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Research Article

Analysis of a Single Server Retrial Queueing System with Finite Capacity, F-Policy, Balking, Reneging and Server Maintenance

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Abstract: In this paper, we determine a single server retrial queuing system with finite capacity, F-policy, balking, reneging and server maintenance. The arriving customer when notices the server becomes free, then they get service immediately. In case server becomes busy, the customers are move to either orbit or may balk the system. In the orbit, after some random time the customer repeats their attempts for service. The customers are waiting in the orbit for longer time become impatient and leaves the orbit without getting service. Once the number of customers in the orbit reaches its full capacity then the arrival of customer is not allowed to enter into the orbit. The new customer is allowed into the orbit when the F-policy level is reached. If no more customer in the orbit, server will move to the vacation. By using recursive technique, the steady state distributions are obtained and we have developed various performance measures.

***Keywords*:** retrial queue, F-policy, vacation, discouragement, recursive technique

MSC: 60K25, 60K30, 60KM20, 90B22, 90B36

1. Introduction

In real life we can see many customers are waiting for service and they form the queue, because of more demand for service. For example, the customer waiting in line at supermarket to get their service. In today's life, people want to get service immediately but, in some case, server is busy with other customer. In that such situation, when customers notice the server is busy. The customers will move to orbit. From there they will retry for service after some time. For example, In Bank scenario the customers enter into the bank and they noticing server is busy, the customer will move into the orbit (waiting area) and they will retry for the service. When the arriving customer notices the long queue, the customer will not join the queue. For example, ticket counter in theatre, the customer will notice the queue but the customer will not join because of longer queue. In many situations the customer waits for long time in the queue. So, the customers lose the patience leave and the system without getting the service is reneging. For example, customer waiting to buy tickets in the railway station, the customer will leave, after some time customer because of impatience. The concept of F-policy is the arrival control policy. Once the number of customer in the system reaches its capacity, no more customers are allowed to enter into the system until, number of customer reaches the level 'F'. For example, in rubber manufacturing, arrival of raw material as pulp are arrives into the system. When storage is full and no space for new raw material, the arrivals are stopped until each raw material in the orbit will decreases and reaches the level

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'F'. After giving service to all the customers and no customers are present in the orbit, the server will move to the vacation. During vacation server will take secondary job. For example, in petrol bunk when no vehicles are arriving to get petrol, the server will move to the secondary jobs like maintenance work as vacation. [1] have taken single server retrial queueing system with optional service and impatience customers and vacation. By using supplementary variable technique and they solved using general decomposition law. Various performance and numerical examples are illustrated. [2] deals with state dependent rates, the arrival of customers and service are based on the number of customers in the system with single server, the steady state queue size distribution is established by recursive method. The minimal cost of the system is developed by using heuristic method. [3] have taken machine repair problem with single working vacation with F-policy. Matrix method has been used to obtain the steady state probability of the system states and numerical results have been obtained. [4] developed the supplementary variable used to evaluate the steady state queue size distribution. The retrial times are used to provide the exact results, they have taken a recursive method to solve. Quasi-Newton's method and genetic algorithm is used to optimization of the total cost function. [5] analyzed the two-server retrial queueing system with entrance restriction and server maintenance. In the orbit, the customer entry restriction is made by without any collision will get service. The new customer is not allowed to enter the orbit when the maximum level reached. The model performance is analyzed by graphically and numerically. [6] studied the cost function to evaluate the optimal service rate of the system. The numerical results are obtained for this model by using the adaptive neuro fuzzy inference system (ANFIS). [7] using the supplementary variable to the remaining retrial times. The chapman Kolmogorov equations are obtained by the steady state queue size distribution. They performed the numerical simulation and the effects of different parameters. [8] deals with single server feedback queuing model with impatience customers and the vacation. They have derived the steady state probabilities of number of customers in the system, by using stationary distribution obtain the various performance of effectiveness of the model, and they have performed the numerical example and the cost profit. [9] deals with single server queuing system with discouragement and retention of renege customers. The analysis of the model is limited to finite capacity in this model. They have solved in transient state, the results are based on time dependent and then cost profit also to study the economic models. [10] studied a two-server retrial queueing system with vacation and extra server. To increase the service quality the recursive methods are used. The performance measure is analyzed by graphically and numerically. [11] developed the retrial spectacle in a multiple server queueing model. [12] analyzed the retrial M/M/1 queue with dissatisfied jobs and server vacations.

The construction of this article is as follows: Introduction in section 1. Section 2 offers description of the Model. Section 2.2 presents steady-state equations. Section 3 presents the Performance-measures. Section 4 provides the numerical illustration and finally conclusion in Section 5.

2. Model description

Consider an M/M/1/K Retrial Queueing system with F-policy, Impatience and vacation. The server attends the customers at a time by following the First-in-First-out (FIFO) basis. The arrival of customer joins the system by following Poisson process with arrival rate λ. The service time is to follow the exponential distribution $\frac{1}{\mu}$.

In case the server is busy with other customer, they will join to the retrial orbit with probability q or the customer may balk the system with probability of $\overline{q} = (1 - q)$. In the orbit, the customer retry the service with probability of σ or may leave the system without getting a service because of customer lose patience it follows by the probability $\overline{\sigma} = (1 - \sigma)$ and with retrial times following exponential distribution $\frac{1}{\gamma}$. When the orbit reaches its full capacity, no more customers are allowed into the retrial orbit until when each customer in the orbit will reaches the threshold level F. A setup time is taken according to exponential distribution with rate s. When no customers are present in the orbit the server will move to the vacation by following the exponential distribution with rate η. Let random variable $N(t)$ represents the total customer in the system at time t and $S(t)$ defines the state of the system.

$$S(t)\begin{cases} 0, & \text{when the server is on maintenance} \\ 1, & \text{when server is free} \\ 2, & \text{when server is busy, the customer are allowed in the system} \\ 3, & \text{when server is busy, the customer are not allowed in the system} \end{cases}$$

To develop Markov model, the system states probabilities at time t for (j, n) by $P_{j,n}(t) = P\{S(t) = j; N(t) = n\}$. When j takes values 0, 1, 2, 3 and n be the number of customers. $\{S(t), N(t)$: $t \geq 0\}$ is a bi-variate Markov process is with continuous time and discrete state space. Steady states are analyzed by Markov model, when $t \rightarrow \infty$ and $P_{j,n} = \lim_{t\rightarrow\infty} P_{j,n}(t)$ denote the state probability.

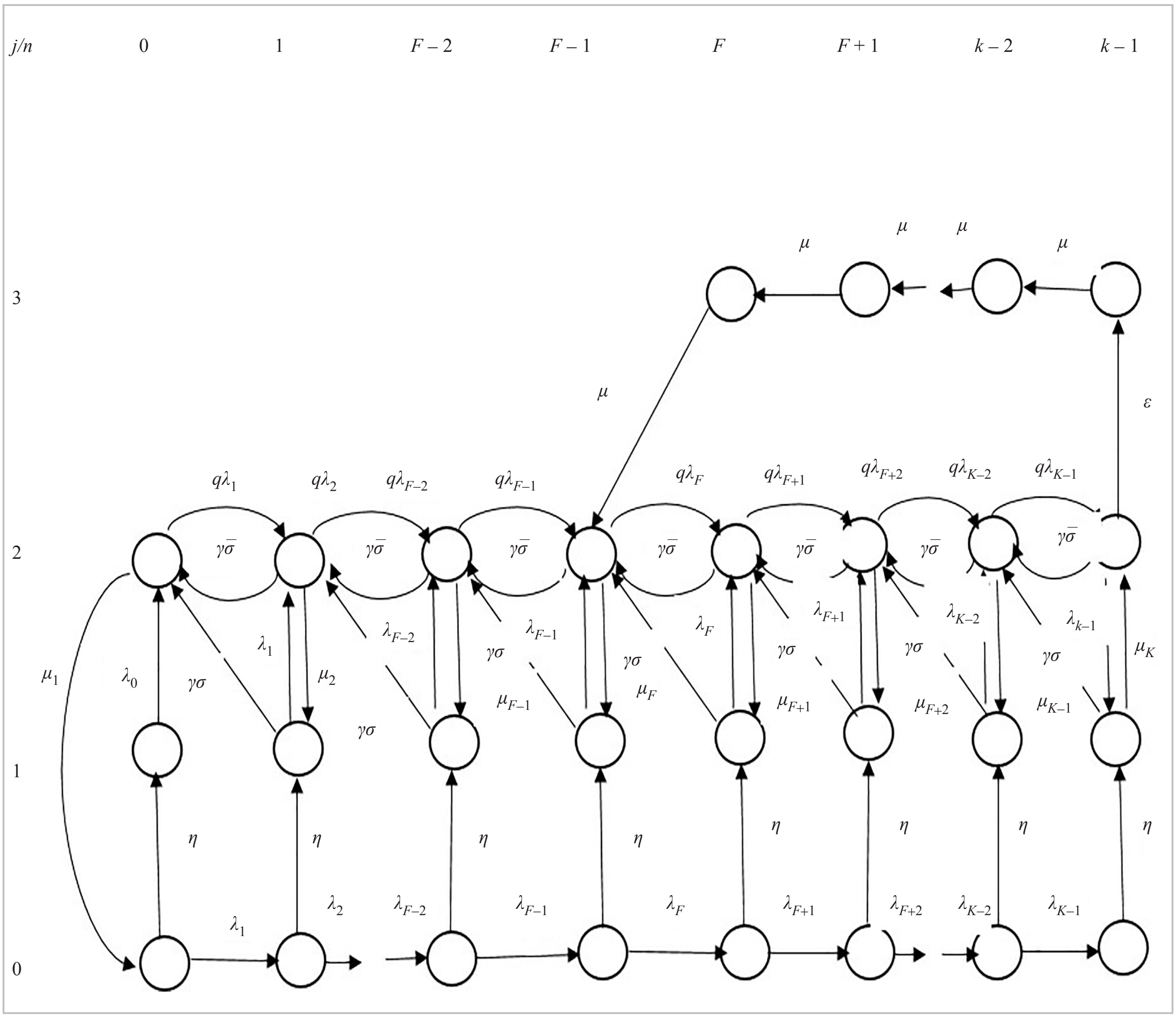

Figure 1. Rate transition diagram

Figure 1 represents the rate transition diagram of single server retrial queueing system with F-policy, balking, reneging and server maintenance.

2.1 *Steady state equations*

To evaluate the single server retrial queueing system, F-policy with vacation and impatience. The steady state Chapman-Kolmogorov equations are obtained for the orbit size related to four different states 0, 1, 2 and 3 respectively. By using the figure 1, we formulate the steady state equations.

$$-(\eta+\lambda_1)P_{0,0}+\mu_1 P_{2,0}=0 \tag{1}$$

$$-(\eta+\lambda_{n+1})P_{0,n}+\lambda_n P_{0,n-1}=0,\ 1\le n\le k-1 \tag{2}$$

$$-\lambda_0 P_{1,0}+\eta P_{0,0}=0 \tag{3}$$

$$-(\lambda_n+\gamma\sigma)P_{1,n}+\mu_{n+1}P_{2,n}+P_{0,n}=0,\ 1\le n\le k-1 \tag{4}$$

$$-(q\lambda_1+\mu_1)P_{2,0}+\lambda_0 P_{1,0}+\gamma\bar{\sigma}P_{2,1}+\gamma\sigma P_{1,1}=0 \tag{5}$$

$$-(q\lambda_{n+1}+\mu_{n+1}+\gamma\bar{\sigma})P_{2,n}+q\lambda_n P_{2,n-1}+\gamma\sigma P_{1,n+1}+\lambda_n P_{1,n}=0;\ 1\le n\le F-2 \tag{6}$$

$$-(q\lambda_F+\mu_F+\mu\bar{\sigma})P_{2,F-1}+\lambda_{F-1}P_{1,F-1}+q\lambda_{F-1}P_{2,F-2}+\gamma\sigma P_{1,F}+\gamma\bar{\sigma}P_{2,F}+P_{3,F}=0 \tag{7}$$

$$-(q\lambda_{n+1}+\mu_{n+1}+\gamma\bar{\sigma})P_{2,n}+\lambda_n P_{1,n}+q\lambda_n P_{2,n-1}+\gamma\bar{\sigma}P_{2,n+1}=0,\ F\le n\le K-2 \tag{8}$$

$$-(\varepsilon+\mu_k+\gamma\bar{\sigma})+P_{2,K-1}+\lambda_{k-1}P_{1,k-1}+q\lambda_{k-1}P_{2,k-2}=0 \tag{9}$$

$$\mu_n P_3=\varepsilon P_{2,k-1},\ F\le n\le k-1 \tag{10}$$

By using recursive technique, from steady state equations (1)-(10) are obtained in terms of $P_{0,0}$. From equation (1), the probability of zero customer in the orbit, when server is busy during allowed customer are obtained,

$$P_{2,0}=\frac{(y+\lambda_1)}{\mu_1}P_{0,0}. \tag{11}$$

From equation (3), the probability of the initial customer in the system, during server in Free State are obtained,

$$P_{1,0}=\frac{y}{\lambda_0}P_{0,0}. \tag{12}$$

From equation (2), In general, the probability of total number of customers are presented, during server is in maintenance period,

$$P_{0,n}=\prod\nolimits_{i=1}^{n}\frac{\lambda_i}{\eta+\lambda_{i+1}}P_{0,0},\ 1\le n\le K-1. \tag{13}$$

From equation (4), by using (13),

$$P_{1,n} = \frac{\mu n+1}{(\lambda n+\gamma\sigma)} P_{2,n} + \frac{\eta}{(\lambda n+\gamma\sigma)} \prod_{i=1}^{n} \frac{\lambda_i}{(\eta+\lambda_{i+1})} P_{0,0},\ 1 \le n \le K-1. \tag{14}$$

From equation (5), using equation (11), (12) and (13), the probability of 1st orbital customer, when during allowed customers of the busy server state,

$$P_{2,1} = \frac{\theta_1}{\delta_1} P_{0,0}, \tag{15}$$

where

$$\theta_1 = q\lambda_1^2\eta^2 + q\lambda_1^3\eta + \lambda_1^2\mu_1\eta + q\lambda_1^2\lambda_2\eta + q\lambda_1^3\lambda_2 + \lambda_1^2\lambda_2\mu_1 + q\lambda_1\eta\gamma\sigma\lambda_2$$

$$+q\lambda_1^2\gamma\sigma\lambda_2 + \mu_1\gamma\sigma\lambda_2 + q\lambda_1\eta^2\gamma\sigma + q\lambda_1^2\gamma\sigma\eta$$

and

$$\delta_1 = \mu_1(\eta+\lambda_2)\left[\lambda + \gamma\sigma(\gamma\bar{\sigma}) + \gamma\sigma\mu_2\right].$$

From equation (4), using equation (14) and (15), the probability of 1st orbital customer, when during server is free state,

$$P_{1,1} = \frac{\psi_1}{\omega_1} P_{0,0}, \tag{16}$$

where

$$\psi_1 = \mu_2(\eta+\lambda_2)\theta_1 + \eta\lambda_1\delta_1$$

and

$$\omega_1 = \delta_1(\lambda_1+\gamma\sigma)(\eta+\lambda_2).$$

From equation (6), using equation (13) and (14), the probabilities of n customers are presented during server in busy period is obtained,

$$P_{2,n} = \frac{\theta_n}{\delta_n} P_{0,0},\ 2 \le n \le F-1 \tag{17}$$

where

$$\theta_n = \left(q\lambda_n + \mu_n + \gamma\bar{\sigma}\right)\left(\lambda_n + \gamma\sigma\right)\theta_{n-1}\omega_n\delta_{n-2}\prod_{i=1}^{n}\left(\eta + \lambda_{i+1}\right)$$

$$-q\lambda_{n-1}\theta_{n-2}\delta_{n-1}\left(\lambda_n + \gamma\sigma\right)\omega_{n-1}\prod_{i=1}^{n}\left(\eta + \lambda_{i+1}\right)\delta_{n-1}\gamma\sigma\eta\omega_{n-1}\prod_{i=1}^{n}\lambda_i$$

$$-\delta_{n-1}\delta_{n-2}\lambda_{n-1}\psi_{n-1}\left(\lambda_n + \gamma\sigma\right)\prod_{i=1}^{n}\left(\eta + \lambda_{i+1}\right),$$

$$\delta_n = \delta_{n-1}\delta_{n-2}\omega_{n-1}\left[\gamma\sigma\mu_{n+1} + \gamma\bar{\sigma}\left(\lambda_n + \gamma\sigma\right)\right]\prod_{i=1}^{n}\left(\eta + \lambda_{i+1}\right).$$

From equation (4), using equation (14) the probability of the customers, when the server in free state are obtained,

$$P_{1,n} = \frac{\psi_n}{\omega_n}P_{0,0},\ 2 \le n \le F-1 \tag{18}$$

where

$$\psi_n = \mu_{n+1}\theta_n\prod_{i+1}^{n}\left(\eta + \lambda_{i+1}\right) + \eta\delta_n\prod_{i=1}^{n}\lambda_i,$$

$$\omega_n = \lambda_n + \gamma\sigma\delta_n\prod_{i=1}^{n}\left(\eta + \lambda_{i+1}\right).$$

From equation (7), the probability of the customer at the level of 'F', when server is busy during the customers are allowed state obtained,

$$P_{2,F} = \frac{\theta_F}{\delta_F}P_{0,0}. \tag{19}$$

Where

$$\theta_F = \left(q\lambda_F + \mu_F + \gamma\sigma\right)\theta_{F-1}\omega_{F-1}\delta_{F-2}\left(\lambda_F + \gamma\sigma\right)\varsigma_{K-F-2}\beta_{K-F-2}\prod_{i=1}^{n}\left(\eta + \lambda_{F+1}\right)$$

$$-\lambda_{F-1}\psi_{F-1}\delta_{F-1}\delta_{F-2}\left(\lambda_F + \gamma\sigma\right)\varsigma_{K-F-2}\beta_{K-F-2}\prod_{i=1}^{n}(\eta +$$

$$\lambda_{F+1}q\lambda_{F-1}\theta_{F-2}\delta_{F-1}\omega_{F-1}\left(\lambda_F + \gamma\sigma\right)\varsigma_{K-F-2}\beta_{K-F-2}$$

$$\prod_{i=1}^{n}\left(\eta + \lambda_{F+1}\right) - \gamma\sigma\eta\delta_{F-1}\omega_{F-1}\delta_{F-2}\varsigma_{K-F-2}\beta_{K-F-2}\prod_{i=1}^{n}\lambda_i + \varepsilon\left(\xi_{K-F-2}\right)$$

$$\left(\alpha_{K-F-2}\right)\delta_{F-1}\omega_{F-1}\delta_{F-2}\left(\lambda_F + \gamma\sigma\right)\prod_{i=1}^{n}\left(\eta + \lambda_{F+1}\right).$$

$$\delta_F = \delta_{F-1}\omega_{F-1}\delta_{F-2}\prod_{i=1}^{n}(\eta+\lambda_{F+1})\beta_{K-F-2}\left[\varepsilon\left(\xi_{K-F-2}\right)\left(\lambda_F+\gamma\sigma\right)+\right.$$

$$\left.\gamma\sigma\mu_{F+1}\varsigma_{K-F-2}+\gamma\bar{\sigma}\left(\lambda_F+\gamma\sigma\right)\right].$$

From equation (4), the probability of the customer in the level 'F', when customers are allowed during in busy server are obtained,

$$P_{1,F} = \frac{\psi_F}{\omega_F}P_{0,0}, \tag{20}$$

where

$$\psi_F = \mu_{F+1}\theta_F\prod_{i+1}^{F}(\eta+\lambda_{i+1})+\eta\delta_F\prod_{i=1}^{F}\lambda_i,$$

$$\omega_F = \left(\lambda_F+\gamma\sigma\right)\delta_F\prod_{i=1}^{F}(\eta+\lambda_{i+1}).$$

From equation (8), using equation (13) and (17), the probability of n customers are presented during server in busy period is obtained,

$$P_{2,n} = \frac{\theta_n}{\delta_n}P_{0,0}, F+1\le n\le K-1 \tag{21}$$

where

$$\theta_n = \left(q\lambda_n+\mu_n+\gamma\bar{\sigma}\right)\left(\lambda_n+\gamma\sigma\right)\theta_{n-1}\omega_{n-1}\delta_{n-2}\prod_{i=1}^{n}(\eta+\lambda_{i+1})$$

$$-q\lambda_{n-1}\theta_{n-2}\delta_{n-1}\left(\lambda_n+\gamma\sigma\right)\omega_{n-1}\prod_{i=1}^{n}(\eta+\lambda_{i+1})-\delta_{n-1}\gamma\sigma\eta\omega_{n-1}\delta_{n-2}\prod_{i=1}^{n}\lambda_i$$

$$-\delta_{n-1}\delta_{n-2}\lambda_{n-1}\psi_{n-1}\left(\lambda_n+\gamma\sigma\right)\prod_{i=1}^{n}(\eta+\lambda_{i+1}).$$

From equation (4), using equation (18), the probability of the customers, when the server in free are obtained,

$$P_{1,n} = \frac{\psi_n}{\omega_n}P_{0,0}, F+1\le n\le K-1 \tag{22}$$

where

$$\psi_n = \mu_{n+1}\theta_n\prod_{i+1}^{n}(\eta+\lambda_{i+1})+\eta\delta_n\prod_{i=1}^{n}\lambda_i,$$

$$\omega_n = \left(\lambda_n+\gamma\sigma\right)\delta_n\prod_{i=1}^{n}(\eta+\lambda_{i+1}).$$

From equation (10), the probabilities of the number customer in the orbit, when customers are not allowed busy during busy server are obtained,

$$P_{3,n} = \frac{\varepsilon\left(\theta_{K-1}\right)}{\mu\left(\delta_{K-1}\right)} P_{0,0}. \tag{23}$$

In normalization condition,

$$\left[\sum_{1}^{K-1} P_{0,n} + P_{1,0} + P_{2,0} + \sum_{n=1}^{F-1} P_{1,n} + \sum_{n=1}^{F-1} P_{2,n} + P_{2,F} + P_{1,F} + \sum_{n=F+1}^{K-1} P_{1,n}\right.$$

$$\left. + \sum_{n=F+1}^{K-1} P_{2,n} + \sum_{n=F}^{K-1} P_{1,n}\right] P_{0,0} = 1.$$

After few algebraic manipulations, we get

$$P_{0,0} = \frac{1}{A_n + B + C_n + D + R_n}, \tag{24}$$

where

$$A_n = \sum_{n=1}^{K-1} P_{0,n},$$

$$B = P_{1,0} + P_{2,0},$$

$$C_n = \sum_{n=1}^{F-1} \left[P_{1,n} + P_{2,n}\right],$$

$$D = P_{2,F} + P_{1,F},$$

and

$$R_n = \sum_{n=F+1}^{K-1} \left[P_{2,n} + P_{1,n}\right].$$

3. Performance measures

The following system are established in terms of probability,
Probability of idle server (P_I)

$$P_I = \sum_{n=0}^{K-1} P_{0,n} + \sum_{n=0}^{K-1} P_{1,n}.$$

Probability of busy server (P_{SB})

$$P_{SB} = \sum_{n=0}^{K-1} P_{2,n} + \sum_{n=F}^{K-1} P_{3,n}.$$

Throughput (TP)

$$TP = \mu_n \sum_{n=0}^{K-1} P_{2,n} + \mu \sum_{n=F}^{K-1} P_{3,n}.$$

Average number of customers in the system $E[N_O]$

$$E\left[N_O\right] = \sum_{n=0}^{K-1} nP_{1,n}.$$

Average number of customers in the system $E[N_S]$

$$E\left[N_S\right] = \sum_{n=0}^{K-1} nP_{0,n} + \sum_{n=0}^{K-1} nP_{1,n} + \sum_{n=0}^{K-1} (n+1) P_{2,n} + \sum_{n=F}^{K-1} (n+1) P_{3,n}.$$

4. Numerical illustration

In this section, we present roughly the numerical instances to validate the various parameters of the model influence the optimal service rate and the performance measures of the system. In Table 1, the retrial rate γ increases then the probability of busy server P_{SB}, probability of idle server P_I and average number of customers in the system $E[N_s]$ also increases.

Table 1. Comparison of system performance measures for different values of γ

γ	P_I	P_{SB}	$E[N_s]$
3	0.03	0.24	1.05
6	0.06	0.32	1.78
9	0.07	0.38	2.37
12	0.09	0.43	3.06
15	0.10	0.47	3.54

The effect of reneging rate σ under different choices of λ, μ, θ and q has been shown in table values. In Table 2, the average number of customers in the system and idle probability decreases when we increase the value of reneging rate. The probability of busy server increases then the values of reneging also increase.

In Table 3, the value of P_I and $E[N_s]$ decreases for increasing reservice probability for different values of λ and μ. When we increase busy period value for the increasing value of reservice probability is also illustrated in addition in table values.

Table 2. Comparison of system performance measures for different values of σ

σ	P_I	P_{SB}	$E[N_s]$
1	0.163	0.40	4.88
2	0.09	0.43	2.73
3	0.06	0.45	1.89
4	0.04	0.45	1.45
5	0.03	0.46	0.99

Table 3. Comparison of system performance measures for different values of probability

Reservice	P_I	P_{SB}	$E[N_s]$
0.30	0.88	0.02	8.004
0.40	0.88	0.02	6.006
0.50	0.87	0.02	3.607
0.60	0.87	0.02	2.009
0.70	0.87	0.03	0.010

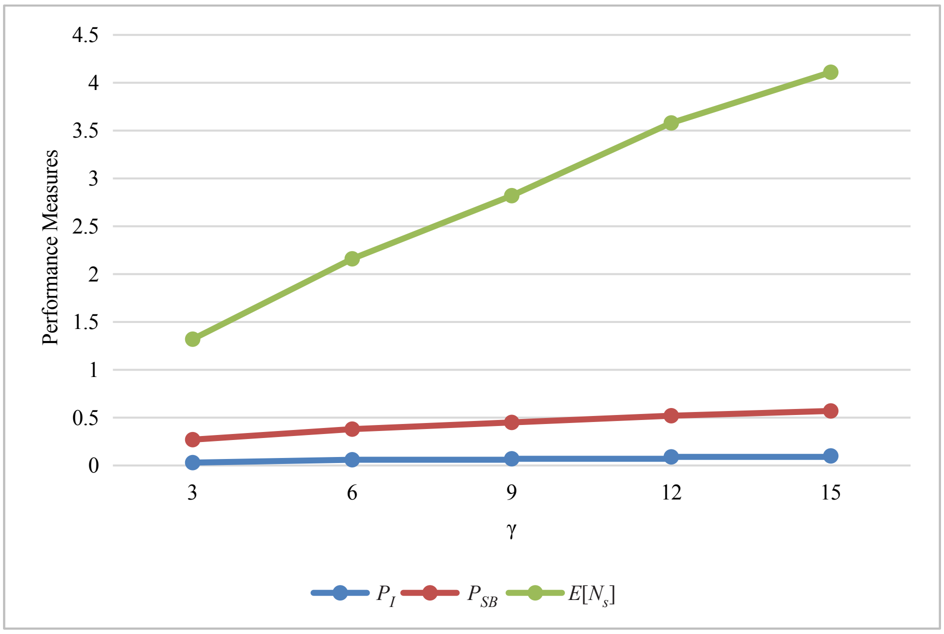

Figure 2. γ compares performance measures P_I, P_{SB}, $E[N_s]$

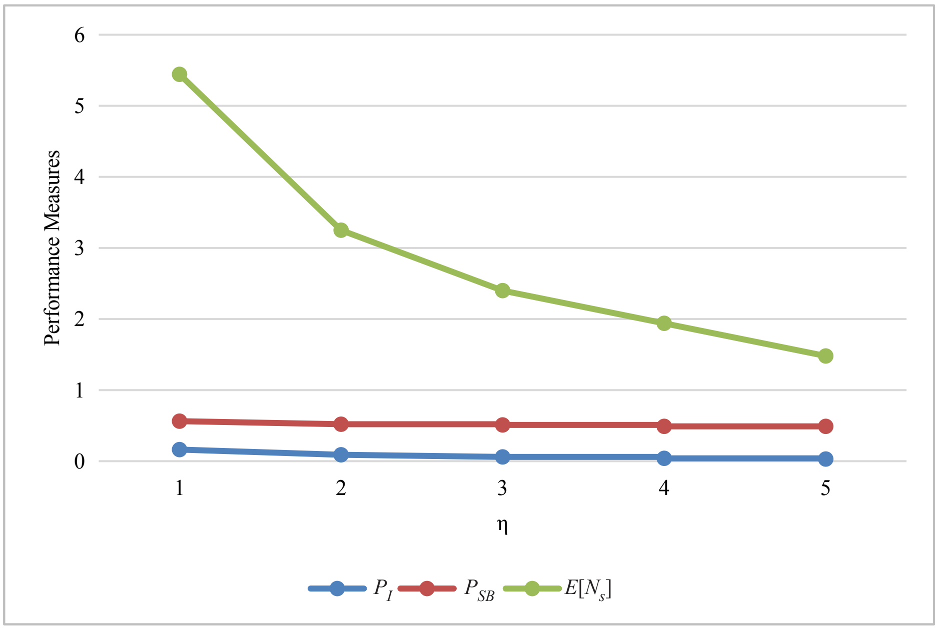

Figure 3. σ compares performance measures P_I, P_{SB}, $E[N_s]$

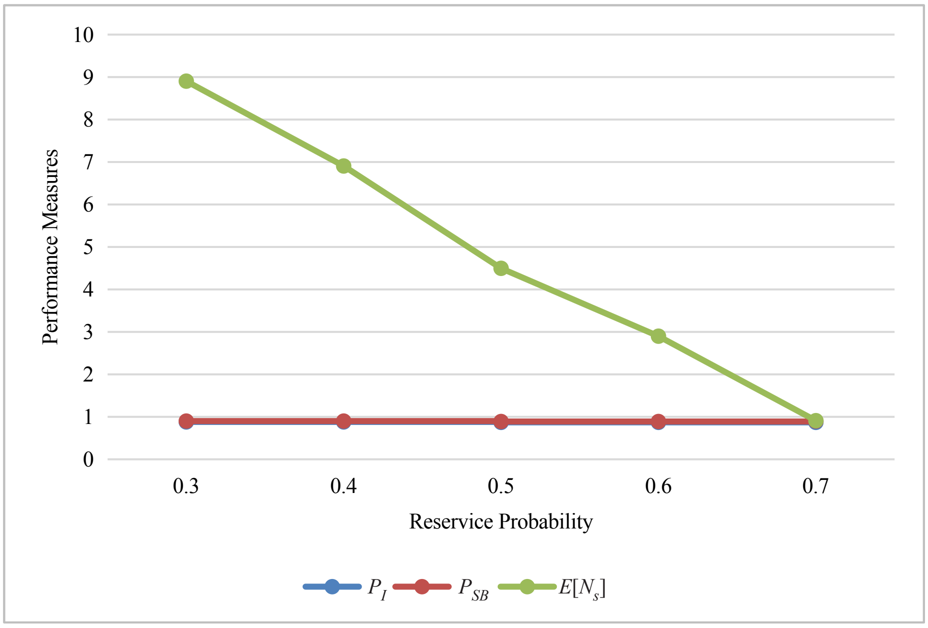

Figure 4. Reservice probability compares performance measures P_I, P_{SB}, $E[N_s]$

In these figures, we have strategized the probability of busy period value at different values of v, σ and reservice probability. The effect of balking and reneging rates on the behaviour of the busy period has also been estimated.

In Figure 2, the probability of busy period, idle probability and $E[N_s]$ is plotted for varying the value of retrial rate γ. When γ increases the probability of busy period P_{SB}, P_I and $E[N_s]$ also increases. We observe that the system performance measures goes to zero when we increase the value of effect of retrial rate.

In Figure 3, the idle probability and $E[N_s]$ decreases for increasing the value of effect of reneging rate but the probability of busy period increases. In Figure 4, the idle probability and $E[N_s]$ decreases for increasing the value of effect of reservice probability value but the probability of busy period value increases.

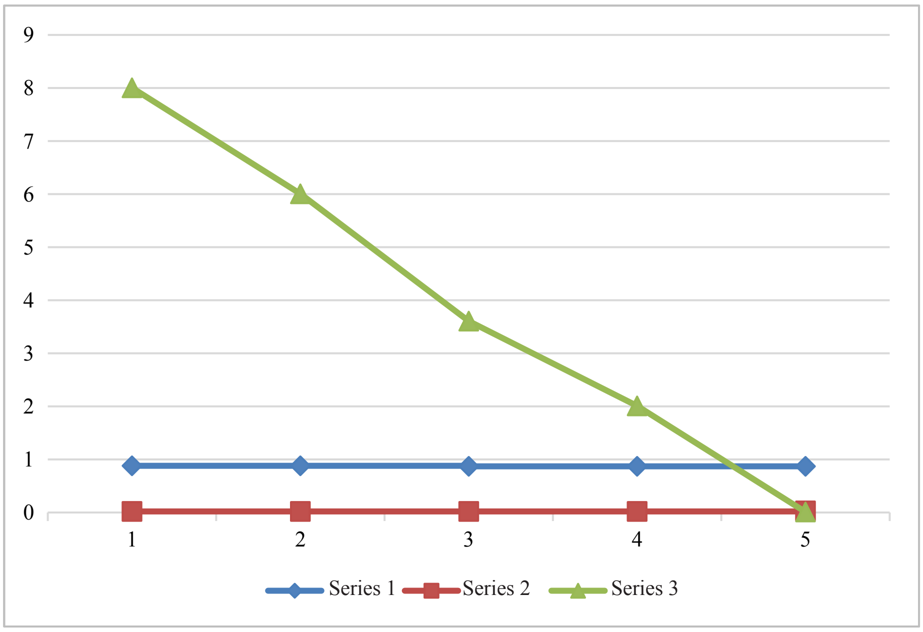

Figure 5. γ, σ versus performance measures $E[N_s]$

In Figure 5, the idle probability P_I, probability of busy server P_{SB} and average number of customers in the system $E[N_s]$ increases for increasing the various values of parameters λ, μ, γ, σ and θ. The above results illustrate the general behaviour for the probability of busy period is taper off faster to zero. This is fathomable since for such values the value. λ decreased as related to service rate.

5. Conclusion

A Markovian single server finite capacity retrial queuing system with F-policy, balking, reneging, and server maintenance is presented in this study. Mathematical expressions such as different states, when server in vacation, server in free state, the customers allowed during busy server and the customers are not allowed during busy server are derived. We have solved steady state queue size distribution by using recursive method. We have developed various performance measures.

Conflict of interest

The authors declare no competing financial interest.

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