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Research Article

Computing Certain Topological Indices of Silicate Triangle Fractal Network Modeled by the Sierpiński Triangle Network

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Abstract: The study of Sierpiński triangle network and extended Sierpiński graph is quite interesting in the field of fractal networks. In nature, fractal networks (FN) and silicate structure networks (Sio_4) play vital roles and in architecture to analyze the dimension of the above-mentioned FN and Sio_4 networks it is necessary to identify the number of copies of the network. In this paper, we introduced a silicate triangle fractal network (Si_n) which is a planar fractal and it is created using a related sequence of graphs named $(Si_n)_{n\geq 0}$, where n is the n^{th} level of silicate triangle fractal network. We analyze the topological indices (TI) of the silicate triangle fractal network Si_n graph and compare the calculated topological indices to the number of copies of silicate structure (Sio_4) in each iteration of Si_n for a sequence of a graph.

***Keywords*:** Sierpiński triangle network, silicate triangle fractal network (Si_n), fractal dimension, topological indices

MSC: 05C92, 92E10, 28A80

1. Introduction

Fractal geometry is a fascinating field that explores self-similar shapes and patterns that can be observed ubiquitously in nature, including trees, rivers, coastlines, mountains, shells, and hurricanes. In 1975, Mandelbrot proposed fractal geometry as a new geometry of nature and developed it as a modern field of pure and applied mathematics. G. F. Cantor, J. H. Poincare, H. Von Koch, W. Sierpinski and others have contributed their ideas to the development of fractal geometry. Fractal geometry has proven to be a powerful tool for measuring the structures of idealized and naturally occurring phenomena. It is being used in a wide range of scientific fields and art fields [1–3]. Fractals are patterns that repeat constantly with approximate or exact self-similar. In [4], iteration is one of the most prevalent sources of self-similarity. This self-similarity property is the persistent distribution of degree under renormalization or the self-repeating pattern for the network structure. It is common practice to generate recursively specified sets using the iterated function system. By Mandelbrot's definition [5], a fractal is a set whose Hausdorff dimension is strictly greater than its topological dimension. The fractal dimension [6] characterizes the fractal property of the network, which is estimated using the power law relation using the number of boxes and their size. Cantor set, Sierpiński gasket, and Koch curve are well-known self-similar fractals. The Sierpiński gasket [7] is a good illustration of a self-similar fractal lattice. It is not like square or honeycomb lattices, which have translational invariance, but self-similar lattices have scaling invariance.

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Sierpiński gasket was introduced by Sierpiński in 1915 [8], it is known as a typical graph and also the first non-trivial "fractal" type graph and known for its crossing number, see [9]. In [10], the average canonical distance between points on the Sierpiński gasket has been studied by Hinz and Schief. It is used in numerous fields of probability, topology, graph theory, and complex networks. In [11], various network covering algorithms have been reviewed and different dimensions are used to describe the fractal property of networks with their application.

A topological index is a connectivity index and it is a form of molecular descriptors. The topological index is a numerical quantity for molecular graphs in a chemical compound, that is helpful to determine the chemical properties of molecular graphs. These descriptors are amply used in the chemical graph theory of QSAR/QSPR investigations. This investigation applies physicochemical properties, medical drugs, and quantum mechanically calculated parameters as descriptors. More than 200 above topological indices are investigated in QSAR/QSPR investigations. In this investigation, several degree based topological indices give better results.

For few authors discussed degree based topological matrix of eigen values and normalized Laplacian matrix. The author [12] defined upper bound of topological matrix in a simple graph G. In [13], the authors established the energy of graphs of some vertex degree based topological indices (TI) and also related the normalized Laplacian energy with TI.

Particularly, degree based Randic and geometric-arithmetic indices are correlated well with alkane physical properties in [14]. The degree based indices are an effective tool to find a positive correlation of cancer drugs [15]. The authors [16] tested standard heats of formation and normal boiling points of octane isomers at the hand of degree-based topological indices and found augmented Zagreb index and the atom-bond connectivity index yield the best results. In [17], the authors investigated Sombor type degree-based topological indices of graph invariants for antiviral drugs and their correlation of the chemical structure of the molecules with different physicochemical properties such as boiling point, enthalpy of vaporization, critical pressure and volume, molar volume, polarizability. Few authors used the method of topological indices for the fractal networks to find their dimensions. Among many chemical structures, only a few structures one can convert as a fractal network and identify their dimension through topological indices.

For specific complex networks various degree based topological indices have been analyzed by [18]. Various Zagreb indices have been studied for a Sierpiński triangle networks by [19]. Various authors [11, 20, 21] have been discussed the entropy of fractal-type networks, topological properties of Sierpiński network, and the fractal dimension of Sierpiński triangle network respectively. The authors [22] have contributed few degrees based topological indices for the Sierpiński rhombus (SR_n) and Koch snowflake (KS_n) graphs.

In this direction, we are motivated to work firstly on various degree based topological indices of silicate triangle fractal network (Si_n) and compare with the copies of each iteration with the obtained topological indices for some n sequence of a fractal graph.

The structure of this article is as follows: In Section 1.1 the preliminaries needed for the construction of a silicate triangle fractal network (Si_n) and fractal dimension of Si_n are presented. In Section 2, the sequence of topological indices are examined for Si_n. In Section 3, the comparison graphs are discussed for the various topological indices of Si_n. Finally, the conclusion is given in Section 4.

1.1 *Preliminaries*

This section begins with a brief discussion about silicate (Sio_3) and the construction of silicate (Sio_3) triangle fractal network using a sequence of graph. Further iterated function for silicate triangle fractal network (Si_n) is presented.

1.2 *Silicate triangle fractal network (Si$_n$)*

Silicate is prepared by fusing metal carbonates or metal oxides with sand. Silicate (Sio_4) contains silicon and oxygen. It is a tetrahedron structure with a center of silicon atoms and oxygen atoms at the corners in a 2D plane (see Figure 1a) that assembles a silicate network. Silicate material is most commonly used for microchip production. It is widely used in electronic products, telecommunications equipment, architecture, and various chemical networks.

A silicate triangle fractal network (Si_n) whose Si_0 initiator is an equilateral triangle (see Figure 1a) connecting with the center of vertex. It resembles Sierpiński triangle. Corner oxygen atoms of three sides produce 3 silicates of the initiator to get Si_1 (see Figure 1b). In this sequence, each iteration of corner oxygen produces 3^n silicates to form a n^{th} level silicate triangle fractal (Si_n) network (see Figure 2). Si_n contains $3(\frac{3^{n+1}-1}{2})+1$ vertices and $3(3^{n+1})-3$ edges. Consequently $Si_0 \subseteq Si_1$, repeat the process until Si_n sets increase with $Si_0 \subseteq Si_1 \subseteq Si_2 \ldots$. Therefore the construction is repeated an infinite number of times.

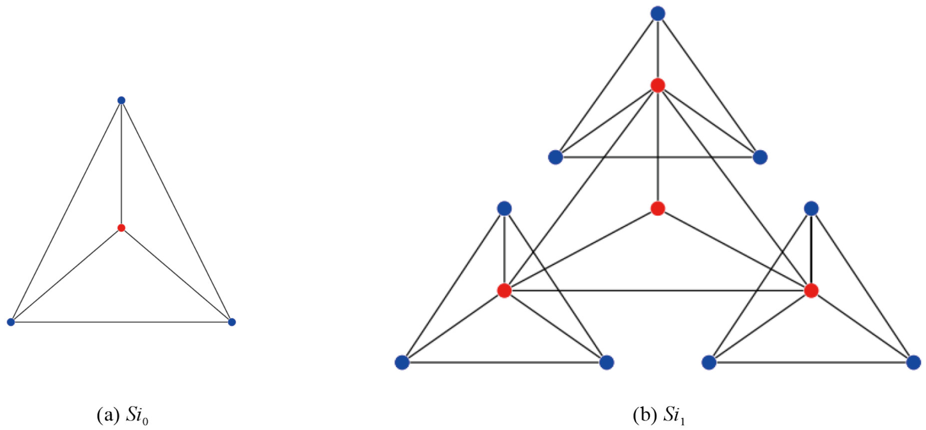

Figure 1. Silicate triangle fractal network (a) intiator Si_0 and (b) first iteration (Si_1)

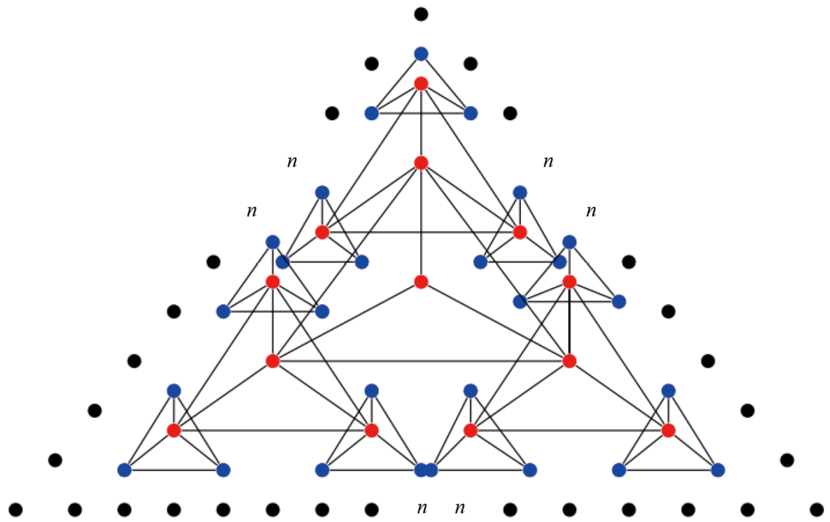

Figure 2. n^{th} iteration of Silicate triangle fractal network Si_n

The fractal dimension (D) is calculated by dividing $logN$ by $log\frac{1}{r}$, where N is the number of boxes intersecting the object and r is the size of the box [23]. Therefore, $D \approx \frac{N}{\frac{1}{r}}$. The fractal dimension of the Sierpiński triangle is $\frac{log3}{log2} \approx 1.585$ and the Sierpiński triangle network is $d_f \approx 1.4556$ [11]. The number of boxes intersecting silicate triangle fractal network is 3 and its size of box is $\frac{1}{2}$ then fractal dimension of silicate triangle fractal network (Si) is estimated by

$$D \approx \frac{N}{\frac{1}{r}} = \frac{3}{2} \approx 1.584$$

Hence, the fractal dimension of Silicate triangle fractal network (Si) is 1.584.

1.3 *Definition of topological indices*

Let G be a graph with vertex set X and edge set Y. The number of edges incident with vertex x is called the degree of vertex x, it is denoted by d_x.

In 1972, Gutman and Trinajstić introduced a first degree index named the first [24] and second Zagreb [25] indices as follows:

$$M_1(G) = \sum_{xy \in Y(G)} (d_x + d_y),$$

$$M_2(G) = \sum_{xy \in Y(G)} (d_x \times d_y).$$

The sum connectivity index ($S(G)$) is proposed by Zhou and Trinajstić [26]

$$S(G) = \sum_{xy \in Y(G)} \frac{1}{\sqrt{d_x + d_y}}.$$

Platt index [27] of a graph G is defined as

$$P(G) = \sum_{xy \in Y(G)} (d_x + d_y - 2).$$

Bollobás et al. [28] and Amić et al. [29] suggested additional information on Randic index and its significant features, it is known as the generalized Randic index

$$R_\alpha(G) = \sum_{xy \in Y(G)} (d_x \times d_y)^\alpha.$$

where α is an arbitrary real number. The Randic index, general Randic index and second Zagreb index are obtained from the generalized Randic index by putting $\alpha = \frac{-1}{2}$, -1 and 1 respectively.

The Milan Randić is developed by Randic ($R_{-1/2}$) index as

$$R_{-1/2}(G) = \sum_{xy \in Y(G)} \frac{1}{\sqrt{d_x \times d_y}},$$

$$R_{-1}(G) = \sum_{xy \in Y(G)} (d_x \times d_y)^{-1},$$

$$R_1(G) = M_2(G) = \sum_{xy \in Y(G)} (d_x \times d_y).$$

Estrada et al. [30] defined the atom bond connectivity (ABC) index, which is

$$ABC(G) = \sum_{xy \in Y(G)} \sqrt{\frac{d_x + d_x - 2}{d_x \times d_y}}.$$

The Harmonic index of G is given by

$$H(G) = \sum_{xy \in Y(G)} \frac{2}{d_x + d_y}.$$

The geometric arithmetic (GA) index is defined by Vukičević et al. [31] as follows

$$GA(G) = \sum_{xy \in Y(G)} \frac{2\sqrt{d_x \times d_y}}{d_x + d_y}.$$

The Forgotten index [32] is defined as

$$F(G) = \sum_{xy \in Y(G)} (d_x^2 + d_y^2).$$

The augmented Zagreb index (AZI) is firstly introduced by Furtula et al. in [33]

$$AZI(G) = \sum_{xy \in Y(G)} \left(\frac{d_x \times d_y}{d_x + d_x - 2}\right)^3.$$

The following is the exponential indices of atom bond connectivity (e^{ABC}) index, second Zagreb index e^{M_2} and Platt index e^P respectively.

$$e^{ABC}(G) = \sum_{xy \in Y(G)} e^{\sqrt{\frac{d_x+d_x-2}{d_x \times d_y}}},$$

$$e^{M_2}(G) = \sum_{xy \in Y(G)} e^{(d_x \times d_y)},$$

$$e^{P}(G) = \sum_{xy \in Y(G)} e^{(d_x+d_y-2)}.$$

2. Topological indices of silicate triangle fractal network (Si_n)

In this section, we computed topological indices of first and second Zagreb index, general Randic index, atom bond connectivity index, Harmonic index, geometric arithmetic index, and forgotten index.

Theorem 1 Let Si_n be the Silicate triangle fractal network. Then

(i) $M_1(Si_n) = 3^{n+3} + 6(3^{n+1}) + 108(3^{n-1}) - 45$,

(ii) $M_2(Si_n) = 27(3^{n+1}) + 324(3^{n-1}) - 162$,

(iii) $S(Si_n) = 0.741(3^{n+1}) - 0.732 + 2.598(3^{n-1})$,

(iv) $R_{-1/2}(Si_n) = 0.569(3^{n+3}) + 1.5(3^{n-1}) - 0.293$,

(v) $R_{-1}(Si_n) = \dfrac{6(3^{n+1}) + 9(3^{n-1})}{36}$,

(vi) $P(Si_n) = 21(3^{n+1}) - 39$,

(vii) $F(Si_n) = 63(3^{n+1}) - 297 + 648(3^{n-1})$,

(viii) $ABC(Si_n) = 1.289(3^{n+1}) - 1.291 + 4.743(3^{n-1})$,

(ix) $GA(Si_n) = 1.941(3^{n+1}) - 3.174 + 9(3^{n-1})$,

(x) $H(Si_n) = 0.556(3^{n+1}) - 0.333 + 1.5(3^{n-1})$,

(xi) $AZI(Si_n) = 28.393(3^{n+1}) - 228.927 + 419.904(3^{n-1})$.

proof. Consider a sequence of silicate triangle fractal network (Si_n) graph. The symbol of X_j and Y_j is assigned to the collection of vertex degrees d_x and edge degrees d_{xy} respectively. The vertex set degrees and edge set degrees of Si_n in each iterations are $X_j = \{3,\ 6\}$ and $Y_j = \{33,\ 36,\ 66\}$ respectively. The cardinality of the vertex sets and edge sets are $3(\dfrac{3^{n+1}-1}{2}) + 1$ and $3(3^{n+1}) - 3$ respectively. The vertex sets are divided into two groups depending on its degrees. For Si_n, we have $|X_3| = 3(3^{n-1}+1)$, $|X_6| = \dfrac{3}{2}[(3^n - 1) - 2(3^{n-1})]$. The edge sets are divided into the corresponding sum of the degrees of end vertices. It is divided by three sets. Y_1 edge set containing end vertices of degree $d_x = 3$ and $d_y = 3$ such that $33 \in Y(Si_n)$, then cardinality of Y_1 is 3^{n+1}. Y_2 edge set contains end vertices of degree $d_x = 3$ and $d_y = 6$ such that $36 \in Y(Si_n)$, then cardinality of Y_2 is $3^{n+1} + 3$. Y_3 edge set contains end vertices of degree $d_x = 6$ and $d_y = 6$ such that $66 \in Y(Si_n)$, then cardinality of Y_3 is $3[1 - 3(1 - 3^{n-1})]$. By definition of M_1 index of Si_n,

$$M_1(Si_n) = \sum_{xy \in Y(Si_n)} (d_x + d_y)$$

$$= \sum_{33 \in Y(Si_n)} (3+3) + \sum_{36 \in Y(Si_n)} (3+6)$$

$$+ \sum_{66 \in Y(Si_n)} (6+6).$$

On simplifying the above form, we obtain the following equation,

$$M_1(Si_n) = 3^{n+3} + 6(3^{n+1}) + 108(3^{n-1}) - 45.$$

By definition, M_2 index of Si_n is computed as follows

$$M_2(Si_n) = \sum_{xy \in Y(Si_n)} (d_x \times d_y)$$

$$= \sum_{33 \in Y(Si_n)} (3 \times 3) + \sum_{36 \in Y(Si_n)} (3 \times 6)$$

$$+ \sum_{66 \in Y(Si_n)} (6 \times 6).$$

After simplifying the above form, we obtain following the equation

$$M_2(Si_n) = 27(3^{n+1}) + 324(3^{n-1}) - 162.$$

By definition, S index of Si_n is computed as follows

$$S(Si_n) = \sum_{xy \in Si_n} \frac{1}{\sqrt{d_x + d_y}}$$

$$= \sum_{33 \in Y(Si_n)} \frac{1}{\sqrt{3+3}} + \sum_{36 \in Y(Si_n)} \frac{1}{\sqrt{3+6}} + \sum_{66 \in Y(Si_n)} \frac{1}{\sqrt{6+6}}.$$

After simplifying the above form, we have the equation

$$S(Si_n) = 0.741(3^{n+1}) - 0.732 + 2.598(3^{n-1}).$$

By definition, general Randic index R_α of Si_n is computed as follows for $\alpha = -1/2$

$$R_{-1/2}(Si_n) = \sum_{xy \in Si_n} \frac{1}{\sqrt{d_x \times d_y}}$$

$$= \sum_{33 \in Y(Si_n)} \frac{1}{\sqrt{3 \times 3}} + \sum_{36 \in Y(Si_n)} \frac{1}{\sqrt{3 \times 6}} + \sum_{66 \in Y(Si_n)} \frac{1}{\sqrt{6 \times 6}}.$$

On simplify the above form, we obtain the following equation,

$$R_{-1/2}(Si_n) = 0.569(3^{n+3}) + 1.5(3^{n-1}) - 0.293.$$

For $\alpha = -1$

$$R_{-1}(Si_n) = \sum_{xy \in Si_n} (d_x \times d_y)^{-1}$$

$$= \sum_{33 \in Y(Si_n)} (3 \times 3)^{-1} + \sum_{36 \in Y(Si_n)} (3 \times 6)^{-1} + \sum_{66 \in Y(Si_n)} (6 \times 6)^{-1}.$$

We simplify the above, we have the following equation

$$R_{-1}(Si_n) = \frac{6(3^{n+1}) + 9(3^{n-1})}{36}.$$

By definition, P index of Si_n is computed as follows

$$P(Si_n) = \sum_{xy \in Si_n} (d_x + d_y - 2)$$

$$= \sum_{33 \in Y(Si_n)} (3 + 3 - 2) + \sum_{36 \in Y(Si_n)} (3 + 6 - 2) + \sum_{66 \in Y(Si_n)} (6 + 6 - 2).$$

We simplify the above form, we obtain following the result

$$P(Si_n) = 21(3^{n+1}) - 39.$$

By definition, F index of Si_n is computed as follows

$$F(Si_n) = \sum_{xy \in Y(G)} (d_x^2 + d_y^2)$$

$$= \sum_{33 \in Y(Si_n)} (3^2 + 3^2) + \sum_{36 \in Y(Si_n)} (3^2 + 6^2) + \sum_{66 \in Y(Si_n)} (6^2 + 6^2).$$

We obtain following the result after simplifying the above form,

$$F(Si_n) = 63(3^{n+1}) - 297 + 648(3^{n-1}).$$

By definition, ABC index of Si_n is computed as follows

$$ABC(Si_n) = \sum_{xy \in Y(G)} \sqrt{\frac{d_x + d_x - 2}{d_x \times d_y}}$$

$$= \sum_{33 \in Y(Si_n)} \sqrt{\frac{3+3-2}{3 \times 3}} + \sum_{36 \in Y(Si_n)} \sqrt{\frac{3+6-2}{3 \times 6}} + \sum_{66 \in Y(Si_n)} \sqrt{\frac{6+6-2}{6 \times 6}}.$$

We obtain following the result after simplifying the above form,

$$ABC(Si_n) = 1.289(3^{n+1}) - 1.291 + 4.743(3^{n-1}).$$

By definition, GA index of Si_n is computed as follows

$$GA(Si_n) = \sum_{xy \in Y(G)} \frac{2\sqrt{d_x \times d_y}}{d_x + d_y}$$

$$= \sum_{33 \in Y(Si_n)} \frac{2\sqrt{3 \times 3}}{3+3} + \sum_{36 \in Y(Si_n)} \frac{2\sqrt{3 \times 6}}{3+6} + \sum_{66 \in Y(Si_n)} \frac{2\sqrt{6 \times 6}}{6+6}.$$

We obtain following the result after simplifying the above form,

$$GA(Si_n) = 1.942(3^{n+1}) - 3.174 + 9(3^{n-1}).$$

By definition, H index of Si_n is computed as follows

$$H(Si_n) = \sum_{xy \in Y(G)} \frac{2}{d_x + d_y}$$

$$= \sum_{33 \in Y(Si_n)} \frac{2}{3+3} + \sum_{36 \in Y(Si_n)} \frac{2}{3+6} + \sum_{66 \in Y(Si_n)} \frac{2}{6+6}.$$

We obtain following the result after simplifying the above form,

$$H(Si_n) = 0.556(3^{n+1}) - 0.333 + 1.5(3^{n-1}).$$

By definition, AZI index of Si_n is computed as follows

$$AZI(Si_n) = \sum_{xy \in Y(G)} (\frac{d_x \times d_y}{d_x + d_x - 2})^3$$

$$= \sum_{33 \in Y(Si_n)} (\frac{3 \times 3}{3+3-2})^3 + \sum_{36 \in Y(Si_n)} (\frac{3 \times 6}{3+6-2})^3 + \sum_{66 \in Y(Si_n)} (\frac{6 \times 6}{6+6-2})^3.$$

We obtain following the result after simplifying the above form,

$$AZI(Si_n) = 28.393(3^{n+1}) - 228.927 + 419.904(3^{n-1}).$$

$\square$

Theorem 2 Let Si_n be the Silicate triangle fractal network. Then,
(i) $e^{ABC}(Si_n) = 3.431(3^{n+1}) + 11.907(3^{n-1}) - 3.507$,
(ii) $e^{M_2}(Si_n) = e^9(3^{n+1}) + 3.e^{18} + 3^{n+1}e^{18} - 6.e^{36} + 3(3^{n-1})e^{36}$,
(iii) $e^P(Si_n) = 1151.23(3^{n+1}) - 128868.87 + 198238.14(3^{n-1})$.

Proof. Consider a sequence of silicate triangle fractal network (Si_n) graphs. The vertex is divided into two groups depending on its degrees. we have $|X_3| = 3(3^{n-1}+1)$, $|X_6| = \frac{3}{2}[(3^n - 1) - 2(3^{n-1})]$. Edge set is divided by three sets. Y_1 edge set containing end vertices degree $d_x = 3$ and $d_y = 3$ such that $33 \in Y(Si_n)$, then cardinality of Y_1 is 3^{n+1}. Y_2 edge set contains end vertices of degree $d_x = 3$ and $d_y = 6$ such that $36 \in Y(Si_n)$, then cardinality of Y_2 is $3^{n+1} + 3$. Y_3 edge set contains end vertices of degree $d_x = 6$ and $d_y = 6$ such that $66 \in Y(Si_n)$, then cardinality of Y_3 is $3[1 - 3(1 - 3^{n-1})]$. By definition, e^{ABC} index of Si_n is computed as follows

$$e^{ABC}(Si_n) = \sum_{xy \in Y(G)} e^{\sqrt{\frac{d_x + d_x - 2}{d_x \times d_y}}}$$

$$= \sum_{33 \in Y(Si_n)} e^{\sqrt{\frac{(3+3-2)}{3 \times 3}}} + \sum_{36 \in Y(Si_n)} e^{\sqrt{\frac{(3+6-2)}{3 \times 6}}} + \sum_{66 \in Y(Si_n)} e^{\sqrt{\frac{(6+6-2)}{6 \times 6}}}.$$

On simplifying the above form, we obtain the following equation,

$$e^{ABC}(Si_n) = 3.431(3^{n+1}) + 11.907(3^{n-1}) - 3.507.$$

By definition, e^{M_2} index of Si_n is computed as follows

$$\begin{aligned}
e^{M_2}(Si_n) &= \sum_{xy \in Y(G)} e^{(d_x \times d_y)} \\
&= \sum_{xy \in Y(Si_n)} (d_x \times d_y) \\
&= \sum_{33 \in Y(Si_n)} e^{(3\times3)} + \sum_{36 \in Y(Si_n)} e^{(3\times6)} + \sum_{66 \in Y(Si_n)} e^{(6\times6)}.
\end{aligned}$$

On simplifying the above form, we obtain the following equation,

$$e^{M_2}(Si_n) = e^9(3^{n+1}) + 3.e^{18} + 3^{n+1}e^{18} - 6.e^{36} + 3(3^{n-1})e^{36}.$$

By definition, e^P index of Si_n is computed as follows

$$\begin{aligned}
e^P(Si_n) &= \sum_{xy \in Y(G)} e^{(d_x + d_y - 2)} \\
&= \sum_{33 \in Y(Si_n)} e^{(3+3-2)} + \sum_{36 \in Y(Si_n)} e^{(3+6-2)} + \sum_{66 \in Y(Si_n)} e^{(6+6-2)}.
\end{aligned}$$

We obtain following the result after simplifying the above form,

$$e^P(Si_n) = 1151.23(3^{n+1}) - 128868.87 + 198238.14(3^{n-1}).$$

□

3. Comparison of topological indices for *Si_n*

The comparison is made between the copies of each iteration of the silicate triangle fractal network (Si_n) with the obtained topological indices (TI) for some sequence n and results are presented in Figure 3, where x-axis represents the number of iterations (n) and y-axis represents the TIs' for silicate triangle fractal network (Si_n) corresponding to the number of iterations n. The obtained degree sum topological indices and exponential degree sum topological indices of the silicate triangle network (Si_n) is plotted in Figure 3 using the Matlab tool The graph shows degree based topological indices and exponential degree based TIs' namely $M_1(Si_n)$, $M_2(Si_n)$, $R_{-1}(Si_n)$, $R_{-1/2}(Si_n)$, $F(Si_n)$, $ABC(Si_n)$, $GA(Si_n)$,

$H(Si_n)$, $S(Si_n)$, $P(Si_n)$ and $e^{M_2}(Si_n)$, $e^{ABC}(Si_n)$, $e^{P}(Si_n)$ respectively. As the number of iteration increases (n), the value of the topological indices also increases accordingly. Increasing topological indices relates copies of silicate structure in each iteration of Si_n network, we have represented a stem graph of obtained degree sum topological indices and exponential degree sum topological indices in Figure 3(a) and 3(b) respectively.

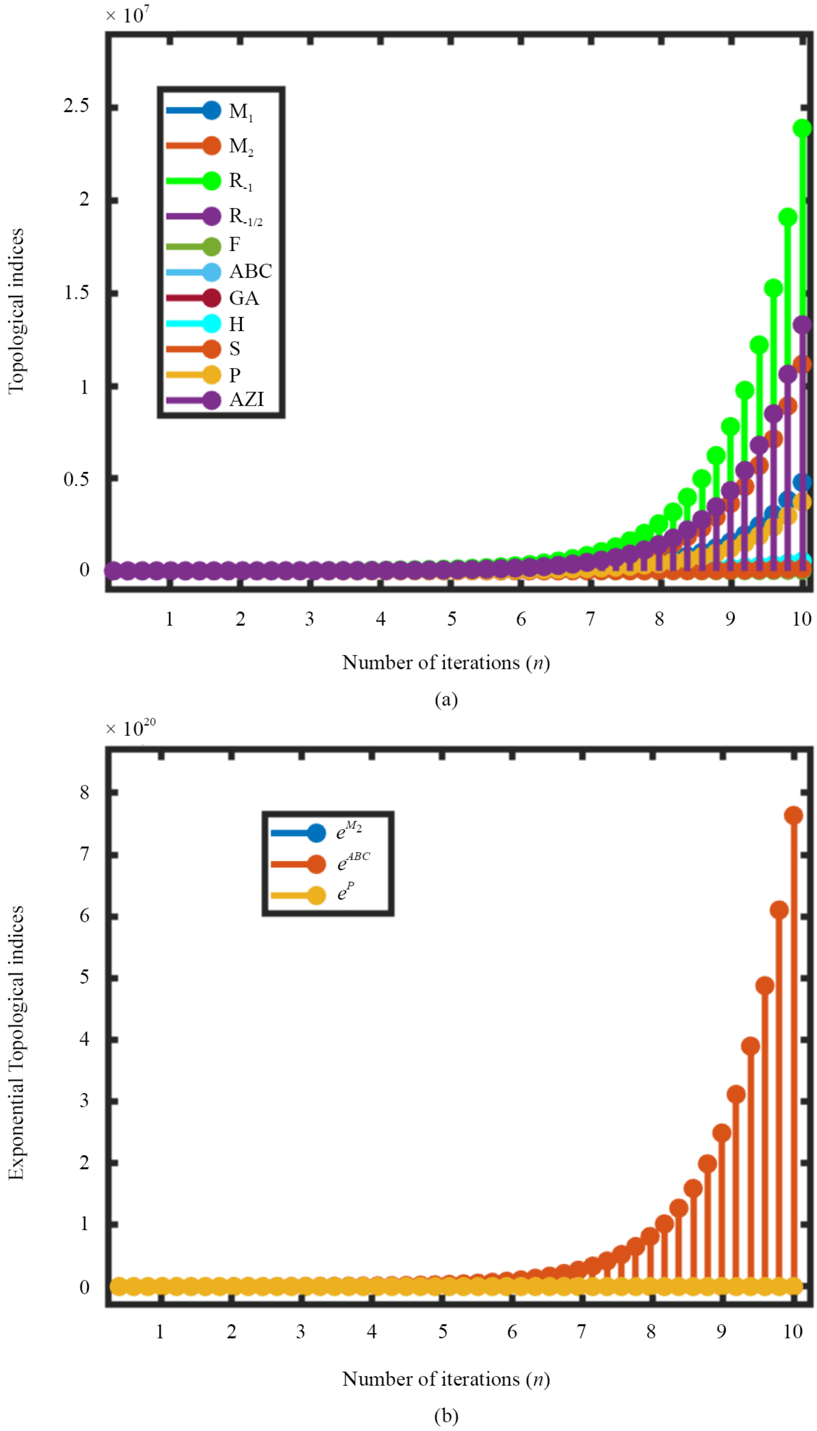

Figure 3. Graphs for (a) degree sum and (b) exponential degree sum

Table 1 shows the calculated values of TIs with 3 silicate copies in each iteration n and $R_{-1}(Si_n)$ index gives 3 copies in the first iteration ($n = 1$) itself while other indices give 3 copies in different iterations n. So, it is successfully proposed that the $R_{-1}(Si_n)$ index gives the fractional dimension of the Si_n more than other indices.

Table 1. Topological indices of 3 silicate copies of Si_n

S.No	Topological indices	3 copies of silicate in iteration n
1.	$M_2(Si_n)$	$n = 14$
2.	$F(Si_n)$	$n = 14$
3.	$S(Si_n)$	$n = 13$
4.	$GA(Si_n)$	$n = 13$
5.	$R_{-1}(Si_n)$	$n = 1$
6.	$AZI(Si_n)$	$n = 14$
7.	$e^{ABC}(Si_n)$	$n = 13$
8.	$e^{P}(Si_n)$	$n = 15$

Further it is noticed that in Figure 3(a) and 3(b), the general Randic index ($R_{-1}(Si_n)$) attains a maximum in Figure 3(a) and exponential atom bond connectivity index ($e^{ABC}(Si_n)$) attains a maximum in Figure 3(b). While comparing all the indices it is observed that those maximum values are closely related to the number of the copies of the silicate structure (Sio_4) in silicate triangle fractal network (Si_n). In [16] augmented Zagreb index and the atom-bond connectivity index yield the best results in standard heats of formation and normal boiling points of octane isomers. But in this paper, from all the obtained topological indices the general Randic index gives ($R_{-1}(Si_n)$) the better approximation with a smaller number of iterations to find the number of copies of silicate structure (Sio_4) and fractal dimension of silicate triangle fractal network (Si_n).

4. Conclusion

In this study, various types of topological indices namely atom-bond connectivity index, geometric arithmetic index, harmonic index, general Randic index, first and second Zagreb index, forgotten index, Platt index, exponential ABC index, exponential second Zagreb index and exponential Platt index are discussed for a sequence of silicate triangle fractal network (Si_n). Further, the comparison of all topological indices with respect to the number of iterations are presented in Figure 3. It is noted that few topological indices give three silicate copies such as the second Zagreb index ($M_2(Si_n)$), Forgotten index ($F(Si_n)$), sum connectivity ($S(Si_n)$) index, geometric arithmetic ($GA(Si_n)$) index, generalized Randic index ($R_\alpha(Si_n)$), and Platt index ($P(Si_n)$) among all the obtained topological indices. However if $\alpha = -1$, the general Randic index gives the first three silicate copies for the graph Si_n.

This paper is an eye-opener for researchers in the sense that the same Si_n network can be derived from neighborhood degree, eigen degree matrix and Laplacian matrix to the number of copies of silicate structure in each iteration, fractal dimension of Si_n. In this sequel, the properties of various topological indices will be investigated for complex fractal networks with large values for n.

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Conflict of interest

The authors declare that there is no conflict of interest.

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