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Research Article

Optical Solitons for the Dispersive Concatenation Model with Power-Law Nonlinearity by the Complete Discriminant Approach

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Abstract: The present paper is a sequel to the paper on the topic with Kerr law nonlinearity. For power-law nonlinearity, it was derived through balancing principle that the solitons would exist for two values of the power-law parameter, namely $n = 1$ or $n = 2$. The current paper derives dark and singular solitons for the latter value of n since the first case was already covered in a previous report that was dedicated to address the model with Kerr law.

***Keywords*:** dark, singular, plane, cnoidal

MSC: 78A60

1. Introduction

An interesting governing model to study the propagation of solitons through optical fibers was proposed a decade ago in 2014 [1, 2]. This is a concatenation of the well-known nonlinear Schrödinger's equation (NLSE), the Lakshmanan-Porsezian-Daniel (LPD) model and the Sasa-Satsuma equation. This model is now well-studied and well-understood. However, the following year, in 2015, a dispersive version of the concatenation model was proposed and reported [3-5]. This one is a combination of the Schrödinger-Hirota equation (SHE), the LPD and the dispersive fifth-order NLSE. The presence of third-order dispersion and fifth-order dispersion terms in the SHE and the fifth-order NLSE components make it truly a dispersive concatenation model. Later, this model started gaining attention too [6-20]. This is the model that is going to be the focus of the current work.

Several features of the concatenation model and dispersive concatenation model were disseminated with time. They are the study of the concatenation model in magneto-optic waveguides, the retrieval of the quiescent optical solitons for nonlinear chromatic dispersion (CD) by the usage of Lie symmetry and otherwise, extension of the model to

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power-law nonlinearity, locating the conservation laws and also analyzing the model with the spatio-temporal dispersion in addition to CD to reduce the Internet bottleneck effect are now known. The numerical simulations of the model using the Laplace-Adomian decomposition scheme are also recovered [6-20].

The dispersive concatenation model, with Kerr law of self-phase modulation (SPM) effect, has been earlier studied by the complete discriminant approach [18]. The current paper is a sequel to its previous counterpart and considers power-law of SPM. This paper will therefore focus on the retrieval of optical soliton solutions to the dispersive concatenation model with power-law SPM by the aid of complete discriminant approach. It will be observed that the power-law nonlinearity parameter values, for which the solitons would exist, are $n = 1$ and $n = 2$ only where n represents the power-law parameter. Therefore, it is just necessary to focus on the later value of n for this paper since the former value represents Kerr law of SPM whose results are recently reported [18]. The detailed analysis of the retrieval of the solitons and other solutions are exhibited in the rest of the paper after a quick and succinct introduction to the model.

Our paper extends previous research on Kerr law nonlinearity to power-law nonlinearity ($n = 2$). Validation involves comparing our results for power-law ($n = 2$) with our earlier work on Kerr law ($n = 1$) and verifying theoretical predictions through numerical simulations. This approach ensures the uniqueness of our contributions and their alignment with established principles.

This research is established through key steps: We reviewed existing work on Kerr law nonlinearity to understand soliton behavior. Recognizing a gap, we focused on power-law nonlinearity ($n = 2$) and formulated our research question. We rigorously derived solutions for power-law nonlinearity using the balancing principle, emphasizing $n = 2$. Results were compared with Kerr law nonlinearity ($n = 1$) from our previous work to showcase novel contributions. Findings were validated through numerical simulations, ensuring stability of dark and singular solitons. Results were discussed in the context of existing knowledge, highlighting implications in nonlinear optics.

The motivation behind this paper lies in addressing a crucial gap in understanding soliton dynamics by extending our analysis from Kerr law to power-law nonlinearity ($n = 2$). Motivated by the ubiquity of power-law phenomena in various scientific domains, our research contributes to nonlinear optics, opening new avenues for applications. The exploration of soliton solutions for the unique power-law parameter of $n = 2$ enriches our understanding and offers practical insights for fields such as signal processing and communication.

1.1 *Governing model*

The dimensionless form of the dispersive concatenation model with power-law nonlinearity reads [20]:

$$
\begin{aligned}
& iq_t + aq_{xx} + b|q|^{2n} q - i\delta_1\left[\sigma_1 q_{xxx} + \sigma_2 |q|^{2n} q_x\right] \\
& +\delta_2\left[\sigma_3 q_{xxxx} + \sigma_4 |q|^{2n} q_{xx} + \sigma_5 |q|^{2n+2} q + \sigma_6 |q_x|^2 q + \sigma_7 q_x^2 q^* + \sigma_8 q_{xx}^* q^2\right] \\
& -i\delta_3\left[\sigma_9 q_{xxxxx} + \sigma_{10} |q|^{2n} q_{xxx} + \sigma_{11} |q|^{2n+2} q_x + \sigma_{12} q q_x q_{xx}^* + \sigma_{13} q^* q_x q_{xx} + \sigma_{14} q q_x^* q_{xx} + \sigma_{15} q_x^2 q_x^*\right] = 0.
\end{aligned}
\tag{1}
$$

In equation (1), the dependent variable is $q(x, t)$ that is a complex-valued function and represents the wave amplitude with x and t being the independent variables that stand for the spatial and temporal coordinates respectively. The first term is the linear temporal evolution with its coefficient being $i = \sqrt{-1}$, while the second and third terms account for CD and SPM whose coefficients are a and b respectively. The coefficient of δ_1 represents the extension of NLSE to formulate the SHE. Then, the coefficients of δ_2 and δ_3 stand for the LPD components and the fifth-order NLSE that brings in the dispersive effect from the fifth-order dispersion term in it.

2. Mathematical analysis

In order to integrate Eq. (1), the following traveling wave transformation is assumed:

$$q(x,t) = Q(\xi)e^{i\phi(x,t)}, \tag{2}$$

where

$$\xi = k(x - vt),\ \ \phi(x,t) = -hx + \omega t + \iota. \tag{3}$$

Here, v, h, ω, ι correspond to the velocity, frequency, wave number and phase constant respectively. Inserting Eqs. (2) and (3) into Eq. (1), the real and imaginary parts are respectively expressed as

$$\begin{aligned}
&(-\omega + \delta_1\sigma_1 h^3 + \delta_2\sigma_3 h^4 - \delta_3\sigma_9 h^5 - ah^2)Q + (-(\sigma_6 + \sigma_7 + \sigma_8)\delta_2 h^2 + (\sigma_{12} + \sigma_{13} - \sigma_{14} - \sigma_{15})\delta_3 h^3)Q^3 \\
&+(b - \delta_1\sigma_2 h + \delta_2\sigma_4 h^2 + \delta_3\sigma_{10} h^3)Q^{2n+1} + (\delta_2\sigma_5 - \delta_3\sigma_{11} h)Q^{2n+3} + (-3\delta_1\sigma_1 k^2 h - 6\delta_2\sigma_3 k^2 h^2 \\
&+10\delta_3\sigma_9 k^2 h^3 + ak^2)Q'' + (\delta_2\sigma_8 k^2 + (-\sigma_{12} - \sigma_{13} + \sigma_{14})\delta_3 k^2 h)Q^2 Q'' + (\delta_2\sigma_4 k^2 - 3\delta_3\sigma_{10} k^2 h)Q^{2n} Q'' \\
&+((\sigma_6 + \sigma_7)\delta_2 k^2 + (2\sigma_{12} - 2\sigma_{13} - 2\sigma_{14} - \sigma_{15})\delta_3 k^2 h)Q(Q')^2 + (\delta_2\sigma_3 k^4 - 5\delta_3\sigma_9 k^4 h)Q'''' = 0,
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
&(-kv + 3\delta_1\sigma_1 kh^2 + 4\delta_2\sigma_3 kh^3 - 2ahk - 5\delta_3\sigma_9 kh^4)Q' - (\delta_1\sigma_2 k + \delta_2\sigma_4 kh)Q^{2n}Q' + (2(\sigma_8 - \sigma_6 - \sigma_7)\delta_2 kh \\
&-(\sigma_{12} - 3\sigma_{13} + \sigma_{14} + \sigma_{15})\delta_3 kh^2)Q^2 Q' + 3\delta_3\sigma_{10} kh^2 Q^{2n} Q' - \delta_3\sigma_{11} k Q^{2n+2} Q' + (-\delta_1\sigma_1 k^3 - 4\delta_2\sigma_3 k^3 h \\
&+10\delta_3\sigma_9 k^3 h^2)Q''' - \delta_3\sigma_{10} k^3 Q^{2n} Q''' - \delta_3\sigma_9 k^5 Q^{(v)} - (\sigma_{12} + \sigma_{13} + \sigma_{14})\delta_3 k^3 QQ'Q'' - \delta_3\sigma_{15} k^3 (Q')^3 = 0.
\end{aligned} \tag{5}$$

From Eq. (5), we obtain the velocity

$$v = \frac{3\delta_1\sigma_1 kh^2 + 4\delta_2\sigma_3 kh^3 - 2ahk - 5\delta_3\sigma_9 kh^4}{k}, \tag{6}$$

along with the constraint conditions

$$\delta_1\sigma_2 = -\delta_2\sigma_4 h,$$

$$2(\sigma_8 - \sigma_6 - \sigma_7)\delta_2 = (\sigma_{12} - 3\sigma_{13} + \sigma_{14} + \sigma_{15})\delta_3 h,$$

$$\delta_3\sigma_{10} kh^2 = 0,$$

$$\delta_3\sigma_{11} k = 0,$$

$$-\delta_1\sigma_1 - 4\delta_2\sigma_3 h + 10\delta_3\sigma_9 h^2 = 0,$$

$$\delta_3\sigma_{10} k^3 = 0,$$

$$\delta_3\sigma_9 k^5 = 0,$$

$$(\sigma_{12} + \sigma_{13} + \sigma_{14})\delta_3 k^3 = 0,$$

$$\delta_3\sigma_{15} k^3 = 0. \tag{7}$$

Simplify the real part (4) as

$$A_1 Q + A_2 Q^3 + A_3 Q^{2n+1} + A_4 Q^{3n+1} + A_5 Q'' + A_6 Q^2 Q'' + A_7 Q^{2n} Q'' + A_8 Q(Q')^2 + A_9 Q'''' = 0, \tag{8}$$

where

$$A_1 = -\omega + \delta_1\sigma_1 h^3 + \delta_2\sigma_3 h^4 - \delta_3\sigma_9 h^5 - ah^2,$$

$$A_2 = -(\sigma_6 + \sigma_7 + \sigma_8)\delta_2 h^2 + (\sigma_{12} + \sigma_{13} - \sigma_{14} - \sigma_{15})\delta_3 h^3,$$

$$A_3 = b - \delta_1\sigma_2 h + \delta_2\sigma_4 h^2 + \delta_3\sigma_{10} h^3,$$

$$A_4 = \delta_2\sigma_5 - \delta_3\sigma_{11} h,$$

$$A_5 = -3\delta_1\sigma_1 k^2 h - 6\delta_2\sigma_3 k^2 h^2 + 10\delta_3\sigma_9 k^2 h^3 + ak^2,$$

$$A_6 = \delta_2\sigma_8 k^2 + (-\sigma_{12} - \sigma_{13} + \sigma_{14})\delta_3 k^2 h,$$

$$A_7 = \delta_2\sigma_4 k^2 - 3\delta_3\sigma_{10} k^2 h,$$

$$A_8 = (\sigma_6 + \sigma_7)\delta_2 k^2 + (2\sigma_{12} - 2\sigma_{13} - 2\sigma_{14} - \sigma_{15})\delta_3 k^2 h,$$

$$A_9 = \delta_2\sigma_3 k^4 - 5\delta_3\sigma_9 k^4 h. \tag{9}$$

Setting

$$Q = U^{\frac{1}{n}}, \tag{10}$$

Eq. (8) is transformed into

$$A_1 n^4 U^4 + A_2 n^4 U^4 U^{\frac{2}{n}} + A_3 n^4 U^6 + A_4 n^4 U^6 U^{\frac{2}{n}} + A_5 n^3 U^3 U'' + (A_6(n^2 - n^3) + A_8 n^2) U^2 U^{\frac{2}{n}} (U')^2$$

$$+ A_6 n^3 U^3 U^{\frac{2}{n}} U'' + A_5 (n^2 - n^3) U^2 (U')^2 + A_7 (n^2 - n^3) U^4 (U')^2 + A_7 n^3 U^5 U'' + (6n - 18n^2 + 12n^3) A_9 U (U')^2 U''$$

$$+(3A_9(n^2-n^3))U^2(U'')^2+4A_9(n^2-n^3)U^2U'U'''+A_9(1+11n^2-6n^3-6n)(U')^4+A_9n^3U^3U''''=0. \tag{11}$$

Take the trial equation as follows

$$(U')^2=\sum_{i=0}^{j}a_iU^i(\xi), \tag{12}$$

where j is a positive integer, which can be obtained by balancing the highest order derivative term with the highest order nonlinear term. For n in Eq. (11), we only consider it as 1 or 2 in order to effectively combine the trial equation method with the complete discriminant system for a polynomial for better solving. In this paper, we take n to be equal to 2, and Eq. (11) becomes

$$16A_1U^4+16A_2U^5+16A_3U^6+16A_4U^7+8A_5U^3U''-4A_5U^2(U')^2+(-4A_6+4A_8)U^3(U')^2+8A_6U^4U''$$

$$-4A_7U^4(U')^2+8A_7U^5U''+36A_9U(U')^2U''-12A_9U^2(U'')^2-16A_9U^2U'U'''-15A_9(U')^4+8A_9U^3U''''=0. \tag{13}$$

Substituting Eq. (12) into Eq. (13), and balancing U^5U'' with U^7 yields $j = 3$. Thus, Eq. (12) turns into

$$(U')^2=a_3U^3+a_2U^2+a_1U+a_0, \tag{14}$$

where

$$a_3=\frac{4A_4+3A_7}{A_7},$$

$$a_2=-\frac{4A_3A_7^2-156A_4^2A_9-4A_4A_6A_7+4A_4A_7A_8-144A_4A_7A_9+3A_7^2A_8-27A_7^2A_9}{A_7^3},$$

$$a_1=\frac{1}{4A_7^5\left(24A_4A_9-A_7A_8+9A_7A_9\right)}(16A_1A_7^6+16A_3^2A_7^4A_9-1248A_3A_4^2A_7^2A_9^2-32A_3A_4A_6A_7^3A_9$$

$$+32A_3A_4A_7^3A_8A_9-1152A_3A_4A_7^3A_9^2-16A_3A_5A_7^5+24A_3A_7^4A_8A_9-216A_3A_7^4A_9^2+24336A_4^4A_9^3$$

$$+1248A_4^3A_6A_7A_9^2-1248A_4^3A_7A_8A_9^2+44928A_4^3A_7A_9^3+624A_4^2A_5A_7^3A_9+16A_4^2A_6^2A_7^2A_9-32A_4^2A_6A_7^2A_8A_9$$

$$+1152A_4^2A_6A_7^2A_9^2+16A_4^2A_7^2A_8^2A_9-2088A_4^2A_7^2A_8A_9^2+29160A_4^2A_7^2A_9^3+16A_4A_5A_6A_7^4-16A_4A_5A_7^4A_8$$

$$+576A_4A_5A_7^4A_9-24A_4A_6A_7^3A_8A_9+216A_4A_6A_7^3A_9^2+24A_4A_7^3A_8^2A_9-1080A_4A_7^3A_8A_9^2+7776A_4A_7^3A_9^3$$

$$-12A_5A_7^5A_8+108A_5A_7^5A_9+9A_7^4A_8^2A_9-162A_7^4A_8A_9^2+729A_7^4A_9^3),$$

$$a_0=0, \tag{15}$$

and a_2 and a_3 satisfy the restriction

$$16A_2 + 12A_5 - 4A_5a_3 + (-4A_6 + 4A_8)a_2 + 8A_6a_2 + 18A_9a_2 - 58A_9a_3a_2 + 8A_9\left(6a_2a_3 + \frac{3}{2}a_2\right) = 0. \tag{16}$$

Simplify Eq. (14) to the integral form

$$\pm\sqrt{a_3}\xi - \xi_0 = \int \frac{dU}{\sqrt{F(U)}}, \tag{17}$$

where

$$F(U) = U\left(U^2 + \frac{a_2}{a_3}U + \frac{a_1}{a_3}\right). \tag{18}$$

According the second order polynomial discriminant system

$$\Delta = \left(\frac{a_2}{a_3}\right)^2 - \frac{4a_1}{a_3}, \tag{19}$$

we divide the roots of $F(U)$ by the discriminant system and solve the integral (17).

3. Exact solutions

The wide spectrum of solutions that emerge from the scheme depending on the sign of the discriminant, are now enlisted and classified:

Case 1 $\Delta = 0$. For $U > 0$, if $a_2a_3 < 0$, the dark and singular solitons evolve as

$$q_1 = \left[-\frac{a_2}{2a_3}\tanh^2\left\{\frac{1}{2}\sqrt{-\frac{a_2}{2a_3}}(\sqrt{a_3}\xi - \xi_0)\right\}\right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}, \tag{20}$$

and

$$q_2 = \left[-\frac{a_2}{2a_3}\coth^2\left(\frac{1}{2}\sqrt{-\frac{a_2}{2a_3}}(\sqrt{a_3}\xi - \xi_0)\right)\right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}, \tag{21}$$

if $a_2a_3 > 0$, the singular periodic wave is recovered as

$$q_3 = \left[\frac{a_2}{2a_3}\tan^2\left\{\frac{1}{2}\sqrt{\frac{a_2}{2a_3}}(\sqrt{a_3}\xi - \xi_0)\right\}\right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}, \tag{22}$$

if $a_2 = 0$, the rational wave emerges:

$$q_4 = \left[\frac{4}{(\sqrt{a_3}\xi - \xi_0)^2} \right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}. \tag{23}$$

Case 2 $\Delta > 0$ and $a_1 = 0$. For $U > -\dfrac{a_2}{a_3}$, if $a_2 a_3 > 0$, the dark and singular solitons stick out as

$$q_5 = \left[\frac{a_2}{2a_3} \tanh^2 \left\{ \frac{1}{2} \sqrt{\frac{a_2}{2a_3}} (\sqrt{a_3}\xi - \xi_0) \right\} - \frac{a_2}{a_3} \right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}, \tag{24}$$

and

$$q_6 = \left[\frac{a_2}{2a_3} \coth^2 \left\{ \frac{1}{2} \sqrt{\frac{a_2}{2a_3}} (\sqrt{a_3}\xi - \xi_0) \right\} - \frac{a_2}{a_3} \right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}, \tag{25}$$

if $a_2 a_3 < 0$, the singular periodic wave shapes up as

$$q_7 = \left[-\frac{a_2}{2a_3} \tan^2 \left\{ \frac{1}{2} \sqrt{-\frac{a_2}{2a_3}} (\sqrt{a_3}\xi - \xi_0) \right\} - \frac{a_2}{a_3} \right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}. \tag{26}$$

Case 3 $\Delta > 0$ and $\dfrac{a_1}{a_3} \neq 0$. Suppose that $r_1 < r_2 < r_3$, one of them is zero, and others two are roots of $U^2 + \dfrac{a_2}{a_3}U + \dfrac{a_1}{a_3}$. For $r_1 < U < r_2$, the snoidal wave is introduced as below

$$q_8 = \left[r_1 + (r_2 - r_1)\mathrm{sn}^2 \left\{ \frac{\sqrt{r_3 - r_1}}{2} (\sqrt{a_3}\xi - \xi_0), m \right\} \right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}, \tag{27}$$

and for $U > r_3$, the combo snoidal-cnoidal wave comes out as

$$q_9 = \left[\frac{r_3 - r_2 \mathrm{sn}^2 \left\{ \frac{\sqrt{r_3 - r_1}}{2} (\sqrt{a_3}\xi - \xi_0), m \right\}}{\mathrm{cn}^2 \left\{ \frac{\sqrt{r_3 - r_1}}{2} (\sqrt{a_3}\xi - \xi_0), m \right\}} \right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}, \tag{28}$$

where

$$m^2 = \frac{r_2 - r_1}{r_3 - r_1}.$$

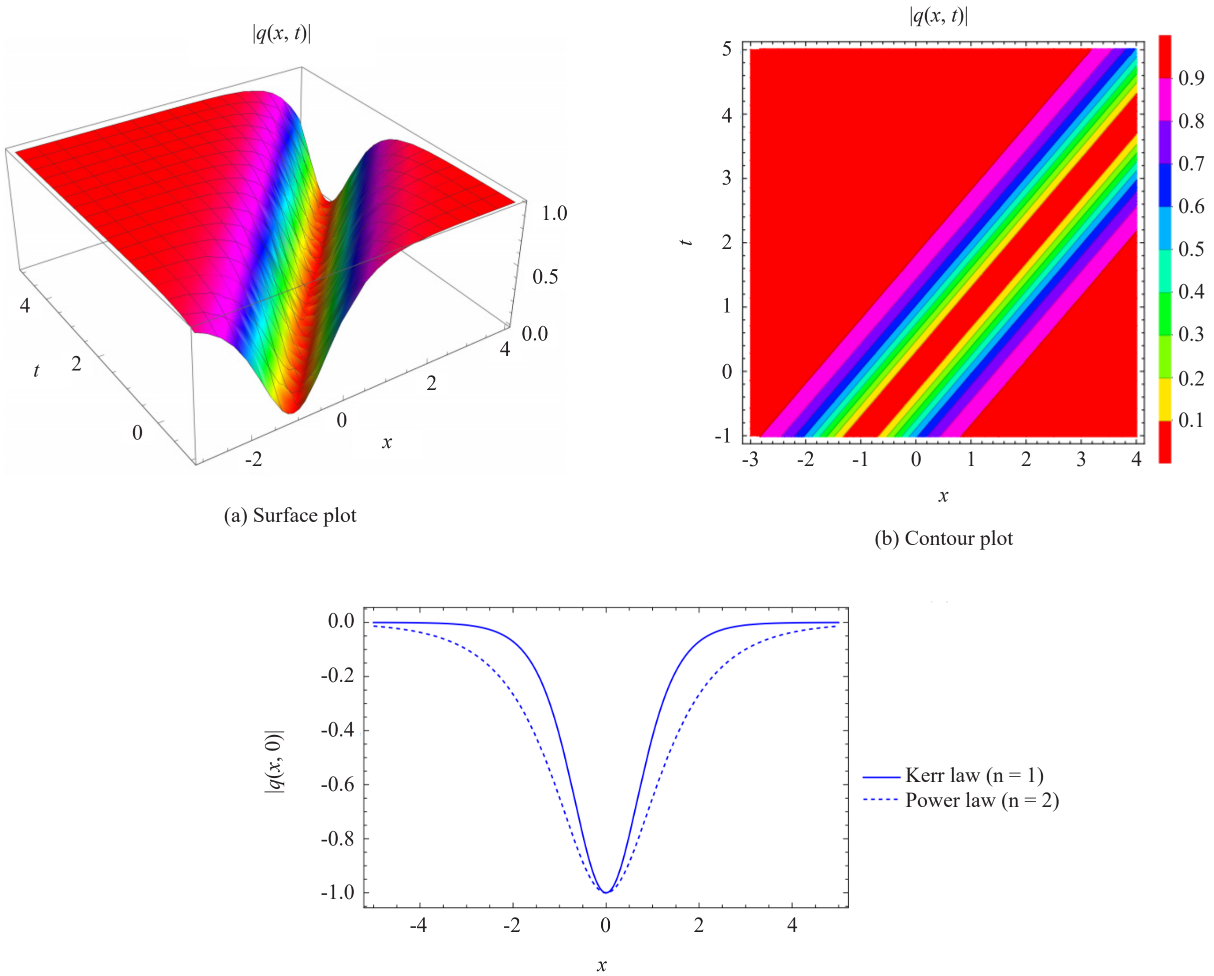
Case 4 $\Delta < 0$, for $U > 0$, the cnoidal wave is recovered as

$$q_{10} = \left[\frac{2\sqrt{\frac{a_1}{a_3}}}{1 + \mathrm{cn}\left\{ \left(\frac{a_1}{a_3} \right)^{\frac{1}{4}} (\sqrt{a_3}\xi - \xi_0), m \right\}} - \sqrt{\frac{a_1}{a_3}} \right]^{\frac{1}{2}} e^{i(-hx+\omega t+\iota)}, \tag{29}$$

where $a_1 a_3 > 0$ and

$$m^2 = \frac{1}{2} - \frac{a_2}{4\sqrt{a_1 a_3}}.$$

Figure 1 represents the surface plot, contour plot and 2D plot of a dark soliton (20). The parameter values that have been chosen are $a = 1$, $b = 1$, $\delta_1 = 1$, $\delta_2 = 1$, $\delta_3 = 1$, $\sigma_1 = 1$, $\sigma_2 = 1$, $\sigma_3 = 1$, $\sigma_4 = 1$, $\sigma_5 = 1$, $\sigma_6 = 1$, $\sigma_7 = 1$, $\sigma_8 = 1$, $\sigma_9 = 1$, $\sigma_{10} = 1$, $\sigma_{11} = 1$, $\sigma_{12} = 1$, $\sigma_{13} = 1$, $\sigma_{14} = 1$, $\sigma_{15} = 1$, $h = 1$, $k = 1$, and $\xi_0 = 1$.



(a) Surface plot

(b) Contour plot

(c) Effect of SPM

Figure 1. Profile of a dark soliton solution (20)

4. Conclusions

The current paper studied the dispersive concatenation model with power-law of SPM by the aid of the complete discriminant approach. The integration scheme yielded dark and singular solitons for two values of the power-law parameter n. The special case $n = 1$ that falls back to Kerr law was addressed earlier [18]. Thus the current work considered the other permissible value of n namely $n = 2$ and the dark and singular soliton solutions have emerged using this integration scheme. As a byproduct, the scheme also yielded plane waves, singular periodic waves, cnoidal and snoidal waves and these are enlisted. The existence criteria for all such waves are listed as parameter constraints. The results are indeed promising and the future holds strong for this model.

Later, the model will be extended and studied with differential group delay and eventually extended to dispersion-flattened fibers. The results would subsequently be revealed and reported. Additionally, the dispersive concatenation model would be studied in magneto-optic waveguides, the element of stochasticity would be included. The corresponding quiescent optical solitons and gap solitons would also be considered using techniques apart from Lie symmetry. The consideration of the model with additional forms of waveguide such as optical metamaterials is also on the table. The plethora of results are soon to be reported and will be made visible all over after aligning them with the pre-existing works [21-39].

Conflict of interest

The authors claim there is no conflict of interest.

References

[1] Ankiewicz A, Nail A. Higher-order integrable evolution equation and its soliton solutions. *Physics Letters*. 2014; 378(4): 358-361. Available from: doi: 10.1016/j.physleta.2013.11.031.

[2] Ankiewicz A, Wang Y, Wabnitz S, Nail A. Extended nonlinear Schrödinger equation with higher-order odd and even terms and its rogue wave solutions. *Physical Review E*. 2014; 89(1): 012907. Available from: doi: 10.1103/physreve.89.012907.

[3] Amdad C, Kedziora DJ, Ankiewicz A, Nail A. Soliton solutions of an integrable nonlinear Schrödinger equation with quintic terms. *Physical Review E*. 2014; 90(3): 032922. Available from: doi: 10.1103/physreve.90.032922.

[4] Amdad C, Kedziora DJ, Ankiewicz A, Nail A. Breather-to-soliton conversions described by the quintic equation of the nonlinear Schrödinger hierarchy. *Physical Review E*. 2015; 91(3): 032928. Available from: doi: 10.1103/physreve.91.032928.

[5] Amdad C, Kedziora DJ, Ankiewicz A, Nail A. Breather solutions of the integrable quintic nonlinear Schrödinger equation and their interactions. *Physical Review E*. 2015; 91(2): 022919. Available from: doi: 10.1103/physreve.91.022919.

[6] Abdullahi RA, Biswas A, Yakup Y, Asim A. Implicit quiescent optical solitons for the ispersive concatenation model with nonlinear chromatic dispersion by lie symmetry. *Contemporary Mathematics*. 2023; 4(4): 666-674. Available from: doi: 10.37256/cm.4420233575.

[7] Abdullahi RA, Biswas A, Yakup Y, Asim A. Implicit quiescent optical solitons for the concatenation model with nonlinear chromatic dispersion and power-law of self-phase modulation by Lie symmetry. *Journal of Optics*. 2023. Available from: doi: 10.1007/s12596-023-01443-z.

[8] Abdullahi RA, Biswas A, Yakup Y, Asim A. Implicit quiescent optical solitons for the concatenation model with Kerr law nonlinearity and nonlinear chromatic dispersion by Lie symmetry. *Journal of Optics*. 2023. Available from: doi: 10.1007/s12596-023-01450-0.

[9] Abdullahi RA, Biswas A, Yakup Y, Asim A. Implicit quiescent optical solitons for the concatenation model with nonlinear chromatic dispersion and in absence of self-phase modulation by lie symmetry. *Journal of Optics*. 2023. Available from: doi: 10.1007/s12596-023-01451-z.

[10] Moraru L. Optical solitons and conservation laws for the concatenation model with spatio-temporal dispersion (internet traffic regulation). *Journal of the European Optical Society: Rapid Publications*. 2023; 19(2): 35. Available from: doi: 10.1051/jeos/2023031.

[11] Arnous AH, Biswas A, Yakup Y, Moraru L, Cătălina I, Puiu LG, et al. Optical solitons and complexitons for the concatenation model in birefringent fibers. *Ukrainian Journal of Physical Optics*. 2023; 24(4): 04060-04086. Available from: doi: 10.3116/16091833/24/4/04060/2023.
[12] Arnous AH, Biswas A, Kara AH, Yakup Y, Moraru L, Cătalina I, et al. Optical solitons and conservation laws for the concatenation model: Power-law nonlinearity. *Ain Shams Engineering Journal*. 2023; 102381. Available from: doi: 10.1016/j.asej.2023.102381.
[13] Biswas A, José V-G, Kara AH, Khan S, Houria T, González-Gaxiola O, et al. Optical solitons and conservation laws for the concatenation model: Undetermined coefficients and multipliers approach. *Universe*. 2022; 9(1): 15. Available from: doi: 10.3390/universe9010015.
[14] Biswas A, José V-G, Yakup Y, Moshokoa SP, Aphane M, Alghamdi AS. Optical solitons for the concatenation model with power-law nonlinearity: undetermined coefficients. *Ukrainian Journal of Physical Optics.* 2023; 24(3): 185-192. Available from: doi: 10.3116/16091833/24/3/185/2023.
[15] González-Gaxiola O, Biswas A, Ruíz J, Asim A. Bright and dark optical solitons for the concatenation model by the Laplace-Adomian decomposition scheme. *Ukrainian Journal of Physical Optics*. 2023; 24(3): 222-234. Available from: doi: 10.3116/16091833/24/3/222/2023.
[16] Shohib Reham MA, Alngar Mohamed EM, Biswas A, Yakup Y, Houria T, Moraru L, et al. Optical solitons in magneto-optic waveguides for the concatenation model. *Ukrainian Journal of Physical Optics*. 2023; 24(3): 248-261. Available from: doi: 10.3116/16091833/24/3/248/2023.
[17] Wang MY, Biswas A, Yakup Y, Moraru L, Moldovanu S, Alshehri HM. Optical solitons for a concatenation model by trial equation qpproach. *Electronics*. 2022; 12(1): 19. Available from: doi: 10.3390/electronics12010019.
[18] Wang MY, Biswas A, Yildirim Y, Alshomrani AS. Optical solitons for dispersive concatenation model with Kerr law nonlinearity by the complete discriminant method. *Journal of Optics*.
[19] Yakup Y, Biswas A, Moraru L, Alghamdi AA. Quiescent optical solitons for the concatenation model with nonlinear chromatic dispersion. *Mathematics*. 2023; 11(7): 1709. Available from: doi: 10.3390/math11071709.
[20] Zayed EME, Gepreel KA, El-Horbaty M, Biswas A, Yildirim Y, Triki H, et al. Optical solitons for the dispersive concatenation model. *Contemporary Mathematics*. 2023; 4(3): 592-611. Available from: doi: 10.37256/cm.4320233321.
[21] Abu-AlShaeer MJ, Jawad AJM. Highly dispersive optical solitons with cubic law and cubic-quintic-septic law nonlinearities by two methods. *Al-Rafidain Journal of Engineering Sciences*. 2023; 1(1): 1-9.
[22] Jihad NJ, Almuhsan MAA. Evaluation of impairment mitigations for optical fiber communications using dispersion compensation techniques. *Al-Rafidain Journal of Engineering Sciences*. 2023; 1(1): 95-108.
[23] Jawad A, Biswas A. Solutions of resonant nonlinear Schrödinger's equation with exotic Non-Kerr Law nonlinearities. *Al-Rafidain Journal of Engineering Sciences*. 2024; 2(1): 43-50. Available from: doi: 10.61268/2bz73q95.
[24] Wang S. Novel soliton solutions of CNLSEs with Hirota bilinear method. *Journal of Optics*. 2023; 52(3): 1602-1607. Available from: doi: 10.1007/s12596-022-01065-x.
[25] Bahadır K, Emrullah Y. The investigation of unique optical soliton solutions for dual-mode nonlinear Schrödinger's equation with new mechanisms. *Journal of Optics*. 2022; 52(3): 1513-1527. Available from: doi: 10.1007/s12596-022-00998-7.
[26] Tang L. Bifurcations and optical solitons for the coupled nonlinear Schrödinger equation in optical fiber Bragg gratings. *Journal of Optics*. 2022; 52. Available from: doi: 10.1007/s12596-022-00963-4.
[27] Thủy NT, Lanh CV. Supercontinuum generation based on suspended core fiber infiltrated with butanol. *Journal of Optics*. 2023; 52(4): 2296-2305. Available from: doi: 10.1007/s12596-023-01323-6.
[28] Li Z, Zhu EY. Optical soliton solutions of stochastic Schrödinger-Hirota equation in birefringent fibers with spatiotemporal dispersion and parabolic law nonlinearity. *Journal of Optics*. 2023. Available from: doi: 10.1007/s12596-023-01287-7.
[29] Han T, Zhao L, Li C, Zhao L. Bifurcations, stationary optical solitons and exact solutions for complex Ginzburg-Landau equation with nonlinear chromatic dispersion in non-Kerr law media. *Journal of Optics*. 2022; 52(2): 831-844. Available from: doi: 10.1007/s12596-022-01041-5.
[30] Ghatak A. The optical fiber: A historical perspective. *Journal of Optics*. 2004; 33(3): 131-143. Available from: doi: 10.1007/bf03354759.
[31] Hasegawa A. Application of optical solitons for information transfer in fibers - A tutorial review. *Journal of Optics*. 2004; 33(3): 145-156. Available from: doi: 10.1007/bf03354760.
[32] Xu SL, Petrović N, Belić MR. Two-dimensional dark solitons in diffusive nonlocal nonlinear media. *Journal of Optics*. 2015; 44(2): 172-177. Available from: doi: 10.1007/s12596-015-0243-z.

[33] Zeliha K, Dumitru B, Bayram M. Theory and application for the time fractional Gardner equation with Mittag-Leffler kernel. *Journal of Taibah University for Science*. 2019; 13(1): 813-819. Available from: doi: 10.1080/16583655.2019.1640446.

[34] Cinar M, Secer A, Ozisik M, Bayram M. Derivation of optical solitons of dimensionless Fokas-Lenells equation with perturbation term using Sardar sub-equation method. *Optical and Quantum Electronics*. 2022; 54(7). Available from: doi: 10.1007/s11082-022-03819-0.

[35] Hashemi MS, Bayram M. Symmetry properties and exact solutions of the time fractional Kolmogorov Petrovskii-Piskunov equation. *Revista Mexicana De Fisica*. 2019; 65: 529-535. Available from: doi: 10.31349/revmexfis.65.529.

[36] Akinlar MA, Seçer A, Bayram M. Stability, synchronization control and numerical solution of fractional shimizu-morioka dynamical system. *Applied Mathematics & Information Sciences*. 2014; 8(4): 1699-1705. Available from: doi: 10.12785/amis/080426.

[37] Yusuf A, Sulaıman TA, Khalil EM, Bayram M, Ahmad H. Construction of multi-wave complexiton solutions of the Kadomtsev-Petviashvili equation via two efficient analyzing techniques. *Results in Physics*. 2021; 21: 103775. Available from: doi: 10.1016/j.rinp.2020.103775.

[38] Farman M, Akgül A, Garg H, Băleanu D, Hınçal E, Shahzeen S. Mathematical analysis and dynamical transmission of monkeypox virus model with fractional operator. *Expert Systems*. 2023. Available from: doi: 10.1111/exsy.13475.

[39] Farman M, Akgul A, Alshaikh N, Azeem M, Asad J. Fractional order newton raphson method for nonlinear equation with convergence and stability analysis. *Fractals*. 2023. Available from: doi: 10.1142/s0218348x23400790.