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Research Article

ANFIS-Enhanced $M/M/2$ Queueing Model Investigation in Heterogeneous Server Systems with Catastrophe and Restoration

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Abstract: In real-life applications like production systems, health care systems, and the automobile industry, queueing systems with balking, catastrophes, Restoration, intermittent services have gained great significance in recent years. Two varying service rates (ω_1 and ω_2) have been employed based on the critical value of the customer count within the system, with $\omega_1 \neq \omega_2$. When a customer arrives and observes other customers within the system, they make a decision to either join the system with probability β or decline with a consistent probability $\bar{\beta}$. Unsatisfied customers may return to the service provider after receiving their initial service to seek further services, with a probability $\bar{\eta}$. The possibility that η considers satisfied customers. Inspired by the above-mentioned applications, the current study investigates the mean quantity of patrons in system and queue (L_s and L_q). The constructed queueing structure is determined by the matrix geometric method (MGM) method, and the evaluation of the cost function is undertaken to ascertain the optimal values of the system's decision variables. Also, the adaptive neuro-fuzzy inference system (ANFIS) methodology is used to check the accuracy of the obtained results.

***Keywords*:** intermittently available server, feedback, balking, catastrophes, restoration, matrix geometric method, cost analysis, ANFIS

MSC: 68M20, 90B22, 60K25

Nomenclature

λ	Arrival rate
ω_1	Initial server service rate
ω_2	Secondary server service rate
β	Balking probability for joining the queue
$\overline{\beta}$	Balking probability for leaving the queue
η	Feedback probability for a satisfied customer
$\overline{\eta}$	Feedback probability for a unsatisfied customer
ν	The retrieval capacity rate
ξ	Catastrophic rate

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κ	Restoration rate
$X(t)$	Two-state Markov process at time t
$\mathscr{N}(t)$	No. of customers in the system at time t
$\mathscr{S}(t)$	Server 2 status at time t
Q	Infinitesimal generator matrix
$\begin{bmatrix} D_0, & D_1 & \\ D_2, & B_0 & \\ A_0, & A_1, & A_2 \end{bmatrix}$	Sub-matrices of Q
E	Steady-state probability vector
R	Rate matrix
A	Least generator matrix
e	Column vector of 1's

1. Introduction

Queueing theory focuses its attention on comprehending the diverse intricacies involved in waiting lines and developing strategies to enhance efficiency. This theory, along with its associated models, is employed to simulate real-life queueing scenarios, enabling a mathematical exploration of queueing performance.

Within operational hours, disruptions occasionally arise during the work period. These disruptions can manifest as server malfunctions or unavailability. Queueing Systems (QS) that incorporate service disruptions have found extensive applications across manufacturing, computer, and telecommunication systems for over two decades. In numerous real-world queueing scenarios, an environment frequently emerges where servers contend with service interference.

Frequently, servers exhibit varying service rates despite providing the same service. This has spurred numerous researchers to explore multi-server queueing models featuring servers of differing capabilities. Several scholars have delved into the analysis of multi-server queues characterized by heterogeneous service rates.

The current research is committed to exploring the increasing fascination with the $M/M/2$ queueing structure, featuring Poisson arrival processes and the utilization of dual distinct servers. Server 1 remains consistently accessible, while Server 2 experiences periodic availability or periods of inactivity. Service interruptions transpire for varying, random duration, but exclusively after the ongoing service is finalized. This category of service is denoted as intermittently obtainable service.

The transition from an idle state to a working state occurs when the server is activated to serve customers. Similarly, the transition from an idle state to an intermittently obtainable state takes place when the server is made available to assist with certain rapid and diverse tasks, specifically when the queue length is non-empty (i.e., non-exhaustive service). These intermittently obtainable servers are commonly present in computer and communication networks.

1.1 *Significant statement*

In our model, we consider feedback and balking customers in order to more accurately reflect real-world queueing systems. Feedback, where customers re-enter the system after service completion, and balking, where customers refuse to join the queue under certain conditions, are common phenomena in many service environments. By incorporating these elements, our model captures essential dynamics that impact system performance. Addressing situations such as catastrophes and restoration, is crucial for evaluating system resilience and planning for contingencies. By examining how the system behaves during and after disruptive events, we gain insights into its ability to recover and maintain service levels under adverse conditions. The use of matrix-geometric techniques allows for the efficient derivation of steady-state solutions for complex Markovian queueing systems. This approach enables us to obtain analytical insights into the system's behavior, facilitating a deeper understanding of its performance under various scenarios. The insights gained from our model have practical implications for system design, optimization, and management in various domains, including telecommunications, healthcare, transportation, and manufacturing. Understanding the effects of feedback, balking, catastrophes, and restoration can inform decision-making processes aimed at improving service quality, resource allocation, and overall system efficiency.

1.2 *Literature review*

Using the previously mentioned data compilations and descriptions, the following sentences delve into the literature to perform a review of essential surveys related to our Model -QS's, as detailed below:

Morse [1] was the pioneer in formulating the theory of heterogeneity within the realm of service. A solitary server with intermittent availability delivers the service for incoming batches, which follow a Poisson distribution and have varying sizes, while outgoing batches with variable sizes follow a general distribution formed by Sharda [2]. The system characterised by heterogeneity is contrasted with its analogous homogeneous counterpart, which was discussed by Singh [3].

Intermittent timeframes are designated for server service situations arising from server unavailability, and these instances were managed by Federgrune [4]. In a balanced $M/M/N$ QS, every server is prone to interruptions in service, a situation addressed by Mitrany et al. [5]. Nain [6] dealt with a QS involving servers that are intermittently available. Utilizing intermittently accessible servers, Seenivasan et al. [7] incorporated the MGM to analyze the QS.

Retrial queues have found broad applications in replicating various complexities and contextual factors in phone device systems, call centres, network communications, digital connectivity, and everyday life. The crucial approach to steady-state probabilities has been derived through the utilization of the matrix geometric technique, which was extensively discussed in Seenivasan et al. [8] retrial queueing research article.

Various functional waiting systems, particularly those featuring balking and reneging, have found extensive application in numerous real-world scenarios. Examples include scenarios with impatient customers at telephone switchboards, medical emergency departments managing seriously ill patients, and material management systems dealing with perishable goods storage. They find application in the design and management of various systems, such as data transmission and the emergency wards of the healthcare industry, where patient hesitation is common and the chance of revisiting treatments is elevated.

In the realm of queueing literature, feedback denotes customer discontent arising from inadequate quality of service. When feedback occurs, customers seek service again after receiving partial or incomplete assistance. In computer communication, there are situations in which protocol data units are re-transmitted due to errors, often resulting from inadequate service quality.

QS's that involve balking have received significant attention due to the correlation between losing a customer and experiencing a loss in profits. Conversely, QS's featuring feedback depict scenarios in which a server must re-engage in providing service based on customer requests or in response to the customer deeming the initial service unsatisfactory. The concept of customer impatience was initially introduced in Haight's [9] research. Takacs [10] later examined a queue system with a single server and feedback. In order to investigate potential uses, references to studies conducted by Santhakumaran and Thangaraj [11], Choudhury and Paul [12], Kumar [13], Varalakshmi et al. [14], and Bouchentouf et al. [15] can be consulted. These works delve into the historical background and contributions of researchers within the realm of QS's that incorporate balking and feedback.

Bouchentouf [16] examined the QS with balking, reneging, and feedback made up of two heterogeneous servers, additionally exhibit how the different model parameters impact the system's behaviour. Singh [17] examined the steady-state properties of a bulk queueing model featuring a solitary server, encompassing balking capabilities, and inclusive of optional vacations. In this setup, a queueing arrangement consists of a lone server, incorporating retrials, instances of balking, and feedback. The system functions within a multi-layered working vacation framework formed by Rajadurai et al. [18]. In 2023, George Mytalas [19] conducted a study investigating the interaction among vacations, Bernoulli feedback, and occurrences of disasters followed by subsequent repair processes.

This paper examines a straightforward Markovian QS incorporating environmental, catastrophic, and restorative influences. Given the uncomplicated nature of Markovian queues, multiple researchers have explored them and derived solutions for both time-dependent and steady-state scenarios. Over the past four decades, particular emphasis has been placed on comprehending the impact of catastrophes.

These catastrophic events, occurring randomly, result in the destruction of all present customers. Numerous references can be found in the literature concerning queueing models that incorporate the potential for catastrophes (Brockwell [20], Jain [21], Chao [22]). Such queueing models involving catastrophes have the potential to be effectively utilized in various real-world scenarios, including computer communication, biological sciences, and agricultural sciences.

Until now, the assumption has been that when a catastrophe occurs, the system promptly resumes operation. However, in numerous real-world scenarios, it has been observed that the system doesn't return to normal functioning immediately after being impacted by a catastrophe.

Our findings in 2007 revealed that the system requires a certain duration for restoration. This period of repair is termed the restoration time. Throughout this repair interval, the decision of whether a customer chooses to join the system or not becomes contingent upon individual preferences several real-world instances of catastrophes involve: Situations such as the abrupt illness or unexpected resignation of a specialized employee within the service field, and more instances where a server unexpectedly crashes within the manufacturing sector, potentially leading waiting units to abandon the ongoing process.

Chowdhury [23] employs a matrix-geometric methodology to outline the steady-state solutions of the feedback queue with catastrophic events and balking. The entire system occasionally experiences a catastrophic breakdown under any environmental conditions. This results in the removal and loss of all current customers from the system. Following this incident, a repair process promptly starts, and Bura et al. [24] incorporated the term "restoration time" to describe the length of the repair. Ayyappan et al. [25] adeptly managed both transient solution and steady-state analysis in the QS, considering factors such as catastrophic events and server maintenance.

Catastrophes and restoration times exhibit exponential distribution characteristics within the $M/M/2$ QS examined by Seenivasan et al. [26]. Furthermore, the assessment of performance metrics is conducted through the utilization of probability vectors corresponding to diverse parameter values inherent to the queueing model. Within the queueing framework of $Geo/Geo/1$ design, encompassing catastrophes, balking, and service dependent on the system's state, handled by Lumb et al. [27]. Seenivasan et al. [28] explored a dual-server queueing model with heterogeneous characteristics involving distinct service rates and applied the MGM for restoration purposes.

To acquire equilibrium solutions for the aforementioned queueing system using the matrix geometric approach. When addressing intricate queueing problems, the matrix geometric approach proves notably superior to the conventional probability-generating method. Neuts (1981) [29] was the one who first developed this strategy. This method offers the ability to formulate robust algorithms for various real-world problems featuring distinct structural attributes, regardless of their higher-dimensional nature. In our study, we have employed the matrix-geometric technique to derive steady-state solutions for a Markovian queue with feedback and balking customers under the different situations like catastrophes and restoration.

Listed below are the main contributions of this study:

- The model description outlines both the construction and geometric structure of our model. Subsequent sections provide the matrix solution and develop the stability criteria.
- In the following section, a variety of assessment formulas will be elucidated, along with a comprehensive rationale for practical application and exploration of model scenarios in specialized cases.
- The following section is dedicated to the numerical study, where meticulous outcome estimates have been accurately computed. An in-depth explanation of a sensitivity analysis related to the previously mentioned model is provided.
- In the next section, delve into the process of deriving the total cost accumulation across activities and cost elements.
- Concerned with the validation of analytical results and the implementation of the ANFIS technique, this will be demonstrated through a numerical example.
- An overall discussion on the key findings of the paper has been addressed.
- The conclusion section is focused on accentuating the distinguished attributes.

2. Model description

The design of a dual-server markovian $M/M/2$ QS with catastrophes and restoration events on heterogeneous servers, incorporating balking and feedback mechanisms, is as follows:

The Rate of Arrival Process: Customer arrivals within the system adhere to the Poisson process, characterized by the arrival rate denoted as λ.

The Rate of Service Process: The servers are providing services to incoming customers in the subsequent manner. In the context of the Markovian queueing framework, a system consisting of two separate servers is under consideration. The customers entering the system initiate the formation of a waiting queue as they arrive. In the event that an arriving customer perceives an unoccupied server, they promptly initiate their service session. Conversely, if the customer observes all servers to be occupied, they opt to join the queue for subsequent service. The two servers in operation exhibit distinct service rates: denoted as ω_1 for the initial server and ω_2 for the secondary server. It is important to note that ω_1 is not equivalent to ω_2 signifying inequality between their respective service capabilities.

The duration of service provided by the two servers adheres to an exponential distribution. While the first server remains consistently accessible, the second server's availability is sporadic. The second server reallocates its focus to execute specific and significant tasks during instances of queue expansion. However, prior to undertaking these specialized tasks, the second server must complete its ongoing service obligations.

Balking: When arriving consumers make their entrance, they face a choice: they can either decide to join the line with a probability of β (the probability of queueing), provided that the line is not empty, or they can opt out with a probability of $\bar{\beta}$ (where $\beta + \bar{\beta} = 1$). As a result, during the period of joining, the rate of customer arrivals is $\beta\lambda$.

Feedback: After receiving a service, a consumer faces two situations. They may either prefer another service with the probability $\bar{\eta}$ or exit the system permanently with the probability η (which is equal to $1-\bar{\eta}$). It's essential to emphasize that the sum of η and $\bar{\eta}$ equals 1.

Retrieval Rate: The intervals at which the second server becomes available follow an exponential distribution, characterized by the parameter ν.

Catastrophic and Restoration Rates: Catastrophic incidents unfold in accordance with a Poisson process characterized by the parameter ξ. However, these incidents only transpire when the systems are occupied, resulting in the immediate elimination of all customers present and inflicting damage upon the system itself. Subsequently, a certain span of time is required for the system to recommence operation in a standard manner; this interval is referred to as the restoration time. The restoration time exhibits independent and identically distributed characteristics, governed by the parameter κ, which remains non-negative. The unpredictable events within the system occur without any influence from one another. The Figure 1 illustrates the way this model's transitions are organized.

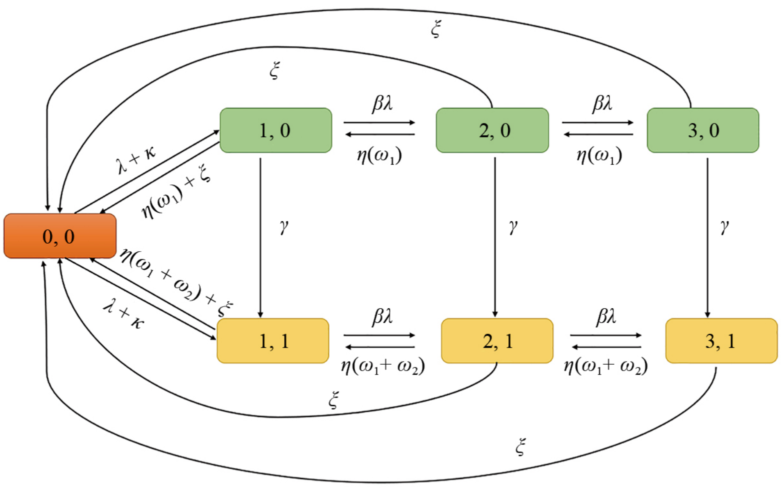

Figure 1. Transition diagram of the model

2.1 *Governing equation*

By using QBD process, Governing equations can be formulated as follows:

$$2(\lambda+\kappa)E_{0,\,0}=(\eta\omega_1+\xi)E_{1,\,0}+(\eta\omega_1+\eta\omega_2+\xi)E_{1,\,1}+\xi\sum_{i=2}^{\infty}E_{i,\,0}+\xi\sum_{i=2}^{\infty}E_{i,\,1}$$

$$(\beta\lambda+\eta\omega_1+\xi+\nu)E_{1,\,0}=(\lambda+\kappa)E_{0,\,0}+\eta\omega_1E_{2,\,0}$$

$$(\beta\lambda+\eta\omega_1+\xi+\nu)E_{i,\,0}=(\lambda+\kappa)E_{i-1,\,0}+\eta\omega_1E_{i+1,\,0}\qquad i=2,\,3,\,\ldots$$

$$(\eta\omega_1+\eta\omega_2+\xi+\beta\lambda)E_{1,\,1}=(\lambda+\kappa)E_{0,\,0}+\nu E_{1,\,0}+(\eta\omega_1+\eta\omega_2)E_{2,\,1}$$

$$(\eta\omega_1+\eta\omega_2+\xi+\beta\lambda)E_{i,\,1}=\beta\lambda E_{1,\,1}+(\eta\omega_1+\eta\omega_2)E_{i+1,\,1}\qquad i=2,\,3,\,\ldots$$

Using the aforementioned difference-differential equations, calculate the steady state probabilities of the model.

2.2 *Transition rates of the process*

A mathematical framework is created to represent a dual-server Markovian QS with two phases of distinct services in this research. This system includes feedback loops and customers who might leave due to impatience, all operating within the context of catastrophic events and service restoration with intermittently available servers.

Matrix analytic techniques are utilized to ascertain the distribution of queue lengths. This is accomplished by applying the MGM and considering probabilities in the stationary state of a bivariate Markov process.

This approach is employed to derive outcomes concerning the probability vector in the steady state. The probability vector, denoted as E, is calculated based on the generator matrix Q, which is expressed as follows:

Consideration and Presentation of a Quasi-Birth-Death Process with Infinitesimal Generator Matrix Q:

$$Q=\begin{bmatrix} D_0 & D_1 & 0 & 0 & 0 & 0 & \cdots \\ D_2 & A_1 & A_2 & 0 & 0 & 0 & \cdots \\ B_0 & A_0 & A_1 & A_2 & 0 & 0 & \cdots \\ B_0 & 0 & A_0 & A_1 & A_2 & 0 & \cdots \\ B_0 & 0 & 0 & A_0 & A_1 & A_2 & \cdots \\ B_0 & 0 & 0 & 0 & A_0 & A_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ldots \end{bmatrix}$$

Where

$$D_0=\begin{bmatrix}-2\lambda-2\kappa\end{bmatrix};\qquad D_1=\begin{bmatrix}\lambda+\kappa & \lambda+\kappa\end{bmatrix};\qquad D_2=\begin{bmatrix}\eta\omega_1+\xi \\ \eta(\omega_1+\omega_2)+\xi\end{bmatrix};\qquad B_0=\begin{bmatrix}\xi\end{bmatrix};$$

$$A_2=\begin{bmatrix}\beta\lambda & 0 \\ 0 & \beta\lambda\end{bmatrix};\quad A_0=\begin{bmatrix}\eta\omega_1 & 0 \\ 0 & \eta(\omega_1+\omega_2)\end{bmatrix}\text{ and }A_1=\begin{bmatrix}-(\beta\lambda+\eta\omega_1+\nu+\xi) & \nu \\ 0 & -(\beta\lambda+\eta(\omega_1+\omega_2)+\xi))\end{bmatrix}$$

2.3 *The matrix geometric approach*

A stochastic variable is established and denoted as $X(t)$, which characterizes the server's condition at time 't' as given below:

Let $X(t) = \{(\mathscr{N}(t),\ \mathscr{S}(t));\ t \geq 0\}$ be a Markov process (MP) with a state space at time t. where $\mathscr{N}(t)$ represents the number of consumers in the system, $\mathscr{S}(t)$ represents the condition of server 2.

$$\mathscr{S}(t) = \begin{cases} 0, & \text{when the server 2 is intermittently obtainable state} \\ 1, & \text{when the server 2 is working state} \end{cases}$$

$\mathscr{N}(t)$ and $\mathscr{S}(t)$ are the indicators for the system state. Let $\{\mathscr{N}(t),\ \mathscr{S}(t);\ t \geq 0\}$ be the Markovian process.

The Quasi-Birth-Death Process and its State Space are arranged in lexicographical manner is given below.

$$\Omega = \bigcup \{(i,\ j);\ \ i \geq 0,\ \ j = 0,\ 1\}$$

The notation P_{ij} is defined as $P_{ij} = \{N = i,\ S = j\}$ where i represent the overall number of consumers in the queueing line also j indicates the state of server. The probability vector is described as follows:

$$E = (E_0,\ E_1,\ E_2,\ \ldots) \text{ and } E_i = (E_{i,\,0},\ E_{i,\,1}),\ \ i = 0,\ 1,\ 2,\ \ldots$$

The static probability matrix E is solved by using $EG = 0$. If the steady-state criterion is achieved, then the sub probability vectors E_i satisfies following equations:

$$E_0D_0 + E_1D_2 + E_2B_0 + E_3B_0 + E_4B_0 + \ldots = 0 \tag{1}$$

$$E_0D_1 + E_1A_1 + E_2A_0 = 0 \tag{2}$$

$$E_1A_2 + E_2A_1 + E_3A_0 = 0 \tag{3}$$

$$E_2A_2 + E_3A_1 + E_4A_0 = 0 \tag{4}$$

$$\vdots$$

$$E_iA_2 + E_{i+1}A_1 + E_{i+2}A_0 = 0 \tag{5}$$

$$E_i = E_1R^{i-1}, \tag{6}$$

where $i \geq 2$. Let assume matrix R be rate matrix. From eqns. (1) to (5) using (6) we get

$$E_0D_0 + E_1[D_2 + R(I - R)^{-1}B_0] = 0 \tag{7}$$

$$E_0D_1 + E_1[A_1 + RA_0] = 0 \tag{8}$$

$$E_1[A_2 + RA_1 + R^2A_0] = 0 \tag{9}$$

$$E_1R[A_2 + RA_1 + R^2A_0] = 0. \tag{10}$$

$$E_1R^{i-1}[A_2 + RA_1 + R^2A_0] = 0, \quad i \geq 2. \tag{11}$$

The normalizing equation is stated below,

$$E_0e + E_1(I - R)^{-1}e = 1. \tag{12}$$

In this case, e is a column vector where every elements are 1's in the corresponding column. The matrix R is the quadratic matrix equation's least non-negative solution.

$$A_2 + RA_1 + R^2A_0 = 0 \tag{13}$$

$$R = -A_1^{-1}[R^2A_0 + A_2], \tag{14}$$

where $R \geq 0$ and it's an irreducible non-negative matrix of radius smaller than one. The matrix R can be estimated by subsequent substitution in the recurrence relationship,

$$R_0 = 0 \tag{15}$$

$$R_{n+1} = -A_1^{-1}[R_n^2A_0 + A_2] \qquad n \geq 1 \tag{16}$$

All values of R will expand monotonically, and R is converging to $-A_1^{-1}[R_n^2A_0 + A_2]$ which is non-negative.

2.4 *Stability condition*

The stability condition, referred to as the Neuts mean drift condition, introduced by Neuts in 1981, asserts that a Markov process Q achieves stability if and only if

$$EA_2e \leq EA_0e \tag{17}$$

The steady state is attained via the MGM technique. Where row vector $E = (E_0, E_1, E_2 \dots)$ is obtained from the infinitesimal generator Matrix $A = A_2 + A_1 + A_0$ is given bellow

$$A = \begin{bmatrix} -\nu - \xi & \nu \\ 0 & -\xi \end{bmatrix}. \tag{18}$$

In a Markov process, the matrix A is the least generator and irreducible. Unique equations are also satisfied by E

$$EA = 0; \quad Ee = 1. \tag{19}$$

Solving (18) and (19) we get

$$E_0 = \frac{\xi}{\xi + \nu} \tag{20}$$

$$E_1 = \frac{\nu}{\xi + \nu}. \tag{21}$$

Now we applying the stability condition $EA_2e \le EA_0e$ for QBD procedure in matrix G. we get

$$\beta\lambda[E_0 + E_1] \le \eta\omega_1[E_0 + E_1] + \eta\omega_2 E_1 \tag{22}$$

3. System performance measures

A variety of performance metrics for the scrutinized queueing model are presented, articulated in the context of steady-state probabilities.

3.1 *Analyzing the probabilities of system states*

In order to comprehend the distribution of customers within the system, the probabilities of various system states are assessed in the following manner:

1. In an empty state, the probability of the servers

$$P_{emp} = E_{0,\,0}$$

2. Intermittent state probability of Server 2's availability

$$P_{IO} = \sum_{i=0}^{\infty} E_{i,\,0}$$

3. In the working state, the probability of Server 2

$$P_W = \sum_{i=0}^{\infty} E_{i,\,1}$$

3.2 *Expected customer quantities within the system and queue*

1. If Server 2 operates intermittently, the average number of customers within the system is

$$ES_{IO} = \sum_{i=0}^{\infty} iE_{i,\,0}$$

2. Average number of users in the system if server 2 is in working state

$$ES_W = \sum_{i=0}^{\infty} iE_{i,\,1}$$

3. If server 2 is only intermittently available, then the average number of customers in the queue

$$EQ_{IO} = \sum_{i=1}^{\infty} (i-1)E_{i,\,0}$$

4. The average number of customers in the queue if server 2 is in a working state

$$EQ_W = \sum_{i=1}^{\infty} (i-1)E_{i,\,1}$$

5. Mean number of consumers identified by the system

$$L_s = ES_{IO} + ES_W$$

6. Mean number of consumers identified by the queue

$$L_q = EQ_{IO} + EQ_W$$

Here from this queueing model system the above key performance metrics such as the average customer count in both the system and the queue, idle probability, busy probability values are calculated, and the sensitivity analysis by using calculated table values.

3.3 *Practical justification of the model*

The real-life use of the model would possibly be apparent in a scenario of the comprehensive overview of restaurant management by incorporating the key concepts of intermittently obtainable servers, feedback mechanisms, balking customers, handling catastrophes, and restoration scenarios.

• Effective Restaurant Management in the Face of Catastrophes, Restoration, and Customer Dynamics: In the realm of restaurant management, various factors come into play, often simultaneously. Restaurants need to navigate intermittently obtainable servers, customer feedback, balking behaviors, catastrophes, and the complex process of restoration to ensure their sustained success.

• Intermittently Obtainable Servers: In pandemics, intermittent service can result from staffing shortages or periodic closures for cleaning and safety measures. Customers, cautious about dining in crowded spaces, may shift their behaviour and opt for takeout or delivery at different times, creating intermittent demand. Restaurants must optimise staffing and delivery services while consistently implementing safety measures during restoration.

• Catastrophes and Restoration: Pandemic Outbreaks: The COVID-19 pandemic exemplified a catastrophic event, resulting in revenue loss and restrictions on restaurant operations. Restoration includes adapting to new health guidelines, implementing contact-less dining options, and finding creative ways to maintain revenue through takeout and delivery services.

• Feedback and Balking Customers: Natural Disasters, Pandemics, and Food Safety Incidents: Catastrophic events can trigger balking behavior as customers may be reluctant to visit the restaurant due to safety concerns. Feedback from customers who do visit is crucial in understanding their evolving preferences and concerns. Restaurants should communicate transparently about corrective actions taken, actively address concerns, and rebuild customer confidence during restoration efforts.

In the face of these multifaceted challenges, successful restaurant management requires a holistic approach that incorporates customer feedback, adapts to balking behaviors, prepares for catastrophes, and diligently executes restoration plans. The ability to navigate these dynamics effectively is pivotal to a restaurant's long-term viability and customer satisfaction.

3.4 *Special cases*

Case (i): No Restoration, $\kappa = 0$. Our model acts as an abbreviated version of prior studies on queue stability with balking using MGM tactics which is identical to [23].

Case (ii): No feedback, $\eta = 0$ and No balking $\beta = 0$. Our model becomes the distilled form of the analysis of intermittent server queue with restoration in the markovian model which coincides with [28].

Case (iii): No Server 2 $\omega_2 = 0$, and No balking $\beta = 0$ and No feedback $\eta = 0$ with partial breakdown and repair. Our model becomes the distilled form of the study of catastrophe, restoration, and partial breakdown in a single-server queue which coincides with [30].

4. Numerical experiments

Analytical findings are affirmed through the utilization of numerical illustrations. Subsequently, a sensitivity analysis is undertaken to assess the impact of various descriptors on system characteristics with respect to the stability conditions at the times of arrival rate changes, service rate changes, feedback rates of customers, and changes in balking rates. The following values are calculated and these results are clearly mentioned in the Tables from 1 to 5 are listed below:

• P_{emp}: Probability values of the servers are being in empty stage.
• P_{IO}: Probability of the server 2 being Intermittently Obtainable state.
• P_W: Probability of the server 2 is in working state.
• ES_{IO}: If server 2 is in intermittently obtainable, then average no. of consumers presented in the system,

• ES_W: If server 2 is in working, then average no. of consumers presented in the system and the corresponding value of the mean number of consumers presented in the system L_S,

• EQ_{IO}: If server 2 is in intermittently obtainable, then average no. of consumers presented in the queue.

• EQ_W: If server 2 is in working, then average no. of consumers presented in the queue and the followed value of the mean number of consumers presented in the queue L_Q.

Table 1. The effect of arrival rate (λ) on performance measures

λ	ω_1	ω_2	η	β	L_s	L_q	P_{emp}	P_{IO}	P_W	ES_{IO}	ES_W	TC
0.1	0.7	0.3	0.7	0.8	0.2677	0.0296	0.7618	0.8034	0.1965	0.0474	0.2203	301.246
0.2	-	-	-	-	0.5602	0.1306	0.5702	0.7210	0.2788	0.1998	0.3604	314.576
0.3	-	-	-	-	0.8766	0.3062	0.4294	0.6533	0.3465	0.3462	0.5304	326.487
0.4	-	-	-	-	1.2443	0.5685	0.3235	0.5916	0.4077	0.4867	0.7576	338.165
0.5	-	-	-	-	1.5751	0.9474	0.2430	0.5309	0.4689	0.4874	1.0877	351.259

Table 2. The effect of service rate 1 (ω_1) on performance measures

λ	ω_1	ω_2	η	β	L_s	L_q	P_{emp}	P_{IO}	P_W	ES_{IO}	ES_W	TC
0.6	0.8	0.3	0.7	0.8	2.0163	1.2150	0.1975	0.5355	0.4633	0.8333	1.1830	354.845
-	0.9	-	-	-	1.7594	0.9850	0.2252	0.5781	0.4215	0.8040	0.9554	348.475
-	1.0	-	-	-	1.5588	0.8114	0.2517	0.6069	0.3922	0.7521	0.8067	344.219
-	1.1	-	-	-	1.4067	0.6842	0.2770	0.6289	0.3706	0.7006	0.7060	341.807
-	1.2	-	-	-	1.2829	0.5845	0.3011	0.6459	0.3536	0.6501	0.6328	340.260

Table 3. The effect of service rate 2 (ω_2) on performance measures

λ	ω_1	ω_2	η	β	L_s	L_q	P_{emp}	P_{IO}	P_W	ES_{IO}	ES_W	TC
0.7	1.5	0.6	0.5	0.7	1.5185	0.7758	0.2566	0.6177	0.3816	0.7515	0.7670	359.823
-	-	0.7	-	-	1.4919	0.7557	0.2630	0.6306	0.3686	0.7663	0.7256	360.147
-	-	0.8	-	-	1.4702	0.7397	0.2689	0.6416	0.3578	0.7766	0.6936	360.917
-	-	0.9	-	-	1.4533	0.7283	0.2744	0.6503	0.3491	0.7829	0.6704	361.998
-	-	1	-	-	1.4411	0.7211	0.2794	0.6568	0.3426	0.7861	0.6550	363.451

Table 4. The effect of feedback (η) on performance measures

λ	ω_1	ω_2	η	β	L_s	L_q	P_{emp}	P_{IO}	P_W	ES_{IO}	ES_W	TC
0.1	0.7	0.3	0.4	0.4	0.3698	0.0337	0.6638	0.7204	0.2795	0.0628	0.3070	313.737
-	-	-	0.5	-	0.3143	0.0237	0.7093	0.7600	0.2399	0.0555	0.2588	307.697
-	-	-	0.6	-	0.2731	0.0173	0.7440	0.7897	0.2101	0.0493	0.2238	303.138
-	-	-	0.8	-	0.2170	0.0106	0.7935	0.8314	0.1685	0.0402	0.1768	296.856
-	-	-	0.9	-	0.1967	0.0085	0.8117	0.8466	0.1533	0.0367	0.1600	294.555

Table 5. The effect of balking (β) on performance measures

λ	ω_1	ω_2	η	β	L_s	L_q	P_{emp}	P_{IO}	P_W	ES_{IO}	ES_W	TC
0.2	0.7	0.3	0.4	0.5	0.7222	0.1703	0.4479	0.6338	0.366	0.2448	0.4774	328.053
-	-	-	-	0.6	0.7832	0.2193	0.4357	0.6232	0.3764	0.2615	0.5217	330.053
-	-	-	-	0.7	0.8514	0.2751	0.4233	0.6122	0.3874	0.2795	0.5719	332.261
-	-	-	-	0.8	0.9281	0.3391	0.4106	0.6006	0.3990	0.2987	0.6294	334.641
-	-	-	-	0.9	1.0112	0.4095	0.3978	0.5884	0.4111	0.3174	0.6938	337.135

5. Sensitivity analysis

To inspect the sensitivity of the system framework described, numerical results were generated through MATLAB software. Tables 1 to 5, as well as Figures 2(a) to 2(e), demonstrate the effect of diverse key metrics. By assuming arbitrary parameter values, to ensure that the stability criterion is met.

Arrival Rate from $\lambda = 0.1$ to 0.5, remaining values ω_1, ω_2, η, β, ξ, κ are fixed with $\omega_1 = 0.7$, $\omega_2 = 0.3$, $\eta = 0.7$, $\beta = 0.8$, $\xi = 0.05$, $\kappa = 0.07$, and $\nu = 0.07$. Table 1 provides a clear depiction of how a mount in the arrival rate λ leads to a corresponding escalation in the values of L_s and L_q.

Service rate 1 from $\omega_1 = 0.8$ to 1.2, remaining values λ, ω_2, η, β, ξ, κ are fixed with $\lambda = 0.6$, $\omega_2 = 0.5$, $\eta = 0.6$, $\beta = 0.7$, $\xi = 0.04$, $\kappa = 0.05$, and $\nu = 0.05$.

Service rate 2 from $\omega_2 = 0.6$ to 1.0, remaining values λ, ω_1, η, β, ξ, κ are fixed with $\lambda = 0.7$, $\omega_1 = 1.5$, $\eta = 0.5$, $\beta = 0.7$, $\xi = 0.04$, $\kappa = 0.05$, and $\nu = 0.05$. Tables 2 and 3 clearly demonstrate that as the service rates ω_1 and ω_2 increase, L_s and L_q values decrease accordingly.

Feedback Rate from $\eta = 0.4$ to 0.9, remaining values λ, ω_1, ω_2, β, ξ, κ are fixed with $\lambda = 0.1$, $\omega_1 = 0.7$, $\omega_2 = 0.3$, $\beta = 0.4$, $\xi = 0.05$, $\kappa = 0.07$, and $\nu = 0.05$.

Balking Rate from $\beta = 0.5$ to 0.9, remaining values λ, ω_1, ω_2, η, ξ, κ are fixed with $\lambda = 0.2$, $\omega_1 = 0.7$, $\omega_2 = 0.3$, $\eta = 0.4$, $\xi = 0.05$, $\kappa = 0.07$, and $\nu = 0.05$. Table 4 vividly illustrates that higher feedback rates (η) result in a proportional decrease in the values of L_s and L_q, next the L_s and L_q values are on the rise, as clearly indicated in Table 5, instead of the expected increase in the balking rate.

In Figure 2(a), we can discern that the rise in L_s and L_q is a natural consequence of the increased arrival rate (λ), which is a reasonable observation.

Figures 2(b) and 2(c) illustrate the diminishing impact on L_s and L_q as service rates increase.

Figure 2(d) clearly depict that elevated feedback rates (η) lead to a corresponding reduction in the values of L_s and L_q.

The L_s and L_q values are experiencing an increase, as clearly depicted in Figure 2(e), rather than the anticipated rise in the balking rate (β).

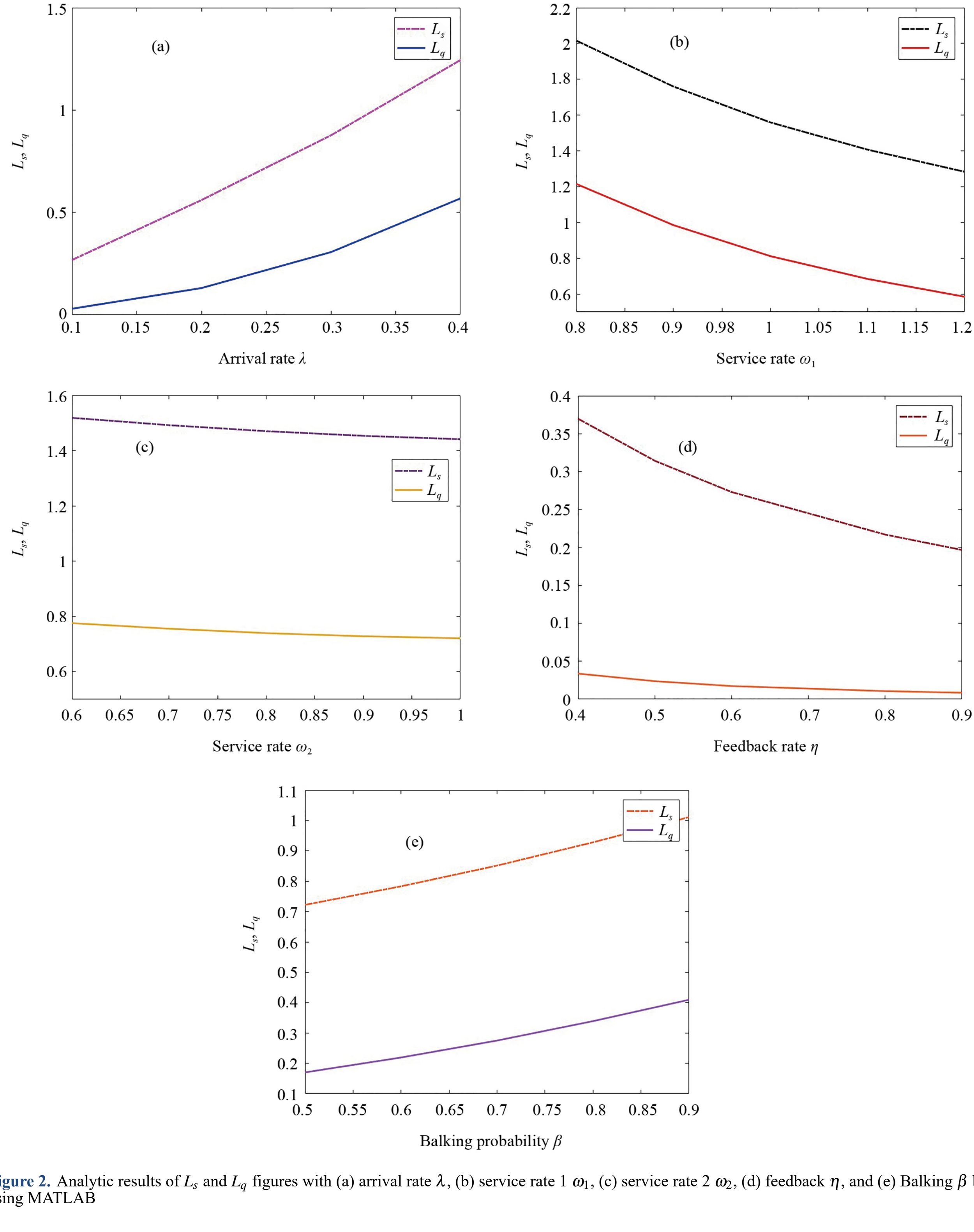

Figure 2. Analytic results of L_s and L_q figures with (a) arrival rate λ, (b) service rate 1 ω_1, (c) service rate 2 ω_2, (d) feedback η, and (e) Balking β by using MATLAB

6. Cost analysis

Cost analysis is an essential component of queuing systems, as it offers an economic framework that can be applied to a wide range of technical and industrial situations. Conducting a cost analysis of the model is necessary for optimizing its economic viability and aiding in the future enhancement of the system's design. Service delays are unavoidable, but the focus can be on assessing the system's efficiency in a cost-effective and economically optimal manner. This portion of the text outlines techniques for establishing a Markovian Queueing System with servers that are intermittently accessible, as previously mentioned. Once the cost components are fixed, $(\mathscr{A}_h, \mathscr{A}_i, \mathscr{A}_w, \mathscr{A}_1, \mathscr{A}_2)$ means that the below ensuing equation provides the definition of the cost function per unit time.

$$TC = \mathscr{A}_h \cdot L_q + \mathscr{A}_i \cdot P_{IO} + \mathscr{A}_w \cdot P_w + \mathscr{A}_1 \cdot \omega_1 + \mathscr{A}_2 \cdot \omega_2 \tag{23}$$

Where,

$\mathscr{A}_h$ represents the cost incurred by each consumer per unit of time spent within the system.

$\mathscr{A}_i$ stands for the cost per unit of time when the server is operating under normal conditions.

$\mathscr{A}_w$ represents the cost per unit time for server intermittently obtainable periods.

$\mathscr{A}_1$ denotes the cost incurred for each consumer served when the server is in a working state.

$\mathscr{A}_2$ represents the cost per consumer served in the server's intermittently accessible state.

Sensitivity analysis can be performed on select parameters within the system. By keeping the base values constant, individual parameters can be adjusted one at a time, and the corresponding cost function values can be calculated, as outlined in Tables 1 to 5 in the last columns above. Herein, the data pertaining to different cost components and various factors is presented, with specific reference to $\mathscr{A}_h = 10$, $\mathscr{A}_i = 350$, $\mathscr{A}_w = 400$, $\mathscr{A}_1 = 20$, $\mathscr{A}_2 = 25$, $\lambda = 0.1$, $\omega_1 = 0.7$, $\omega_2 = 0.3$, $\eta = 0.7$, $\beta = 0.8$, $\xi = 0.05$, $\kappa = 0.07$ and $\nu = 0.07$ the corresponding total expected cost per unit of time is TC = 381.586. The effects of the combinations of $(\mathscr{A}_h, \mathscr{A}_i)$, $(\mathscr{A}_i, \mathscr{A}_w)$, $(\mathscr{A}_1, \mathscr{A}_2)$ on the predicted cost function are shown in Table 6. Using surface graphs to illustrate the total cost function (TC) improves the practicality of this study when viewed from a cost-benefit standpoint. Figure 3(a-c) depicts the effects of alterations in the parameters λ, ω_1, and η on the aggregate cost function designated as (TC).

Table 6. Effects of $(\mathscr{A}_h, \mathscr{A}_i)$, $(\mathscr{A}_i, \mathscr{A}_w)$, $(\mathscr{A}_1, \mathscr{A}_2)$ on Total Cost (TC)

Sets/TC	Set 1/TC	Set 2/TC	Set 3/TC	Set 4/TC	Set 5/TC
$(\mathscr{A}_h, \mathscr{A}_i)$	(10, 250)	(10, 300)	(10, 350)	(20, 250)	(30, 250)
TC	301.246	341.416	381.586	301.542	301.838
$(\mathscr{A}_i, \mathscr{A}_w)$	(250, 450)	(250, 500)	(250, 550)	(300, 400)	(350, 400)
TC	311.071	320.896	330.721	341.416	381.586
$(\mathscr{A}_1, \mathscr{A}_2)$	(20, 30)	(20, 35)	(20, 40)	(25, 35)	(30, 35)
TC	302.746	304.246	305.746	307.746	311.246

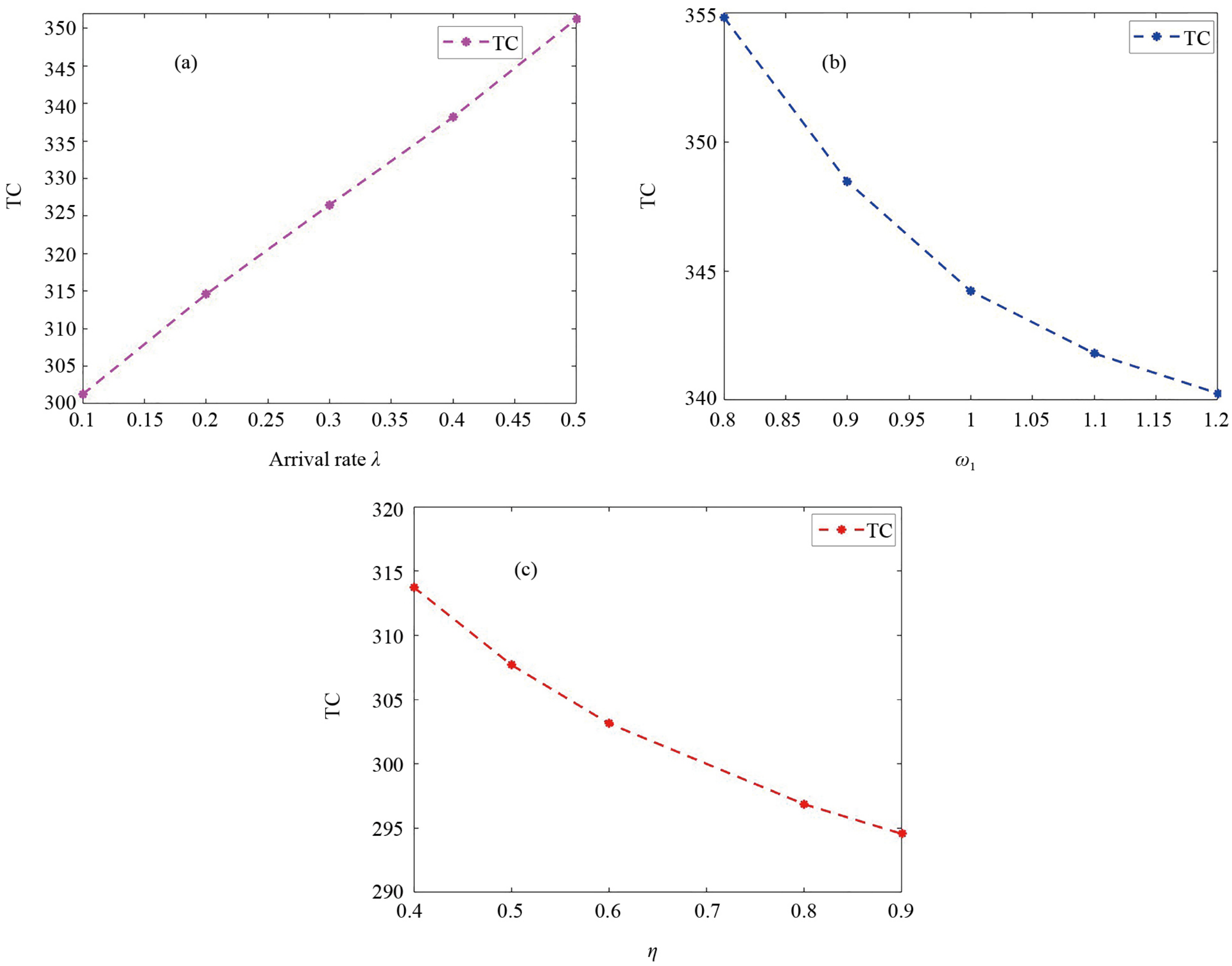

Figure 3. Total Cost (TC) variation for different arrival rates (λ), service rates-1(ω_1), and feedback's (η) values

7. Adaptive neuro-fuzzy inference system (ANFIS)

Jang [31] introduced ANFIS, a hybrid soft computing approach founded upon a multilayer adaptive network-based fuzzy inference system. ANFIS represents a synthesis of two distinct domains, uniting the principles of fuzzy inference systems (FIS) with artificial neural networks (ANN). The main way ANFIS works is by using neuro-adaptive learning techniques to improve membership functions (MF) over and over again until the best solution is found. In this paper, ANFIS is mainly incorporated to validate our results with its neurofuzzy values. By comparing the results from the proposed method with those generated by ANFIS, the paper ensures the robustness and reliability of the findings. Moreover, QS often exhibits nonlinear and dynamic behaviours that may be challenging to capture using traditional analytical approaches. ANFIS, with its ability to validate such nonlinear relationships and uncertainty, can provide a more accurate representation of system dynamics. By validating both traditional methods and advanced data-driven techniques, the paper demonstrates a rigorous and multifaceted approach to research, which enhances the credibility and impact of the findings. Further, by incorporating insights from queueing theory and system dynamics, ANFIS-based models can maintain theoretical rigour while addressing practical considerations specific to the system being analyzed. This ensures that the model aligns with established theoretical frameworks while accounting for real-world complexities. Additionally, ANFIS can be used to optimize system parameters and decision-making processes, leading to improved performance and resource allocation. Overall, our use of ANFIS addresses both practical and theoretical challenges, offering a promising solution with significant implications for queueing theory and beyond.

ANFIS comprises multiple inference rules utilized for decision-making and predictive purposes are highlighted in Kumar's et al. [32] research paper. ANFIS possesses a rapid learning capability and enables the adaptive interpretation of intricate patterns in both linear and nonlinear relationships. ANFIS employs Sugeno-type systems to acquire knowledge, formulating fuzzy if-then rules utilizing Gaussian functions for membership. The training process involves utilizing pairs of input and output data derived from an analytical approach, as also reflected in the article of Thakur et al. [33].

Table 7. Deriving membership function values in linguistics from input parameters λ, ω_1, ω_2

Input parameters	No. of membership function	Linguistic Values
λ	5	extremely low, low, moderate, high, and extremely high
ω_1	5	extremely low, low, moderate, high, and extremely high
ω_2	5	extremely low, low, moderate, high, and extremely high

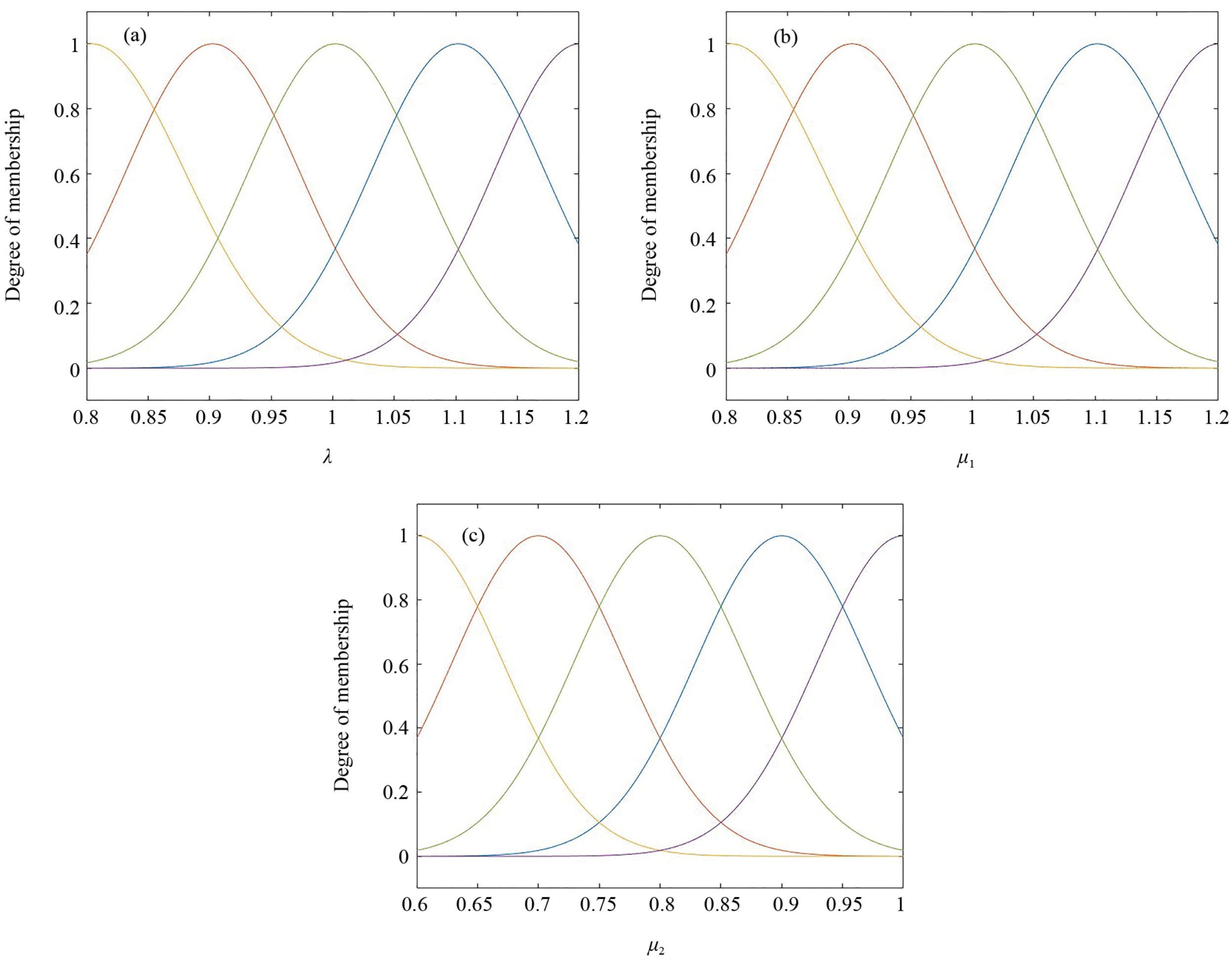

Figure 4. Membership functions for (a) arrival rate λ, (b) service rate 1 (ω_1), (c) service rate 2 (ω_2)

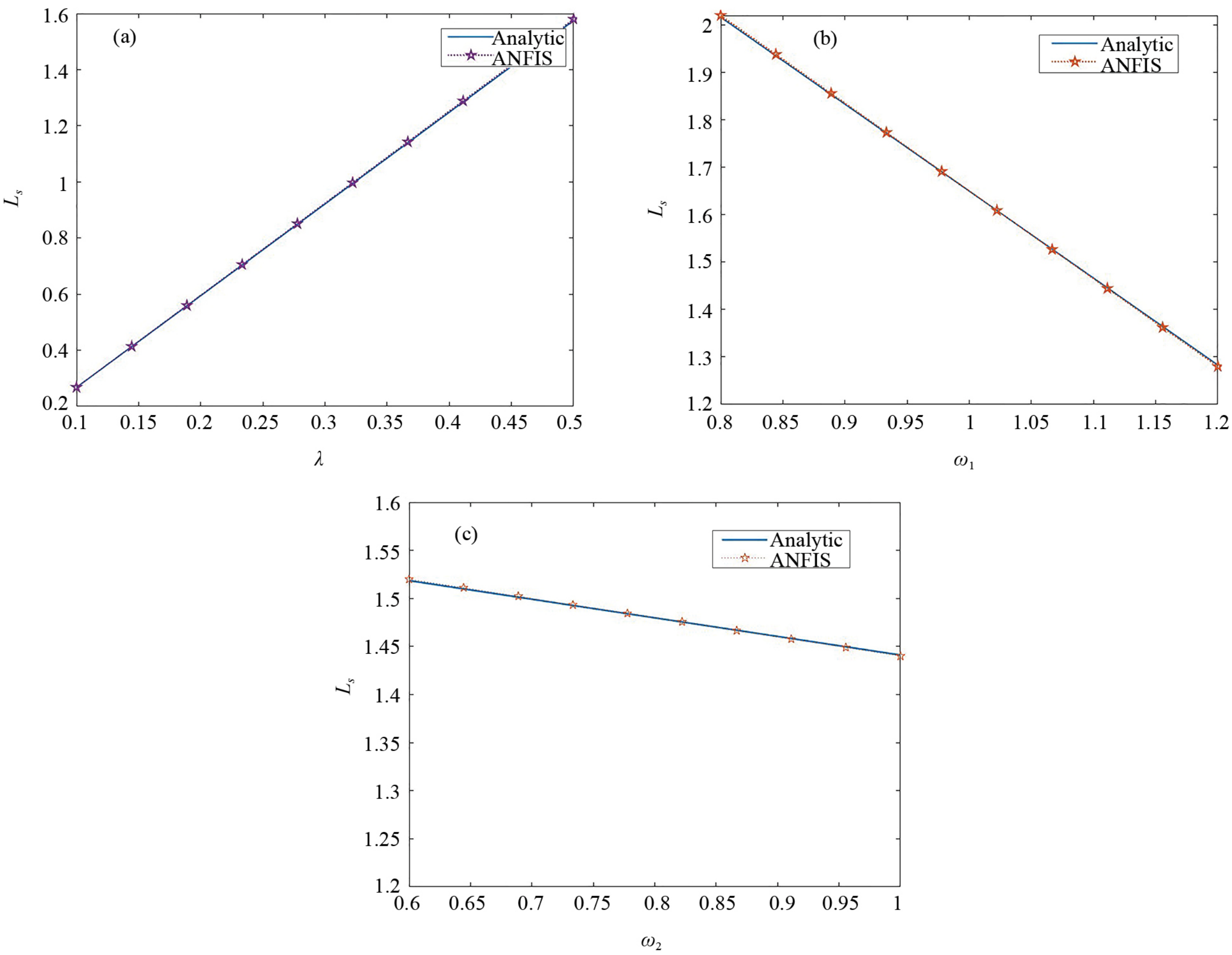

Figure 5. Analytic and ANFIS result for (a) arrival rate λ, (b) service rate 1 (ω_1), (c) service rate 2 (ω_2)

ANFIS modeling has practical utility in a diverse array of fields, including but not limited to transportation, traffic management, telecommunications, and atmospheric research, where precise predictive capabilities are essential for tasks like classification, optimization, and control. To calculate the ANFIS outcomes, the MATLAB neuro-fuzzy tool is employed. Subsequently, a comparison is made between the ANFIS results and the numerical outcomes derived from analytical methods. The neuro-fuzzy methodology is employed for computing ANFIS outcomes, with the application of Gaussian membership functions to the three input parameters λ, ω_1, ω_2 and one output parameter L_s. A few aspects are observed as linguistic terms and are taken into account as inputs when connecting a fuzzy method to ANFIS networks. In Figure 5(a-c), the fuzzification process is demonstrated for the variables λ, ω_1, ω_2. Further, to obtain the final result, we make use of five linguistic values which are viewed as linguistic variables and also these variables has a membership function whose degree have been defines as follows, (i) extremely low, (ii) low, (iii) moderate, (iv) high, and (v) extremely high. Moreover, the range of L_s from 0.8 to 1 has been estimated in reference to the values λ, ω_1, ω_2 by having fixed L_s. Table 7 showcases the structural representation, and Figure 5(a-c) provides a graphical depiction of the variation.

The system's customer count is determined through various arrival rate calculations employing the MGM, and these results align with the customer count estimations obtained through the ANFIS methodology. Both outcomes demonstrate remarkable similarity, highlighting the practicality of ANFIS in addressing complex real-time QSs.

In Figures 5(a-c), the influence of different parameters on the average system queue length, denoted as L_s is described. Additionally, these findings are compared with the numerical results obtained from the ANFIS model implemented in

MATLAB, indicated by (⋆). Within Figure 5(a), a notable observation is the rise in L_s as the arrival rate λ increases, a trend that aligns with expectations. Figures 5(b) and 5(c) delineate a diminishing trajectory in service rates ω_1, ω_2.

8. Discussion

• While many studies focus on single server systems, this paper explores the more realistic scenario of heterogeneous servers where service rates differ. Moreover, traditional queueing models often do not account for catastrophic events that can wipe out all customers in the system or the process of system restoration. This research introduces and analyzes these events, providing a more comprehensive understanding of system resilience and recovery.

• Furthermore, for the proposed model, numerical and sensitivity analysis have been carried out by finding the mean number of customers in the system and in the queue, and further by assessing the impact of various system parameters. This analysis helps in understanding how changes in arrival rates, service rates, feedback and balking probabilities influence overall system performance.

• Moreover, the inclusion of special cases demonstrates the robustness of our model, showing strong alignment with existing literature when specific system parameters are modified. This reaffirms the reliability and versatility of our approach.

• The results and insights from the numerical and sensitivity analysis are thoroughly explained using 2D graphs. These graphs effectively illustrate the relationships and impacts of various system parameters on the mean number of customers in the system and in the queue.

• A comprehensive cost analysis has been conducted to identify the optimal cost structure for the system. This analysis considers various cost components related to the system. Overall, the cost analysis ensures that the system operates not only efficiently but also within a financially optimal framework, offering valuable insights for decision-makers aiming to enhance both service quality and cost-effectiveness.

• ANFIS serves as a validation tool, offering a data-driven approach to validate the findings derived from the analytical model. By comparing the outputs of ANFIS-generated results with those obtained analytically, the study enhances the models' robustness and credibility.

9. Conclusion

In conclusion, this paper successfully addresses the challenges posed by catastrophes and restoration incidents within the $M/M/2$ QS, particularly in the context of heterogeneous servers featuring balking and consumer feedback. It has effectively calculated the steady-state probabilities for system size and queue size. These probabilities have, in turn, enabled the derivation of key performance measures for the system. This research contributes valuable insights into the efficient management of such complex queueing systems. Subsequently, cost optimization analyses are presented, studying the impact of the system parameters and cost elements. The utilisation of fuzzy descriptors through the ANFIS technique implementation and the proposed approach seem to offer significant advantages for system engineers.

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Conflict of interest

The authors declare no competing financial interest.

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