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Research Article

Building Renovation by Using Maximum Edge Distance Method

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Abstract: In this competitive world, it is becoming more difficult to manipulate the entities for project managers and organizers to come up with successful project management strategies. Here, we introduce the Maximum Edge Distance (MED) methodology to determine the adequate solution for finding an intuitionistic fuzzy critical path (IFCP) for building renovation work. This methodology incorporates the use of intuitionistic fuzzy sets, allowing for the consideration of uncertainty and vagueness in the analysis. The Maximum Edge Distance approach involves assigning weights or distances to edges in the project network, optimizing the renovation work schedule by considering factors such as time, resource availability, and uncertainty. We used data from a construction company called Mega Star Technical, which specializes in renovations and building construction at Nigeria. This investigation will use the IFCP. Forward pass and backward pass computations are used for verification. MATLAB is used to manifest the simulation results. Finally, the results of this method demonstrate that the project is likely to be accomplish within the stated time range.

***Keywords*:** acyclic network, critical path problem, optimization, intuitionistic fuzzy triangular number

MSC: 90B10, 90B15, 90B50, 90C06, 90C35

Abbreviation

(ab) or a-b	Activity (a, b) with tail event a and head event b
TE_a	Earliest occurrence time of 'a' event
TL_b	Latest occurrence time of 'b' event
ED_{ab}	Estimated completion duration of (a, b) activity
ES_{ab}	Earliest starting duration of (a, b) activity
EF_{ab}	Earliest finish duration of (a, b) activity
LS_{ab}	Latest starting duration of (a, b) activity
LF_{ab}	Latest finishing duration of (a, b) activity
FS	Fuzzy Set
IFS	Intuitionisitc Fuzzy Set
IFCP	Intuitionistic Fuzzy Critical Path
TFN	Triangular Fuzzy Number
ITFN	Intuitionistic Triangular Fuzzy Number

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1. Introduction

Project management entails the scientific application of current tools and procedures in the planning, scheduling, funding, implementing, auditing, governing, and correlating of specific activities or tasks in order to generate desired outputs within time and cost limitations. Any organization's backbone is its projects. Project management is becoming more popular as a technique since it strives to maximize resource usage. Project management is something that everyone does on a daily basis. When the government wants to create an express highway connecting two major cities over a distance of many kilometers, it is a far larger project than building a house. As a project grows in size, the challenges it faces in terms of planning, scheduling, implementing, managing, and monitoring increase. Systematically created strategies are used to effectively manage larger and more complicated projects. Since traditional management strategies have been proved unable to handle projects efficiently, project management has expanded into its own area of management.

But because of the intricacy of certain projects that arose in the late 1950s, new methods had to be adopted that would be more suitable and efficient. Then, two approaches were able to flourish simultaneously thanks to operations research. The techniques are Project Evaluation and Review Technique (PERT) and Critical Path Method (CPM). PERT was initially developed by the US Navy in 1957 to support the Polaris nuclear submarine project's convoy [1]. In contrast, Morgan R. Walker of DePont and James E. Kelley Jr. of Remington Rand developed the CPM as a project modeling approach [2]. The World Trade Center Twin Towers, a well-known skyscraper at the time, were constructed in New York City using CPM for the first time in 1966. Mauchly Associates adopted CPM's additional benefits. PERT and CPM are primarily time-oriented techniques. The use of time estimates was one of the most notable differences between PERT and CPM. Even though they were deterministic in CPM, the time value was assigned to be probabilistic in PERT [3]. It was well recognized as an important tool for programming and appearance for large groups. Consistently, the project manager may employ the key route to manage cost overruns and enhance the effectiveness of resource allocation to guarantee project quality. Both PERT and CPM would be used to operate various projects which including construction works, roadways, bridges, refineries (including planning and maintenance), and so on. They are also deployed ship maintenance and other major activities, as well as the development of new weapons and the manufacturing of large goods like airplanes and ships. They can also be employed for routine things like home modeling, cleaning, and painting, as well as relocating to a new apartment. Construction, aerospace and defence, software development, research projects, product development, engineering, and plant maintenance are just a few of the industries that use Intuitionistic Fuzzy Critical Path (IFCP) analysis [4]. This type of mathematical analysis can be applied to any project, including interdependent activities. The main objective of this study will be on the use of Maximum edge distance method to find intuitionistic fuzzy critical path to handle Mega Star Technical and Construction Company's renovation/building construction project at 48 Forces Avenue in Port Harcourt, Nigeria [5].

It is censorious for organizers and decision makers to approach initiatives in competing domains without using adequate project management procedures. Project managers must keep an eye out for methods that will enable them to complete projects and do so within a specific time frame [6]. It is very difficult to determine the precise activity time because of data ambiguity and differences in the management structure. Consequently, Lofti Asker Zadeh developed the fuzzy set theory, which is important in this kind of decision-making environment [7]. Many approaches to solving the fuzzy critical path (FCP) problem have been published in the public literature. The initial technique for identifying the optimal path was created by Chanas and Kamburowski [8] and is known as the Fuzzy Program Evaluation and Review Technique (FPERT). While project managers can utilize the Fuzzy Critical Path Method (FCPM) when they have predictable data to determine the critical path, FPERT considers that finding the critical path will take some time. A technique for analyzing the critical route in a project network with octagonal fuzzy numbers was provided by Stephen Dinakar and Rameshan [9]. A novel approach to determining the critical path where the network diagram's imprecise parameters lead to the intuitionistic fuzzy triangular numbers (IFTN) rather than crisp numbers was put out by Balaganesan and Ganesan [10]. A novel technique known as the new JOSE Algorithm discover FCP was proposed by Jose Parvin Praveena et al. Using the alpha cut ranking technique, Euclidean ranking method, and dynamic encoding critical path recursion in terms of triskaidecagonal and Triskaidecagonal fuzzy number, this methodology was developed to find the fuzzy critical path using 13 parameters [11]. A novel analytical technique for identifying the crucial path was proposed by Ravi Shankar Nowpada et al. using a fuzzy project network. By using their newly suggested trapezoidal fuzzy number defuzzification method on the float time of each task, they were able to identify the crucial path [12]. The fuzzy critical route issue has seen the publication of several works. Some of the best are [13–16].

The following is how the paper is framed: The basic definitions of fuzzy set theory are reviewed in Section 2. Section 3 focuses on two different techniques for identifying the intuitionistic fuzzy critical path with a compared to the conventional method (Forward and backward pass computations). The Maximum Edge Distance approach was used to develop a solution for estimating building renovation. An algorithm that was constructed in MATLAB is shown in the simulation result. The findings produced by the suggested methodology and the existing method were discussed in Section 4. Under results and discussions (Forward and backward pass computations). The paper comes to a close with Section 5.

2. Preliminaries

2.1 *Fuzzy set [7]*

The set which has limits that express a degree of membership function in the closed unit interval [0, 1] are known as fuzzy sets. Consider P as a non-empty set, then a fuzzy set X is therefore a set with ordered pairings in the form $X = \{(p,\ \alpha_A(p)) : p \in P\}$ where the membership function is written as $\alpha_x : P \to [0,\ 1]$ and the degree of membership is denoted as $\alpha_x(p)$, for each element $p \in P$.

2.2 *Intuitionisitc fuzzy set [17]*

An intuitionistic fuzzy set X with the form $X = \{(p,\ \alpha_A(p),\ \beta_A(p)) : p \in P\}$ where the function $\alpha_x : P \to [0,\ 1]$ is the degree of membership and $\beta_x : P \to [0,\ 1]$ is the degree of non-membership of the element $p \in P$ to the set X and $p \in P$, $0 \le \alpha_A(p) + \beta_A(p) \le 1$.

2.3 *Fuzzy number [7]*

If the following three criteria are met, then the number is said to be fuzzy number, Piecewise continues, Convex and Normal.

2.4 *Triangular fuzzy number [7]*

A triplet $(p,\ q,\ r;\ 1)$, where $p < q < r$; $p,\ q,\ r \in \Re$ may be used to construct a triangular fuzzy number, X. This is how the membership function $\alpha_x(p)$ is given as follows (Figure 1):

$$\alpha_x(p) = \begin{cases} \dfrac{x-p}{q-p}, & p \le x < q \\ 1, & x = q \\ \dfrac{r-x}{r-q}, & q < x \le r \\ 0, & \text{otherwise} \end{cases}$$

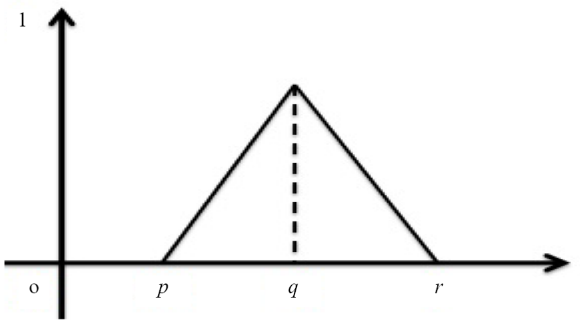

Figure 1. Triangular fuzzy number

2.5 *Intuitionistic triangular fuzzy number [17]*

A triplet $(p', q, r'; 1)$ can be used to define an intuitionistic triangular fuzzy number, $p' < p < q < r < r'$; $p', q, r' \in \Re$. As stated in Definition 2.4, the membership function is similar. The following is the non-membership function $\beta_x(p)$ (Figure 2):

$$\beta_x(p) = \begin{cases} \dfrac{x - \bar{p}}{q - \bar{p}}, & \bar{p} \le x < q \\ 1, & x = q \\ \dfrac{\bar{r} - x}{\bar{r} - q}, & q < x \le \bar{r} \\ 0, & \text{otherwise} \end{cases}$$

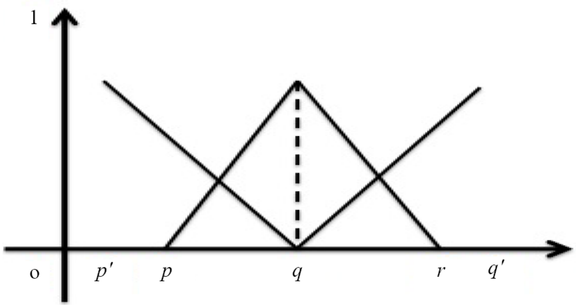

Figure 2. Intuitionisitc triangular fuzzy number

2.6 *Operation on ITFN [17]*

Let $A = (p_1, q_1, r_1)\ (p_1^{'}, q_1, r_1^{'})$ and $B = (p_2, q_2, r_2)\ (p_2^{'}, q_2, r_2^{'})$ be two ITFN, then
$A \oplus B = (p_1 + p_2, q_1 + q_2, r_1 + r_2)\ (p_1^{'} + p_2^{'}, q_1 + q_2, r_1^{'} + r_2^{'})$.
$A \ominus B = (p_1 - p_2, q_1 - q_2, r_1 - r_2)\ (p_1^{'} - p_2^{'}, q_1 - q_2, r_1^{'} - r_2^{'})$.
$L_{max} = \max[(p_1, p_2), (q_1, q_2), (r_1, r_2)], \max[(p_1^{'}, p_2^{'}), (q_1, q_2), (r_1^{'}, r_2^{'})]$.

$L_{min} = \min[(p_1, p_2), (q_1, q_2), (r_1, r_2)], \min[(p_1', p_2'), (q_1, q_2), (r_1', r_2')].$

2.7 *Acyclic network [18]*

A graph with directed edges is referred to as a digraph. Consequently, a directed graph without a cycle is an acyclic digraph.

2.8 *Signed distance measure*

The signed distance measure of $[a, b]$ measured from '0' is defined as $d[a, b] = \frac{1}{2}(a+b)$ for any closed interval $[a, b]$.

2.9 *Traditional forward and backward pass algorithm*

Intuitionistic Fuzzy Critical Path Method (IFCPM) using IFTN

Utilizing the IFCPM for scheduling is as follows:

Step 1 Determine the activities and events that are specific to project:

These are the various jobs or tasks that must be completed in order for the project to be completed. On the other hand, events are a defined point in time that marks the begining and end of one or more actions.

Step 2 Pinpoint the suitable order of the activities:

In this scenario, the appropriate activity sequence must be established. This can be done in addition to the previous point. Significant occupations' sequences are self-evident, while others may necessitate some investigation to determine their right order. The task could be listed in a tabular manner with information on sequence and duration for ease of implementation.

Step 3 Design a network diagram:

A network diagram is a graphical communication and planning tool made up of a number of nodes connected by arcs or arrows. It allows you to optimize your time effectively. Each activity is a node, and the arcs or arrows show how they are connected. The network diagram is created using the information about the activity sequence and duration acquired from the table generated from points 1 and 2 [6].

Step 4 Predict the time to complete each activity:

Although most time predictions are made in weeks, other consistent time units may be considered in rare instances. FCPM is a deterministic data-based model. We can estimate how long the job will take to complete. We are represented as IFTN in this case.

Step 5 Calculate the intuitionistic fuzzy critical path:

The entire schedule time necessary for the project is determined by the critical path. This path is created by adding up the individual times for each activity in the sequence and using that information to create the longest route in the entire project.

The following criteria should be considered while determining the Intuitionistic Fuzzy Critical Path:

• The project's total completion time.

• The earliest possible start time for each activity.

• The latest time that each activity can begin without causing the project to be delayed.

• Float for each activity, i.e. the amount of time that an activity's completion can be postponed without delaying the project's overall completion.

• Evaluate the intuitionistic fuzzy critical activities and IFCP.

The two ways for computing the above mentioned duration: forward pass and backward pass computation.

Forward computation

1. Set the earliest occurrence time of the original event to zero.
2. The earliest activity starting time (a, b) is the earliest event of the tail end event, that is to express $[ES_{ab}] = TE_a$.

3. The sum of the earliest starting time and the activity duration is equal to the earliest finish time of activity $(a,\ b)$. $EF_{ab} = ES_b + ED_{ab}$, in other words.

4. The earliest event time for event b is the sum of all activities ending in that event's earliest finish times. To put it another way, if you're looking for a unique approach to express

$$TE_b = \text{maximum}[\{EF_{ab}\} \text{ for all immediate predecessors } (a,\ b)].$$

Backward pass computation

1. Assume $TE = TL$ for the final event. Take account of all ES were computed using forward pass computation.

2. The most recent finish time for activity $(a,\ b)$ is the same as the most recent event time for event b, $LF_b = TL_b$.

3. The difference between the latest completion time of $(a,\ b)$ and the activity time is equal to the latest starting time activity $(a,\ b)$. $ED_{ab} = LS_{ab} = LF_{ab}$.

4. The minimum of the latest start time of all activities coming from that event is the latest event time for event a. That is to say,

$$L_i = \text{minimum}[\{LS_{ab}\} \text{ for all immediate successors of } (a,\ b)].$$

Computation of float and slack time

The floats are calculated as follows when calculating the earliest and latest occurrence times: The difference between the activity's latest finish and its earliest finish, or the difference between the activity's latest start and its earliest start, is the activity's total float $[Tf]$.

$$Tf = Tf_{ab} = LS_{ab} - ES_{ab} \text{ or } Tf_{ab} = LF_{ab} - EF_{ab}.$$

Because it is concerned with the whole project length, this is the most essential form of float. The total float of an activity is the amount of time that it can be delayed without influencing the project's overall duration.

In this paper we referred Mega Star Technical and Construction Company Limited is a Nigerian civil engineering firm with a license to operate. The case study was based on data from Renovaworks for renovation/building construction project at 48 Forces Avenue in Port Harcourt, Rivers State, Nigeria (which took place from August to December 2017). To begin, the raw data was plotted and analyzed for project management purposes, as mentioned in Table 1. They used PERT method to find the critical path [3].

Table 1. Duration of project

Activity	Description	Predecessors	Duration (days)
1	Main building	0	(75, 89, 91) (70, 89, 95)
2	Start	1	(0, 0, 0) (0, 0, 0)
3	Demolition works	2	(8, 10, 12) (6, 10, 14)
4	New wall construction	3	(12, 14, 16) (11, 14, 17)
5	New roof trusses	4	(8, 9, 11) (7, 9, 13)
6	New roof covering	5	(4, 7, 9) (3, 7, 10)
7	Mep conduit piping	6	(10, 15, 17) (9, 15, 20)
8	Doors and windows	9	(8, 11, 13) (7, 11, 14)
9	Plastering (Internal)	6	(9, 14, 16) (8, 14, 17)

Table 1. (cont.)

Activity	Description	Predecessors	Duration (days)
10	Plastering (External)	9	(10, 14, 16) (9, 14, 17)
11	Handrails/Burglary bars	9	(8, 10, 12) (7, 10, 13)
12	Electrical cabling	10	(10, 12, 14) (9, 12, 15)
13	Ceiling works	12	(11, 14, 16) (10, 14, 17)
14	Floor screening/Tile	13	(9, 12, 14) (8, 12, 15)
15	Door and window panels	14	(3, 7, 9) (2, 7, 10)
16	Electrical/plumbing	14	(7, 10, 12) (5, 10, 13)
17	A/c installation	16	(4, 7, 9) (3, 7, 10)
18	Internal painting	17	(6, 10, 12) (5, 10, 13)
19	External painting	18	(5, 7, 9) (4, 7, 10)
20	End of main building	19	(0, 0, 0) (0, 0, 0)
21	Boys quarter	1	(40, 60, 64) (39, 60, 63)
22	Demolition and alteration	4	(3, 5, 7) (2, 5, 8)
23	Substructure	22	(4, 7, 9) (3, 7, 10)
24	New block walls	23	(4, 7, 9) (3, 7, 10)
25	Roof trusses	24	(3, 7, 9) (2, 7, 10)
26	Roof covering	25	(2, 3, 5) (1, 3, 6)
27	Mep conduit piping	26	(2, 4, 6) (1, 4, 7)
28	Door and window	27	(1, 2, 4) (0, 2, 5)
29	Plastering (Internal)	28	(2, 3, 5) (1, 3, 6)
30	Plastering (External)	29	(2, 3, 4) (1, 3, 5)
31	Electrical cabling	30	(3, 4, 5) (2, 4, 6)
32	Ceiling works	31	(2, 3, 5) (1, 3, 7)
33	Floor screening	32	(2, 3, 5) (1, 3, 7)
34	Door and window panels	33	(3, 4, 5) (2, 4, 7)
35	Electrical/plumbing	34	(1, 3, 5) (0, 3, 7)
36	Internal painting	35	(2, 3, 5) (1, 3, 6)
37	External painting	36	(1, 2, 4) (0, 2, 5)
38	Gate house	1	(22, 32, 34) (21, 32, 35)
39	Demolition and alteration	27	(1, 2, 4) (0, 2, 5)
40	Substructure	39	(3, 4, 6) (2, 4, 7)
41	New block walls	40	(2, 3, 5) (1, 3, 6)
42	Roof trusses	41	(2, 3, 5) (1, 3, 6)
43	Roof covering	42	(1, 2, 4) (0, 2, 5)
44	Mep conduit piping	43	(1, 2, 4) (0, 2, 5)
45	Door and window	44	(1, 2, 4) (0, 2, 5)
46	Plastering (Internal)	45	(1, 2, 4) (0, 2, 5)
47	Plastering (External)	46	(1, 2, 3) (0, 2, 5)
48	Electrical cabling	47	(1, 2, 4) (0, 2, 5)
49	Ceiling works	48	(1, 2, 4) (0, 2, 5)
50	Floor screening	49	(1, 2, 3) (0, 2, 4)
51	Door and window panels	50	(1, 2, 3) (0, 2, 4)
52	Electrical/plumbing	51	(1, 2, 4) (0, 2, 5)
53	Internal painting	52	(1, 2, 4) (0, 2, 5)
54	External painting	53	(1, 3, 5) (0, 3, 6)
55	End of gate house	54	(0, 0, 0) (0, 0, 0)

3. Proposed algorithm

Theorem 3.1 If (m, n, o) (m', n', o') be the IFTN then magnitude measure is defined as,

$$Mag = \frac{m+7n+o}{12}, \frac{2m'+5n'+2o'}{12}.$$

Proof. Let $S^{\gamma} = \{[L(\gamma), R(\gamma)], [L'(\gamma), R'(\gamma)]\}$ be a IFTN, then the signed distance of $[L(\gamma), R(\gamma)]$ which is measured from zero is defined as $d([L(\gamma), R(\gamma)], 0) = \frac{1}{2}[L(\gamma), R(\gamma)]$ [2]

Left measure $L(\gamma)$,

$$\frac{x-m}{n-m} = \gamma$$

$$x - m = \gamma(n-m)$$

$$x = m + (n-m)\gamma \tag{1}$$

Right measure $R(\gamma)$,

$$\frac{o-x}{o-n} = \gamma$$

$$o - x = \gamma(o-n)$$

$$x = o - (n-m)\gamma \tag{2}$$

From equation (1) and (2),

$$S^{\gamma} = [L(\gamma), R(\gamma)] = [m + (n-m)\gamma, o - \gamma(o-n)]$$

$$mag = \int_1^0 \frac{1}{2}[L(\gamma) + R(\gamma) + n] \times (\gamma) d\gamma$$

$$= \int_1^0 \frac{1}{2}[m + (n-m)\gamma + o - \gamma(o-n) + n] \times (\gamma) d\gamma$$

$$= \frac{1}{2}\int_1^0 [m\gamma + n\gamma^2 - m\gamma^2 + o\gamma - o\gamma^2 + n\gamma^2 + n\gamma] d\gamma$$

After intergrating and applying the limit we get,

$$mag = \frac{m+7n+o}{12}$$

Left measure $L^{'}(\gamma)$,

$$\frac{m^{'}-x}{n-m^{'}} = \gamma$$

$$m^{'}-x = \gamma(n-m^{'})$$

$$x = m^{'}+\gamma(n-m^{'}) \tag{3}$$

Right measure $R^{'}(\gamma)$,

$$\frac{x-n}{o^{'}-n} = \gamma$$

$$x-n = \gamma(o^{'}-n)$$

$$x = n+\gamma(o^{'}-n) \tag{4}$$

From equation (3) and (4),

$$S^{'\gamma} = [L^{'}(\gamma),\ R^{'}(\gamma)] = [m^{'}+\gamma(n-m^{'}),\ n+\gamma(o^{'}-n)]$$

$$mag^{'} = \int_{1}^{0}\frac{1}{2}[L^{'}(\gamma)+R^{'}(\gamma)+n]\times(\gamma)d\gamma$$

$$= \int_{1}^{0}\frac{1}{2}[m^{'}+\gamma(n-m^{'}), n+\gamma(o^{'}-n)]\times(\gamma)d\gamma$$

$$= \frac{1}{2}\int_{1}^{0}[m^{'}\gamma+n\gamma^{2}-m^{'}\gamma^{2}+n\gamma+o^{'}\gamma^{2}-n\gamma^{2}+n\gamma]d\gamma$$

After intergrating and applying the limit we get,

$$mag^{'} = \frac{m^{'}+5n+2o^{'}}{12}$$

Magnitude measure for intuitionistic fuzzy is,

$$Mag = \left\{ \frac{m+7n+o}{12}, \frac{m^{'}+5n+2o^{'}}{12} \right\}$$

3.1 *Maximum Edge Distance (MED) algorithm for IFCPM using IFTN*

(i) Forward procedure to calculate the IFCP

Step 1 Place all the vertices in Q = priority queue $(1, 2, \cdots\cdots, n-1, n)$.

Step 2 Choose $s = u = 1$, choose the source node as permanent node. Set $EL[u] = (0, 0, 0)$ $(0, 0, 0)$.

Step 3 Extract the maximum edge distance.

For all $v \in Adj[u]$ that is for all edges emerging from u, calculate the following:

1. If u is incident to only one node v then, $EV[v] = EL[u] \oplus d[u, v]$ using Definition 2.3.

2. If u is incident to more than one node v then, $EL[v] = \max_{v \in s}[(EL[u] \oplus d[u, v])]$ using Definition 2.3.

The new permanent node = v. Now, form the new priority queue by removing the source node $s = u = 1$ and the other nodes adjacent to u which are different from v.

Repeat Step 3, until the permanent node = t. If so, terminate the execution of the algorithm.

Step 4 The intuitionistic fuzzy distance along the intuitionistic fuzzy critical path P namely intuitionistic fuzzy critical path length is denoted by $D(P)$ and is defined as $D(P) = \sum_{(u, v) \in P} l_{uv}$, where l_{uv} is the path length. It is calculated using Definition 2.6 and the corresponding path is the IFCP.

(ii) Backward procedure to calculate the IFCP

Step 1 Place all the vertices in Q = priority queue $(n, n-1, \cdots\cdots, 2, 1)$.

Step 2 Choose $t = u = n$, that is choose the destination node as permanent node. Set $EL[u] = (0, 0, 0)$.

Step 3 Extract the maximum edge distance.

For all $v \in Adj[u]$ that is for all edges incident on u, calculate the following:

1. If u is incident to only one node v then, $EL[u] = EL[v] \oplus d[u, v]$ using Definition 2.3.

2. If u is incident to more than one node v then, $EL[u] = max_{v \in s}[(EL[v] \oplus d[u, v])]$ using Definition 2.3.

The new permanent node = v. Now remove the destination node $u = t$ from the priority queue and the other nodes incident to u other than v.

Repeat Step 3, until the permanent node = s. If so, terminate the execution of the algorithm.

Step 4 Calculate the edge distance by using Step 4 as given in forward procedure to calculate the IFCP.

4. Results

The result section contain the real time example of proposed algorithm and its solution. Verification using traditional forward and backward pass computation (2.1). The defuzzification result is given using Theorem 3.1 for the purpose of verification. Simulation result for proposed algorithm using MATLAB.

Numerical computation

Using Definition 2.4, generate an acyclic network for the data in Table 1 which is displayed in Figure 3.

Figure 3 shows the network diagram for the data given in Table 1 using MATLAB.

Consider Figure 3 to find the IFCPM, Backward procedure of an algorithm 3.1 to calculate the IFCPM will not work here, since the edges incident to the destination node as the same path length. Hence we apply forward procedure of an algorithm 3.1.

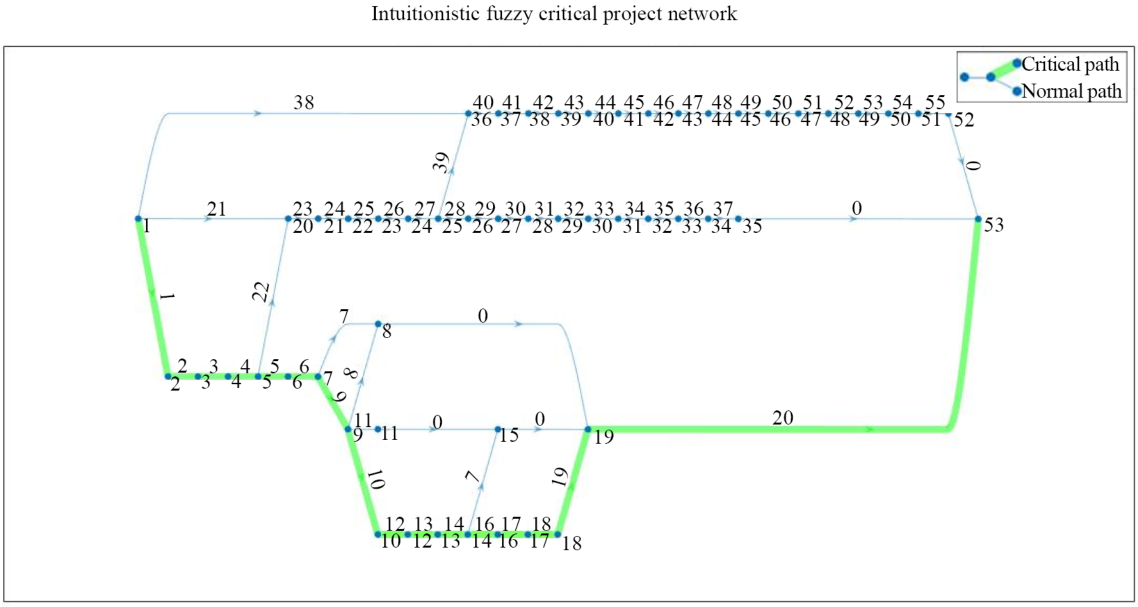

Figure 3. Intuitionistic fuzzy critical project network

Step 1 $Q =$ priority queue (1, 2, 3, 4, 5, 6, 7, $\cdots\cdots$, 55).
Step 2 $S = u = 1$ (source node) $EL[1] =$ (0, 0, and 0) (0, 0, and 0).
Step 3 $2 \in Adj[1]$, $20 \in Adj[1]$ and $36 \in Adj[1]$
$EL[2] = EL[1] \oplus d[1, 2]$ = (0, 0, 0) (0, 0, 0) + (75, 89, 91) (70, 89, 95) = (75, 89, 91) (70, 89, 95).
$EL[20] = EL[1] \oplus d[1, 20]$ = (0, 0, 0) (0, 0, 0) + (40, 60, 62) (39, 60, 63) = (40, 60, 62) (39, 60, 63).
$EL[36] = EL[1] \oplus d[1, 36]$ = (0, 0, 0) (0, 0, 0) + (24, 32, 34) (22, 32, 35) = (24, 32, 34) (22, 32, 35).
Max$(EL[1] \oplus d[1, 2])$, $(EL[1] \oplus d[1, 20])$, $(EL[1] \oplus d[1, 36])$ = (75, 89, 91) (70, 89, 95) = $EL[2]$.
The new permanent node = 2.
Remove source node 1, node 20 and node 36 from the priority queue.
New priority queue is $Q =$ Priority queue (3, 4, 5, $\cdots\cdots$, 19, 21, 22, $\cdots\cdots$, 35, 37, 38, $\cdots\cdots$, 55).
$3 \in Adj[2]$
$EL[3] = EL[2] \oplus d[2, 3]$ = (75, 89, 91) (70, 89, 95) + (0, 0, 0) (0, 0, 0) = (75, 89, 91) (70, 89, 95).
Remove source node 2 from the priority queue.
The new permanent node = 3.
New priority queue is $Q =$ Priority queue (3, 4, 5, $\cdots\cdots$, 19, 21, 22, $\cdots\cdots$, 35, 37, 38, $\cdots\cdots$, 55).
$4 \in Adj[3]$
$EL[4] = EL[3] \oplus d[3, 4]$ = (75, 89, 91) (70, 89, 95) + (8, 10, 12) (6, 10, 14) = (83, 99, 103) (76, 99, 109).
The new permanent node = 4.
Remove source node 3 from the priority queue.
New priority queue is $Q =$ Priority queue (4, 5, $\cdots\cdots$, 19, 21, 22, $\cdots\cdots$, 35, 37, 38, $\cdots\cdots$, 55).
$5 \in Adj[4]$, $21 \in Adj[4]$
$EL[5] = EL[4] \oplus d[4, 5]$ = (83, 99, 103) (76, 99, 109) + (12, 14, 16) (11, 14, 17) = (95, 113, 119) (87, 113, 126).
$EL[21] = EL[4] \oplus d[4, 5]$ = (83, 99, 103) (76, 99, 109) + (3, 5, 7) (2, 5, 8) = (86, 104, 110) (78, 104, 117).
Max$(EL[5],\ EL[21])$ = (95, 113, 119) (87, 113, 126), (86, 104, 110) (78, 104, 117).
Max$(EL[5],\ EL[21])$ = (95, 113, 119) (87, 113, 126) = $EL[5]$.
The new permanent node = 5.
Remove source node 4 and 21 from the priority queue.
New priority queue is $Q =$ Priority queue (5, 6, $\cdots\cdots$, 19, 22, $\cdots\cdots$, 35, 37, 38, $\cdots\cdots$, 55).

$6 \in Adj[5]$
$EL[6] = EL[5] \oplus d[5, 6]$ = (95, 113, 119) (87, 113, 126) + (8, 9, 11) (7, 9, 13) = (103, 122, 130) (94, 122, 139).
The new permanent node = 6.
Remove source node 5 from the priority queue.
New priority queue is $Q =$ Priority queue (6, 7, $\cdots\cdots$, 19, 22, $\cdots\cdots$, 35, 37, 38, $\cdots\cdots$, 55).
$7 \in Adj[6]$
$EL[7] = EL[6] \oplus d[6, 7]$ = (103, 122, 130) (94, 122, 139) + (4, 7, 9) (3, 7, 10) = (107, 129, 139) (97, 129, 149).
The new permanent node = 7.
Remove source node 6 from the priority queue.
New priority queue is $Q =$ Priority queue (7, 8, $\cdots\cdots$, 19, 22, $\cdots\cdots$, 35, 37, 38, $\cdots\cdots$, 55).
Repeat the Step 3, till the permanent node reaches the destination node.
The critical path is 1-2-3-4-5-6-9-10-12-13-14-15-16-17-18-19-20.
Verification
Intuitionistic fuzzy critical path method:
Using the Method 2.1, from the Table 2, it shows that the path 1-2-3-4-5-6-9-10-12-13-14-15-16-17-18-19-20 is the intuitionistic fuzzy critical path.

Table 2. Result of project using Method 2.1

Activity	Duration	EFT	LFT	TF
1	(75, 89, 91) (70, 89, 95)	(75, 89, 91) (70, 89, 95)	(75, 89, 91) (70, 89, 95)	0
2	(0, 0, 0) (0, 0, 0)	(75, 89, 91) (70, 89, 95)	(75, 89, 91) (70, 89, 95)	0
3	(8, 10, 12) (6, 10, 14)	(83, 99, 103) (76, 99, 109)	(83, 99, 103) (76, 99, 109)	0
4	(12, 14, 16) (11, 14, 17)	(95, 113, 119) (87, 113, 126)	(95, 113, 119) (87, 113, 126)	0
5	(8, 9, 11) (7, 9, 13)	(103, 122, 130) (94, 122, 139)	(103, 122, 130) (94, 122, 139)	0
6	(4, 7, 9) (3, 7, 10)	(107, 129, 139) (97, 129, 149)	(107, 129, 139) (97, 129, 149)	0
7	(10, 15, 17) (9, 15, 20)	(117, 114, 156) (106, 114, 169)	(178, 229, 257) (158, 229, 276)	(61, 85, 101) (52, 85, 107)
8	(8, 11, 13) (7, 11, 14)	(124, 154, 168) (112, 154, 180)	(178, 229, 257) (158, 229, 276)	(54, 75, 89) (36, 75, 76)
9	(9, 14, 16) (8, 14, 17)	(116, 143, 155) (105, 143, 166)	(116, 143, 155) (105, 143, 166)	0
10	(10, 14, 16) (9, 14, 17)	(126, 157, 171) (114, 151, 183)	(126, 157, 171) (114, 157, 183)	0
11	(8, 10, 12) (7, 10, 13)	(124, 153, 167) (112, 153, 179)	(178, 229, 257) (158, 229, 276)	(54, 76, 90) (46, 76, 97)
12	(10, 12, 14) (9, 12, 15)	(136, 169, 185) (123, 169, 198)	(136, 169, 185) (123, 169, 198)	0
13	(11, 14, 16) (10, 14, 17)	(147, 183, 201) (133, 183, 215)	(147, 183, 201) (133, 183, 215)	0
14	(9, 12, 14) (8, 12, 15)	(156, 195, 215) (141, 195, 230)	(156, 195, 215) (141, 195, 230)	0
15	(3, 7, 9) (2, 7, 10)	(159, 202, 224) (143, 202, 240)	(178, 229, 257) (158, 229, 276)	(19, 27, 33) (15, 27, 36)
16	(7, 10, 12) (5, 10, 13)	(163, 205, 227) (146, 205, 243)	(163, 205, 227) (146, 205, 243)	0
17	(4, 7, 9) (3, 7, 10)	(167, 212, 236) (149, 212, 253)	(167, 212, 236) (149, 212, 253)	0
18	(6, 10, 12) (5, 10, 13)	(173, 222, 248) (154, 222, 266)	(173, 222, 248) (154, 222, 266)	0
19	(5, 7, 9) (4, 7, 10)	(178, 229, 257) (158, 229, 276)	(178, 229, 257) (158, 229, 276)	0
20	(0, 0, 0) (0, 0, 0)	(178, 229, 257) (158, 229, 276)	(178, 229, 257) (158, 229, 276)	0
21	(40, 60, 64) (39, 60, 63)	(40, 60, 62) (39, 60, 63)	(143, 164, 163) (144, 164, 150)	(103, 104, 101) (105, 104, 87)
22	(3, 5, 7) (2, 5, 8)	(86, 104, 110) (78, 104, 117)	(143, 164, 163) (144, 164, 150)	(57, 60, 53) (66, 60, 33)
23	(4, 7, 9) (3, 7, 10)	(90, 111, 119) (81, 111, 127)	(147, 171, 172) (147, 171, 106)	(57, 60, 53) (66, 60, 33)
24	(4, 7, 10) (3, 7, 10)	(94, 118, 128) (84, 118, 137)	(151, 178, 181) (150, 178, 170)	(57, 60, 53) (66, 60, 33)
25	(3, 7, 9) (2, 7, 10)	(97, 125, 137) (87, 125, 147)	(154, 185, 190) (152, 185, 180)	(57, 60, 53) (66, 60, 33)
26	(2, 3, 5) (1, 3, 6)	(99, 128, 142) (88, 128, 153)	(156, 188, 195) (153, 188, 186)	(57, 60, 53) (66, 60, 33)
27	(2, 4, 6) (1, 4, 7)	(101, 132, 148) (89, 132, 160)	(158, 192, 191) (154, 192, 193)	(57, 60, 53) (66, 60, 33)
28	(1, 2, 4) (0, 2, 5)	(102, 34, 152) (89, 134, 165)	(159, 194, 195) (154, 194, 198)	(57, 60, 53) (66, 60, 33)
29	(2, 3, 5) (1, 3, 6)	(104, 137, 157) (90, 137, 171)	(160, 204, 211) (149, 204, 216)	(56, 67, 62) (59, 67, 45)

Table 2. (cont.)

Activity	Duration	EFT	LFT	TF
30	(2, 3, 4) (1, 3, 5)	(106, 140, 161) (91, 140, 176)	(162, 207, 223) (150, 207, 221)	(56, 67, 62) (59, 67, 45)
31	(3, 4, 5) (2, 4, 6)	(109, 144, 166) (93, 144, 182)	(165, 211, 228) (152, 211, 237)	(56, 67, 62) (59, 67, 55)
32	(2, 3, 5) (1, 3, 7)	(112, 147, 171) (95, 147, 189)	(168, 214, 233) (154, 214, 244)	(56, 67, 62) (59, 67, 55)
33	(2, 3, 5) (1, 3, 7)	(115, 115, 176) (97, 150, 196)	(171, 217, 238) (155, 217, 251)	(56, 67, 62) (58, 67, 55)
34	(3, 4, 5) (2, 4, 7)	(118, 154, 181) (99, 154, 103)	(174, 221, 243) (157, 224, 265)	(56, 67, 62) (58, 67, 55)
35	(1, 3, 5) (0, 3, 7)	(119, 157, 186) (99, 157, 110)	(175, 224, 248) (157, 224, 265)	(56, 67, 62) (58, 67, 55)
36	(2, 3, 5) (1, 3, 6)	(121, 160, 191) (100, 160, 116)	(177, 227, 253) (158, 227, 271)	(56, 67, 62) (58, 67, 55)
37	(1, 2, 4) (0, 2, 5)	(122, 162, 195) (100, 162, 121)	(178, 229, 257) (158, 229, 256)	(56, 67, 62) (58, 67, 55)
38	(22, 32, 34) (21, 32, 35)	(22, 32, 34) (22, 32, 35)	(159, 194, 195) (154, 194, 198)	(137, 162, 161) (132, 162, 163)
39	(1, 2, 4) (0, 2, 5)	(102, 134, 152) (89, 134, 165)	(159, 194, 195) (154, 194, 198)	(57, 60, 43) (65, 60, 334)
40	(3, 4, 6) (2, 4, 7)	(105, 138, 158) (91, 138, 172)	(162, 198, 201) (156, 198, 205)	(57, 60, 43) (65, 60, 33)
41	(2, 3, 5) (1, 3, 6)	(107, 141, 164) (92, 141, 178)	(164, 201, 206) (157, 201, 211)	(57, 60, 42) (65, 60, 33)
42	(2, 3, 5) (1, 3, 6)	(109, 144, 169) (93, 144, 185)	(166, 204, 211) (158, 204, 217)	(57, 60, 42) (65, 60, 33)
43	(1, 2, 4) (0, 2, 5)	(110, 146, 173) (93, 146, 190)	(167.206, 215) (158, 206, 222)	(57, 60, 42) (65, 60, 33)
44	(1, 2, 4) (0, 2, 5)	(111, 148, 177) (93, 148, 195)	(168, 208, 219) (158, 208, 227)	(57, 60, 42) (65, 60, 33)
45	(1, 2, 4) (0, 2, 5)	(112, 150, 181) (93, 150, 200)	(169, 210, 223) (158, 210, 232)	(57, 60, 42) (65, 60, 33)
46	(1, 2, 4) (0, 2, 5)	(113, 152, 185) (93, 152, 205)	(170, 212, 227) (158, 212, 237)	(57, 60, 42) (65, 60, 33)
47	(1, 2, 3) (0, 2, 5)	(114, 154, 188) (193, 154, 210)	(117, 214, 230) (158, 214, 242)	(57, 60, 42) (65, 60, 33)
48	(1, 2, 4) (0, 2, 5)	(115, 156, 192) (93, 156, 215)	(172, 216, 234) (158, 216, 247)	(57, 60, 42) (65, 60, 33)
49	(1, 2, 4) (0, 2, 5)	(116, 158, 196) (93, 158, 220)	(173, 218, 238) (158, 218, 252)	(57, 60, 42) (65, 60, 33)
50	(1, 2, 3) (0, 2, 4)	(117, 160, 199) (93, 160, 224)	(174, 220, 241) (158, 220, 156)	(57, 60, 42) (65, 60, 33)
51	(1, 2, 3) (0, 2, 4)	(118, 162, 202) (93, 162, 228)	(175, 222, 244) (158, 222, 260)	(57, 60, 42) (65, 60, 33)
52	(1, 2, 4) (0, 2, 5)	(119, 164, 206) (93, 164, 233)	(176, 224, 248) (158, 224, 265)	(57, 60, 42) (65, 60, 33)
53	(1, 2, 4) (0, 2, 5)	(120, 166, 210) (93, 166, 238)	(177, 226, 252) (158, 226, 270)	(57, 60, 42) (65, 60, 33)
54	(1, 3, 5) (0, 3, 6)	(121, 169, 215) (93, 169, 238)	(178, 229, 257) (158, 229, 276)	(57, 60, 42) (65, 60, 33)
55	(0, 0, 0) (0, 0, 0)	(121, 169, 215) (93, 169, 144)	(178, 229, 257) (158, 229, 276)	(57, 60, 42) (65, 60, 33)

Using Theorem 3.1, the path length are shown and the path length is defuzzified using magnitude mesure formula which is derived in Theorem 3.1.

Table 3. Result for defuzzification using Theorem 3.1

Path length	Magnitude measure	Ranking
(117, 144, 156) (106, 144, 169)	(106.75, 105.83)	7
(124, 154, 168) (112, 154, 180)	(114.167, 112.83)	5
(178, 229, 257) (158, 229, 276)	**(169.83, 167.75)**	**1**
(159, 202, 224) (143, 202, 240)	(149.75, 148)	2
(124, 153, 167) (112, 153, 179)	(113.5, 112.25)	6
(76, 118, 147) (60, 118, 167)	(87.4167, 87)	9
(122, 162, 195) (99, 162, 221)	(120.917, 120.833)	4
(75, 125, 166) (53, 125, 189)	(93, 92.4167)	8
(43, 67, 96) (26, 67, 113)	(50.67, 51.08)	10
(121, 169, 214) (92, 169, 243)	(126.5, 126.25)	3

From Table 3, the intuitionistic fuzzy critical path is 1-2-3-4-5-6-9-10-12-13-14-15-16-17-18-19-20.

Simulation result using MATLAB

Using MED algorithm the code was developed and the result is displayed in Figure 4 and 5. Figure 5 shows the bar chart for path and path length. From Figure 5 it is clear that path 3 1-2-3-4-5-6-9-10-12-13-14-15-16-17-18-19-20 is identified as intuitionistic fuzzy critical path.

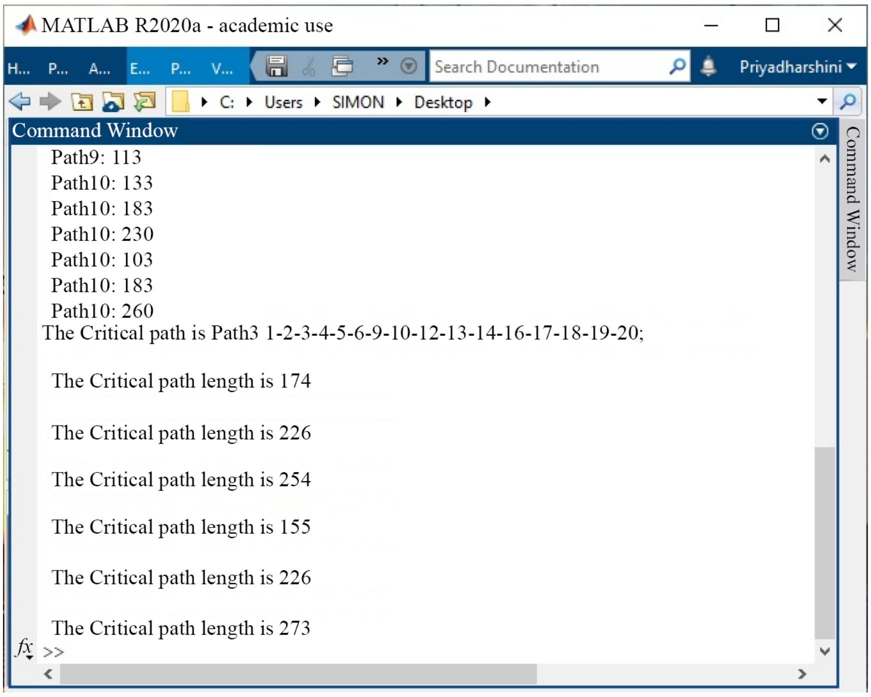

Figure 4. Result of MED using MATLAB

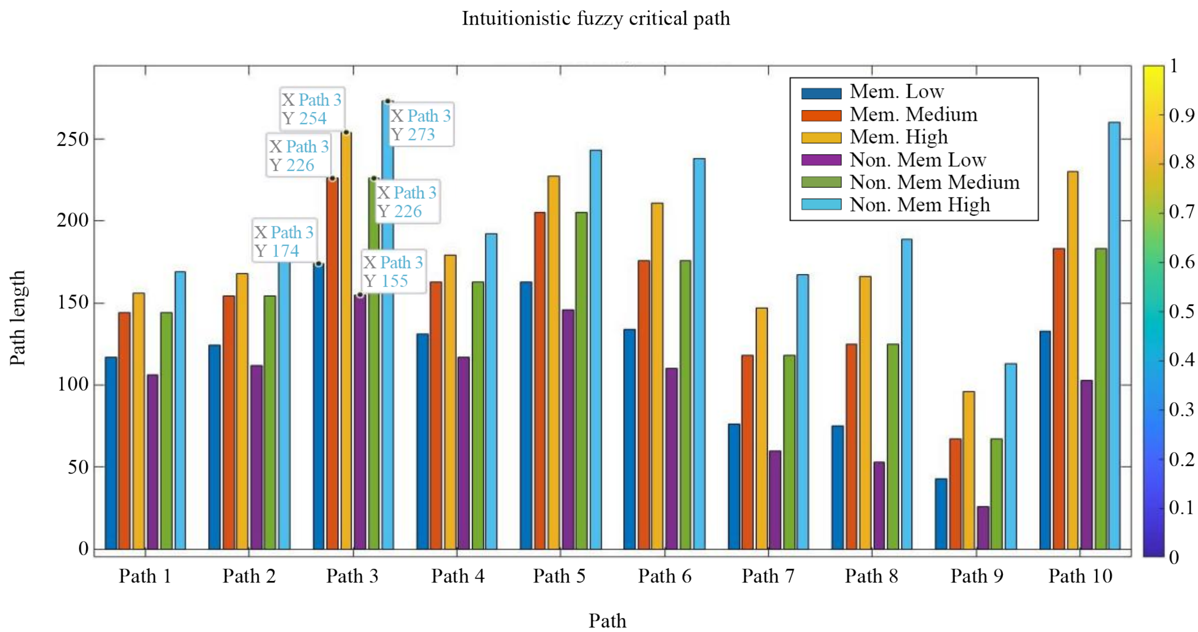

Figure 5. Result of path and path length

5. Discussions

The method proposed in this paper is an alternative way to identify the intuitionistic fuzzy critical path in the fuzzy environment. This method has turned down the iteration. The approximation of the project can be done effortlessly through this "Maximum edge distance" method. The completion time of the project given by this method will be optimized at its best as shown in the solution illustrated in numerical computation. Obviously, this method reduces the time consumption when compared to the regular methods used already. Comparison was done between traditional forward and backward pass calculations and Maximum Edge Distance Method, which is displayed in Table 2 and 3. It is found that the result obtained in this paper, coincides with the result obtained through the existing methods, that is in both the traditional forward and backward pass computation (Intuitionistic fuzzy Critical Path Method (2.1)), Defuzzification (Theorem 3.1) and Maximum Edge Distance Method. The path P3: 1-2-3-4-5-6-9-10-12-13-14-15-16-17-18-19-20 is identified as the intuitionistic fuzzy critical path. Simulation result using MATLAB is shown in Figure 4 and 5.

6. Conclusion

The method put forward in this paper is an alternative way to identify the critical path in the fuzzy environment. The contribution is, a new method is propose a different algorithm and ranking technique to find the optimal path in a intuitionistic fuzzy acyclic network with its edge weights as IFTN. The recurrence has been reduced using this strategy. The approximation of the project can be done facile through the "Maximum edge distance" method. Verification is done using ranking technique and tradition intuitionistic fuzzy critical path Method. Defuzzification is done using magnitude measure, which results in the same critical path. Concluding with the result obtained through this method, the time consumed is much effectively reduced when compared to the existing methods.The limitation of this study is, if the edge incident to succeeding node has same path length then this method has some complexity to slove the problem. In future we may try to find case studies more and different methodology to find critical path in cyclic network. We try to extend this method for time cost trade off problem. The scope of this study extends to projects of different sizes and complexities, making it a valuable technique in the field of project management.

Conflict of interest

The authors declare no conflicts of interest.

References

[1] Jack L, Okeke OC, Okechukwu SI, Akinola AO. Project management: A system approach to planning, implementation, monitoring and evaluation. *International Journal of Advanced Academic Research: Social & Management Sciences*. 2016; 2(11): 65-79.

[2] Kelley J, Walker M. Critical-path planning and scheduling. In *1959 Proceedings of the Eastern Joint Computer Conference*. Boston, Massachusetts: The National Joint Computer Committee; 1959. p.160-173.

[3] Lin FT. A shortest path network problem in a fuzzy environment. In *10th IEEE International Conference on Fuzzy Systems*. Melbourne, VIC, Australia: IEEE; 2001. p.1096-1100.

[4] Atanassov K, Sotirova E, Andonov V. Generalized net model of multicriteria decision making procedure using intercriteria analysis. In: Kacprzyk J, Szmidt E, Zadrożny S, Atanassov K, Krawczak M. (eds.) *Advances in Fuzzy Logic and Technology*. Cham: Springer International Publishing; 2018. p.99-111.

[5] Nsikan PA, Gregory OA. Modelling building renovation using PERT. *Asian Research Journal of Mathematics*. 2020; 16(4): 25-38.

[6] Elizabeth S, Sujatha L, Bharathi K. On searching critical path in a project network under fuzzy environment. *National Conference on Recent Approaches of Mathematics to Science and Technology*. 2015.

[7] Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8: 338-353.

[8] Chanas S, Kamburowski J. The use of fuzzy variables in PERT. *Fuzzy Sets and Systems*. 1981; 5: 11-19.

[9] Stephen Dinagar D, Rameshan N. A new approach for fuzzy critical path method using octogonal fuzzy numbers. *International Journal of Pure and Applied Mathematics*. 2018; 119(13): 357-364.

[10] Balaganesan M, Ganesan K. An approach to find critical path in intuitionistic fuzzy environment. *International Journal of Pure and Applied Mathematics*. 2018; 119(9): 395-403.

[11] Jose Parvin Praveena N, Sagaya Nathan Stalin C, Rajkumar A. Critical path problem through intuitionistic triskaidecagonal fuzzy number using two different algorithms. In: Chiplunkar NN, Fukao T. (eds.) *Advances in Artificial Intelligence and Data Engineering*. Singapore: Springer Nature Singapore; 2021. p.159-167.

[12] Ravi Shankar N, Sireesha V, Phani Bushan Rao P. An analytical method for finding critical path in a fuzzy project network. *International Journal of Contemporary Mathematical Sciences*. 2010; 5(20): 953-962.

[13] Anushya V, Balasowandari P. Fuzzy critical path with area measure. *International Journal of Pure and Applied Mathematics*. 2018; 118(6): 167-173.

[14] Elizabeth S, Sujatha L. Project scheduling method using triangular intuitionistic fuzzy numbers and triangular fuzzy numbers. *Applied Mathematical Sciences*. 2015; 9(4): 185-198.

[15] Salimian S, Mousavi SM, Tupenaite L, Antucheviciene J. An integrated multi-criteria decision model to select sustainable construction projects under intuitionistic fuzzy conditions. *Buildings*. 2023; 13(4): 848.

[16] Sireesha V, Ravishankar N. A new approach to find total float time and critical path in a fuzzy project network. *International Journal of Engineering Science and Technology*. 2010; 2(4): 600-609.

[17] Atanassov K. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*. 1986; 20(1): 87-96. Available from: http://dx.doi.org/10.1016/s0165-0114(86)80034-3.

[18] Blue M, Bush B, Puckett J. Unified approach to fuzzy graph problems. *Fuzzy Sets and Systems*. 2002; 125: 355-368.