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Research Article

The Dynamic Behavior of Conjugate Multipliers on Some Reflexive Banach Spaces of Analytic Functions

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Abstract: In this paper, we characterize the hypercyclic, mixing and chaotic conjugate multipliers on some reflexive spaces of analytic functions. Our results extend several well-known results in the existing literature. We present some nontrivial examples to show the validity of our results.

***Keywords*:** hypercyclic operators, mixing operators, chaotic operators, conjugate multipliers

MSC: 47B37, 47A16, 46A45

1. Introduction

Throughout this article, let $\mathbb{N}$ denote the set of nonnegative integers. Let $\mathbb{K}$ denote the real number field $\mathbb{R}$ or the complex number field $\mathbb{C}$. Let $\mathbb{Q}$ denote the rational number field. If $z \in \mathbb{C}$ and $r > 0$ are fixed then define $B(z, r) = \{\lambda \in \mathbb{C} : |\lambda - z| < r\}$. Let $\mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$ and $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$.

A continuous linear operator T on a Banach space X is called hypercyclic if there is an element x in X whose orbit $\{T^n x : n \in \mathbb{N}\}$ under T is dense in X; topologically transitive if for any pair U, V of nonempty open subsets of X, there exists some nonnegative integer n such that $T^n(U) \cap V \neq \varnothing$; mixing if for any pair U, V of nonempty open subsets of X, there exists some nonnegative integer N such that $T^n(U) \cap V \neq \varnothing$ for all $n \geq N$; and chaotic if T is topologically transitive and T has a dense set of periodic points. It is well known that a continuous linear operator on a separable Banach space is topologically transitive if and only if it is hypercyclic (see page 10 in [1]).

The historical interest in hypercyclicity is related to the invariant subset problem. The invariant subset problem, which is open to this day, asks whether every continuous linear operator on any infinite dimensional separable Hilbert space possesses an invariant closed subset other than the trivial ones given by $\{0\}$ and the whole space. Counterexamples do exist for continuous linear operators on non-reflexive spaces like l^1. After a simple observation, a continuous linear operator T on a Banach space X has no nontrival invariant closed subsets if and only if every nonzero vector x is hypercyclic (i.e., the orbit $\{T^n x : n \in \mathbb{N}\}$ under T is dense in X).

The best known examples of hypercyclic operators are due to Birkhoff [2], MacLane [3] and Rolewicz [4]. Each of these papers had a profound influence on the literature on hypercyclicity. Birkhoff's result on the hypercyclicity of the translation operator $T_a(f)(z) = f(z + a)$, $a \neq 0$, on the space $H(\mathbb{C})$ of entire functions has led to an extensive study of

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hypercyclic composition operators (see pages 110-118 in [1, 4-7]), while MacLane's result on the hypercyclicity of the differentiation operator $Df = f'$ on $H(\mathbb{C})$ initiated the study of hypercyclic differential operators (see pages 104-110 in [1], [8]).

Recently Godefroy and Shapiro [8] have studied the dynamic properties of conjugate multipliers on some Hilbert spaces of analytic functions, characterizing hypercyclic, mixing and chaotic conjugate multipliers on such spaces. It is therefore very natural to try to characterize hypercyclic, mixing and chaotic conjugate multipliers on arbitrary reflexive Banach spaces of analytic functions. In this paper we will characterize the hypercyclic, mixing and chaotic conjugate multipliers on some reflexive spaces of analytic functions, generalizing Theorem 4.5, Theorem 4.9, Theorem 6.2 in [8].

Theorem 1.1 Let $\Omega \subseteq \mathbb{C}$ be a nonempty open connected subset. Let $X \neq \{0\}$ be a reflexive Banach space of analytic functions on Ω such that each point evaluation $k_\lambda : X \to \mathbb{C}(\lambda \in \Omega)$ is continuous on X, where $k_\lambda(f) = f(\lambda)(f \in X)$. Suppose further that every bounded analytic function ψ on Ω defines a multiplication operator $M_\psi : X \to X$ with $\|M_\psi\| \leq \sup_{z\in\Omega} |\psi(z)|$, where $M_\psi(f) = \psi f(f \in X)$. Let φ be a nonconstant bounded analytic function on Ω and M_φ^* the conjugate of M_φ. Then the following assertions are equivalent:

(1) M_φ^* is hypercyclic;

(2) M_φ^* is mixing;

(3) M_φ^* is chaotic;

(4) $\varphi(\Omega) \bigcap \mathbb{T} \neq \varnothing$.

This paper is organized as follows. In Section 2 we characterize the hypercyclic, mixing and chaotic conjugate multipliers on some reflexive spaces of analytic functions, generalizing Theorem 4.5, Theorem 4.9, Theorem 6.2 in [8]. Furthermore, we exhibit several hypercyclic, mixing and chaotic conjugate multipliers on H^p spaces for $p > 1$. These examples show that our generalizations are more effective.

2. The hypercyclic, mixing and chaotic conjugate multipliers

Recall the notion of annihilator introduced in page 163 in [9].

Definition 2.1 Let X be a normed linear space. If $A \subseteq X$, the annihilator $A^\perp$ of A is the set

$$A^\perp = \{x' \in X^* : x'(x) = 0 \text{ for all } x \in A\},$$

where X^* is the set of continuous linear functionals on X.

If $F \subseteq X^*$, the annihilator $F^\perp$ of F is the set

$$F^\perp = \{x \in X : x'(x) = 0 \text{ for all } x' \in F\}.$$

The following technical results will help us characterize hypercyclic, mixing and chaotic conjugate multipliers on some reflexive Banach spaces of analytic functions.

The following proposition is well known (see page 164 in [9]).

Proposition 2.2 A normed linear space X is a reflexive Banach space if and only if every norm-closed linear subspace in X^* is $\sigma(X^*, X)$-closed, where $\sigma(X^*, X)$ is the *weak** topology on X^*.

We need the following proposition (see pages 163-164 in [9]).

Proposition 2.3 Let X be a normed linear space. If F is a nonempty subset of X^*, then $F^{\perp\perp}$ is the $\sigma(X^*, X)$-closed linear subspace generated by F, where $F^{\perp\perp} = (F^\perp)^\perp$.

The following basic result is well known in complex analysis.

The Identity Principle for analytic functions Let $\Omega \subseteq \mathbb{C}$ be a nonempty open connected subset. Let $f : \Omega \to \mathbb{C}$

be an analytic function on Ω. Then the following are equivalent statements:

(1) $f \equiv 0$;

(2) $\{z \in \Omega : f(z) = 0\}$ has a limit point in Ω.

We need the following Godefroy-Shapiro criterion (see pages 69-70 in [1]).

Proposition 2.4 Let T be a continuous linear operator on a separable Banach space X. Suppose that the subspaces

$$X_0 = span\{x \in X : Tx = \lambda x \text{ for some } \lambda \in \mathbb{K} \text{ with } |\lambda| < 1\},$$

$$Y_0 = span\{x \in X : Tx = \lambda x \text{ for some } \lambda \in \mathbb{K} \text{ with } |\lambda| > 1\}$$

are dense in X. Then T is mixing, and in particular hypercyclic.

If, moreover, X is a complex space and also the subspace

$$Z_0 = span\{x \in X : Tx = e^{\alpha\pi i}x \text{ for some } \alpha \in \mathbb{Q}\}$$

is dense in X, then T is chaotic.

The following is the major technique we need.

Lemma 2.5 Let $\Omega \subseteq \mathbb{C}$ be a nonempty open connected subset. Let $X \neq \{0\}$ be a reflexive Banach space of analytic functions on Ω such that each point evaluation $k_\lambda : X \to \mathbb{C}(\lambda \in \Omega)$ is continuous on X, where $k_\lambda(f) = f(\lambda)(f \in X)$. Let $\Lambda \subseteq \Omega$ be a set with a limit point in Ω. Then the set $span\{k_\lambda : \lambda \in \Lambda\}$ is dense in X^*.

Proof. First we will show that $(span\{k_\lambda : \lambda \in \Lambda\})^\perp = \{0\}$. Let $f \in (span\{k_\lambda : \lambda \in \Lambda\})^\perp$. We will show that $f \equiv 0$. Since $f \in (span\{k_\lambda : \lambda \in \Lambda\})^\perp$, we have $k_\lambda(f) = 0$ for all $\lambda \in \Lambda$. Notice that $k_\lambda(f) = f(\lambda)$. Then $f(\lambda) \equiv 0$ for all $\lambda \in \Lambda$. This implies that

$$\Lambda \subseteq \{z \in \Omega : f(z) = 0\}.$$

Since Λ has a limit point in Ω, $\{z \in \Omega : f(z) = 0\}$ has a limit point in Ω. By the Identity Principle for analytic functions we have $f \equiv 0$.

Next we will show that $\overline{span\{k_\lambda : \lambda \in \Lambda\}} = X^*$. Since

$$(\overline{span\{k_\lambda : \lambda \in \Lambda\}})^\perp = (span\{k_\lambda : \lambda \in \Lambda\})^\perp$$

and $(span\{k_\lambda : \lambda \in \Lambda\})^\perp = \{0\}$, $(\overline{span\{k_\lambda : \lambda \in \Lambda\}})^\perp = \{0\}$. Hence

$$(\overline{span\{k_\lambda : \lambda \in \Lambda\}})^{\perp\perp} = \{0\}^\perp = X^*.$$

Since X is reflexive and $\overline{span\{k_\lambda : \lambda \in \Lambda\}}$ is norm-closed, by Proposition 2.2 we have $\overline{span\{k_\lambda : \lambda \in \Lambda\}}$ is $\sigma(X^*, X)$-closed. Finally by Proposition 2.3 we have

$$(\overline{span\{k_\lambda : \lambda \in \Lambda\}})^{\perp\perp} = \overline{span\{k_\lambda : \lambda \in \Lambda\}}.$$

Therefore $\overline{span\{k_\lambda : \lambda \in \Lambda\}} = X^*$. □

Remark 2.6 Let $\Omega \subseteq \mathbb{C}$ be a nonempty open connected subset. Let $X \neq \{0\}$ be a reflexive Banach space of analytic functions on Ω such that each point evaluation $k_\lambda : X \to \mathbb{C}(\lambda \in \Omega)$ is continuous on X. Then by Lemma 2.5 we have X^* is separable.

Next we prove Theorem 1.1.

Proof of Theorem 1.1 (4) $\Rightarrow$ (2) First we will show that $M_\varphi^*(k_\lambda) = \varphi(\lambda)k_\lambda$ for all $\lambda \in \Omega$. Let $\lambda \in \Omega$ and $f \in X$. Notice that

$$M_\varphi^*(k_\lambda)(f) = k_\lambda(M_\varphi(f)) \tag{1}$$

$$= k_\lambda(\varphi f) \tag{2}$$

$$= (\varphi f)(\lambda) \tag{3}$$

$$= \varphi(\lambda) f(\lambda) \tag{4}$$

$$= (\varphi(\lambda)k_\lambda)(f) \tag{5}$$

Therefore $M_\varphi^*(k_\lambda) = \varphi(\lambda)k_\lambda$ for all $\lambda \in \Omega$.

Next we will show that $\{\lambda \in \Omega : |\varphi(\lambda)| < 1\}$ is nonempty and has a limit point in Ω. Since $\varphi(\Omega) \cap \mathbb{T} \neq \emptyset$, we may choose $z_0 \in \Omega$ with $|\varphi(z_0)| = 1$. By the Open Mapping Theorem, $\varphi(\Omega)$ is open. Hence there exists $\delta > 0$ such that $B(\varphi(z_0), \delta) \subseteq \varphi(\Omega)$. Since $|\varphi(z_0)| = 1$, we may choose $\lambda_0 \in B(\varphi(z_0), \delta)$ with $|\lambda_0| < 1$. Since $B(\varphi(z_0), \delta) \subseteq \varphi(\Omega)$, we have $\lambda_0 \in \varphi(\Omega)$. Therefore there exists $w_0 \in \Omega$ such that $\lambda_0 \in \varphi(w_0)$. This implies that

$$w_0 \in \{\lambda \in \Omega : |\varphi(\lambda)| < 1\}$$

and $\{\lambda \in \Omega : |\varphi(\lambda)| < 1\}$ is nonempty. Since φ is continuous, $\{\lambda \in \Omega : |\varphi(\lambda)| < 1\}$ is a nonempty open subset. Hence $\{\lambda \in \Omega : |\varphi(\lambda)| < 1\}$ has a limit point in Ω.

Similarly we can show that $\{\lambda \in \Omega : |\varphi(\lambda)| > 1\}$ is nonempty and has a limit point in Ω. Since $\varphi(\Omega) \cap \mathbb{T} \neq \emptyset$, we may choose $z_0 \in \Omega$ with $|\varphi(z_0)| = 1$. By the Open Mapping Theorem, $\varphi(\Omega)$ is open. Hence there exists $\delta > 0$ such that $B(\varphi(z_0), \delta) \subseteq \varphi(\Omega)$. Since $|\varphi(z_0)| = 1$, we may choose $\lambda_0 \in B(\varphi(z_0), \delta)$ with $|\lambda_0| > 1$. Since $B(\varphi(z_0), \delta) \subseteq \varphi(\Omega)$, we have $\lambda_0 \in \varphi(\Omega)$. Therefore there exists $w_0 \in \Omega$ such that $\lambda_0 \in \varphi(w_0)$. This implies that

$$w_0 \in \{\lambda \in \Omega : |\varphi(\lambda)| > 1\}$$

and $\{\lambda \in \Omega : |\varphi(\lambda)| > 1\}$ is nonempty. Since φ is continuous, $\{\lambda \in \Omega : |\varphi(\lambda)| > 1\}$ is a nonempty open subset. Hence $\{\lambda \in \Omega : |\varphi(\lambda)| > 1\}$ has a limit point in Ω.

Finally we will show that M_φ^* is mixing. Let

$$A = span\{x^* \in X^* : M_\varphi^* x^* = \lambda x^* \text{ for some } \lambda \in \mathbb{C} \text{ with } |\lambda| < 1\},$$

$$B = span\{x^* \in X^* : M_\varphi^* x^* = \lambda x^* \text{ for some } \lambda \in \mathbb{C} \text{ with } |\lambda| > 1\}.$$

Since $M_\varphi^*(k_\lambda) = \varphi(\lambda)k_\lambda$ for all $\lambda \in \Omega$, $spank\{k_\lambda : \lambda \in \Omega$ and $|\varphi(\lambda)| < 1\} \subseteq A$ and $spank\{k_\lambda : \lambda \in \Omega$ and $|\varphi(\lambda)| > 1\} \subseteq B$. Since $\{\lambda \in \Omega : |\varphi(\lambda)| < 1\}$ and $\{\lambda \in \Omega : |\varphi(\lambda)| > 1\}$ both have a limit point in Ω, by Lemma 2.5 we have $spank\{k_\lambda : \lambda$

$\in \Omega$ and $|\varphi(\lambda)| < 1\}$ and $spank\{k_\lambda : \lambda \in \Omega$ and $|\varphi(\lambda)| > 1\}$ are both dense in X^*. Hence A and B are both dense in X^*. By Proposition 2.4, we have M_φ^* is mixing.

(2) $\Rightarrow$ (1) This is trivial.

(1) $\Rightarrow$ (4) Let us suppose that $\varphi(\Omega)$ does not intersect the unit circle. Since $\varphi(\Omega)$ is open and connected, it must lie entirely inside or entirely outside $\mathbb{D}$. If $\varphi(\Omega) \subseteq \mathbb{D}$ then

$$\| M_\varphi^* \| = \| M_\varphi \| \leq \sup_{z \in \Omega} |\varphi(z)| \leq 1, \tag{6}$$

and hence M_φ^* cannot be hypercyclic. If $\varphi(\Omega) \subseteq \mathbb{C} \backslash \overline{\mathbb{D}}$ then $\psi = \frac{1}{\varphi}$ is a bounded analytic function on Ω with $\psi(\Omega) \subseteq \mathbb{D}$, which implies that M_ψ^* cannot be hypercyclic. But M_φ is the inverse of M_ψ and therefore M_φ^* is the inverse of M_ψ^*. Hence M_φ^* cannot be hypercyclic.

(4) $\Rightarrow$ (3) First we will show that $\{\lambda \in \Omega : \varphi(\lambda)$ is a root of unity$\}$ has a limit point in Ω. Since $\varphi(\Omega) \cap \mathbb{T} \neq \emptyset$, we may choose $z_0 \in \Omega$ with $|\varphi(z_0)| = 1$. Since Ω is open, there is $r > 0$ such that $\overline{B(z_0, r)} \subseteq \Omega$. By the Open Mapping Theorem, $\varphi(B(z_0, r)$ is open. Since $\varphi(z_0) \in \varphi(B(z_0, r))$, there is $\delta > 0$ such that $B(\varphi(z_0), \delta) \subseteq \varphi(B(z_0, r)$. Since $|\varphi(z_0)| = 1$, $B(\varphi(z_0), \delta)$ contains infinitely many roots of unity. This implies that infinitely many preimages of roots of unity lie in the compact subset $\overline{B(z_0, r)}$ of Ω. Therefore $\{\lambda \in \Omega : \varphi(\lambda)$ is a root of unity$\}$ has a limit point in Ω.

Next we will show that M_φ^* is chaotic. Let

$$C = span\{x^* \in X^* : M_\varphi^* x^* = e^{\alpha \pi i} x^* \text{ for some } \alpha \in \mathbb{Q}\}.$$

Since $M_\varphi^*(k_\lambda) = \varphi(\lambda) k_\lambda$ for all $\lambda \in \Omega$, $spank\{k_\lambda : \lambda \in \Omega$ and $\varphi(\lambda)$ is a root of unity$\} \subseteq C$. Since $\{\lambda \in \Omega : \varphi(\lambda)$ is a root of unity$\}$ has a limit point in Ω, by Lemma 2.5 we have $spank\{k_\lambda : \lambda \in \Omega$ and $\varphi(\lambda)$ is a root of unity$\}$ is dense in X^*. Hence C is dense in X^*. By (4) $\Rightarrow$ (2), we have M_φ^* is mixing. By Proposition 2.4, taking $Z_0 = C$, we have M_φ^* is chaotic.

(3) $\Rightarrow$ (1) This is trivial. □

Godefroy and Shapiro [8] proved Theorem 1.1 in the case of Hilbert spaces of analytic functions, thus Theorem 1.1 generalizes Theorem 4.5, Theorem 4.9, Theorem 6.2 in [8].

Example 2.7 For $1 \leq p < +\infty$, let H^p denote the space of all analytic functions on $\mathbb{D}$ for which

$$\sup_{0 \leq r < 1} \left(\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta\right)^{\frac{1}{p}} < +\infty.$$

For any $f \in H^p$, let

$$\|f\|_p = \sup_{0 \leq r < 1} \left(\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta\right)^{\frac{1}{p}}.$$

Then $(H^p, \|\cdot\|_p)$ is a Banach space.

In this example we will characterize hypercyclic, mixing and chaotic conjugate multipliers on H^p for $1 < p < +\infty$.

First we will show that each point evaluation $k_\lambda : H^p \to \mathbb{C} (\lambda \in \mathbb{D})$ is continuous on H^p for $1 \leq p < +\infty$, where $k_\lambda(f) = f(\lambda) (f \in H^p)$. Let $\lambda \in \mathbb{D}$, $\{f_n\}_{n=1}^\infty$ be a sequence in H^p, $f \in H^p$ and $\lim_{n \to \infty} \|f_n - f\|_p = 0$. We will show that $\lim_{n \to \infty} f_n(\lambda) = f(\lambda)$. Since $\lambda \in \mathbb{D}$, we may choose $r, R \in (0, 1)$ with $|\lambda| < r < R < 1$. Notice that

$$\| f_n - f\|_p \geq (\frac{1}{2\pi}\int_0^{2\pi} |f_n(Re^{i\theta}) - f(Re^{i\theta})|^p \, d\theta)^{\frac{1}{p}} \tag{7}$$

$$= \frac{1}{(2\pi)^{\frac{1}{p}}}(\int_0^{2\pi} |f_n(Re^{i\theta}) - f(Re^{i\theta})|^p \, d\theta)^{\frac{1}{p}} \tag{8}$$

$$\geq \frac{1}{(2\pi)^{\frac{1}{p}}} \frac{1}{(2\pi)^{\frac{1}{q}}} \int_0^{2\pi} |f_n(Re^{i\theta}) - f(Re^{i\theta})| \, d\theta \text{ (where } \frac{1}{p} + \frac{1}{q} = 1) \tag{9}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} |f_n(Re^{i\theta}) - f(Re^{i\theta})| \, d\theta. \tag{10}$$

By Cauchy's integral formula, we have

$$(f_n - f)(\lambda) = \frac{1}{2\pi i}\int_\gamma \frac{(f_n - f)(\omega)}{\omega - \lambda} d\omega \text{ (where } \gamma(t) = Re^{i\theta}, \ 0 \leq \theta \leq 2\pi) \tag{11}$$

$$= \frac{1}{2\pi i}\int_0^{2\pi} \frac{f_n(Re^{i\theta}) - f(Re^{i\theta})}{Re^{i\theta} - \lambda} d(Re^{i\theta}) \tag{12}$$

$$= \frac{R}{2\pi}\int_0^{2\pi} \frac{f_n(Re^{i\theta}) - f(Re^{i\theta})}{Re^{i\theta} - \lambda} e^{i\theta} d\theta. \tag{13}$$

Therefore

$$\frac{1}{2\pi}\int_0^{2\pi} |f_n(Re^{i\theta}) - f(Re^{i\theta})| \, d\theta \tag{14}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} |\frac{f_n(Re^{i\theta}) - f(Re^{i\theta})}{Re^{i\theta} - \lambda} e^{i\theta}| \cdot |Re^{i\theta} - \lambda| \, d\theta \tag{15}$$

$$\geq \frac{R - r}{2\pi}\int_0^{2\pi} |\frac{f_n(Re^{i\theta}) - f(Re^{i\theta})}{Re^{i\theta} - \lambda} e^{i\theta}| \, d\theta \tag{16}$$

$$\geq \frac{R - r}{2\pi} |\int_0^{2\pi} \frac{f_n(Re^{i\theta}) - f(Re^{i\theta})}{Re^{i\theta} - \lambda} e^{i\theta} d\theta| \tag{17}$$

$$= \frac{R - r}{2\pi} | \frac{2\pi}{R}(f_n(\lambda) - f(\lambda))| \tag{18}$$

$$= \frac{R-r}{R} | f_n(\lambda) - f(\lambda) |. \tag{19}$$

Hence $\| f_n - f \|_p \geq \frac{R-r}{R} | f_n(\lambda) - f(\lambda) |$. Since $\lim_{n\to\infty} \| f_n - f \|_p = 0$, $\lim_{n\to\infty} f_n(\lambda) = f(\lambda)$.

Next we will show that every bounded analytic function ψ on $\mathbb{D}$ defines a multiplication operator $M_\psi : H^p \to H^p$ with $\| M_\psi \| \leq \sup_{z\in\mathbb{D}} |\psi(z)|$ for $1 \leq p < +\infty$. Let ψ be a bounded analytic function on $\mathbb{D}$. Then ψ defines a multiplication operator M_ψ on H^p. Furthermore, for any $f \in H^p$ we have

$$\| M_\psi(f) \|_p = \sup_{0\leq r<1} \left(\frac{1}{2\pi} \int_0^{2\pi} \left| \psi(re^{i\theta}) f(re^{i\theta}) \right|^p d\theta \right)^{\frac{1}{p}} \tag{20}$$

$$\leq \sup_{z\in\mathbb{D}} |\psi(z)| \cdot \sup_{0\leq r<1} \left(\frac{1}{2\pi} \int_0^{2\pi} \left| f(re^{i\theta}) \right|^p d\theta \right)^{\frac{1}{p}} \tag{21}$$

$$= \sup_{z\in\mathbb{D}} |\psi(z)| \cdot \| f \|_p. \tag{22}$$

Therefore $\| M_\psi \| \leq \sup_{z\in\mathbb{D}} |\psi(z)|$.

It is already well-known that the Hardy spaces $(H^p, \|\cdot\|_p)$ are reflexive Banach spaces for $1 < p < +\infty$ (see pages 112-113 in [10]). By Theorem 1.1, for any nonconstant bounded analytic function φ on $\mathbb{D}$ and $1 < p < +\infty$, $M_\varphi^* : (H^p)^* \to (H^p)^*$ is hypercyclic if and only if $M_\varphi^* : (H^p)^* \to (H^p)^*$ is mixing if and only if $M_\varphi^* : (H^p)^* \to (H^p)^*$ is chaotic if and only if $\varphi(\mathbb{D}) \cap \mathbb{T} \neq \emptyset$.

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Conflict of interest

The author declares no competing financial interest.

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