^*$, according to the definition of the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other),

$$\tilde{x}_i(c_i,\, c_{-i}^*) \leq \tilde{x}_i(c_i^*,\, c_{-i}^*).$$

Therefore, player i won't betray coalition C_{c_i} and join the corresponding coalition $C_{c_i^*}$ in situation $\left(c_i,\, c_{-i}^*\right)$ through deviation. When player i escapes from coalition C_{c_i} via deviation and join coalition $C_{c_i^*}$, his escape-payoff deriving from deviation satisfies:

$$Mv_i(C_{c_i^*}) \leq w_i^{-C_{c_i}}(c_i,\, c_{-i}^*).$$

Coalition $C_{c_i^*}$ wouldn't give a distribution more than $Mv_i(C_{c_i^*})$ to player i, because $Mv_i(C_{c_i^*})$ is the marginal contribution of player i to coalition $C_{c_i^*}$ through his deviation. According to the competitive distribution condition, we have:

$$\tilde{x}(c_i^*,\, c_{-i}^*) \leq Mv_i(C_{c_i^*}).$$

Therefore,

$$w_i^{-C_{c_i^*}}(c_i^*,\, c_{-i}^*) \leq \tilde{x}_i(c_i^*,\, c_{-i}^*) \leq w_i^{-C_{c_i}}(c_i,\, c_{-i}^*).$$

So,

$$w_i^{-C_{c_i^*}}(c_i^*,\ c_{-i}^*) \leq w_i^{-C_{c_i}}(c_i,\ c_{-i}^*),$$

$$c_i^* = \arg\ \max \tilde{x}_i(c_i,\ c_{-i}^*) \Rightarrow c_i^* = \arg\ \min w_i^{-C_{c_i}}(c_i,\ c_{-i}^*).$$

(2) If $c_i^* = \arg\min w_i^{-c_{c_i}}\left(c_i,\ c_{-i}^*\right)$, according to the definition, coalition situation $c^* = \left(c_i^*,\ c_{-i}^*\right)$ is the coalition equilibrium under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other). In this coalition equilibrium, the escape-payoff deriving from deviation of player *i*, $w_i^{-C_{c_i^*}}(c_i^*,\ c_{-i}^*)$, satisfies the competitive distribution condition:

$$\tilde{x}_i(c_i^*, c_{-i}^*) \geq w_i^{-C_{c_i^*}}(c_i^*, c_{-i}^*).$$

Assume that player *i* selects another coalition-choosing strategy $c_i,\ c_i \neq c_i^*$, according to the definition of coalition equilibrium under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other),

$$w_i^{-C_{c_i}}(c_i,\ c_{-i}^*) \geq w_i^{-C_{c_i^*}}(c_i^*,\ c_{-i}^*).$$

In coalition situation $\left(c_i^*,\ c_{-i}^*\right)$ when other players keep their coalition-choosing strategies unchanged, and player *i* selects coalition-choosing strategy c_i,

$$Mv_i\left(C_{c_i}\right) > w_i^{-c_{c_i}},$$

player *i*'s cooperative payoff distribution which he obtains from the corresponding coalition C_{c_i} which he joins meets:

$$\tilde{x}_i(c_i, c_{-i}^*) \leq w_i^{-C_{c_i^*}}(c_i^*, c_{-i}^*),$$

because $w_i^{-C_{c_i^*}}(c_i^*,\ c_{-i}^*)$ is player *i*'s marginal contribution to coalition C_{c_i} through deviation, we have:

$$\tilde{x}_i(c_i^*, c_{-i}^*) \geq \tilde{x}_i(c_i, c_{-i}^*).$$

So,

$$c_i^* = \arg\ \min w_i^{-C_{c_i}}(c_i,\ c_{-i}^*) \Rightarrow c_i^* = \arg\ \max \tilde{x}_i(c_i,\ c_{-i}^*).$$

That is to say,

$$c_i^* = \arg\max \tilde{x}_i(c_i,\ c_{-i}^*),\ \text{if at least for a certain } c_i \neq i,\ \tilde{x}_i(i,\ c_{-i}^*) < \tilde{x}_i(c_i,\ c_{-i}^*),$$

$$\text{and } Mv_i(C_{c_i}) > w_i^{-C_{c_i}},\ Mv_{T_h}(C_{c_i}) > \sum\nolimits_{j\in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}.$$

$$\Leftrightarrow c_i^* = \arg\min w_i^{-C_{c_i}}(c_i,\ c_{-i}^*),\ \text{if at least for a certain } c_i \neq i,\ w_i^{-C_i}(i,\ c_{-i}^*) > w_i^{-C_{c_i}}(c_i,\ c_{-i}^*),$$

$$\text{and } Mv_i(C_{c_i}) > w_i^{-C_{c_i}},\ Mv_{T_h}(C_{c_i}) > \sum\nolimits_{j\in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}.$$

□

Therefore, ignoring the opportunistic behaviors in the cooperative payoff distribution process, in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, there exists the coalition equilibrium under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other). If in a feasible coalition situation, for some coalition C_j, the sum of the escape-payoffs deriving from deviation of the members is more than its cooperative payoff, that is to say, $\sum_{i\in C_j} u_i(s^*) \leq \sum_{i\in C_j} w_i^{-C_j}$, even though the coalition members have a chance to arrive at an agreement implemented by a third party (that is, $\sum_{i\in C_j} u_i(s^*) \geq \sum_{i\in C_j} w_i(C_j)$, where the equation holds if and only if there is one and only member in coalition C_{j}), this agreement can't be self-implemented. If there is no third party to guarantee the implementation of such an agreement, coalition C can't come forth.

Example 2.1 Assume that in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented,

(1) the set of players is $N = \{1,\ 2\}$;

(2) the strategy set of each player i is $S_i = \{s_i^1, s_i^2\},\ i = 1,\ 2$;

(3) each player is risk neutral, and the utility of his income r_i is:

$$u_i(r_i) = r_i.$$

The payoff function of the players can be expressed as the following payoff matrix.

Table 1. The payoff matrix of game $\Gamma(N,\ \{S_i\},\ \{u_i\})$

		Player 2	
		s_2^1	s_2^2
Player 1	s_1^1	0, 50	150, 100
	s_1^2	100, 100	50, 0

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, we will examine the coalition equilibrium of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented.

Without loss of generality, assume that the coalition-choosing strategy of player 1 is $c_1 = 1$, and the coalition-choosing strategy set of player 2 is $\{1,\ 2\}$.

When $c_2 = 1$, in coalition situation $c^1 = (1,\ 1)$, players 1 and 2 will form a non-empty coalition $C_1 = \{1,\ 2\}$, the optimal combination strategy of coalition C_1 is $s^* = (s_1^1,\ s_2^2)$, and its cooperative payoff $V_{C_1} = 250$.

At this time, if player 1 escapes from coalition C_1 through deviation, his target is coalition C_2, his strategy choice is s_1^1 when he deviates. In strategic situation $\left(s_1^1,\ s_2^2\right)$, the cooperative payoff of coalition $C_2\bigcup\{1\}$ is $V_{C_2\bigcup\{1\}}=150$, that's to say, the marginal contribution of player 1 to coalition C_2 is $Mv_1\left(C_2\right)=150$, and this is his escape-payoff deriving from deviation in coalition situation c^1:

$$w_1^{-C_1}\left(c^1\right)=150.$$

If player 2 escapes from coalition C_1 through deviation, his target is coalition C_2, his strategy choice is s_2^2 when he deviates. In strategic situation $\left(s_1^1,\ s_2^2\right)$, the cooperative payoff of coalition $C_2\bigcup\{2\}$ is $V_{C_2U\{2\}}=100$, that's to say, the marginal contribution of player 2 to coalition C_2 is $Mv_2\left(C_2\right)=100$, and this is his escape-payoff deriving from deviation in coalition situation c^1:

$$w_2^{-C_1}\left(c^1\right)=100.$$

In another coalition situation $c^2=(1,\ 2)$, there exist two non-empty coalition $C_1=\{1\}$ and $C_2=\{2\}$, the Nash equilibrium of the non-cooperative game between coalitions C_1 and C_2 is $s^*=\left(\frac{2}{3}s_1^1+\frac{1}{3}s_1^2,\ \frac{1}{2}s_2^1+\frac{3}{2}s_2^2\right)$, the cooperative payoffs of coalitions C_1 and C_2 are $V_{C_1}=75$ and $V_{C_2}=\frac{200}{3}$.

At this time, if player 1 escapes form coalition C_1 through deviation, the target coalition is coalition C_2, the optimal strategy combination of coalition $C_2\cup\{1\}$ is $\left(s_1^1,\ s_2^2\right)$, and the cooperative payoff of coalition $C_2\cup\{1\}$ is $V_{C_2\cup\{1\}}=250$, the marginal contribution of player 1 to coalition C_2 is $M_{v_1}\left(C_2\right)=250-\frac{200}{3}=\frac{550}{3}$, and this is the escape-payoff deriving from deviation of player 1 in coalition situation c^2:

$$w_1^{-C_1}\left(c^2\right)=\frac{550}{3}.$$

If player 2 escapes form coalition C_2 through deviation, the target coalition is coalition C_1, the optimal strategy combination of coalition $C_1\cup\{2\}$ is $\left(s_1^1,\ s_2^2\right)$, and the cooperative payoff of coalition $C_1\cup\{2\}$ is $V_{C_1\cup\{2\}}=250$, the marginal contribution of player 2 to coalition C_1 is $Mv_2\left(C_1\right)=250-75=175$, and this is the escape-payoff deriving from deviation of player 2 in coalition situation c^2:

$$w_2^{-C_2}\left(c^2\right)=175.$$

Given player 1's coalition-choosing strategy $c_1=1$, now we get the players' escape-payoff deriving from deviation matrix shown as follows.

Table 2. The players' escape-payoff deriving from deviation matrix

	$c_2=1$	$c_2=2$
$c_1=1$	150, 100	$\frac{550}{3}$, 175

Obviously, under the criterion of minimum escape-payoff deriving from deviation, the equilibrium coalition-choosing strategy of player 2 is $c_2^* = 1$. In coalition equilibrium $c^* = (1,\ 1)$, the cooperative payoff of coalition C_1 satisfies:

$$V_{C_1} > w_i^{-C_1},\ i = 1,\ 2;$$

$$V_{C_1} \geq \sum_{i=1}^{2} w_i^{-C_1}.$$

Therefore, in coalition equilibrium $c^* = (1,\ 1)$, the members of coalition C_1 trust each other, that's to say, c^* is a feasible coalition situation.

2.4 *Non-cooperative bargaining game on the distribution of the cooperative payoff of a coalition*

Next, we'll investigate the non-cooperative bargaining game of a coalition and the distribution of its cooperative payoff in the coalition equilibrium in a cooperative game with agreements self-implemented. If the agreement of a coalition is self-implemented, the cooperative payoff distribution scheme must meet the requirements of its members for their escape-payoffs deriving from deviation as their reservation distributions, and the cooperative payoff surplus (that is, the difference between the cooperative payoff of the coalition and the sum of the escape-payoffs deriving from deviation of all the coalition members) must be distributed to the coalition members through the bargaining game on the distribution of the cooperative payoff surplus of the coalition, in order to guarantee the collective rationality condition.

In a coalition with a self-implemented agreement, how the cooperative payoff is distributed to the members is very important, as it not only affects their interests, but also affects their strategic choices in the coalition-choosing game (if the competitive distribution condition is not met). Obviously, the Nash equilibrium of the bargaining game of a coalition on the distribution of its cooperative payoff surplus determines the cooperative payoff distribution scheme of the coalition.

We'll examine the bargaining game on the distribution of the cooperative payoff surplus of a coalition. First, we assume that the coalition members are non-cooperative in the bargaining game; Secondly, we assume that the coalition members are cooperative in the game.

2.4.1 *Distribution of the common payoff of a member set*

Assume that the members of a coalition are non-cooperative in the bargaining game, the distribution scheme of cooperative payoff surplus of the coalition is determined by the equilibrium situation of the non-cooperative bargaining game.

In the non-cooperative bargaining game on the distribution of the cooperative payoff surplus of a coalition, members are unallied but solely participate in the game, each member will establish his own threat point, that is, his lowest requirement for the cooperative payoff surplus distribution, if the possible distribution of his is lower than his threat point, he will withdraw from the coalition and join another coalition.

In order to get the Nash equilibrium of the non-cooperative bargaining game on the distribution of the cooperative payoff surplus of a coalition, first we examine the bargaining game on the distribution of the common payoff of a member set.

Chen [26] defined the common payoff of a member set of a coalition when agreements are implemented by a third party in the cooperative game. Similarly, we can define the common payoff of a member set of a coalition when agreements are self-implemented in the cooperative game.

Denote the member set composed of members $m_1,\ m_2,\ \cdots,\ m_k$ of coalition C as $M_{m_1,\ m_2,\ \cdots,\ m_k}$, in the coalition equilibrium of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, now we define the common payoff of set $M_{m_1,\ m_2,\ \cdots,\ m_k}$.

Definition 2.1 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition equilibrium c^* of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, denote the member set composed of members $m_1, m_2, \cdots, m_k$ of coalition C as $M_{m_1, m_2, \cdots, m_k}$ (that is, a subset of member set M of coalition C, $M_{m_1, m_2, \cdots, m_k} \subseteq M(k \leq m)$), the common payoff of member set $M_{m_1, m_2, \cdots, m_k}$ is defined as follows:

$$\theta\left(M_{m_1, m_2, \cdots, m_k}\right) = V_{M_{m_1, m_2, \cdots, m_k}} - \sum_{i=1}^{k} w_{m_i}^{-C} - \sum \theta_{(2)}\left(M_{m_1, m_2, \cdots, m_k}\right) - \cdots - \sum \theta_{(k-1)}\left(M_{m_1, m_2, \cdots, m_k}\right),$$

where $V_{M_{m_1, m_2, \cdots, m_k}}$ represents coalition C's cooperative payoff when other members except those in set $M_{m_1, m_2, \cdots, m_k}$ have escaped from the coalition and as a whole join some coalition which can maximize the sum of their escape-payoffs, while keeping the coalition-choosing strategies of the players outside coalition C unchanged; $\sum \theta_{(j)}\left(M_{m_1, m_2, \cdots, m_k}\right)$ represents the sum of the common payoffs of all subsets of set $M_{m_1, m_2, \cdots, m_k}$ consisting of j members; $\sum_{i=1}^{k} w_{m_i}^{-c}$ represents the sum of the escape-payoffs deriving from deviation of the members in set $M_{m_1, m_2, \cdots, m_k}$.

The cooperative payoff surplus of a coalition can be decomposed into the common payoffs of all subsets of the member set.

Theorem 2.4 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition equilibrium c^* of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ when agreements are self-implemented, the cooperative payoff surplus can be decomposed into the common payoffs of all subsets of the member set of coalition C:

$$V_C - \sum_{i=1}^{m} w_i^{-C} = \sum \theta_{(2)}\left(M_{1, 2, \cdots, m}\right) + \sum \theta_{(3)}\left(M_{1, 2, \cdots, m}\right) + \cdots + \sum \theta_{(m-1)}\left(M_{1, 2, \cdots, m}\right) + \theta\left(M_{1, 2, \cdots, m}\right),$$

where V_C is the cooperative payoff of coalition C in the coalition equilibrium.

According to Definition 2.1, we can easily draw the conclusion in Theorem 2.4 and the proof is omitted.

In a coalition, the cooperative payoff distribution gotten by each member should be the result of bargaining between members. However, in the bargaining game of a coalition, if each member asks for his marginal contribution to the coalition as his distribution requirement, the cooperative payoff of the coalition is not sufficient to meet all the distribution requirements of the members. If each member asks for his escape-payoffs deriving from deviation as his distribution requirement, the cooperative payoff surplus of the coalition has not been distributed.

The reason why we define the common payoff of a member set of a coalition is that it has an important property, that is, when any member withdraws from the coalition, the common payoff of the member sets to which this member belongs will disappear from the cooperative payoff of the coalition. That is to say, if a member withdraws from the coalition, the common payoffs of all member sets to which this member belongs will become 0. This property of the common payoff of a member set is very helpful for us to obtain the distribution equilibrium of the cooperative payoff in a coalition.

Theorem 2.4 shows that the cooperative payoff of a coalition can be decomposed into the common payoffs of all subsets of the member set of this coalition:

$$V_C = \sum_{i=1}^{m} w_{m_i}^{-C} + \sum \theta_{(2)}(M_{m_1, m_2, \cdots, m_m}) + \cdots + \sum \theta_{(m-1)}(M_{m_1, m_2, \cdots, m_m}) + \theta(M_{m_1, m_2, \cdots, m_m})$$

$$= \sum_{i=1}^{m} w_{m_i}^{-C} + \sum_{i_1=1}^{m} \sum_{i_2=1}^{i_1-1} \theta(M_{m_{i_1}, m_{i_2}}) + \sum_{i_1=1}^{m} \sum_{i_2=1}^{i_1-1} \sum_{i_3=1}^{i_2-1} \theta(M_{m_{i_1}, m_{i_2}, m_{i_3}}) + \cdots$$

$$+\sum_{i_1=1}^{m}\cdots\sum_{i_{m-2}=1}^{i_{m-3}-1}\sum_{i_{m-1}=1}^{i_{m-2}-1}\theta(M_{m_{i_1},\,\cdots,\,m_{i_{m-1}}})+\theta(M_{m_1,\,m_2,\,\cdots,\,m_m})$$

$$=\sum_{i=2}^{m}w_{m_i}^{-C}+\sum_{i_1=2}^{m}\sum_{i_2=2}^{i_1-1}\theta(M_{m_{i_1},\,m_{i_2}})+\sum_{i_1=2}^{m}\sum_{i_2=2}^{i_1-1}\sum_{i_3=2}^{i_2-1}\theta(M_{m_{i_1},\,m_{i_2},\,m_{i_3}})+\cdots$$

$$+\sum_{i_1=2}^{m}\cdots\sum_{i_{m-2}=2}^{i_{m-3}-1}\sum_{i_{m-1}=2}^{i_{m-2}-1}\theta(M_{m_{i_1},\,\cdots,\,m_{i_{m-1}}})+\theta(M_{m_2,\,\cdots,\,m_m})$$

$$+w_{m_1}^{-C}+\sum_{i_2=2}^{m}\theta(M_{m_1,\,m_{i_2}})+\sum_{i_2=2}^{m}\sum_{i_3=2}^{i_2-1}\theta(M_{m_1,\,m_{i_2},\,m_{i_3}})+\cdots$$

$$+\sum_{i_2=2}^{m}\cdots\sum_{i_{m-2}=2}^{i_{m-3}-1}\sum_{i_{m-1}=2}^{i_{m-2}-1}\theta(M_{m_1,\,m_{i_2},\,\cdots,\,m_{i_{m-1}}})+\theta(M_{m_1,\,m_2,\,\cdots,\,m_m}).$$

Without losing generality, assume that some member m_1 withdraws from the coalition, the member set of the coalition changes to be

$$M_{m_2,\,\cdots,\,m_m}=M_{m_1,\,m_2,\,\cdots,\,m_m}\backslash\{m_1\},$$

and the cooperative payoff of coalition C changes to be:

$$V_{M_{m_2,\,\cdots,\,m_m}}=V_{M_{m_1,\,m_2,\,\cdots,\,m_m\backslash\{m_1\}}}$$

$$=\sum_{i=2}^{m}w_{m_i}^{-C}+\sum\theta_{(2)}(M_{m_2,\,\cdots,\,m_m})+\cdots+\sum\theta_{(m-2)}(M_{m_2,\,\cdots,\,m_m})+\theta(M_{m_2,\,\cdots,\,m_m})$$

$$=\sum_{i=2}^{m}w_{m_i}^{-C}+\sum_{i_1=2}^{m}\sum_{i_2=2}^{i_1-1}\theta(M_{m_{i_1},\,m_{i_2}})+\sum_{i_1=2}^{m}\sum_{i_2=2}^{i_1-1}\sum_{i_3=2}^{i_2-1}\theta(M_{m_{i_1},\,m_{i_2},\,m_{i_3}})+\cdots$$

$$+\sum_{i_1=2}^{m}\cdots\sum_{i_{m-2}=2}^{i_{m-3}-1}\sum_{i_{m-1}=2}^{i_{m-2}-1}\theta(M_{m_{i_1},\,\cdots,\,m_{i_{m-2}}})+\theta(M_{m_2,\,\cdots,\,m_m}).$$

When member m_1 withdraws from the coalition, in cooperative payoff of coalition C, $w_{m_1}^{-C}$, and all items in expression $\sum_{i_2=2}^{m}\theta(M_{m_1,\,m_{i_2}})+\sum_{i_2=2}^{m}\sum_{i_3=2}^{i_2-1}\theta(M_{m_1,\,m_{i_2},\,m_{i_3}})+\cdots+\sum_{i_2=2}^{m}\cdots\sum_{i_{m-2}=2}^{i_{m-3}-1}\sum_{i_{m-1}=2}^{i_{m-2}-1}\theta(M_{m_1,\,m_{i_2},\,\cdots,\,m_{i_{m-1}}})+\theta(M_{m_1,\,m_2,\,\cdots,\,m_m})$ disappear. That's to say, when any member m_1 in the member set withdraws from coalition C, the escape-payoffs deriving from deviation of member m_1, and all the common payoffs of member sets that m_1 is involved disappear completely. This property of common payoff makes that the members in the member set evenly distribute the common payoff (Theorem 2.5). When we consider the cooperative payoff surplus distribution of a member as the sum of his common payoff distributions of all the member sets to which he belongs, applying this conclusion to the bargaining game on the distribution of the cooperative payoff of coalition C , we can get the distribution equilibrium in coalition C.

In Theorem 2.5, we will show that common-payoff $\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$ is distributed on average among the members in member set $M_{m_1,\, m_2,\, \cdots,\, m_k}$ through the non-cooperative bargaining game.

Theorem 2.5 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, assume that member set $M_{m_1,\, m_2,\, \cdots,\, m_k}$ is a subset of member set M of coalition C, which is composed of members $m_1,\, m_2,\, \cdots,\, m_k$, $M_{m_1,\, m_2,\, \cdots,\, m_k} \subseteq M (k \leq m)$, in coalition equilibrium c^* of cooperative game $\Gamma(N,\, \{S_i\},\, \{u_i\})$ with agreements self-implemented, the Nash equilibrium of the non-cooperative bargaining game on the distribution of common payoff $\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$ of member subset $M_{m_1,\, m_2,\, \cdots,\, m_k}$ is:

$$t_{m_i}^* = \frac{1}{k}\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right),\; i = 1,\, 2,\, \cdots,\, k$$

That's to say, all players in member set $M_{m_1,\, m_2,\, \cdots,\, m_k}$ will be evenly distributed the common payoff.

Proof. In the bargaining game between members $m_1,\, m_2,\, \cdots,\, m_k$ on the distribution of common payoff $\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$, members in set $M_{m_1,\, m_2,\, \cdots,\, m_k}$ must be non-cooperative. If members are allied in the bargaining game, the only reason why some members form a coalition is that when they form a coalition to escape from the member set, as a whole, their marginal contribution to the target coalition is greater than the sum of their respective marginal contributions when they escape independently. That is to say, the coalition they forms is beneficial for raising their bargaining chip. However, according to Definition 2.1, when measuring the common payoff of a member set, it is assumed that all the escaping members withdraws towards the same target coalition as a whole. Therefore, in the distribution process of common payoff of a member set, it is meaningless for the members to cooperate. In the bargaining game on the distribution of common payoff $\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$, each member establishes his threat point, when his distribution is below the threat point, he will escape from coalition C.

Assume that the final requirements for distribution proposed by members $m_1,\, m_2,\, \cdots,\, m_k$ are $s_{m_1},\, s_{m_2},\, \cdots,\, s_{m_k}$, respectively; and that $t_{m_1},\, t_{m_2},\, \cdots,\, t_{m_k}$ are their threat points, respectively. It is obvious that $t_{m_1},\, t_{m_2},\, \cdots,\, t_{m_k},\, s_{m_1},\, s_{m_2},\, \cdots,\, s_{m_k}$ are all continuous on the interval $\left[0,\; \theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)\right]$. Obviously,

$$s_{m_1} + s_{m_2} + \cdots + s_{m_k} \geq \theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right).$$

If $t_{m_1} + t_{m_2} + \cdots + t_{m_k} < \theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$, $t_{m_1},\, t_{m_2},\, \cdots,\, t_{m_k}$ will not become the final requirements for distribution, at $s_{m_1} + s_{m_2} + \cdots + s_{m_k} = \theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$, members in set $M_{m_1,\, m_2,\, \cdots,\, m_k}$ will come to an agreement.

In this bargaining game, when each member m_i selects his strategy s_{m_i}, strategic situation $\left(s_{m_1},\, s_{m_2},\, \cdots,\, s_{m_k}\right)$ is formed. Note that if the agreement between the members cannot be reached, say, $s_{m_1} + s_{m_2} + \cdots + s_{m_k} > \theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$, the common payoff of this member set will disappear, then in strategic situation $\left(s_{m_1},\, s_{m_2},\, \cdots,\, s_{m_k}\right)$, the payoff function of any member m_i is as follows:

$$u_{m_i}\left(t_{m_i}\right) = \begin{cases} s_{m_i}, & \text{if } s_{m_1} + s_{m_2} + \cdots + s_{m_k} = \theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right); \\ 0, & \text{if } s_{m_1} + s_{m_2} + \cdots + s_{m_k} > \theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right). \end{cases}$$

If $s_{m_1} + s_{m_2} + \cdots + s_{m_k} > \theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$, members in set $M_{m_1,\, m_2,\, \cdots,\, m_k}$ cannot come to an agreement. At this point, the cooperative relationship cannot be formed, such a situation cannot occur. therefore,

$$u_{m_i}(t_{m_i}) = s_{m_i}\left(s_{m_1} + s_{m_2} + \cdots + s_{m_k} = \theta\left(M_{m_1,\, m_2,\, \cdots m_k}\right)\right).$$

It's easy to know that players with identical strategy set and identical payoff function can get identical payoff in the Nash equilibrium of a non-cooperative game. Therefore, in the above bargaining game,

$$s^*_{m_1} = s^*_{m_2} = \cdots = s^*_{m_k}.$$

Thus,

$$s^*_{m_1} = s^*_{m_2} = \ldots = s^*_{m_k} = \frac{1}{k}\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$$

Obviously, that the final requirements for distribution of all members is common knowledge, therefore each member in set $\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$ will set an identical threat point,

$$t^*_{m_i} = \frac{1}{k}\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right),\ i = 1,\ 2,\ \cdots,\ k.$$

□

Next, applying the distribution rule of the common payoff of a member set mentioned above, we will examine the distribution equilibrium of the cooperative payoff of a coalition.

2.4.2 *Distribution of the cooperative payoff surplus of a coalition*

According to the distribution rule of the common payoff of a member set shown in Theorem 2.5, we can draw the distribution rule of the cooperative payoff surplus of a coalition in the coalition equilibrium of a cooperative game with agreements self-implemented.

In coalition C with a self-implemented agreement, the distribution of cooperative payoff surplus of some member $m_i\,(m_i \in C)$ is:

$$\tilde{y}_{m_i} = \frac{1}{2}\sum_{\substack{m_j=1\\ m_j \neq m_i}}^{m} \theta\left(M_{m_i,\, m_j}\right) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j \neq m_i}}^{m}\ \sum_{\substack{m_k=1\\ m_k \neq m_i}}^{m_j-1} \theta\left(M_{m_i,\, m_j,\, m_k}\right) + \cdots + \frac{1}{m}\theta\left(M_{1,\, 2,\, \cdots,\, m}\right),$$

where $\theta\left(M_{m_i,\, m_j}\right)$, $\theta\left(M_{m_i,\, m_j,\, m_k}\right)$, $\ldots$, $\theta\left(M_{1,\, 2,\, \cdots,\, m}\right)$ are the common payoffs of the corresponding member sets, respectively.

The total distribution that member m_i can get is:

$$\tilde{x}_{m_i} = \tilde{y}_{m_i} + w_{m_i}^{-C}$$

$$= w_{m_i}^{-C} + \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta\left(M_{m_i,\, m_j}\right) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta\left(M_{m_i,\, m_j,\, m_k}\right) + \cdots + \frac{1}{m}\theta\left(M_{1,\, 2,\, \cdots,\, m}\right).$$

Member m_i's marginal contribution to the coalition is no less than his cooperative payoff distribution:

$$Mv_{m_i} = w_{m_i}^{-C} + \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \theta(M_{1,\, 2,\, \cdots,\, m})$$

$$\geq \bar{x}_{m_i}.$$

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, when the agreements are self-implemented, by applying the distribution rule of common payoff, we obtained a distribution scheme of the cooperative payoff surplus of coalition C. Theorem 2.6 shows that this distribution scheme is just the equilibrium of the bargaining game of coalition C on the distribution of its cooperative payoff surplus.

Theorem 2.6 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition equilibrium c^* of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, assume that in the bargaining game of coalition C on the distribution of its cooperative payoff surplus, members are non-cooperative, the distribution obtained by each member $m_i\,(m_i \in C)$,

$$\tilde{y}_{m_i} = \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta(M_{1,\, 2,\, \cdots,\, m}),$$

$$m_i = 1,\ 2,\ \cdots,\ m,$$

is the pure-strategic equilibrium of the bargaining game of coalition C, where $\theta\left(M_{m_i,\, m_j}\right),\ \theta\left(M_{m_i,\, m_j,\, m_k}\right),\ \ldots,\ \theta\left(M_{1,\, 2,\, \cdots,\, m}\right)$ are the common payoffs of the corresponding member sets, respectively.

Proof. It's easy to show that the cooperative payoff surplus distribution scheme,

$$\tilde{y}_{m_i} = \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta\left(M_{m_i,\, m_j}\right) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta\left(M_{m_i,\, m_j,\, m_k}\right) + \cdots + \frac{1}{m}\theta\left(M_{1,\, 2,\, \cdots,\, m}\right),$$

$$m_i = 1,\ 2,\ \cdots,\ m,$$

meets both the individual rationality condition and the collective rationality condition.

Denote the final requirement for the cooperative payoff surplus distribution (that is, his strategy in the bargaining game) of each member $m_i\,(m_i \in C)$ as t_{m_i}, t_{m_i} is a continuous variable on the interval $\left[0,\ V_C - \sum_{m_i=1}^{m} w_{m_i}^{-C}\right]$. In the following, we will show that the above distribution scheme is the distribution equilibrium, that's to say,

$$t_{m_i}^* = \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta(M_{1,\, 2,\, \cdots,\, m}),$$

$$m_i = 1,\ 2,\ \cdots,\ m,$$

is the equilibrium of the bargaining game of coalition C, and this equilibrium is unique.

Keeping the strategy combination $t_{-m_i}^*$ of other members unchanged, if the strategy t_m played by player m_i meets:

$$t_{m_i} > \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta(M_{1,\, 2,\, \cdots,\, m}),$$

the cooperation agreement between member m_i and coalition C cannot be reached; on the other hand, if the strategy t_{m_i} played by player m_i meets:

$$t < \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta(M_{1,\, 2,\, \cdots,\, m}),$$

although the cooperation agreement between member m_i and coalition C cannot be reached, member m_i will get a distribution less than $\tilde{y}_{m_i}$, because

$$\tilde{y}_{m_i} < \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m} \theta\left(M_{m_i, m_j}\right) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1} \theta\left(M_{m_i,\, m_j,\, m_k}\right) + \cdots + \frac{1}{m}\theta\left(M_{1,\, 2,\, \cdots,\, m}\right).$$

Therefore, member m_i will not unilaterally deviate from the strategic situation $\left(t_{m_i}^*,\ t_{-m_i}^*\right)$, according to the definition of Nash equilibrium, situation $\left(t_{m_i}^*,\ t_{-m_i}^*\right)$ is the Nash equilibrium of the bargaining game of coalition C.

Debreu [27], Glicksberg, Burgess and Gochberg [28] and Fan [29] have shown that if the payoffs of all players are continuous, quasi-concave functions, and the strategic space is continuous and quasi-concave, the Nash equilibrium of a non-cooperative game is pure-strategic rather than mixed-strategic.

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition equilibrium c^* of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, it is easy to verify that in the bargaining game of coalition C on the distribution of its cooperative payoff surplus, the strategic space and the payoff functions of all players satisfy the conditions mentioned above. Therefore, Nash equilibrium of the bargaining game is a pure-strategic one. □

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in the Nash equilibrium of the bargaining game of coalition C, the competitive distribution condition must be met, that is to say,

$$Mv_i(C) \geq \tilde{x}_i \geq w_i^{-C},\ i \in C.$$

Theorem 2.7 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, assume that in the Nash equilibrium,

$$t_{m_i}^* = \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \theta(M_{1,\, 2,\, \cdots,\, m}),$$

$$m_i = 1,\ 2,\ \cdots,\ m,$$

of the bargaining game of coalition C on the distribution of its cooperative payoff surplus members are non-cooperative, in the non-cooperative bargaining game of coalition C, the competitive distribution condition must be met, that is to say,

$$Mv_{m_i}(C) \geqslant \tilde{x}_{m_i} \geqslant w_{m_i}^{-c},\ m_i \in C,$$

where the equations hold if and only if there is one and only member in coalition C.

Proof. When members are non-cooperative in the bargaining game of coalition C on the distribution of its cooperative payoff surplus, in the pure-strategic equilibrium,

$$t_{m_i}^* = \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta(M_{1,\, 2,\, \cdots,\, m}),$$

$$m_i = 1,\ 2,\ \cdots,\ m,$$

the cooperative payoff surplus distribution obtained by each member $m_i\,(m_i \in C)$ is

$$\tilde{y}_{m_i} = \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta(M_{1,\, 2,\, \cdots,\, m}).$$

And his total cooperative payoff distribution is the sum of his reservation payoff and his cooperative payoff surplus distribution:

$$\tilde{x}_{m_i} = w_{m_i}^{-C} + \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta\left(M_{m_i,\, m_j}\right) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta\left(M_{m_i,\, m_j,\, m_k}\right) + \cdots + \frac{1}{m}\theta\left(M_{1,\, 2,\, \cdots,\, m}\right).$$

Because common payoff $\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$ is a part of the synergy among the members in set $M_{m_1,\, m_2,\, \cdots,\, m_k}$, obviously,

$$\theta_{(j)}\left(M_{m_1, m_2,\, \cdots,\, m_k}\right) \geq 0,$$

then,

$$\tilde{x}_{m_i} \geq w_{m_i}^{-C}.$$

Member m_i's marginal contribution of to his coalition should meet:

$$Mv_{m_i}(C) = w_{m_i}^{-C} + \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \theta(M_{1,\, 2,\, \cdots,\, m})$$

$$\geq w_{m_i}^{-C} + \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta(M_{1,\, 2,\, \cdots,\, m})$$

$$= \tilde{x}_{m_i}(C).$$

□

Example 2.2 Take the previous example, we'll examine the distribution of the cooperative payoff of coalition C_1 under the coalition equilibrium c^* of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented.

According to the definition of the common payoff of a member set, the common payoff of member set $M = \{1,\ 2\}$ of coalition C_1 is:

$$\theta_M = V_{C_1} - \sum_{i=1}^{2} w_i^{-C_i} = 250 - 150 - 100 = 0.$$

Therefore, in the coalition equilibrium c^*, the cooperative payoff distributions that players 1 and 2 get are

$$\tilde{x}_1 = w_1^{-C_1} + \frac{1}{2}\theta_M = 150,$$

$$\tilde{x}_2 = w_1^{-C_1} + \frac{1}{2}\theta_M = 100,$$

respectively.

2.5 *Cooperative bargaining game on the distribution of the cooperative payoff of a coalition*

In this section, we'll investigate the Nash equilibrium of the cooperative bargaining game on the distribution of the cooperative payoff surplus among the members of some coalition C, when the members of the coalition are cooperative and the agreement of the coalition is self-implemented. In the cooperative bargaining game of coalition C, the m coalition members will form m coalitions (including empty ones) to take part in the bargaining game as the players of the game.

2.5.1 *Distribution of the common payoff of a coalition set*

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in the coalition equilibrium of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, assume that members are cooperative in the bargaining game of coalition C, what is distribution equilibrium?

In coalition situation t of cooperative bargaining game of coalition C, denote a coalition set consisting of coalitions $m_1, m_2, \cdots, m_k$ as $M_{m_1, m_2, \cdots, m_k}$, the common payoff $\theta\left(M_{m_1, m_2, \cdots, m_k}\right)$ of coalition set $M_{m_1, m_2, \cdots, m_k}$ is defined as follows:

$$\theta\left(M_{m_1, m_2, \cdots, m_k}\right) = V_{M_{m_1, m_2, \cdots, m_k}} - \sum_{i=1}^{k} w_{m_i}^{-C} - \sum \theta_{(2)}\left(M_{m_1, m_2, \cdots, m_k}\right) - \cdots - \sum \theta_{(k-1)}\left(M_{m_1, m_2, \cdots, m_k}\right),$$

where $V_{M_{m_1, w_2, \ldots, m_k}}$ represents the cooperative payoff that coalition C gets when all players except members in the coalitions in set $M_{m_1, m_2, \cdots, m_k}$ have escaped from coalition C and as a whole join another coalition which can bring them the largest escape-payoff, keeping the coalition-choosing strategies of the players outside coalition C unchanged; $\sum \theta_{(j)}\left(M_{m_1, m_2, \ldots, m_k}\right)$ represents the sums of the common payoffs of all coalitions in subsets of set $M_{m_1, m_2, \cdots, m_k}$ composed of j coalitions; $\sum_{i=1}^{k} w_{m_i}^{-C}$ represents the sums of the coalition members' escape- payoffs deriving from deviation in the coalitions in set $M_{m_1, m_2, \cdots, m_k}$.

Theorem 2.8 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in the coalition equilibrium of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, in coalition situation t of the cooperative bargaining game of coalition C, the distribution equilibrium is that all coalitions $m_1, m_2, \cdots, m_k$ will distribute common payoff $\theta\left(M_{m_1, m_2, \cdots, m_k}\right)$ equally:

$$\tilde{y}_{m_i}^{*} = \frac{1}{k}\theta\left(M_{m_1, m_2, \cdots, m_k}\right), \ i = 1, 2, \cdots, k.$$

Similar to the proof of Theorem 2.5, it is easy to draw the conclusion in Theorem 2.8, which is omitted here.

Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition situation t of cooperative bargaining game $\Gamma(M, \{T_i\}, \{\tilde{x}_i\})$ of coalition C on its cooperative payoff surplus distribution, when the agreement is self-implemented, according to Theorem 2.8, the distribution of cooperative payoff surplus each coalition m_i obtains in the distribution equilibrium is:

$$\tilde{y}_{m_i} = \frac{1}{2}\sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta\left(M_{m_i, m_j}\right) + \frac{1}{3}\sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j - 1} \theta\left(M_{m_i, m_j, m_k}\right) + \cdots + \frac{1}{m}\theta\left(M_{1, 2, \cdots, m}\right).$$

And its total cooperative payoff distribution is:

$$\tilde{x}_{m_i} = w_{m_i}^{-C} + \frac{1}{2} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta\left(M_{m_i,\, m_j}\right) + \frac{1}{3} \sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta\left(M_{m_i,\, m_j,\, m_k}\right) + \cdots + \frac{1}{m} \theta\left(M_{1,\, 2,\, \cdots,\, m}\right).$$

2.5.2 *Coalition equilibrium of the cooperative bargaining game*

In this section, we will examine the coalition equilibrium of the bargaining game of coalition C. Theorem 2.9 indicates the existence of the coalition equilibrium of cooperative bargaining game $\Gamma(M,\, \{T_i\},\, \{\tilde{x}_i\})$ of coalition C.

Theorem 2.9 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition equilibrium c^* of cooperative game $\Gamma(N,\, \{S_i\},\, \{u_i\})$ with agreements self-implemented, there exists the coalition equilibrium under the criterion of maximum cooperative payoff distribution in cooperative bargaining game $\Gamma(M,\, \{T_i\},\, \{\tilde{x}_i\})$ of coalition C,

$$\forall i = 1,\, 2,\, \cdots,\, m,$$

$$t_i^* = \begin{cases} i, \text{ if for any } t_i,\; \tilde{x}_i\left(i,\, t_{-i}^*\right) \geq \tilde{x}_i\left(t_i,\, t_{-i}^*\right); \\ \arg\max \tilde{x}_i\left(t_i,\, t_{-i}^*\right), \text{ if at least for a certain } t_i \neq i,\; \tilde{x}_i\left(i,\, t_{-i}^*\right) < \tilde{x}_i\left(t_i,\, t_{-i}^*\right), \end{cases}$$

and also there exists the coalition equilibrium under the criterion of minimum escape-payoff,

$$\forall i = 1,\, 2,\, \cdots,\, m,$$

$$t_i^* = \begin{cases} i, \text{ if for any } t_i,\; w_i\left(T_{t_i}\right)\left(i,\, t_{-i}^*\right) \leq w_i\left(T_{t_i}\right)\left(t_i,\, t_{-i}^*\right); \\ \arg\min w_i\left(T_{t_i}\right)\left(t_i,\, t_{-i}^*\right), \text{ if at least for a certain } t_i \neq i,\; w_i\left(T_{t_i}\right)\left(i,\, t_{-i}^*\right) > w_i\left(T_{t_i}\right)\left(t_i,\, t_{-i}^*\right). \end{cases}$$

If the distribution scheme of each coalition meets the competitive distribution condition, the above two coalition equilibria are equivalent, that's to say,

$$\forall i = 1,\, 2,\, \cdots,\, m,$$

$$t_i^* = \begin{cases} i, \text{ if for any } t_i,\; \tilde{x}_i\left(i,\, t_{-i}^*\right) \geq \tilde{x}_i\left(t_i,\, t_{-i}^*\right); \\ \arg\max \tilde{x}_i\left(t_i,\, t_{-i}^*\right), \text{ if at least for a certain } t_i \neq i,\; \tilde{x}_i\left(i,\, t_{-i}^*\right) < \tilde{x}_i\left(t_i,\, t_{-i}^*\right). \end{cases}$$

$$\Leftrightarrow t_i^* = \begin{cases} i, \text{ if for any } t_i,\ w_i(T_{t_i})(i,\ t_{-i}^*) \le w_i(T_{t_i})(t_i,\ t_{-i}^*); \\ \\ \arg\min w_i(T_{t_i})(t_i,\ t_{-i}^*), \text{ if at least for a certain } t_i\ \neq i,\ w_i(T_{t_i})(i,\ t_{-i}^*) > w_i(T_{t_i})(t_i,\ t_{-i}^*). \end{cases}$$

Chen [26] proved that in the coalition equilibrium of a cooperative game with agreements implemented third party, there exist the coalition equilibrium under the criterion of maximum cooperative payoff distribution, and also the one under the criterion of minimum escape-payoff in the cooperative bargaining game of a coalition, and indicated the above two equilibria are equivalent. Similarly, we can prove Theorem 2.9 when agreements are self-implemented. For the proof of Theorem 2.9, please refer to Chen [26].

Now we examine the distribution equilibrium in cooperative bargaining game $\Gamma(M,\ \{T_i\},\ \{\tilde{x}_i\})$ of coalition C. Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition equilibrium t^* of cooperative bargaining game $\Gamma(M,\ \{T_i\},\ \{\tilde{x}_i\})$ of coalition C, the distribution of cooperative payoff surplus obtained by each coalition m_i^* $(m_i^* \in C)$ is:

$$\tilde{y}_{m_i^*} = \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \neq m_i^*}}^{m} \theta(M_{m_i^*,\ m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \neq m_i^*}}^{m}\ \sum_{\substack{m_k^*=1 \\ m_k^* \neq m_i^*}}^{m_j^*-1} \theta(M_{m_k^*,\ m_j^*,\ m_k^*}) + \cdots + \frac{1}{m}\theta(M_{1,\ 2,\ \cdots,\ m}).$$

And its total cooperative payoff distribution is:

$$\begin{aligned} \tilde{x}_{m_i^*} &= \tilde{y}_{m_i^*} + w_{m_i^*}^{-C} \\ &= w_{m_i^*}^{-C} + \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \neq m_i^*}}^{m} \theta(M_{m_i^*,\ m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \neq m_i^*}}^{m}\ \sum_{\substack{m_k^*=1 \\ m_k^* \neq m_i^*}}^{m_j^*-1} \theta(M_{m_k^*,\ m_j^*,\ m_k^*}) + \cdots + \frac{1}{m}\theta(M_{1,\ 2,\ \cdots,\ m}). \end{aligned}$$

Theorem 2.10 Ignoring the opportunistic behaviors in the cooperative payoff distribution process, in coalition equilibrium t^* of the cooperative bargaining game of coalition C, the distribution of cooperative payoff surplus obtained by each coalition m_i^* $(m_i^* \in C)$ is,

$$\tilde{y}_{m_i^*} = \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \neq m_i^*}}^{m} \theta(M_{m_i^*,\ m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \neq m_i^*}}^{m}\ \sum_{\substack{m_k^*=1 \\ m_k^* \neq m_i^*}}^{m_j^*-1} \theta(M_{m_k^*,\ m_j^*,\ m_k^*}) + \cdots + \frac{1}{m}\theta(M_{1,\ 2,\ \cdots,\ m}),$$

where $\theta\left(M_{m_i^*,\ m_j^m}\right),\ \theta\left(M_{m_i^*,\ m_j^*,\ m_k^*}\right),\ \ldots,\ \theta\left(M_{1,\ 2,\ \cdots,\ m}\right)$ represent the common payoffs of the corresponding coalition sets, respectively.

And the final requirement for the cooperative payoff surplus distribution of each coalition m_i^*,

$$t^*_{m^*_i}=\frac{1}{2}\sum_{\substack{m^*_j=1\\ m^*_j\neq m^*_i}}^{m}\theta(M_{m^*_i,\, m^*_j})+\frac{1}{3}\sum_{\substack{m^*_j=1\\ m^*_j\neq m^*_i}}^{m}\sum_{\substack{m^*_k=1\\ m^*_k\neq m^*_i}}^{m^*_j-1}\theta(M_{m^*_i,\, m^*_j,\, m^*_k})+\cdots+\frac{1}{m}\theta(M_{1,\,2,\,\cdots,\,m}),$$

$$m^*_i=1,\ 2,\ \cdots,\ m,$$

is the pure-strategic equilibrium of the cooperative bargaining game.

Similar to the proof of Theorem 2.6, it is easy to draw the conclusion in Theorem 2.10, which is omitted here.

Through the above analysis, we can obtain the distribution equilibrium of the cooperative bargaining game of coalition C on its cooperative payoff distribution when its coalition equilibrium is reached. However, at this time what we get is the cooperative payoff surplus distribution of each coalition rather than the cooperative payoff distribution of each member of coalition C.

If there are more than 2 members in some coalition, we cannot directly get the distribution of cooperative payoff surplus of each member. We need to further examine the (second level) cooperative bargaining game of this coalition in order to obtain the cooperative payoff surplus of each (second level) coalition under the coalition equilibrium of the (second level) cooperative bargaining game. Step by step, we will finally get the cooperative payoff surplus distribution of each member of coalition C in the coalition equilibrium of the cooperative bargaining game of the coalition.

3. Self-implemented agreements: coalitions centralize all the payoffs that their members get in the game

In the above discussion on self-implemented agreements, it's assumed that, in the cooperative payoff distribution process, the coalition members carry no opportunistic behavior. But, as we have pointed out, in a cooperative game a member may betray his coalition if his immoral behavior can bring more distribution from another coalition, then, we have reasons to believe that, in the cooperative payoff distribution process, players may engage in opportunistic behaviors too. The distribution of cooperative payoff of a coalition member isn't just what he obtains in the game, although the sum of the cooperative payoff distributions equals to the sum of the payoffs they get in the game, this means that some members should get higher distributions than the payoffs they get in the game, at the same time, other members' distributions are less than the payoffs they get in the game. Obviously, if a coalition member's distribution is more than the payoff he gets in the game, he'll carry no opportunistic behavior in the distribution process. However, if a coalition member's distribution is less than the payoff he gets in the game, he may carry opportunistic behavior (that is, refusing to hand in the surplus payoff in excess of his distribution) to seek more benefit.

Of course, the possibility of opportunistic behaviors of the players in the distribution process doesn't mean that all the self-implemented agreements cannot be reached, because the players' opportunistic behaviors in the distribution process can be inhibited:

(1) If a coalition can centralize all the payoffs of its members, the execution of its distribution scheme of cooperative payoff can be guaranteed;

(2) Even if a coalition cannot centralize all the payoffs of its members, it can also ensure that its members carry no opportunistic behavior of refusing the distribution scheme through the prior distribution.

However, in either case, in a coalition situation in a cooperative game, the escape-payoff deriving from deviation a player can get from his deviation behavior will be different from that when the opportunistic behaviors of players in the distribution process are ignored. If a coalition centralizes all the payoffs gotten by its members to ensure its distribution scheme can be implemented, if some member $i(i\in C)$ doesn't belong to a 1-member coalition, when he betrays coalition C and escapes through deviation, player i will no longer take away the payoff that he obtains in the game, and at the same time, he cannot get his distribution of cooperative payoff from the coalition which he originally belongs to. Assume that $i \in C$, when the opportunistic behaviors in the distribution process are ignored,

$$w_i^{-C} = \max Mv_i(C_k)$$

$$= \max\{V_{C_k \cup \{i\}}(s_{C_k}^{\circ},\ s_i^{\circ},\ s_{-C_k,\ -i}^{*}) - V_{C_k}(s_{C_k}^{*},\ s_{-C_k}^{*})\}$$

$$= \max\{\sum_{\substack{j \in C_k \\ j \neq i}} u_j(s_{C_k}^{\circ},\ s_i^{\circ},\ s_{-C_k,\ -i}^{*}) + u_i(s_{C_k}^{\circ},\ s_i^{\circ},\ s_{-C_k,\ -i}^{*}) - \sum_{\substack{j \in C_k \\ j \neq i}} u_j(s_{C_k}^{*},\ s_{-C_k}^{*})\},$$

$$C_k \neq C,$$

where $\left(s_{C_k}^{\circ},\ s_i^{\circ}\right) = \arg \max_{\left(s_{C_k},\ s_i\right)} V_{C_k \cup \{i\}}\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^{*}\right) = \arg\max_{\left(s_{C_k},\ s_i\right)} \left\{\sum_{\substack{j \in C_k \\ j \neq i}} u_j\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^{*}\right) + u_i\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^{*}\right)\right\}$.

And if the opportunistic behaviors in the distribution process are considered, assume that each coalition centralizes all the payoffs gotten by its members, then,

$$\hat{w}_i^{-c} = \max Mv_i\left(C_k\right) = \begin{cases} \max\left\{V_{c_k}\left(s_{c_k}^{\circ},\ s_i^{\circ},\ s_{-c_k-i}^{*}\right) - V_{c_k}\left(s_{c_k}^{*},\ s_{-c_k}^{*}\right)\right\} \\ = \max\left\{\sum_{\substack{j \in c_k \\ j \neq i}} u_j\left(s_{c_k}^{\circ},\ s_i^{\circ},\ s_{-c_k-i}^{*}\right) - \sum_{\substack{j \in c_k \\ j \neq i}} u_j\left(s_{c_k}^{*},\ s_{-c_k}^{*}\right)\right\},\ \text{if}\ C\backslash\{i\} \neq \phi; \\ \max\left\{V_{c_k}\left(s_{c_k}^{\circ},\ s_i^{\circ},\ s_{-c_k-i}^{*}\right) = \hat{w}_i^{-c},\ \ \text{if}\ C\backslash\{i\} = \phi, \end{cases}$$

$$C_k \neq C,$$

where $\left(s_{C_k}^{\circ},\ s_i^{\circ}\right) = \underset{\left(s_{C_k},\ s_i\right)}{\text{argmax}} V_{C_k}\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^{*}\right) = \underset{\left(s_{C_k},\ s_i\right)}{\text{argmax}} \sum_{\substack{j \in C_k \\ j \neq i}} u_j\left(s_{C_k},\ s_i,\ s_{-C_k,\ -i}^{*}\right)$.

And the marginal loss of coalition C when some member i takes opportunistic action and withdraws from C through deviation is

$$\hat{M}o_i(C) = V_C - V_{C\backslash\{i\}} = \begin{cases} \sum_{\substack{j \in C \\ j \neq i}} u_j(s_C^{*},\ s_{-C}^{*}) + u_i(s_C^{*},\ s_{-C}^{*}) - \sum_{\substack{j \in C \\ j \neq i}} u_j(s_{C_k}^{\circ},\ s_i^{\circ},\ s_{-C_k,\ -i}^{*}) - u_i(s_{C_k}^{\circ},\ s_i^{\circ},\ s_{-C_k,\ -i}^{*}),\ \ \text{if}\ C\backslash\{i\} \neq \phi; \\ u_i(s_C^{*},\ s_{-C}^{*}),\ \ \text{if}\ C\backslash\{i\} = \phi. \end{cases}$$

If the opportunistic behaviors in the distribution process are considered, and the coalitions distribute their cooperative payoffs before the game begins, the escape-payoff deriving from deviation a coalition member obtains may be more or less than that he gets when the opportunistic behaviors in the distribution process are ignored. Because each member i of coalition C can withdraw from his coalition through deviation with his prior distribution $(\tilde{x}_i - u_i)$ [Here, the benchmark for the pre-distribution taken by each coalition C is its cooperative payoff distribution scheme when opportunistic behaviors in the cooperative payoff distribution process are ignored, $\tilde{x}_C$. Such a criterion seems to be very inappropriate. When coalitions distribute their cooperative payoffs before the game begins, because the escape-payoffs deriving from deviation of the players will change, the coalition equilibrium of the cooperative game with agreements self-implemented and the equilibrium of the bargaining game on the distribution of the cooperative payoff of each coalition will change too. Therefore, at this time a reasonable benchmark for the pre-distribution is the equilibrium distribution scheme, $\not{x}_C$]. At this point, in some coalition situation c, the escape-payoff deriving from deviation gotten by member $i(i \in C)$ through deviation is:

$$\begin{aligned}
\not{w}_i^{-C} &= \max Mv_i(C_k) \\
&= \max\{V_{C_k\cup\{i\}}(s^{\circ}_{C_k}, s^{\circ}_i, s^*_{-C_k,-i}) - V_{C_k}(s^*_{C_k}, s^*_{-C_k}) + \tilde{x}_i(s^*_{C_k}, s^*_{-C_k}) - u_i(s^*_{C_k}, s^*_{-C_k})\} \\
&= \max\{\sum_{\substack{j\in C_k\\ j\neq i}} u_j(s^{\circ}_{C_k}, s^{\circ}_i, s^*_{-C_k,-i}) + u_i(s^{\circ}_{C_k}, s^{\circ}_i, s^*_{-C_k,-i}) + \tilde{x}_i(s^*_{C_k}, s^*_{-C_k}) - u_i(s^*_{C_k}, s^*_{-C_k}) - \sum_{\substack{j\in C_k\\ j\neq i}} u_j(s^*_{C_k}, s^*_{-C_k})\} \\
&= w_i^{-C} + \tilde{x}_i(s^*_{C_k}, s^*_{-C_k}) - u_i(s^*_{C_k}, s^*_{-C_k}) \\
&= w_i^{-C} + \tilde{x}_i(s^*) - u_i(s^*),
\end{aligned}$$

$$C_k \neq C,$$

where $(s^{\circ}_{C_k}, s^{\circ}_i) = \underset{(s_{C_k}, s_i)}{\arg\max} V_{C_k\cup\{i\}}(s_{C_k}, s_i, s^*_{-C_k,-i}) = \underset{(s_{C_k}, s_i)}{\arg\max}\{\sum_{\substack{j\in C_k\\ j\neq i}} u_j(s_{C_k}, s_i, s^*_{-C_k,-i}) + u_i(s_{C_k}, s_i, s^*_{-C_k,-i})\}$, $\tilde{x}_i$ represents the cooperative payoff distribution of member i, according to the prior distribution scheme of coalition C.

And the marginal loss of cooperative payoff in coalition C when member i takes opportunistic action and withdraws from C through deviation is:

$$\begin{aligned}
\not{M}o_i(C) &= V_C - V_{C\setminus\{i\}} \\
&= \sum_{\substack{j\in C\\ j\neq i}} u_j(s^*_C, s^*_{-C}) + u_i(s^*_C, s^*_{-C}) - \{\sum_{\substack{j\in C\\ j\neq i}} u_j(s^{\circ}_{C_k}, s^{\circ}_i, s^*_{-C_k,-i}) - [\not{x}_i(s^*_C, s^*_{-C}) - u_i(s^*_C, s^*_{-C})]\} \\
&= \sum_{\substack{j\in C\\ j\neq i}} u_j(s^*_C, s^*_{-C}) - \sum_{\substack{j\in C\\ j\neq i}} u_j(s^{\circ}_{C_k}, s^{\circ}_i, s^*_{-C_k,-i}) + \tilde{x}_i(s^*_C, s^*_{-C}).
\end{aligned}$$

Next, we will investigate the coalition equilibrium of a cooperative game with agreements self-implemented and the cooperative payoff distribution in a coalition, when the opportunistic behaviors of the players in the distribution process are considered.

When the possible opportunistic behaviors in the distribution process are considered, first we assume that the coalitions centralize all the payoffs gotten by their members.

In Theorem 3.1, we'll prove that, when coalitions distribute their cooperative payoffs before the game begins, if each coalition C takes equilibrium distribution scheme x_C as the benchmark for pre-distribution, then in cooperative game $\Gamma(N,\ S_i,\ u_i)$ with agreements self-implemented, the coalition equilibrium doesn't exist.

Theorem 3.1 In cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, when coalitions distribute their cooperative payoff before the game begins, if each coalition C takes equilibrium distribution scheme x_C as the benchmark for pre-distribution, the coalition equilibrium of the game doesn't exist.

Proof. Assume that in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, when coalitions distribute their cooperative payoff before the game begins, each coalition C takes equilibrium distribution scheme x_C as the benchmark for pre-distribution.

If there are synergies between some players and they trust each other, cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented won't degenerates into a non-cooperative game. Assume that there is synergy between at least two players and they trust each other. Without losing generality, assume that there is synergy between player 1 and player 2 and they trust each other, then coalition situation $c = (1,\ 1,\ 3,\ \cdots,\ i,\ \cdots,\ n)$ is feasible.

In coalition situation $c = (1,\ 1,\ 3,\ \cdots,\ i,\ \cdots,\ n)$, player 1 and player 2 form a 2-member coalition $C_1 = \{1,\ 2\}$. When coalition $C_1 = \{1,\ 2\}$ distributes its cooperative payoff before the game begins and takes equilibrium distribution scheme x_{C_1} as the benchmark for pre-distribution, assume that the payoff vector is $(u_1(s^*),\ u_2(s^*))$, and that the equilibrium distribution scheme $x_{C_1}{=}(x_1,\ x_2)$, where s^* is the Nash equilibrium of the non-cooperative game among coalitions in coalition c.

Assume that when the opportunistic behaviors in the distribution process are ignored, in coalition situation c, the vector of escape-payoff deriving from deviation is $(\tilde{w}_1^{-C_1},\ \tilde{w}_2^{-C_1})$ When coalition C_1 takes equilibrium distribution scheme as the benchmark for pre-distribution, the pre-distribution vector is $(x_1 - u_1(s^*),\ x_2 - u_2(s^*))$, and the vector of escape-payoff deriving from deviation is $(w_1^{-C_1},\ w_2^{-C_1}){=}(\tilde{w}_1^{-C_1} + x_1 - u_1(s^*),\ \tilde{w}_2^{-C_1} + x_2 - u_2(s^*))$, then the common payoff of coalition C_1 is:

$$
\begin{aligned}
\theta_{\{1,\ 2\}} &= V_C(s^*) - w_1^{-C_1} - w_2^{-C_1} = V_C(s^*) - (\tilde{w}_1^{-C_1} + x_1 - u_1(s^*)) - (\tilde{w}_2^{-C_1} + x_2 - u_2(s^*)) \\
&= V_C(s^*) - \tilde{w}_1^{-C_1} - \tilde{w}_2^{-C_1} \\
&= \theta_{\{1,\ 2\}}.
\end{aligned}
$$

According to the distribution rule of common payoff, the above common payoff will be distributed evenly to both members, therefore, the total cooperative payoff distributions gotten by these two players are:

$$
x_1 = w_1^{-C_1} + \frac{1}{2}\theta_{\{1,\ 2\}} = \tilde{w}_1^{-C_1} + x_1 - u_1(s^*) + \frac{1}{2}\theta_{\{1,\ 2\}},
$$

$$
x_2 = w_2^{-C_1} + \frac{1}{2}\theta_{\{1,\ 2\}} = \tilde{w}_2^{-C_1} + x_2 - u_2(s^*) + \frac{1}{2}\theta_{\{1,\ 2\}}.
$$

That's to say, when coalition $C_1 = \{1,\ 2\}$ distributes its cooperative payoff before the game begins and takes equilibrium distribution scheme x_{c_1} as the benchmark for pre-distribution, situations $u_1(s^*) = \tilde{w}_1^{-c_1} + \frac{1}{2}\theta_{\{1,\ 2\}}$, $u_2(s^*) = \tilde{w}_2^{-c_1} + \frac{1}{2}\theta_{\{1,\ 2\}}$ must be satisfied, namely, when the possible opportunistic behaviors are ignored in the cooperative payoff distribution process, in coalition situation c, the cooperative payoff distributions of both members of coalition C_1 must be the payoffs they get in the game. However, the above two conditions are usually not satisfied, which means that, coalition $C_1 = \{1,\ 2\}$ cannot take x_{c_1} as the benchmark for pre-distribution.

In cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, if in some coalition situation c the vector of cooperative payoff distributions of all the players doesn't exist, the equilibrium of the coalition-choosing game doesn't exist, that is to say, in the cooperative game with agreements self-implemented, the coalition equilibrium doesn't exist. □

3.1 *Coalition equilibrium*

If the opportunistic behaviors in the distribution process are considered, assume that the coalitions centralize all the payoffs gotten by their members, in some coalition C in a cooperative game with agreements self-implemented, only if the marginal contributions of each member and each member set meet the conditions below, the agreement of the coalition would be self-implemented (the equations hold if and only if there is one and only member in the coalition):

$$Mv_i(C) \geq \hat{w}_i^{-C},\ i \in C;$$

$$Mv_{T_h}(C) \geq \sum\nolimits_{j \in T_h} \hat{w}_j^{-C},\ i \in T_h \subseteq C.$$

Assume that the coalitions centralize all the payoffs gotten by their members, in coalition C in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, the cooperative payoff distribution vector of coalition C must satisfy the following conditions:

(1) $Mv_i(C) \geq \hat{x}_i \geq w_i^{-c},\ i \in C$ (the equations hold if and only if there is one and only member in the coalition);

(2) $\sum_{i \in C} \hat{x}_i = V_C$,

where condition (1) is called the individual rationality condition (or competitive distribution condition), condition (2) is called the collective rationality condition.

When the possible opportunistic behaviors in the distribution process are considered, assume that the coalitions centralize all the payoffs which their members get, in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented there exists the coalition equilibrium under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other):

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i,\ \text{if for any } c_i \neq i,\ \hat{w}_i^{-C_i}(i,\ c_{-i}^*) \leq \hat{w}_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \\ \text{or, } Mv_i(C_{c_i}) \leq \hat{w}_i^{-C_{c_i}} \text{or, } Mv_{T_h}(C_{c_i}) \leq \sum\nolimits_{j \in T_h} \hat{w}_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\min \hat{w}_i^{-C_{c_i}}(c_i,\ c_{-i}^*),\ \text{if at least for a certain } c_i \neq i,\ \hat{w}_i^{-C_i}(i,\ c_{-i}^*) > \hat{w}_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \\ Mv_i(C_{c_i}) > \hat{w}_i^{-C_{c_i}} \text{ and } Mv_{T_h}(C_{c_i}) \geq \sum\nolimits_{j \in T_h} \hat{w}_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}, \end{cases}$$

and also there exists the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other):

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i, \ \text{ if for any } c_i \neq i,\ \hat{x}_i\left(i,\ c_{-i}^*\right) \geq \hat{x}_i\left(c_i,\ c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq \hat{w}_i^{-c_{c_i}}, \ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum\limits_{j\in T_h} \hat{w}_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\max \hat{x}_i\left(c_i,\ c_{-i}^*\right), \ \text{if at least for a certain } c_i \neq i,\ \hat{x}_i\left(i,\ c_{-i}^*\right) < \hat{x}_i\left(c_i, c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > \hat{w}_i^{-C_{c_i}} \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum\limits_{j\in T_h} \hat{w}_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}, \end{cases}$$

where c_i^* represents player i's equilibrium coalition-choosing strategy; $\hat{w}_i^{-C_i}(\cdot)$ represents player i's escape-payoff deriving from deviation in the corresponding coalition situation, $\left(i,\ c_{-i}^*\right)$; $Mv_i\left(C_{c_i}\right)$, $\hat{w}_i^{-c_{c_i}}$ represent player $i's$ marginal contribution to coalition C_{c_i}, and his escape-payoff deriving from deviation in the corresponding coalition situation, $\left(c_i,\ c_{-i}^*\right)$, respectively; $Mv_{T_h}(C_{c_i})\sum_{j\in T_h}\hat{w}_j^{-C_{c_i}}$ represent the marginal contribution to coalition C_{c_i} of member set T_h, which player i belongs to, and the sum of the escape-payoffs deriving from deviation of the members in member set T_h, in the corresponding coalition situation, $\left(c_i,\ c_{-i}^*\right)$, respectively.

If the competitive distribution condition is met in the distribution process of each coalition, $Mv_i(C) \geq \hat{x}_i \geq \hat{w}_i^{-C}$, $i \in C$ (the equations hold if and only if there is one and only member in the coalition), in the cooperative game with agreements self-implemented, the above two equilibria are equivalent:

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i, \ \text{ if for any } c_i \neq i,\ \hat{w}_i^{-C_i}\left(i,\ c_{-i}^*\right) \leq \hat{w}_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right), \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq \hat{w}_i^{-C_{c_i}}, \ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum\limits_{j\in T_h} \hat{w}_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\min \hat{w}_i^{-C_{c_i}}\left(c_i,\ c_{-i}^*\right), \ \text{ if at least for a certain } c_i \neq i,\ \hat{w}_i^{-C_i}\left(i,\ c_{-i}^*\right) > \hat{w}_i^{-c_{c_i}}\left(c_i,\ c_{-i}^*\right), \\ Mv_i\left(C_{c_i}\right) > \hat{w}_i^{-C_{c_i}} \text{ and } Mv_{T_h}\left(C_{c_i}\right) \geq \sum\limits_{j\in T_h} \hat{w}_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}. \end{cases}$$

$$
\Leftrightarrow c_i^* = \begin{cases} i, \ \text{ if for any } c_i \neq i, \ \hat{x}_i\left(i, \ c_{-i}^*\right) \geq \hat{x}_i\left(c_i, \ c_{-i}^*\right), \\ \\ \text{or, } Mv_i\left(C_{c_i}\right) \leq \hat{w}_i^{-c_{c_i}}, \ \text{ or, } Mv_{T_h}\left(C_{c_i}\right) \leq \sum_{j \in T_h} \hat{w}_j^{-c_{c_i}}, \ i \in T_h \subseteq C_{c_i}; \\ \\ \arg\max \hat{x}_i\left(c_i, \ c_{-i}^*\right), \ \text{ if at least for a certain } c_i \neq i, \ \hat{x}_i\left(i, \ c_{-i}^*\right) < \hat{x}_i\left(c_i, \ c_{-i}^*\right), \\ \\ Mv_i\left(C_{c_i}\right) > \hat{w}_i^{-c_{c_i}} \text{ and } Mv_{T_h}\left(C_{c_i}\right) > \sum_{j \in T_h} \hat{w}_j^{-C_{c_i}}, \ i \in T_h \subseteq C_{c_i}. \end{cases}
$$

Example 3.1 Take the previous example, assume that coalition centralize all the payoffs gotten by their members to inhibit the possible opportunistic behaviors in the distribution process, we'll examine the coalition equilibrium of cooperative game $\Gamma(N, \ \{S_i\}, \ \{u_i\})$ with agreements self-implemented.

Without loss of generality, assume that the coalition-choosing strategy of player 1 is $c_1 = 1$, when $c_2 = 1$, in coalition situation $c^1 = (1, \ 1)$, players 1 and 2 will form a non-empty coalition $C_1 = \{1, \ 2\}$, the optimal combination strategy of coalition C_1 is $s^* = \left(s_1^1, \ s_2^2\right)$, and its cooperative payoff $V_{C_1} = 250$.

At this time, if player 1 escapes from coalition C_1 through deviation, his target is coalition C_2, his strategy choice is s_1^1 when he deviates, in strategic situation $\left(s_1^1, \ s_2^2\right)$, the cooperative payoff of coalition $C_2 \bigcup \{1\}$ is $V_{C_2 \bigcup \{1\}} = 0$, that's to say, the marginal contribution of player 1 to coalition C_2 is $Mv_1\left(C_2\right) = 0$, and this is his escape-payoff deriving from deviation in coalition situation c^1:

$$
\hat{w}_1^{-C_1}\left(c^1\right) = 0.
$$

If player 2 escapes from coalition C_1 through deviation, his target is coalition C_2, his strategy choice is s_2^2 when he deviates. In strategic situation $\left(s_1^1, \ s_2^2\right)$, the cooperative payoff of coalition $C_2 \bigcup \{2\}$ is $V_{C_2 \cup \{2\}} = 0$, that's to say, the marginal contribution of player 2 to coalition C_2 is $Mv_2\left(C_2\right) = 0$, and this is his escape-payoff deriving from deviation in coalition situation c^1:

$$
\hat{w}_2^{-C_1}\left(c^1\right) = 0.
$$

Obviously, for some player who belongs to a 1-member coalition, whether the coalition centralizes the payoff gotten by its member or not has no impact on his escape-payoff deriving from deviation. If this player escapes from the 1-member coalition through deviation, what the coalition gains will always be owned by this player. Therefore, in coalition situation c^2, the escape-payoffs deriving from deviation of players 1 and 2 are

$$
\hat{w}_1^{-C_1}\left(c^2\right) = \frac{550}{3}, \ \hat{w}_2^{-C_2}\left(c^2\right) = 175
$$

respectively.

Given player l's coalition-choosing strategy $c_1 = 1$, now we get the players' escape-payoff deriving from deviation matrix shown as follows.

Table 3. The players' escape-payoff deriving from deviation matrix

	$c_2 = 1$	$c_2 = 2$
$c_1 = 1$	0, 0	$\frac{550}{3}$, 175

Obviously, under the criterion of minimum escape-payoff deriving from deviation, the equilibrium coalition-choosing strategy of player 2 is $c_2^* = 1$. In coalition equilibrium $c^* = (1,\ 1)$, the cooperative payoff of coalition C_1 satisfies:

$$V_{C_1} > \hat{w}_i^{-C_1},\ i = 1,\ 2;$$

$$V_{C_1} > \sum_{i=1}^{2} \hat{w}_i^{-C_1}.$$

Therefore, in coalition equilibrium $c^* = (1,\ 1)$, the members of coalition C_1 trust each other, that is, c^* is a feasible coalition situation.

3.2 *Non-cooperative bargaining game on the distribution of the cooperative payoff of a coalition*

When coalitions centralize all the payoffs that its members get in the game, in coalition equilibrium c^* of the cooperative game, if the members of coalition C are non-cooperative in the bargaining game, denote a subset of member set M of coalition C as $M_{m_1,\ m_2,\ \cdots,\ m_k}$, $M_{m_1,\ m_2,\ \cdots,\ m_k} \subseteq M(k \leq m)$, which is composed of members $m_1,\ m_2,\ \cdots,\ m_k$, the common payoff of member set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ is defined as follows:

$$\hat{\theta}\left(M_{m_1,\ m_2,\ \cdots,\ m_k}\right) = V_{M_{m_1,\ m_2,\ \cdots,\ m_k}} - \sum_{i=1}^{k} \hat{w}_{m_i}^{-C} - \sum \hat{\theta}_{(2)}\left(M_{m_1,\ m_2, \cdots,\ \cdots m_k}\right) - \cdots - \sum \hat{\theta}_{(k-1)}\left(M_{m_1,\ m_2,\ \cdots,\ m_k}\right)$$

where $V_{M_{m_1}},\ \ m_2,\ \cdots,\ m_k$ represents the cooperative payoff of coalition C when other members except those in set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ have all escaped from the coalition C and as a whole join some coalition which can bring them maximum escape-payoff, keeping the coalition-choosing strategies of the players in other coalitions unchanged; $\sum \hat{\theta}_{(j)}\left(M_{m_1,\ m_2,\ \cdots,\ m_k}\right)$ represents the sum of the common payoffs of all subsets of set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ consisting of j members; $\sum_{i=1}^{k} \hat{w}_{m_i}^{-C}$ represents the sum of the escape-payoffs deriving from deviation of the members in set $M_{m_1,\ m_2,\ \cdots,\ m_k}$.

Through analysis similar to Theorem 2.5, the common-payoff $\hat{\theta}\left(M_{m_1,\ m_2,\ \cdots m_k}\right)$ is distributed on average among the members in set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ through their bargaining game:

$$\hat{y}_{m_i}^* = \frac{1}{k}\hat{\theta}\left(M_{m_1,\ m_2,\ \cdots,\ m_k}\right),\ i = 1,\ 2,\ \cdots,\ k.$$

Therefore, when the members are non-cooperative in the bargaining game, in coalition C, the distribution of cooperative payoff surplus of each member $m_i\,(m_i \in C)$ is:

$$\hat{y}_{m_i} = \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\hat{\theta}\left(M_{m_i,\, m_j}\right) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1}\hat{\theta}\left(M_{m_i,\, m_j,\, m_k}\right) + \cdots + \frac{1}{m}\hat{\theta}\left(M_{1,\, 2,\, \cdots,\, m}\right),$$

where $\hat{\theta}\left(M_{m_i,\, m_j}\right),\ \hat{\theta}\left(M_{m_i,\, m_j,\, m_k}\right),\ \ldots,\ \hat{\theta}\left(M_{1,\, 2,\, \cdots,\, m}\right)$ represent the common payoffs of the corresponding member sets, respectively.

His total cooperative payoff distribution is:

$$\begin{aligned}\hat{x}_{m_i} &= \hat{y}_{m_i} + \hat{w}_{m_i}^{-C}\\ &= \hat{w}_{m_i}^{-C} + \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\hat{\theta}\left(M_{m_i,\, m_j}\right) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1}\hat{\theta}\left(M_{m_i,\, m_j,\, m_k}\right) + \cdots + \frac{1}{m}\hat{\theta}\left(M_{1,\, 2,\, \cdots,\, m}\right).\end{aligned}$$

Example 3.2 Take the previous example, assume that coalitions centralize all the payoffs gotten by their members to inhibit the possible opportunistic behaviors in the distribution process, we'll examine the distribution equilibria of the cooperative payoffs of the coalitions in the coalition equilibrium of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented.

According to the definition of the common payoff of a member set, the common payoff of member set $M = \{1,\ 2\}$ of coalition C_1 is:

$$\hat{\theta}_M = V_{C_1} - \sum_{i=1}^{2}\hat{w}_i^{-C_i} = 250 - 0 - 0 = 250.$$

Therefore, in the coalition equilibrium c^*, the cooperative payoff distributions that players 1 and 2 get are

$$\hat{x}_1 = \hat{w}_1^{-C_1} + \frac{1}{2}\hat{\theta}_M = 125,$$

$$\hat{x}_2 = \hat{w}_1^{-C_1} + \frac{1}{2}\hat{\theta}_M = 125,$$

respectively.

3.3 *Cooperative bargaining game on the distribution of the cooperative payoff of a coalition*

In cooperative bargaining game $\Gamma(M,\ \{T_i\},\ \{\hat{x}_i\})$ of coalition C, the m members will compose m coalitions which compete with each other. Because in the bargaining game, members can't betray his coalition, its coalition equilibrium must be the one under the criterion of minimum escape-payoff, t^*.

When coalitions centralize all the payoffs gotten by their members, in cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, in some coalition situation t in the cooperative bargaining game of coalition C, the total distribution gotten by each coalition m_i is:

$$\hat{x}_{m_i}=\hat{y}_{m_i}+\hat{w}_{m_i}^{-C}$$

$$=\hat{w}_{m_i}^{-C}+\frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\hat{\theta}(M_{m_i,\, m_j})+\frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1}\hat{\theta}(M_{m_i,\, m_j,\, m_k})+\cdots+\frac{1}{m}\hat{\theta}(M_{1,\, 2,\, \cdots,\, m}),$$

where $M_{m_1,\, m_2,\, \cdots,\, m_k}$ represents a coalition set composed of coalitions $m_1,\, m_2,\, \cdots,\, m_k$; $\hat{\theta}\left(M_{m_1,\, m_2,\, \ldots,\, m_k}\right)$ represents the common payoff of set $M_{m_1,\, m_2,\, \cdots,\, m_k}$ when coalitions centralize all the payoffs gotten by their members in the game.

When coalitions centralize all the payoffs gotten by their members, in cooperative game $\Gamma(N,\, \{S_i\},\, \{u_i\})$ with agreements self-implemented, there exists the coalition equilibrium under the criterion of maximum cooperative payoff distribution in cooperative bargaining game $\Gamma(M,\, \{T_i\},\, \{\hat{x}_i\})$ of coalition C:

$$\forall i=1,\, 2,\, \cdots,\, m,$$

$$t_i^*=\begin{cases} i, \text{ if for any } t_i,\ \hat{x}_i\left(i,\, t_{-i}^*\right)\geq \hat{x}_i\left(t_i,\, t_{-i}^*\right); \\ \arg\max \hat{x}_i\left(t_i,\, t_{-i}^*\right), \text{ if at least for a certain } t_i\neq i,\ \hat{x}_i\left(i,\, t_{-i}^*\right)<\hat{x}_i\left(t_i,\, t_{-i}^*\right), \end{cases}$$

and also there exists the one under the criterion of minimum escape-payoff:

$$\forall i=1,\, 2,\, \cdots,\, m,$$

$$t_i^*=\begin{cases} i, \text{ if for any } t_i,\ w_i(T_{t_i})(i,\, t_{-i}^*)\leq w_i(T_{t_i})(t_i,\, t_{-i}^*); \\ \arg\min w_i(T_{t_i})(t_i,\, t_{-i}^*), \text{ if at least for a certain } t_i\neq i,\ w_i(T_{t_i})(i,\, t_{-i}^*)>w_i(T_{t_i})(t_i,\, t_{-i}^*). \end{cases}$$

If the competitive distribution condition is met in the distribution process, that's to say, $w_i\left(T_{t_i}\right)\leq \hat{x}_i\left(T_{t_i}\right)\leq Mv_i\left(T_{t_i}\right)$ (the equations hold if and only if there is one and only member in the coalition), the above two equilibria are equivalent,

$$\forall i=1,\, 2,\, \cdots,\, m,$$

$$t_i^*=\begin{cases} i, \text{ if for any } t_i,\ \hat{x}_i\left(i,\, t_{-i}^*\right)\geq \hat{x}_i\left(t_i,\, t_{-i}^*\right); \\ \arg\max \hat{x}_i\left(t_i,\, t_{-i}^*\right), \text{ if at least for a certain } t_i\neq i,\ \hat{x}_i\left(i,\, t_{-i}^*\right)<\hat{x}_i\left(t_i,\, t_{-i}^*\right). \end{cases}$$

$$\Leftrightarrow t_i^* = \begin{cases} i, \text{ if for any } t_i, \; w_i(T_{t_i})(i, \; t_{-i}^*) \le w_i(T_{t_i})(t_i, \; t_{-i}^*); \\ \\ \arg\min w_i(T_{t_i})(t_i, \; t_{-i}^*), \text{ if at least for a certain } t_i \ne i, \; w_i(T_{t_i})(i, \; t_{-i}^*) > w_i(T_{t_i})(t_i, \; t_{-i}^*). \end{cases}$$

When coalitions centralize all the payoffs their members get, in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, in the cooperative bargaining game of coalition C, the original competition between the m members of coalition C is now replaced by the competition between the m coalitions because the members are cooperative rather than non-cooperative.

When coalitions centralize all the payoffs their members get in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, denote a coalition set composed of coalitions $m_1^*, m_2^*, \cdots, m_k^*$ in coalition equilibrium t^* of cooperative bargaining game $\Gamma(M, \{T_i\}, \{\hat{x}_i\})$ as $M_{m_1^*, m_2^*, \cdots, m_k^*}$, the common payoff of coalition set $M_{m_1^*, m_2^*, \cdots, m_k^*}$ is defined as follows:

$$\hat{\theta}\left(M_{m_1^*, m_2^*, \cdots, m_k^*}\right) = V_{M_{m_1^*, m_2^*, \cdots, m_k^*}} - \sum_{i=1}^{k} \hat{w}_{m_i^*}^{-c} - \sum \hat{\theta}_{(2)}\left(M_{m_1^*, m_2^*, \cdots, m_k^*}\right) - \cdots - \sum \hat{\theta}_{(k-1)}\left(M_{m_1^*, m_2^*, \cdots, m_k^*}\right).$$

The Nash equilibrium of the bargaining game is:

$$\hat{y}_{m_i^*}^* = \frac{1}{k}\hat{\theta}\left(M_{m_1^*, m_2^*, \cdots, m_k^*}\right), \; i = 1, \; 2, \; \cdots, \; k.$$

When coalitions centralize all the payoffs their members get in the game, in coalition equilibrium t^* of cooperative bargaining game $\Gamma(M, \{T_i\}, \{\hat{x}_i\})$ of coalition C on cooperative payoff surplus distribution, the cooperative payoff surplus distribution of each coalition m_i^* is:

$$\hat{y}_{m_i^*} = \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \hat{\theta}(M_{m_i^*, m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \sum_{\substack{m_k^*=1 \\ m_k^* \ne m_i^*}}^{m_j^*-1} \hat{\theta}(M_{m_i^*, m_j^*, m_k^*}) + \cdots + \frac{1}{m}\hat{\theta}(M_{1, 2, \cdots, m}).$$

where $\hat{\theta}\left(M_{m_i^*, m_j^*}\right)$, $\hat{\theta}\left(M_{m_i^*, m_j^*, m_k^*}\right)$, $\ldots$, $\hat{\theta}(M_{1, 2, \cdots, m})$ represents the common payoffs of the corresponding coalition sets, respectively.

Its total cooperative payoff distribution is:

$$\begin{aligned} \hat{x}_{m_i^*} &= \hat{y}_{m_i^*} + \hat{w}_{m_i^*}^{-C} \\ &= \hat{w}_{m_i^*}^{-C} + \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \hat{\theta}\left(M_{m_i^*, m_j^*}\right) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \sum_{\substack{m_k^*=1 \\ m_k^* \ne m_i^*}}^{m_j^*-1} \hat{\theta}\left(M_{m_i^* m_i^*, m_j^*, m_k^*}\right) + \cdots + \frac{1}{m}\hat{\theta}(M_{1, 2, \cdots, m}). \end{aligned}$$

If there are three or more than three members in a coalition, we still need to examine the second level, the third level, the forth level cooperative bargaining games and so on, and finally we'll get the cooperative payoff distribution of each member of the coalition.

4. Self-implemented agreements: coalitions distribute their cooperative payoffs before the game begins

When opportunistic behaviors of players in the distribution process are considered, assume that coalitions cannot centralize all the payoffs that their members get in the game, they can ensure that their members carry no opportunistic behaviors of refusing the distribution schemes through the prior distribution of their cooperative payoffs.

4.1 *Coalition equilibrium*

Assume that the coalitions distribute their cooperative payoffs before the game begins, in the coalition equilibrium of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, only if the marginal contributions of each member and each member sets of coalition C meet the conditions below, the agreement of the coalition would be self-implemented (the equations hold if and only if there is one and only member in the coalition):

$$Mv_i(C) \geq w_i^{-C},\ i \in C;$$

$$Mv_{T_h}(C) \geq \sum\nolimits_{j \in T_h} w_j^{-C},\ i \in T_h \subseteq C.$$

Assume that the coalitions distribute their cooperative payoffs before the game begins, in the coalition equilibrium of cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented, the cooperative payoff distribution of the members of coalition C must meet individual rationality condition and the collective rationality condition:

(1) individual rationality condition or competitive distribution condition, $Mv_i(C) \geq x_i \geq w_i^{-C}, i \in C$ (the equations hold if and only if there is one and only member in the coalition);

(2) collective rationality condition, $\sum_{i \in C} x_i = V_C$.

Assume that the coalitions distribute their cooperative payoffs before the game begins, there exists the coalition equilibrium under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other) in cooperative game $\Gamma(N, \{S_i\}, \{u_i\})$ with agreements self-implemented:

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i, \ \text{if for any } c_i \neq i,\ w_i^{-C_i}(i,\ c_{-i}^*) \leq w_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \\ \text{or, } Mv_i(C_{c_i}) \leq w_i^{-C_{c_i}}, \text{ or, } Mv_{T_h}(C_{c_i}) \leq \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\min\ w_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \ \text{if at least for a certain } c_i \neq i,\ w_i^{-C_i}(i,\ c_{-i}^*) > w_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \\ Mv_i(C_{c_i}) > w_i^{-C_{c_i}} \text{ and } Mv_{T_h}(C_{c_i}) > \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}, \end{cases}$$

and also there exists the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other):

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i, \text{ if for any } c_i \neq i,\ x_i(i,\ c_{-i}^*)\ \geq x_i(c_i,\ c_{-i}^*), \\ \text{or, } Mv_i(C_{c_i}) \leq w_i^{-C_{c_i}}, \text{ or, } Mv_{T_h}(C_{c_i}) \leq \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\max x_i(c_i,\ c_{-i}^*), \ \text{ if at least for a certain } c_i \neq i,\ x_i(i,\ c_{-i}^*) < x_i\ (c_i,\ c_{-i}^*), \\ Mv_i(C_{c_i}) > w_i^{-C_{c_i}} \text{ and } Mv_{T_h}\ (C_{c_i}) > \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}, \end{cases}$$

where c_i^* represents player i's equilibrium coalition-choosing strategy; $w_i^{-C_i}(\cdot)$ represents player $i's$ escape-payoff deriving from deviation in the corresponding coalition situation $\left(i,\ c_{-i}^*\right)$; $Mv_i\,(C_{c_i})$, $w_i^{-C_{c_i}}$ represent player i's marginal contribution to coalition C_{c_i} and his escape-payoff deriving from deviation in the corresponding coalition situation, $\left(c_i,\ c_{-i}^*\right)$, respectively; $Mv_{T_h}(C_{c_i}) \sum_{j \in T_h} w_j^{-C_{c_i}}$ represent member set T_h's marginal contribution to coalition C_{c_i}, to which player i belongs, and the sum of the escape-payoffs deriving from deviation of the members in member set T_h, in the corresponding coalition situation $\left(c_i,\ c_{-i}^*\right)$, respectively.

If in all coalitions the competitive distribution condition is met, $Mv_i(C) \geq x_i \geq w_i^{-C},\ i \in C$ (the equations hold if and only if there is one and only member in the coalition), the above two equilibria are equivalent, that's to say,

$$\forall i = 1,\ 2,\ \cdots,\ n,$$

$$c_i^* = \begin{cases} i, \ \text{ if for any } c_i \neq i,\ w_i^{-C_i}(i,\ c_{-i}^*) \leq w_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \\ \text{or, } Mv_i(C_{c_i}) \leq w_i^{-C_{c_i}}, \text{ or, } Mv_{T_h}(C_{c_i}) \leq \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}; \\ \arg\min\ w_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \ \text{ if at least for a certain } c_i \neq i,\ w_i^{-C_i}(i,\ c_{-i}^*) > w_i^{-C_{c_i}}(c_i,\ c_{-i}^*), \\ Mv_i(C_{c_i}) > w_i^{-C_{c_i}} \text{ and } Mv_{T_h}(C_{c_i}) > \sum_{j \in T_h} w_j^{-C_{c_i}},\ i \in T_h \subseteq C_{c_i}. \end{cases}$$

$$
\Leftrightarrow c_i^* = \begin{cases} i, \ \text{ if for any } c_i \neq i, \ \not{x}_i(i, \ c_{-i}^*) \geq \not{x}_i(c_i, \ c_{-i}^*), \\ \\ \text{or, } Mv_i(C_{c_i}) \leq \not{w}_i^{-C_{c_i}}, \ \text{or, } Mv_{T_h}(C_{c_i}) \leq \sum_{j \in T_h} \not{w}_j^{-C_{c_i}}, \ i \in T_h \subseteq C_{c_i}; \\ \\ \arg\max \not{x}_i(c_i, \ c_{-i}^*), \ \text{if at least for a certain } c_i \neq i, \ \not{x}_i(i, \ c_{-i}^*) < \not{x}_i(c_i, \ c_{-i}^*), \\ \\ Mv_i(C_{c_i}) > \not{w}_i^{-C_{c_i}} \text{ and } Mv_{T_h}(C_{c_i}) > \sum_{j \in T_h} \not{w}_j^{-C_{c_i}}, \ i \ \in T_h \subseteq C_{c_i}. \end{cases}
$$

Example 4.1 Take the previous example, assume that the coalition distributes its cooperative payoff before the game begins to inhibit the possible opportunistic behaviors in the distribution process, we'll examine the coalition equilibrium of cooperative game $\Gamma(N, \ \{S_i\}, \ \{u_i\})$ with agreements self-implemented.

Without loss of generality, assume that the coalition-choosing strategy of player 1 is $c_1 = 1$, when $c_2 = 1$, in coalition situation $c^1 = (1, \ 1)$, players 1 and 2 will form a non-empty coalition $C_1 = \{1, \ 2\}$, the optimal combination strategy of coalition C_1 is $s^* = \left(s_1^1, \ s_2^2\right)$, and its cooperative payoff $V_{C_1} = 250$.

At this time, if player 1 escapes from coalition C_1 through deviation, his target coalition is coalition C_2, his strategy choice is s_1^1 when he deviates, in strategic situation $\left(s_1^1, \ s_2^2\right)$, the cooperative payoff of coalition $C_2 \bigcup \{1\}$ is $V_{C_2 \bigcup \{1\}} = 150$, that's to say, the marginal contribution of player 1 to coalition C_2 is $Mv_1\,(C_2) = 150$, and this is his escape-payoff deriving from deviation in coalition situation c^1:

$$
\not{w}_1^{-C_1}(c^1) = Mv_1(C_2) + (\tilde{x}_1 - u_1(s_1^1, \ s_2^2)) = 150 + (150 - 150) = 150.
$$

If player 2 escapes from coalition C_1 through deviation, his target is coalition C_2 and he will play strategy s_2^2. In strategic situation $\left(s_1^1, \ s_2^2\right)$, the cooperative payoff of coalition $C_2 \bigcup \{2\}$ is $V_{C_2 \cup \{2\}} = 100$, that's to say, the marginal contribution of player 2 to coalition C_2 is $\mathrm{Mv}_2\,(C_2) = 100$, and this is his escape-payoff deriving from deviation in coalition situation c^1:

$$
\not{w}_2^{-C_1}(c^1) = Mv_2(C_2) + (\tilde{x}_2 - u_2(s_1^1, \ s_2^2)) = 100 + (100 - 100) = 100.
$$

Obviously, for some player who belongs to a 1-member coalition, whether the coalition distribute its cooperative payoff before the game begins or not has no impact on his escape-payoff deriving from deviation. If this player escapes from the 1-member coalition through deviation, what the coalition gains will always be owned by this player. Therefore, in coalition situation c^2, the escape-payoffs deriving from deviation of players 1 and 2 are

$$
\not{w}_1^{-C_1}(c^2) = \frac{550}{3}, \ \not{w}_2^{-C_2}(c^2) = 175,
$$

respectively.

Given player 1's coalition-choosing strategy $c_1 = 1$, now we get the players' escape-payoff deriving from deviation matrix shown as follows.

Table 4. The players' escape-payoff deriving from deviation matrix

	$c_2=1$	$c_2=2$
$c_1=1$	150, 100	$\frac{550}{3}$, 150

Obviously, under the criterion of minimum escape-payoff deriving from deviation, the equilibrium coalition-choosing strategy of player 2 is $c_2^*=1$. In coalition equilibrium $c^*=(1,\ 1)$, the cooperative payoff of coalition C_1 satisfies:

$$V_{C_1} > w_i^{-C_1},\ i=1,\ 2;$$

$$V_{C_1} \geq \sum_{i=1}^{2} w_i^{-C_1}.$$

Therefore, in coalition equilibrium $c^*=(1,\ 1)$, the members of coalition C_1 trust each other, that is, c^* is a feasible coalition situation.

4.2 *Non-cooperative bargaining game on the distribution of the cooperative payoff of a coalition*

When the coalitions distribute their cooperative payoffs before the game begins, in coalition equilibrium c^* of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, in the non-cooperative bargaining game of coalition C, denote a member set composed of members $m_1,\ m_2,\ \cdots,\ m_k$as$M_{m_1,\ m_2,\ \cdots,\ m_k}$, the common payoff of set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ is defined as follows:

$$\theta(M_{m_1,\ m_2,\ \cdots,\ m_k}) = V_{M_{m_1,\ m_2,\ \cdots,\ m_k}} - \sum_{i=1}^{k} w_{m_i}^{-C} - \sum \theta_{(2)}(M_{m_1,\ m_2,\ \cdots,\ m_k}) - \cdots - \sum \theta_{(k-1)}(M_{m_1,\ m_2,\ \cdots,\ m_k}),$$

where $V_{M_{m_1,\ m_2,\ \ldots,\ m_k}}$ represents coalition C's cooperative payoff when all other members have escaped from the coalition except those in set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ and as a whole join some coalition which can bring them the maximum escape-payoff, while keeping the coalition-choosing strategies of the players outside coalition C unchanged; $\sum \theta_{(j)}(M_{m_1,\ m_2,\ \cdots,\ m_k})$ represents the sum of the common payoffs of all subsets of set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ consisting of j members; $\sum_{i=1}^{k} w_{m_i}^{-C}$ represents the sum of the escape-payoffs deriving from deviation of the members in set $M_{m_1,\ m_2,\cdots,\ m_k}$.

Similarly, in the distribution equilibrium, the common-payoff $\theta(M_{m_1,\ m_2,\ \cdots,\ m_k})$ is distributed on average among the members in set $M_{m_1,\ m_2,\ \cdots,\ m_k}$ through their bargaining game:

$$y_{m_i}^* = \frac{1}{k}\theta(M_{m_1,\ m_2,\ \cdots,\ m_k}),\ i=1,\ 2,\ \cdots,\ k.$$

Therefore, in coalition equilibrium c^* of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented, when the coalitions distribute their cooperative payoffs before the game begins, if the members are non-cooperative in bargaining game $\Gamma(M,\ \{T_i\},\ \{x_i\})$ of coalition C, the cooperative payoff surplus distribution gotten by each member m_i is:

$$\tilde{y}_{m_i} = \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m} \tilde{\theta}(M_{m_i,\, m_j}) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1} \tilde{\theta}(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\tilde{\theta}(M_{1,\, 2,\, \cdots,\, m}),$$

where $\tilde{\theta}(M_{m_i,\, m_j}),\ \tilde{\theta}(M_{m_i,\, m_j,\, m_k}),\ \cdots,\ \tilde{\theta}(M_{1,\, 2,\, \cdots,\, m})$ are the common payoffs of the corresponding member sets, respectively

His total cooperative payoff distribution is:

$$\tilde{x}_{m_i} = \tilde{y}_{m_i} + \tilde{w}_{m_i}^{-C}$$

$$= \tilde{w}_{m_i}^{-C} + \frac{1}{2}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m} \tilde{\theta}(M_{m_i,\, m_j}) + \frac{1}{3}\sum_{\substack{m_j=1\\ m_j\neq m_i}}^{m}\sum_{\substack{m_k=1\\ m_k\neq m_i}}^{m_j-1} \tilde{\theta}(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\tilde{\theta}(M_{1,\, 2,\, \cdots,\, m}).$$

Example 4.2 Take the previous example, assume that coalition distributes its cooperative payoff before the game begins to inhibit the possible opportunistic behaviors in the distribution process, we'll examine the distribution equilibria of the cooperative payoffs of the coalitions in the coalition equilibrium of cooperative game $\Gamma(N,\ \{S_i\},\ \{u_i\})$ with agreements self-implemented.

According to the definition of the common payoff of a member set, the common payoff of member set $M = \{1,\ 2\}$ of coalition C_1 is:

$$\tilde{\theta}_M = V_{C_1} - \sum_{i=1}^{2} \tilde{w}_i^{-C_1} = 0.$$

Therefore, in the coalition equilibrium c^*, the cooperative payoff distributions that players 1 and 2 get are

$$\tilde{x}_1 = \tilde{w}_1^{-C_1} + \frac{1}{2}\tilde{\theta}_M = 150;$$

$$\tilde{x}_2 = \tilde{w}_2^{-C_1} + \frac{1}{2}\tilde{\theta}_M = 100,$$

respectively.

4.3 *Cooperative bargaining game on the distribution of the cooperative payoff of a coalition*

When the agreements are self-implemented, in cooperative bargaining game $\Gamma(M,\ \{T_i\},\ \{\tilde{x}_i\})$ of coalition C, members will compose m coalitions which compete with each other. Because in the bargaining game, no member can betray the coalition through deviation, the coalition equilibrium of the bargaining game is the one under the criterion of minimum escape-payoff, t^*.

In some coalition situation t in the cooperative bargaining game of coalition C, the total cooperative payoff distribution gotten by each coalition m_i is:

$$\begin{aligned}x_{m_i} &= y_{m_i} + w_{m_i}^{-C} \\ &= w_{m_i}^{-C} + \frac{1}{2}\sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m} \theta(M_{m_i,\, m_j}) + \frac{1}{3}\sum_{\substack{m_j=1 \\ m_j \neq m_i}}^{m}\sum_{\substack{m_k=1 \\ m_k \neq m_i}}^{m_j-1} \theta(M_{m_i,\, m_j,\, m_k}) + \cdots + \frac{1}{m}\theta(M_{1,\, 2,\, \cdots,\, m}),\end{aligned}$$

where $M_{m_1,\, m_2,\, \cdots,\, m_k}$ represents the subset composed of coalitions $m_1,\ m_2,\ \cdots,\ m_k$, in coalition situation t of the cooperative bargaining game; $\theta\left(M_{m_1,\, m_2,\, \cdots,\, m_k}\right)$ represents the common payoff of coalition set $M_{m_1,\, m_2,\, \cdots,\, m_k}$.

When the agreements are self-implemented, in cooperative bargaining game $\Gamma(M,\ \{T_i\},\ \{x_i\})$ of coalition C, there exists the coalition equilibrium under the criterion of maximum cooperative payoff distribution:

$$\forall i = 1,\ 2,\ \cdots,\ m,$$

$$t_i^* = \begin{cases} i, \text{ if for any } t_i,\ x_i(i,\ t_{-i}^*) \geq x_i(t_i,\ t_{-i}^*); \\ \arg\max x_i(t_i,\ t_{-i}^*), \text{ if at least for a certain } t_i \neq i,\ x_i(i,\ t_{-i}^*) < x_i(t_i,\ t_{-i}^*). \end{cases}$$

And also there exists the coalition equilibrium under the criterion of minimum escape-payoff (note that in the bargaining game, no member can betray his coalition):

$$\forall i = 1,\ 2,\ \cdots,\ m,$$

$$t_i^* = \begin{cases} i, \text{ if for any } t_i,\ w_i(T_{t_i})(i,\ t_{-i}^*) \leq w_i(T_{t_i})(t_i,\ t_{-i}^*); \\ \arg\min w_i(T_{t_i})(t_i,\ t_{-i}^*), \text{ if at least for a certain } t_i \neq i,\ w_i(T_{t_i})(i,\ t_{-i}^*) > w_i(T_{t_i})(t_i,\ t_{-i}^*). \end{cases}$$

If the competitive distribution condition is met in each coalition, that's to say, $w_i(T_{t_i}) \leq x_i(T_{t_i}) \leq Mv_i(T_{t_i})$ (the equations hold if and only if there is on and only member in the coalition), the above two equilibria are equivalent,

$$\forall i = 1,\ 2,\ \cdots,\ m,$$

$$t_i^* = \begin{cases} i, \text{ if for any } t_i,\ x_i(i,\ t_{-i}^*) \geq x_i(t_i,\ t_{-i}^*); \\ \arg\max x_i(t_i,\ t_{-i}^*), \text{ if at least for a certain } t_i \neq i,\ x_i(i,\ t_{-i}^*) < x_i(t_i,\ t_{-i}^*). \end{cases}$$

$$\Leftrightarrow t_i^* = \begin{cases} i, \text{ if for any } t_i,\ w_i(T_{t_i})(i,\ t_{-i}^*) \le w_i(T_{t_i})(t_i,\ t_{-i}^*); \\ \arg\min\ w_i(T_{t_i})(t_i,\ t_{-i}^*), \text{ if at least for a certain } t_i \ne i,\ w_i(T_{t_i})(i,\ t_{-i}^*) > w_i(T_{t_i})(t_i,\ t_{-i}^*). \end{cases}$$

In coalition equilibrium t^* of the cooperative bargaining game, the competition between the m members is replaced by the competition between the m coalitions.

When the coalitions distribute their cooperative payoffs before the game begins, in coalition equilibrium t^* of the cooperative bargaining game of coalition C, denote the coalition set composed of coalitions $m_1^*,\ m_2^*,\ \cdots,\ m_k^*$ as $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}$, common payoff of set $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}$ is defined as follows:

$$\theta(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}) = V_{M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}} - \sum_{i=1}^{k} w_{m_i^*}^{-C} - \sum \theta_{(2)}(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}) - \cdots - \sum \theta_{(k-1)}(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}),$$

the Nash equilibrium of the cooperative bargaining game between coalitions in set $M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}$ on the distribution of common payoff $\theta(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*})$ is:

$$y_{m_i^*}^* = \frac{1}{k}\theta(M_{m_1^*,\ m_2^*,\ \cdots,\ m_k^*}),\ i = 1,\ 2,\ \cdots,\ k.$$

In coalition equilibrium t^* of cooperative bargaining game $\Gamma(M, \{T_i\},\ \{x_i\})$ on the distribution of the cooperative payoff surplus of coalition C, in the Nash equilibrium, the cooperative payoff surplus distribution gotten by each coalition m_i^* is:

$$y_{m_i^*} = \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \theta(M_{m_i^*,\ m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m}\ \sum_{\substack{m_k^*=1 \\ m_k^* \ne m_i^*}}^{m_j^*-1} \theta(M_{m_i^*,\ m_j^*,\ m_k^*}) + \cdots + \frac{1}{m}\theta(M_{1,\ 2,\ \cdots,\ m}),$$

where $\theta(M_{m_i^*,\ m_j^*}),\ \theta(M_{m_i^*,\ m_j^*,\ m_k^*}),\ \cdots,\ \theta(M_{1,\ 2,\ \cdots,\ m})$ represent the common payoffs of the corresponding coalition sets, respectively.

Its total cooperative payoff distribution is:

$$\begin{aligned} x_{m_i^*} &= y_{m_i^*} + w_{m_i^*}^{-C} \\ &= w_{m_i^*}^{-C} + \frac{1}{2}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m} \theta(M_{m_i^*,\ m_j^*}) + \frac{1}{3}\sum_{\substack{m_j^*=1 \\ m_j^* \ne m_i^*}}^{m}\ \sum_{\substack{m_k^*=1 \\ m_k^* \ne m_i^*}}^{m_j^*-1} \theta(M_{m_i^*,\ m_j^*,\ m_k^*}) + \cdots + \frac{1}{m}\theta(M_{1,\ 2,\ \cdots,\ m}). \end{aligned}$$

Of course, when the coalitions distribute their cooperative payoffs before the game begins, in the Nash equilibrium of the cooperative bargaining game of coalition C on cooperative payoff surplus distribution, what we get is the cooperative payoff surplus distribution of each coalition, rather than the cooperative payoff surplus distribution of each member. If there are more than 2 members in a coalition, we still need to investigate the second level, the third level, the forth level cooperative bargaining games, and so on. Finally we'll obtain the distribution of each member.

5. Conclusion and prospect

We have analyzed the formation of the coalitions and the distribution process of the cooperative payoff of a coalition in a cooperative game with agreements self-implemented in the following three scenarios:

(1) the possible opportunistic behaviors in the distribution process are ignored;

(2) coalitions centralize all the payoffs their members get in the game, to inhibit the possible opportunistic behaviors in the distribution process;

(3) coalitions distribute their cooperative payoffs before the game begins, to inhibit the possible opportunistic behaviors in the distribution process.

Analysis in this paper shows that, in each scenario, in a cooperative game with agreements self-implemented, there must exist the coalition equilibrium under the criterion of maximum cooperative payoff distribution (when the members of each coalition trust each other), and the coalition equilibrium under the criterion of minimum escape-payoff deriving from deviation (when the members of each coalition trust each other) too. If in the coalitions the competitive distribution condition is satisfied, the above coalition equilibria are equivalent.

In the coalition equilibrium of a cooperative game with agreements self-implemented, the cooperative payoff of a coalition can be decomposed into the escape-payoffs deriving from deviation of all members and the common payoffs of all member sets of the coalition. When members are non-cooperative in the bargaining game on the distribution of the common payoff of a member set, the Nash equilibrium of the bargaining game is that all members of the member set will share this common payoff equally. For the cooperative payoff distribution of each member is the sum of his escape-payoff deriving from deviation and the common payoff distributions he gets in the equilibrium of the bargaining games of all member set to which he belongs, therefore there exists the equilibrium in the bargaining game of a coalition on the distribution of its cooperative payoff. When members are cooperative in the bargaining game of a coalition, we can draw a similar conclusion.

Obviously the discussion on the cooperative games with agreements self-implemented has not come the end, and the author believes that there are three important research directions. Firstly, self-implemented agreements exist not only in one-time cooperative games, but also in infinitely-repeated cooperative games. Does there exist the coalition equilibrium in an infinitely-repeated cooperative game with agreements self-implemented? If it does exist, what is it? If it different form the one in a one-time stage game? And how is the cooperative payoff of a coalition been distributed in this coalition equilibrium?

Secondly, an obvious fact is that in the cooperative games with agreements self-implemented, the players are often information asymmetric. In an information-asymmetric cooperative game with agreements self-implemented, does there exist the coalition equilibrium? If it does exist, what is it? What are the conditions for its existence? How will the cooperative payoff of a coalition be distributed in this coalition equilibrium?

Thirdly, in the cooperative games with agreements self-implemented, players are often imperfectly intelligent. Can learning models in game theory come to the coalition equilibrium in an evolutionary game in which the players are imperfectly intelligent and information-asymmetric? If they can, how does a coalition distribute its cooperative payoff among its members under this equilibrium?

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Conflict of interest

The author declares there is no conflict of interest at any point with reference to research findings.

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