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Research Article

Neutrosophic Semi β^F-Contra Continuity in a Neutrosophic Topological Space

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Abstract: The main intention of this article is to propose the theory of a neutrosophic S_β^F contra continuous function, neutrosophic strongly S_β^F continuous function, neutrosophic strongly S_β^F contra continuous function and neutrosophic S_β^F contra irresolute function in neutrosophic topological spaces. Further several properties based on the above concepts have been studied. Many theorems based on the above ideas have been proved with the suitable illustration. Further the relationship connecting neutrosophic S_β^F contra continuous function, neutrosophic strongly S_β^F continuous function, neutrosophic strongly S_β^F contra continuous function and neutrosophic S_β^F contra irresolute function are studied and illustrated. Moreover, various theorems based on compositions of functions have been proved. In addition to this, the product of two neutrosophic sets is defined and related theorems have been proved.

***Keywords*:** neutrosophic topology, neutrosophic S_{β^F} closed set, neutrosophic S_{β^F}-continuity, neutrosophic S_{β^F}-contra continuity, neutrosophic strongly S_{β^F}-contra continuity and neutrosophic S_{β^F}-contra irresolute

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Abbreviation

Neutro TS	Neutrosophic topological space
Neutro S_{β^F} csd st	Neutrosophic S_{β^F}-closed set
Csd st	Closed set
Cty	Continuity
Cts fn	Continuous function
Neutro topo	Neutrosophic topology
Opn st	Open set
Ctra	Contra
Neutro st	Neutrosophic set

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TSs	Topological spaces (Plural)
Fn	Function
topo	Topology
Fzy st	Fuzzy set

1. Introduction

Al-Hamido [1] proposed a new approach of a neutro TS which is more general than neutro TS. Moreover, the neutro concepts are applied in a new kind of a neutro TS to create a new neutro topo. Further some properties of this space were studied and investigated. Ali Abbas et al. [2] proposed a new class of neutro (1, 2)-maps in neutro bi TS and investigated some of its basic properties. In addition, the relationship between various types of classes like neutro (1, 2)-opn, csd, strongly opn maps had been discussed. Al-Omeri et al. [3] proposed a definition of cone metric space in the context of a neutro TS. Further, some fundamental results concerning fixed points for weakly compatible mapping had been proved. Arar and Jafar [4] introduced the concept of $\underline{\mu}$-TS and studied some of its important properties. Further the concept of a $\underline{\mu}$-cty along with $\underline{\mu}$-compactness were studied and investigated. Arokiarani et al. [5] originated the concept of neutro semi opn fn and investigated some of its inheritances. Also, the condition for a fn of a neutro TSs to be a neutro α-cts fn were given and investigated. Basumatary and Basumatary [6] studied the number of neutro TSs having two, three and four open sets are computed for a finite set X^{NT}, whose membership values lies in M^{NT}. Further, the number of a neutro bi TSs and neutro tri TSs having $k(k = 2, 3, 4)$ neutro opn sts on a finite sts are computed. Chandran et al. [7] developed the notion of a smooth neutro TS in a more natural way. Further the concept of product topo was defined and some of its characterizations were investigated. Çoker [8] introduced a concept of a "intuitionistic fzy TS". Also the concept of fzy cty, fzy compactness, fzy connectedness and fzy hausdorff space along with some characterization related to fzy compactness and fzy connectedness was also discussed. Further the concept of "intuitionistic L-fzy sts" are also introduced and investigated some of its properties.

Das et al. [9] originated the theory of a neutro soft st theory with roughness without using full soft st. Moreover, some properties had been studied on roughness with neutro soft st. Das and Tripathy [10] inspected some properties of a neutro multiset topo. Further, the attitude of compactness and connectedness in a neutro multiset topo had been examined. And the characterization of a "neutro multiset TS" had been examined. Dubey and Panwar [11] introduced the theory of $\alpha\psi$-connectedness in a neutro TS and also investigated some properties of $\alpha\psi$-connectedness between sts and subsets of two sts. Gündüz et al. [12] proposed the theory of neutro soft st theory. Further, neutro soft point concept had been introduced. Moreover, the theory of neutro soft T_i-spaces were proposed and the relationship between them were discussed in detail. Ishtiaq et al. [13] discussed topological structures and several fixed-point theorems in the context of generalized neutro cone metric spaces. In addition, the existence and uniqueness of a solution by utilizing a new type of contraction mappings under some circumstances had been discussed. Iswarya and Bageerathi [14] originated the theory of neutro semi-connected spaces which were solved using neutro semi-opn sts and neutro semi-csd sts. Moreover, the relationship between the neutro semi-frontier and neutrosophic semi-irresolute fn had been examined. Kalamani et al. [15] introduced the concept of intuitionistic fzy TS. Also, some of its properties were studied by them. Further, the concept of intuitionistic fzy generalized alpha csd sts and intuitionistic fzy generalized alpha opn sts were proposed and studied. Kumari et al. [16] introduced the concept of n-cylindrical fuzzy neutro TS, n-cylindrical fzy neutro opn sts and n-cylindrical fzy neutro csd st are introduced and studied. Further the relationship between these sts were studied with suitable example.

Lazaar et al. [17] studied the T_0-seperation axioms for a neutro TS. Further, the categorical study of these spaces in the special case of the neutro TS called neutro saturated TSs are discussed. Madhumathi and NirmalaIrudayam [18] proposed the concept of a neutro orbit TS which was symbolized by (X, t_{NO}). Also, some of the important characteristics of a neutro orbit opn sts were discussed and studied. Pushpalatha and Nandhini [19] proposed the concept of a neutro csd st in a neutro TS and studied some of their basic properties. Roja et al. [20] introduced the concept of a intuitionistic fzy ctra cts fn, strongly generalized intuitionistic fzy ctra cts fn and generalized intuitionistic fzy ctra irresolute fns along with some of its characterizations had been studied. Further the concept of generalized intuitionistic fzy filter and intuitionistic fzy convergent was introduced with proper illustration. Salama and Alblowi [21] proposed the concept of fzy TS and intuitionistic fzy TS along with some of their important properties. Savithiri and Janaki [22] introduced a new class of sts called neutro vague *rw*-csd sts, neutro vague *rw*-opn sts, neutro vague *rw* $T_{1/2}$-space and neutro vague *rw* T_w space in a neutro vague TSs. Further, some of its characterizations had been studied with suitable example. Subasree [23] proposed the theory of neutro $\psi\beta$ and neutro $\beta\psi$- csd st in a neutro TS. Further, various properties were also studied. Tran et al. [24] introduced an operator on neutro sts and its properties. Further, the relationship between the topo on neutro sts and fzy soft topologies were studied and investigated. Vadivel and Sundar [25] introduced a new type of cts fns such as N-neutro crisp gamma cts and weakly N-neutro crisp gamma cts fns in a N-neutro crisp TS and also discuss a relation between them in a N-neutro crisp TSs.

2. Preliminaries

Definition 2.1 Accredit S be a non-empty st. A neutro st H is an object having the form $H = \{< s,\ \mu_H(s),\ \sigma_H(s),\ \gamma_H(s) >: s \in S\}$ where $\mu_H(s)$, $\sigma_H(s)$, $\gamma_H(s)$ exhibit the degree of membership, degree of indeterminacy along with the degree of non-membership correspondingly of each element $s \in S$ to the st H.

A neutro st $H = \{< s,\ \mu_H(s),\ \sigma_H(s),\ \gamma_H(s) >: s \in S\}$ can be identified as an arranged triple $< \mu_H(s),\ \sigma_H(s),\ \gamma_H(s) \gg$ in$]-0,\ 1+[$on S .

For a account of clarity, we intend to use the pattern $H =< \mu_H,\ \sigma_H,\ \gamma_H >$ for the neutro st $H = \{< s,\ \mu_H(s),\ \sigma_H(s),\ \gamma_H(s) >: s \in S\}$.

Definition 2.2 Accredit $E = < \mu_E(s),\ \sigma_E(s),\ \gamma_E(s) \gg$ be a neutro st about S, formerly the complement $C(E)$ may be labelled as $C(E) = \{< s,\ \gamma_E(s),\ \sigma_E(s),\ \mu_E(s) >: s \in S\}$.

Definition 2.3 For any two neutro sts $P = \{< s,\ \mu_P(s),\ \sigma_P(s),\ \gamma_P(s) >: s \in S\}$ along with

$Q = \{< s,\ \mu_Q(s),\ \sigma_Q(s),\ \gamma_Q(s) >: s \in S\}$, we have

(1) $P \subseteq Q \Longleftrightarrow \mu_P(s) \le \mu_Q(s),\ \sigma_P(s) \le \sigma_Q(s)$ along with $\gamma_P(s) \ge \gamma_Q(s)\ \forall\ s \in S$.

(2) $P = Q$ if and only if $P \subseteq Q$ along with $Q \subseteq P$.

(3) $P \bigcap Q = \{< s,\ \mu_P(s) \wedge \mu_Q(s),\ \sigma_P(s) \vee \sigma_Q(s),\ \gamma_P(s) \vee \gamma_Q(s) >: s \in S\}$.

(4) $P \bigcup Q = \{< s,\ \mu_P(s) \vee \mu_Q(s),\ \sigma_P(s) \wedge \sigma_Q(s),\ \gamma_P(s) \wedge \gamma_Q(s) >: s \in S\}$.

Definition 2.4 Accredit $\{C_k\ : k \in J\}$ be an arbitrary family of a neutro st in S. Formerly

(1) $\bigcap C_k = \{< s,\ \wedge\mu_{C_k}(s),\ \bigvee \sigma_{C_k}(s),\ \vee\gamma_{C_k}(s) >: s \in S\}$;

(2) $\bigcup C_k = \{< s,\ \bigvee \mu_{C_k}(s),\ \bigwedge \sigma_k(s),\ \bigwedge \gamma_{C_k}(s) >: s \in S\}$;

Therefore our main aim is to fabricate the tools for developing neutro TS, we must propose the neutro sts 0_N along with 1_N in S as follows:

Definition 2.5 $0_N = \{< s,\ 0,\ 0,\ 1 >: s \in S\}$ along with $1_N = \{< s,\ 1,\ 0,\ 0 >: s \in S\}$.

Definition 2.6 A neutro topo on a non-empty set S is a family τ_N of a neutro subsets in S satisfies the subsequent axioms:

(NT1) $0_N,\ 1_N\ \in \tau_N$;

(NT2) $B_1 \bigcap B_2 \in \tau_N$ for any $B_1,\ B_2\ \in \tau_N$;

(NT3) $\bigcup B_i \in \tau_N\ \forall \{B_i : i \in J\} \subseteq \tau_N$.

In the aforementioned case the ordered pair $(S,\ T)$ or candidly S is labelled as a neutro TS (briefly NTS) along with each neutro st in T is labelled as a neutro opn st (briefly NOS). The complement $\overline{V}$ of a neutro opn st V in S is termed as a neutro csd st (briefly NCS) in S.

Definition 2.7 Accredit F be a neutro st in a neutro TS S. Formerly $Nint(F) = \bigcup\{E : E$ is a neutro opn st in Y along with $E \subseteq F\}$ is termed as a neutro interior of F. $NCl(F) = \bigcap\{K : K$ is a neutro csd st in S along with $F \subseteq K\}$ is labelled as a neutro closure of F.

Definition 2.8 Accredit $(M,\ T)$ along with $(N,\ S)$ be any two neutro TS:

(i) A fn $f{:}(M,\ T) \to (N,\ S)$ is termed as a neutro ctra cts in case that the converse image of all neutro opn st in $(N,\ S)$ is a neutro csd st in $(M,\ T)$.

Identically in case that the converse image of all neutro csd st in $(N,\ S)$ is a neutro opn st in $(M,\ T)$.

(ii) A fn f:$(M,\ T) \rightarrow (N,\ S)$ is termed as a generalized neutro ctra cts in case that the converse image of all neutro opn st in $(N,\ S)$ is a generalized neutro csd st in $(M,\ T)$.

Identically in case that the converse image of all neutro csd st in $(N,\ S)$ is a generalized neutro opn st in $(M,\ T)$.

Definition 2.9 Accredit k be a fn from a neutro TS $(M,\ T)$ along with $(N,\ S)$. Formerly k is labeled as

(i) a neutro opn fn in case that $k(L)$ is a neutro opn st in N for all neutro opn st L in M.

(ii) a neutro α-opn fn in case that $k(L)$ is a neutro α-opn st in N for all neutro opn st L in M.

(iii) a neutro preopn fn in case that $k(L)$ is a neutro preopn st in N for all neutro opn st L in M.

(iv) a neutro semi opn fn in case that $k(L)$ is neutro semi opn st in N for all neutro opn st L in M.

3. Neutrosophic semi β^F-contra continuous function

In aforementioned section we propose neutro S_{β^F} ctra cts fn and studied some of its inheritance.

Definition 3.1 A subset H of a neutro TS $(S,\ \tau_N)$ is labeled as a neutro semi β^F csd st if $W \subseteq NCl(H)$ whenever $H \subseteq W$ where W is a neutro β-opn st in $(S,\ \tau_N)$.

Illustration 1 Let $X = \{a,\ b\}, 0_N = \{(0,\ 0,\ 1)\,(0,\ 0,\ 1)\}, 1_N = \{(1,\ 0,\ 0)\,(1,\ 0,\ 0)\}, \tau_N = \{0_N,\ 1_N, A,\ B,\ A\bigcup B,\ A\bigcap B\}$ where $A = \{(0.3,\ 0.6,\ 0.6)\ (0.4,\ 0.5,\ 0.8)\}, B = \{(0.3,\ 0.5,\ 0.8)\ (0.2,\ 0.3,\ 0.7)\}$. Here $C = \{(0.2,\ 0.4,\ 0.8)\ (0.2,\ 0.3,\ 0.9)\}$ is a neutro S_{β^F} -csd st.

Definition 3.2 A fn $g : (X,\ T) \rightarrow (Y,\ S)$ is termed as a neutro S_{β^F} ctra cts if $g^{-1}(B)$ is a neutro S_{β^F} csd st in $(X,\ T)$ for every neutro opn st B in $(Y,\ S)$.

Illustration 2 In illustration 6, it is proved that g is a neutro S_{β^F} ctra cts fn.

Definition 3.3 A fn $f : (D,\ T) \rightarrow (E,\ S)$ is termed as a neutro strongly S_{β^F} cts if $f^{-1}(B)$ is a neutro opn st in $(D,\ T)$ for every neutro S_{β^F} opn st B in $(E,\ S)$.

Illustration 3 Let $S = \{a,\ b\}$. Define a neutro sts G_1 and G_2 in S as follows. Let $G_1 =< (0.6,\ 0.4,\ 0.2)\ (0.8,\ 0.6,\ 0.2) >,\ G_2 =< (0.2,\ 0.4,\ 0.6)\ (0.2,\ 0.6,\ 0.8) >$. Then the families $T = \{0_N,\ 1_N,\ G_1\}$ and $S = \{0_N,\ 1_N,\ G_2\}$ are the neutro topologies on Y. Define a fn $f : (D,\ T) \longrightarrow (E,\ S)$ as follows: $f(a) = b,\ \ f(b) = a$. Then f is a neutro opn st and $f^{-1}(G_2)$ is a neutro S_{β^F} opn st in $(E,\ S)$. Therefore f is a neutro strongly S_{β^F} cts fn.

Definition 3.4 A fn $k : (X,\ T) \rightarrow (Y,\ S)$ is termed as a neutro strongly S_{β^F} ctra cts if $k^{-1}(B)$ is a neutro csd st in $(X,\ T)$ for all neutro S_{β^F} opn st B in $(Y,\ S)$.

Illustration 4 Let $S = \{c,\ d\}$. Define a neutro sts D_1, D_2 and D_3 in S as follows. Let $D_1 =< (0.7,\ 0.5,\ 0.1)\ (0.7,\ 0.4,\ 0.2) >,\ D_2 =< (0.7,\ 0.3,\ 0.2)\ (0.8,\ 0.5,\ 0.3) >$ and $D_3 =< (0.1,\ 0.3,\ 0.7)\ (0.2,\ 0.4,\ 0.8) >$. Then the families $T = \{0_N,\ 1_N,\ D_1,\ D_2,\ D_1\bigcup D_2,\ D_1\bigcap D_2\}$ and $S = \{0_N,\ 1_N,\ D_3\}$ are the neutro topologies on Y. Define a fn $k : (X,\ T) \longrightarrow (Y,\ S)$ as follows: $k(c) = d,\ \ k(d) = c$. Then k is a neutro csd st in $(X,\ T)$ and $k^{-1}(D_3)$ is also a neutro S_{β^F} opn st in $(Y,\ S)$. Therefore, k is a neutro strongly S_{β^F} ctra cts fn.

Definition 3.5 A fn $f : (X,\ T) \rightarrow (Y,\ S)$ is termed as a neutro S_{β^F} ctra irresolute if $f^{-1}(B)$ is a neutro S_{β^F} csd st in $(X,\ T)$ for every neutro S_{β^F} opn st B in $(Y,\ S)$.

Illustration 5 Let $S = \{l,\ m\}$. Define a neutro sts G_1, G_2 and G_3 as follows. Let $G_1 =< (0.2,\ 0.5,\ 0.9)\ (0.2,\ 0.7,\ 0.9) >,\ G_2 =< (0.2,\ 0.7,\ 0.8)\ (0.3,\ 0.6,\ 0.9) >$ and $G_3 =< (0.2,\ 0.5,\ 0.8)\ (0.3,\ 0.6,\ 0.9) >$. Then the families $T = \{0_N,\ 1_N,\ G_1,\ G_2,\ G_1\bigcup G_2,\ G_1\bigcap G_2\}$ and $S = \{0_N,\ 1_N,\ G_3\}$ are the neutro topologies on Y. Define a fn $f : (X,\ T) \longrightarrow (Y,\ S)$ as follows: $f(l) = m,\ f(m) = l$. Then f is a neutro S_{β^F} csd st in $(X,\ T)$ and $f^{-1}(G_3)$ is a neutro S_{β^F} opn st in $(Y,\ S)$. Therefore, f is a neutro S_{β^F} ctra irresolute fn.

Proposition 3.1 For any two neutro TS $(M,\ T)$ along with $(N,\ S)$, if $k : (M,\ T) \rightarrow (N,\ S)$ is a neutro ctra cts fn formerly, k is a neutro S_{β^F} ctra cts fn.

Proof. Accredit L be a neutro opn st in $(N,\ S)$. Therefore k is a neutro ctra cts fn, $k^{-1}(L)$ is a neutro csd st in $(M,\ T)$. Therefore, every neutro csd st is a neutro S_{β^F} csd st, $k^{-1}(L)$ is a neutro S_{β^F} csd st in $(M,\ T)$. So k is a neutro S_{β^F} ctra cts fn.

The inverse of the proposition 3.1 need not be accurate as exhibited in Illustration 6.

Illustration 6 Accredit $S=\{j,\ k\}$. Characterize the neutro sts G_1 along with G_2 in S as follows: $G_1 = <(0.2,\ 0.4,\ 0.8)\ (0.2,\ 0.3,\ 0.9)>$, $G_2 = <(0.3,\ 0.6,\ 0.8)\ (0.2,\ 0.5,\ 0.7)>$. Formerly the families $T=\{0_N,\ 1_N,\ G_1\}$ and $S=\{0_N,\ 1_N,\ G_2\}$ are the neutro topologies on S. Define a fn $g:(M,\ T)\to(N,S)$ as follows $g(j)=k,\ g(k)=j$. Then g is a neutro S_{β^F} ctra cts fn, but $g^{-1}(G_2)$ is not a neutro csd st in $(M,\ T)$. Hence g is not a neutro ctra cts fn.

Proposition 3.2 For any two neutro TS $(M,\ G)$ and $(N,\ H)$, if $k:(M,\ G)\to(N,\ H)$ is a neutro β-ctra cts fn then k is a neutro S_{β^F} ctra cts fn.

Proof. Accredit L be a neutro opn st in $(N,\ H)$. Therefore k is a neutro β ctra cts fn, $k^{-1}(L)$ is a neutro β-csd st in $(M,\ G)$. Therefore every neutro β csd st is a neutro S_{β^F} csd st, $k^{-1}(L)$ is a neutro S_{β^F} csd st in $(M,\ G)$. Hence k is a neutro S_{β^F} ctra cts fn.

The inverse of the proposition 3.2 need not be accurate as exhibited in Illustration 7.

Illustration 7 Accredit $L=\{c,\ d\}$. Define a neutro sts J_1 along with J_2 in L as follows:

$J_1 =<(0.1,\ 0.5,\ 0.8)\ (0.1,\ 0.2,\ 0.9)>$, $J_2 =<(0.2,\ 0.4,\ 0.6)\ (0.1,\ 0.6,\ 0.9)>$. Formerly the families $G=\{0_N,\ 1_N,\ J_1\}$ along with $H=\{0_N,\ 1_N,\ J_2\}$ are the neutro topologies on L. Define a fn $k:(M,\ G)\to(N,\ H)$ as follows $k(d)=c$, $k(c)=d$. Then k is a neutro S_{β^F} ctra cts fn, but $k^{-1}(J_2)$ is not a neutro β csd st in $(M,\ G)$. So k is not a neutro β ctra cts fn.

Proposition 3.3 For any two neutro TS $(D,\ T)$ along with $(E,\ S)$, if $k:(D,\ T)\to(E,\ S)$ is a neutro strongly S_{β^F} ctra cts fn, formerly g is a neutro S_{β^F} ctra cts fn.

Proof. Accredit L be a neutro opn st in $(Y,\ S)$. Every neutro opn st is a neutro S_{β^F} opn st. Now, L is a neutro S_{β^F} opn st in $(Y,\ S)$. Since k is a neutro strongly S_{β^F} ctra cts fn, $k^{-1}(L)$ is a neutro csd st in $(D,\ T)$. Therefore, every neutro csd st is a neutro S_{β^F} csd st, $k^{-1}(L)$ is a neutro S_{β^F} csd st in $(D,\ T)$. Then g is a neutro S_{β^F} ctra cts fn.

The inverse of the proposition 3.3 need not be accurate as exhibited in Illustration 8.

Illustration 8 Accredit $L=\{c,\ d\}$. Define a neutro sts J_1 and J_2 in L as follows: $J_1 =<(0.1,\ 0.5,\ 0.8)\ (0.1,\ 0.4,\ 0.9)>$, $J_2 =<(0.3,\ 0.4,\ 0.9)\ (0.2,\ 0.5,\ 0.7)>$. Formerly the families $T=\{0_N,\ 1_N,\ J_1\}$ and $S=\{0_N,\ 1_N,\ J_2\}$ are the neutro topologies on L. Define a fn $k:(D,\ T)\to(E,\ S)$ as follows $k(d)=c,\ k(c)=d$. Then k is neutro S_{β^F} ctra cts fn. Let $N =<(0.7,\ 0.5,\ 0.2)\ (0.9,\ 0.4,\ 0.2)>$ is a neutro S_{β^F} opn st in $(E,\ S)$, but $k^{-1}(N)$ is not a neutro csd st in $(D,\ T)$. Thus k is not a neutro strongly S_{β^F} ctra cts fn.

Proposition 3.4 For any two neutro TS $(O,\ P)$ and $(Q,\ R)$, if $k:(O,\ P)\to(Q,\ R)$ is a neutro strongly S_{β^F} ctra cts fn, formerly k is a neutro ctra cts fn.

Proof. Accredit L be a neutro opn st in $(Q,\ R)$. Every neutro opn st is a neutro S_{β^F} opn st. Now, L is a neutro S_{β^F} opn st in $(Q,\ R)$. Since k is a neutro strongly S_{β^F} ctra cts fn, $k^{-1}(L)$ is a neutro csd st in $(O,\ P)$. So k is a neutro ctra cts fn.

The inverse of the proposition 3.4 need not be accurate as exhibited in Illustration 9.

Illustration 9 Accredit $Y=\{j,\ k\}$. Define a neutro sts G_1 along with G_2 in Y as follows: $G_1 =<(0.3,\ 0.5,\ 0.8)\ (0.1,\ 0.4, 0.9>$ along with $G_2 =<(0.8,\ 0.5,\ 0.3)\ (0.9,\ 0.4,\ 0.1)>$. Formerly the families $T=\{0_N,\ 1_N,\ G_1\}$ along with $S=\{0_N,\ 1_N,\ G_2\}$ are the neutro topologies on Y. Define a fn $g:(D,\ T)\to(E,\ S)$ as follows $g(j)=k,\ g(k)=j$. Formerly g is a neutro ctra cts fn. Let $N =<(0.8,\ 0.4,\ 0.1)\ (0.9,\ 0.3,\ 0.1)>$ is a neutro S_{β^F} csd st in $(E,\ S)$, but $k^{-1}(N)$ is not a neutro opn st in $(D,\ T)$. So k is not a neutro strongly S_{β^F} ctra cts fn.

4. Interrelations

From the overhead theorems proved, we acquire a figure of implication as exhibited underneath. In the figure $A,\ B,\ C$ along with D denote a neutro ctra cts fn, neutro S_{β^F} ctra cts fn, neutro β ctra cts fn and neutro strongly S_{β^F} ctra cts fns respectively.

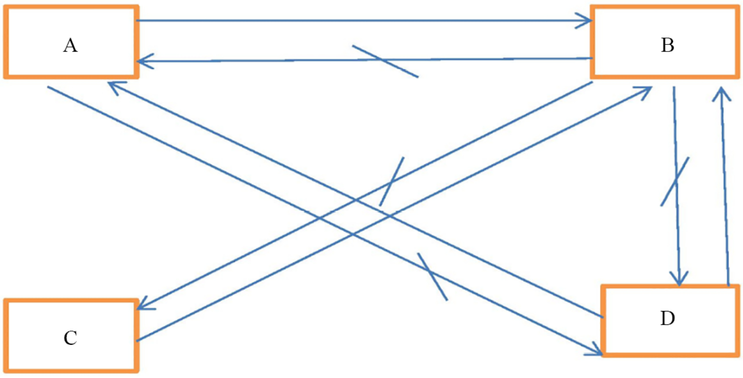

Figure 1. Interrelationship connecting neutro ctra cts fn, neutro $S_{\beta F}$ ctra cts fn, neutro β ctra cts fn and neutro strongly $S_{\beta F}$ ctra cts fns

Preposition 4.1 Accredit (M, G), (N, H) along with (O, P) be any three neutro TS. If a fn $i:(M, G)\to(N, H)$ is a neutro strongly $S_{\beta F}$ cts fn and $j:(N, H)\to(O, P)$ is a neutro $S_{\beta F}$ ctra cts fn then $i\circ j$ is a neutro ctra cts fn.

Proof. Accredit V be a neutro opn st based on (O, P). Therefore j is a neutro β ctra cts fn, $j^{-1}(V)$ is a neutro $S_{\beta F}$ csd st in (N, H). Therefore i is a neutro strongly $S_{\beta F}$ cts fn, $i^{-1}(j^{-1}(V))$ is a neutro csd st in (M, G). Thus $i\circ j$ is a neutro ctra cts fn.

Preposition 4.2 Accredit (M, G), (N, H) along with (O, P) be any three neutro TS. Then the subsequent statement holds:

(i) If k is a neutro $S_{\beta F}$ ctra cts fn along with l is a neutro cts fn, formerly lok is a neutro $S_{\beta F}$ ctra cts fn. (ii) If k is a neutro $S_{\beta F}$ ctra cts fn along with l is a neutro ctra cts fn, formerly lok is a neutro $S_{\beta F}$ ctra cts fn. (iii) If k is a neutro $S_{\beta F}$ ctra irresolute fn along with l is a neutro $S_{\beta F}$ ctra cts fn, formerly lok is a neutro $S_{\beta F}$ cts fn. (iv) If k is a neutro $S_{\beta F}$ irresolute fn along with l is a neutro $S_{\beta F}$ ctra cts fn, formerly lok is a neutro $S_{\beta F}$ ctra cts fn.

Proof. (i) Accredit F be a neutro opn st of (O, P). Therefore l is a neutro cts fn, $l^{-1}(F)$ is a neutro opn st in (N, H). Since k is a neutro $S_{\beta F}$ ctra cts fn, $k^{-1}(l^{-1}(F))$ is a neutro $S_{\beta F}$ csd st in (M, G). Thus lok is a neutro $S_{\beta F}$ ctra cts fn.

(ii) Accredit F be a neutro opn st of (O, P). Therefore l is a neutro ctra cts fn, $l^{-1}(F)$ is a neutro csd st in (N, H). Since k is a neutro $S_{\beta F}$ ctra cts fn, $k^{-1}(l^{-1}(F))$ is a neutro $S_{\beta F}$ opn st in (M, G). Thus lok is a neutro $S_{\beta F}$ cts fn.

(iii) Accredit F be a neutro opn st of (O, P). Therefore l is a neutro ctra cts fn, $l^{-1}(F)$ is a neutro $S_{\beta F}$ csd st in (N, H). Since k is a neutro $S_{\beta F}$ ctra irresolute fn, $k^{-1}(l^{-1}(F))$ is a neutro $S_{\beta F}$ opn st in (M, G). Thus lok is neutro $S_{\beta F}$ cts fn.

(iv) Accredit C be a neutro opn st of (O, P). Therefore l is a neutro ctra cts fn, $l^{-1}(F)$ is neutro $S_{\beta F}$ csd st in (N, H). Since k is a neutro $S_{\beta F}$ irresolute fn, $k^{-1}(l^{-1}(F))$ is a neutro $S_{\beta F}$ csd st in (M, G). Thus lok is a neutro $S_{\beta F}$ ctra cts fn.

Definition 4.1 Accredit (X, T) and (Y, S) be any two neutro TS. Accredit $f:(X, T)\to(Y, S)$ be the fn. The graph $g:X\to X\times Y$ of f is labelled as $g(x)=(x, f(x)), \forall x\in X$.

Definition 4.2 Accredit $C=\{(m, \mu_C(m), \sigma_C(m), \gamma_C(m): m\in M\}$ and $D=\{(n, \mu_D(n), \sigma_D(n), \gamma_D(n): n\in N\}$ be a neutro sts of M and N respectively. The product of the two neutro sts C along with D is labelled as $(C\times D)(m, n) =< (m, n), \min(\mu_C(m), \mu_D(n)), \min(\sigma_C(m), \sigma_D(n)), \max(\gamma_C(m), \gamma_D(n)) >$ for all $m, n\in M\times N$.

Proposition 4.3 Accredit (X, T) along with (Y, S) be any two neutro TS. Accredit $k:(X, T)\to(Y, S)$ be a fn. If the graph $m:X\to X\times Y$ of k is a neutro $S_{\beta F}$ ctra cts fn, then k is also a neutro $S_{\beta F}$ ctra cts fn.

Proof. Accredit B be a neutro csd st in (Y, S). By Definition 4.1, $k^{-1}(L)=1_N\cap k^{-1}(L)=m^{-1}(1_N\times L)$. Since m is a neutro $S_{\beta F}$ ctra cts fn, $m^{-1}(1_N\times L)$ is a neutro $S_{\beta F}$ opn st in (X, T). So k is a neutro $S_{\beta F}$ ctra cts fn.

Proposition 4.4 Accredit (X, T) along with (Y, S) be any two neutro TS. Accredit $f:(X, T)\to(Y, S)$ be a fn. If the graph $g:X\to X\times Y$ of f is a neutro strongly $S_{\beta F}$ ctra cts fn, then f is also a neutro strongly $S_{\beta F}$ ctra cts fn.

Proof. Accredit B be a neutro $S_{\beta F}$ opn st in (Y, S). By definition 4.1, $f^{-1}(B)=1_N\bigcap f^{-1}(B)=g^{-1}(1_N\times B)$. Since g is a strongly $S_{\beta F}$ ctra cts fn, $g^{-1}(1_N\times B)$ is a neutro csd st in (X, T). Now, $f^{-1}(B)$ is a neutro csd st in (X, T). Hence f is a neutro strongly $S_{\beta F}$ ctra cts fn.

Proposition 4.5 Accredit (X, T) along with (Y, S) be any two neutro TS. Accredit $f:(X, T) \to (Y, S)$ be a fn. If the graph $g: X \to X \times Y$ of f is a neutro S_{β^F} ctra irresolute fn, formerly f is also a neutro S_{β^F} ctra irresolute fn.

Proof. Accredit B be a neutro S_{β^F} csd st in (Y, S). By definition 4.1, $f^{-1}(B) = 1_N \cap f^{-1}(B) = g^{-1}(1_N \times B)$. Since g is a neutro S_{β^F} ctra irresolute fn, $g^{-1}(1_N \times B)$ is a neutro S_{β^F} opn st in (X, T). Now, $f^{-1}(B)$ is a neutro S_{β^F} opn st in (X, T). Hence f is a neutro S_{β^F} ctra irresolute fn.

5. Conclusion and future work

This research article is investigating the concepts of a neutrosophic S_β^F contra continuous function, neutrosophic strongly S_β^F continuous function, neutrosophic strongly S_β^F contra continuous function and neutrosophic S_β^F contra irresolute function in neutrosophic topological spaces. In addition, numerous theorems have been proved with the appropriate counter examples. Further the definition of a graph function has been proposed and various theorems are proved. In future, the concepts of a neutrosophic S_β^F contra continuous function, neutrosophic strongly S_β^F continuous function, neutrosophic strongly S_β^F contra continuous function and neutrosophic S_β^F contra irresolute function can be applied in the field of orbit topological spaces. Further, many properties can be studied in the area of orbit topological spaces.

Conflict of interest

Authors declare there is no conflict of interest at any point with reference to research findings.

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