Research Article

Highly Dispersive Optical Gap Solitons with Kundu-Eckhaus Equation Having Multiplicative White Noise


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Abstract: This paper explores the dynamics of highly dispersive gap solitons within the framework of the Kundu-Eckhaus equation, augmented by the inclusion of multiplicative white noise in the Itô sense, a novel addition to the model. Two integration schemes, the extended simplest equation approach and the generalized Riccati equation mapping scheme, are employed to analyze and integrate the model. Despite yielding bright, singular, and dark-singular straddled solitons, both methods independently fail to capture dark optical solitons. Additionally, our investigation highlights that the influence of white noise predominantly affects the phase aspect of the solitons, with negligible impact on their amplitude. Further details regarding the specific physical system or optical medium under study would provide readers with context, aiding in the comprehension of the significance of our findings.

Keywords: gap solitons, Bragg gratings, Kundu-Eckhaus

MSC: 78A60

1. Introduction

The concept of highly dispersive optical solitons made its debut a couple of years ago. This concept was conceived with the absolute need to maintain the delicate balance between chromatic dispersion (CD) and self-phase modulation (SPM) for the soliton propagation to sustain for inter-continental distances. Later, this concept was applied to optical
white noise is modeled for the first time as:

\[ q \]

where

\[ i \alpha_1 q_x + b_1 q_t + \left[ \lambda_1 \left( |q|^2 \right)_x + \sigma \left( q - ib_1 r_x \right) \right] \frac{dW(t)}{dt} = 0, \quad (1) \]

and

\[ i \beta_1 q_x + b_2 q_t + \left[ \lambda_2 \left( |r|^2 \right)_x + \sigma \left( r - ib_2 q_x \right) \right] \frac{dW(t)}{dt} = 0, \quad (2) \]

where \([q(x, t)]\) and \([r(x, t)]\) are complex-valued functions that represent the wave profiles and \([q^*(x, t)]\) and \([r^*(x, t)]\) are their complex-conjugate, \(i = \sqrt{-1}\). The first term is the linear temporal evolution. The constants \(a_{k,j}(k = 1, 2, j = 1 - 6)\) are the coefficients inter-modal dispersion (IMD), chromatic dispersion (CD), third order dispersion (3OD), fourth order dispersion (4OD), fifth order dispersion (5OD) and sixth order dispersion (6OD), respectively. The constants \(b_{j}(j = 1, 2)\) are the coefficients of STD. The constants \(\xi_j, \lambda_j (j = 1, 2)\) are the coefficients of self-phase modulation (SPM), while the constants \(\eta_j, \xi_j, \theta_j (j = 1, 2)\) are the coefficients of cross-phase modulation (XPM). The constants \(\alpha_j, \beta_j, \delta_j (j = 1, 2)\) are the coefficients of noise strength and \(W(t)\) is the standard Wiener process, such that \(dW(t)/dt\) is the white noise.

This article’s primary goal is to use the extended simplest equation approach, and the generalized Riccati equation mapping strategy to locate the bright, singular, straddled dark-singular soliton solutions of Equations (1) and (2).

This article’s structure may be expressed as follows: Section 3 introduces the mathematical preliminaries. We derive the solutions to systems (1) and (2) in Sections 4 and 5. Section 6 concludes the work with a few words on the future plan.
2. Mathematical preliminaries

To analyze the model, we postulate assumption:

\[ q(x, t) = H_1(\xi) \exp \left[ -\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t \right], \]

\[ r(x, t) = H_2(\xi) \exp \left[ -\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t \right], \]

and

\[ \xi = x - Vt, \]

where \( \kappa, \Omega, \theta \) and \( V \) are nonzero real-valued constants which represent frequency of the soliton, wave number, phase constant and soliton velocity, respectively. The functions \( H_1(\xi) \) and \( H_2(\xi) \) are real functions which represent the amplitude portion of the soliton and the phase component of the soliton, respectively. Inserting (3) and (4) into Eqs. (1) and (2) gives the real parts:

\[ a_{16} H_2^{(6)}(\xi) + (a_{14} + 5\kappa a_{15} - 15\kappa^2 a_{16}) H_2^{(4)}(\xi) \]

\[ + \left( a_{12} + 3a_{13}\kappa - 6a_{14}\kappa^2 - 10a_{15}\kappa^3 + 15a_{16}\kappa^4 - b_1 V \right) H_2''(\xi) + (\kappa\alpha_1 - \Omega + \sigma^2) H_1(\xi) \]

\[ + \left[ \beta_1 + a_{11}\kappa - a_{12}\kappa^2 - a_{13}\kappa^3 + a_{14}\kappa^4 + a_{15}\kappa^5 - a_{16}\kappa^6 + b_1 \kappa (\Omega - \sigma^2) \right] H_2(\xi) + \zeta_1 H_1^2(\xi) + \eta_1 H_1^2(\xi) H_2^2(\xi) + \delta_1 H_1(\xi) H_2^2(\xi) = 0, \]

(5)

\[ a_{26} H_1^{(6)}(\xi) + (a_{24} + 5\kappa a_{25} - 15\kappa^2 a_{26}) H_1^{(4)}(\xi) \]

\[ + \left( a_{22} + 3a_{23}\kappa - 6a_{24}\kappa^2 - 10a_{25}\kappa^3 + 15a_{26}\kappa^4 - b_2 V \right) H_1''(\xi) + (\kappa\alpha_2 - \Omega + \sigma^2) H_2(\xi) \]

\[ + \left[ \beta_2 + a_{21}\kappa - a_{22}\kappa^2 - a_{23}\kappa^3 + a_{24}\kappa^4 + a_{25}\kappa^5 - a_{26}\kappa^6 + b_2 \kappa (\Omega - \sigma^2) \right] H_1(\xi) + \zeta_2 H_2^2(\xi) + \eta_2 H_2^2(\xi) H_1^2(\xi) \]

\[ + \delta_2 H_2(\xi) H_1^2(\xi) + 2\lambda_2 H_2^2(\xi) H_1'(\xi) + 2\theta_2 H_2(\xi) H_1(\xi) H_1'(\xi) + \delta_2 H_2(\xi) H_1^2(\xi) = 0, \]

(6)

and the imaginary parts are
(a_1 - V)H'_1(\xi) + \left[ a_{11} - 2a_{12} \kappa - 3a_{13} \kappa^2 + 4a_{14} \kappa^3 + 5a_{15} \kappa^4 - 6a_{16} \kappa^5 + b_1 (\Omega - \sigma^2) + b_1 kV \right] H'_2(\xi)

+ (a_{13} - 4a_{14} \kappa - 10a_{15} \kappa^2 + 20a_{16} \kappa^3) H''_2(\xi) + (a_{15} - 6a_{16} \kappa) H''_2(\xi) = 0, \quad (7)

(a_2 - V)H'_2(\xi) + \left[ a_{21} - 2a_{22} \kappa - 3a_{23} \kappa^2 + 4a_{24} \kappa^3 + 5a_{25} \kappa^4 - 6a_{26} \kappa^5 + b_2 (\Omega - \sigma^2) + b_2 kV \right] H'_1(\xi)

+ (a_{23} - 4a_{24} \kappa - 10a_{25} \kappa^2 + 20a_{26} \kappa^3) H'''_1(\xi) + (a_{25} - 6a_{26} \kappa) H_1''(\xi) = 0, \quad (8)

where \( r = \frac{d}{d\xi}, m = \frac{d^2}{d\xi^2} \), \( a = \frac{d^3}{d\xi^3}, \) \( b = \frac{d^4}{d\xi^4} \) and \( c = \frac{d^5}{d\xi^5} \).

To recover the integer balancing number used in the given integration methods in the current paper, we set

\[ H_2(\xi) = \mu H_1(\xi), \quad (9) \]

where \( \mu \) is a non zero constant, such that \( \mu \neq 1 \). Now, Eqs. (5)-(8) become

\[
a_{16}\mu H''_1(\xi) + \mu \left[ a_{14} + 5\kappa a_{15} - 15\kappa^2 a_{16} \right] H^{(4)}_1(\xi)

+ \mu \left( a_{12} + 3a_{13} \kappa - 6a_{14} \kappa^2 - 10a_{15} \kappa^3 + 15a_{16} \kappa^4 - b_1 V \right) H''_1(\xi)

+ \mu \left[ b_1 + (a_{11} + a_1) \kappa - a_{12} \kappa^2 - a_{13} \kappa^3 + a_{14} \kappa^4 + a_{15} \kappa^5 - a_{16} \kappa^6 - (1 - b_1 \kappa) (\Omega - \sigma^2) \right] H_1(\xi)

+ (\xi_1 + \eta_1 \mu^2 + \xi_2 \mu^4) H''_1(\xi) + 2 (\lambda_1 + \theta_1 \mu^2) H''''_1(\xi) + \delta_1 \mu^2 H''''_1(\xi) = 0, \quad (10)
\]

\[
a_{26}H''_1(\xi) + \left( a_{24} + 5\kappa a_{25} - 15\kappa^2 a_{26} \right) H^{(4)}_1(\xi)

+ \left[ a_{22} + 3a_{23} \kappa - 6a_{24} \kappa^2 - 10a_{25} \kappa^3 + 15a_{26} \kappa^4 - b_2 V \right] H''_1(\xi)

+ \left[ b_2 + (a_{21} + \mu a_2) \kappa - a_{22} \kappa^2 - a_{23} \kappa^3 + a_{24} \kappa^4 + a_{25} \kappa^5 - a_{26} \kappa^6 - (\mu - b_2 \kappa) (\Omega - \sigma^2) \right] H_1(\xi)

+ \mu \left( \mu^3 \xi_1 + \mu^2 \eta_2 + \xi_2 \right) H''''_1(\xi) + 2 \mu (\lambda_2 \mu^2 + \theta_2) H''''_1(\xi) + \delta_2 \mu H''''_1(\xi) = 0, \quad (11)
\]

and
Setting the coefficients of the linearly independent functions of Eqs. (14) and (15) to zero, yields

\[
\left[ \alpha_1 - V + \mu \left( a_{11} - 2a_{12} \kappa - 3a_{13} \kappa^2 + 4a_{14} \kappa^3 + 5a_{15} \kappa^4 - 6a_{16} \kappa^5 + b_1 (\Omega - \sigma^2) + b_1 \kappa V \right) \right] H'_1(\xi)
\]

\[
+ \mu \left( a_{13} - 4a_{14} \kappa - 10a_{15} \kappa^2 + 20a_{16} \kappa^3 \right) H''_1(\xi) + \mu \left( a_{15} - 6a_{16} \kappa \right) H^{(5)}_1(\xi) = 0,
\] (12)

\[
\left[ (\alpha_2 - V) \mu + a_{21} - 2a_{22} \kappa - 3a_{23} \kappa^2 + 4a_{24} \kappa^3 + 5a_{25} \kappa^4 - 6a_{26} \kappa^5 + b_2 (\Omega - \sigma^2) + b_2 \kappa V \right] H'_1(\xi)
\]

\[
+ \left( a_{23} - 4a_{24} \kappa - 10a_{25} \kappa^2 + 20a_{26} \kappa^3 \right) H''_1(\xi) + (a_{25} - 6a_{26} \kappa) H^{(5)}_1(\xi) = 0.
\] (13)

Integrating Eqs. (12) and (13) with zero-integration constants, we have

\[
\left[ \alpha_1 - V (1 - b_1 \kappa) + \mu \left( a_{11} - 2a_{12} \kappa - 3a_{13} \kappa^2 + 4a_{14} \kappa^3 + 5a_{15} \kappa^4 - 6a_{16} \kappa^5 + b_1 (\Omega - \sigma^2) \right) \right] H_1(\xi)
\]

\[
+ \mu \left( a_{13} - 4a_{14} \kappa - 10a_{15} \kappa^2 + 20a_{16} \kappa^3 \right) H''_1(\xi) + \mu \left( a_{15} - 6a_{16} \kappa \right) H^{(4)}_1(\xi) = 0,
\] (14)

\[
\left[ \alpha_2 \mu - (\mu - b_2 \kappa) V + a_{21} - 2a_{22} \kappa - 3a_{23} \kappa^2 + 4a_{24} \kappa^3 + 5a_{25} \kappa^4 - 6a_{26} \kappa^5 + b_2 (\Omega - \sigma^2) \right] H_1(\xi)
\]

\[
+ \left( a_{23} - 4a_{24} \kappa - 10a_{25} \kappa^2 + 20a_{26} \kappa^3 \right) H''_1(\xi) + (a_{25} - 6a_{26} \kappa) H^{(4)}_1(\xi) = 0.
\] (15)

Setting the coefficients of the linearly independent functions of Eqs. (14) and (15) to zero, yields

\[
\kappa = \frac{a_{j5}}{6a_{j6}}, \quad j = 1, 2,
\] (16)

and

\[
V = \frac{\alpha_1 + \mu \left[ a_{11} - 2a_{12} \kappa - 3a_{13} \kappa^2 + 4a_{14} \kappa^3 + 5a_{15} \kappa^4 - 6a_{16} \kappa^5 + b_1 (\Omega - \sigma^2) \right]}{1 - b_1 \kappa},
\] (17)

\[
V = \frac{\alpha_2 \mu + a_{21} - 2a_{22} \kappa - 3a_{23} \kappa^2 + 4a_{24} \kappa^3 + 5a_{25} \kappa^4 - 6a_{26} \kappa^5 + b_2 (\Omega - \sigma^2)}{\mu - b_2 \kappa},
\] (18)

and the constraints conditions

\[
a_{j3} - 4a_{j4} \kappa - 10a_{j5} \kappa^2 + 20a_{j6} \kappa^3 = 0, \quad j = 1, 2,
\] (19)

provided \(a_{j5} \neq 0, \ a_{j6} \neq 0, \ (j = 1, 2), \ b_1 \kappa \neq 1, \ b_2 \kappa \neq \mu\). Eqs. (10) and (11) are equivalent under the constraint conditions:
\(a_{16}\mu = a_{26},\) \hspace{1cm} (20)
\[\delta_1\mu = \delta_2,\] \hspace{1cm} (21)
\[\lambda_1 + \theta_1\mu^2 = \mu (\lambda_2\mu^2 + \theta_2),\] \hspace{1cm} (22)
\[\xi_1 + \eta_1\mu^2 + \nu_1\mu^4 = \mu (\mu^4\xi_2 + \mu^2\eta_2 + \nu_2),\] \hspace{1cm} (23)
\[\mu (a_{14} + 5\kappa a_{15} - 15\kappa^2 a_{16}) = (a_{24} + 5\kappa a_{25} - 15\kappa^2 a_{26}),\] \hspace{1cm} (24)
\[\mu (a_{12} + 3a_{13}\kappa - 6a_{14}\kappa^2 - 10a_{15}\kappa^3 + 15a_{16}\kappa^4 - b_1 V)\]
\[= (a_{22} + 3a_{23}\kappa - 6a_{24}\kappa^2 - 10a_{25}\kappa^3 + 15a_{26}\kappa^4 - b_2 V),\] \hspace{1cm} (25)
\[\mu \left[ \beta_1 + (a_{11} + \alpha_1) - a_{12}\kappa^2 - a_{13}\kappa^3 + a_{14}\kappa^4 + a_{15}\kappa^5 - a_{16}\kappa^6 - (1 - b_1) (\Omega - \sigma^2) \right] \]
\[= \beta_2 + (a_{21} + \mu \alpha_2) - a_{22}\kappa^2 - a_{23}\kappa^3 + a_{24}\kappa^4 + a_{25}\kappa^5 - a_{26}\kappa^6 - (1 - b_2) (\Omega - \sigma^2).\] \hspace{1cm} (26)

From (26), we have the wave number of the soliton:
\[
\Omega = \frac{[a_{21} - \mu (a_{11} + \alpha_1 - \alpha_2)] + (\mu a_{16} - a_{26}) \kappa^6 + (a_{25} - \mu a_{15}) \kappa^5 + (a_{24} - \mu a_{14}) \kappa^4 + (a_{13} \mu - a_{23}) \kappa^3 + (\mu a_{12} - a_{22}) \kappa^2 + \beta_2 - \mu \beta_1 + \sigma^2 \kappa (\mu b_1 - b_2)}{\kappa (\mu b_1 - b_2)},
\] \hspace{1cm} (27)
provided \(\mu b_1 \neq b_2\). From (17) and (18), we deduce that
\[
\sigma^2 = \frac{A (\mu - b_2 \kappa) - B (1 - b_1 \kappa) + \Omega [\mu b_1 (\mu - b_2 \kappa) - b_2 (1 - b_1 \kappa)]}{\mu b_1 (\mu - b_2 \kappa) - b_2 (1 - b_1 \kappa)},
\] \hspace{1cm} (28)
where
\[
A = \alpha_1 + \mu \left[a_{11} - 2a_{12}\kappa - 3a_{13}\kappa^2 + 4a_{14}\kappa^3 + 5a_{15}\kappa^4 - 6a_{16}\kappa^5\right],
\]
\[
B = \alpha_2 \mu + a_{21} - 2a_{22}\kappa - 3a_{23}\kappa^2 + 4a_{24}\kappa^3 + 5a_{25}\kappa^4 - 6a_{26}\kappa^5.
\] \hspace{1cm} (29)

Now, Eq. (10) can be rewritten in the form
\[ H_1^{(6)}(\xi) + L_1 H_1^{(4)}(\xi) + L_2 H_1^{(2)}(\xi) + 2L_3 H_1(\xi) H_1'(\xi) + L_4 H_1(\xi) + L_5 H_1'(\xi) + L_6 H_1''(\xi) = 0, \] (30)

where

\[ L_1 = \frac{a_{14} + 5\kappa a_{15} - 15\kappa^2 a_{16}}{a_{16}}, \]

\[ L_2 = \frac{a_{12} + 3a_{13}\kappa - 6a_{14}\kappa^2 - 10a_{15}\kappa^3 + 15a_{16}\kappa^4 - b_1 V}{a_{16}}, \]

\[ L_3 = \frac{\lambda_1 + \theta_1 \mu^2}{a_{16} \mu}, \]

\[ L_4 = \frac{\beta_1 + (a_{11} + \alpha_1) \kappa - a_{12} \kappa^2 - a_{13} \kappa^3 + a_{14} \kappa^4 + a_{15} \kappa^5 - a_{16} \kappa^6 - (1 - b_1) (\Omega - \sigma^2)}{a_{16}}, \]

\[ L_5 = \frac{\delta_1 \mu}{a_{16}}, \]

\[ L_6 = \frac{\xi_1 + \eta_1 \mu^2 + \zeta_1 \mu^4}{a_{16} \mu}. \] (31)

Now, balancing the terms \( H_1^{(6)}(\xi) \) and \( H_1'(\xi) \) in Eq. (30) yields the balance number \( N = \frac{3}{4} \). Thus, we take the transformation:

\[ H_1(\xi) = Z^2(\xi), \] (32)

where \( Z(\xi) \) is a new positive function of \( \xi \). Next, Eq. (30) changes to:

\[
\begin{align*}
Z^5(\xi)Z^{(6)}(\xi) + 3Z^4(\xi)Z'(\xi)Z^{(5)}(\xi) - \frac{15}{4} \left[ Z^2(\xi) - 2Z(\xi)Z'(\xi) \right] & \left[ Z^3(\xi)Z^{(4)}(\xi) - 2Z^2(\xi)Z'(\xi)Z''(\xi) \right] \\
+ 5Z^4(\xi)Z''^2(\xi) - \frac{15}{4} Z^3(\xi)Z''^2(\xi) + \frac{135}{8} Z^2(\xi)Z^2(\xi)Z''(\xi) + \frac{105}{32} Z^6(\xi) - \frac{225}{16} Z(\xi)Z^6(\xi)Z''(\xi) \\
+ \frac{L_1}{8} \left[ 8Z^3(\xi)Z^{(4)}(\xi) + 16Z^2(\xi)Z'(\xi)Z''(\xi) + 3Z^4(\xi) + 12 \left( Z(\xi)Z''(\xi) - Z^2(\xi) \right) Z(\xi)Z''(\xi) \right] Z^2(\xi) \\
+ \frac{L_2}{2} \left[ Z^2(\xi) + 2Z(\xi)Z''(\xi) \right] Z^4(\xi) + 2L_3Z^2(\xi)Z'(\xi) + \frac{2}{3} L_4 Z^6(\xi) + \frac{2}{3} L_5 Z^6(\xi) + \frac{2}{3} L_6 Z^{12}(\xi) & = 0.
\end{align*}
\] (33)
In Eq. (33), we balance \( Z^5(\xi) Z^{10}(\xi) \) and \( Z^{12}(\xi) \) produces the equilibrium number \( N = 1 \). Eq. (33) will be solved using the following two integration techniques in the next sections.

### 3. Extended simplest equation approach

The formal solution to Equation (33) is [6–8]:

\[
Z(\xi) = \chi_0 + \chi_1 \left[ \frac{\Phi'(\xi)}{\Phi(\xi)} \right] + \rho_0 \left[ \frac{1}{\Phi(\xi)} \right],
\]

and \( \Phi(\xi) \) is the solution of the equation

\[
\Phi''(\xi) + \tau \Phi(\xi) = \nu,
\]

where \( \tau, \nu, \chi_0, \chi_1 \) and \( \rho_0 \) are variables, such \( \chi_1^2 + \rho_0^2 \neq 0 \).

For \( \tau < 0 \), we switch (34) with (33) and apply Eq. (35) with the following relation

\[
\left( \frac{\Phi'(\xi)}{\Phi(\xi)} \right)^2 = T_1 \left( \frac{1}{\Phi(\xi)} \right)^2 - \tau + \frac{2\nu}{\Phi(\xi)},
\]

where \( T_1 = \tau \left( W_1^2 - W_2^2 \right) - \frac{L_6}{\tau} \), while \( W_1 \) and \( W_2 \) are parameters, allows for results

\[
\chi_0 = 0, \chi_1 = 0, \rho_0 = \frac{1}{2} \left[ -\frac{117855\tau^3 \left( W_1^2 - W_2^2 \right)^3}{L_6} \right]^{\frac{1}{2}}, \nu = 0,
\]

and

\[
L_1 = \frac{3247}{84} \tau, L_2 = \frac{135679}{336} \tau^2, L_3 = 0, L_4 = \frac{45873}{64} \tau^3, L_5 = 0,
\]

provided \( (W_1^2 - W_2^2) L_6 > 0 \). Consequently, we obtain bright-singular straddled optical solitons as:

\[
q(x, t) = \left\{ \frac{1}{2} \left[ -\frac{117855\tau^3 \left( W_1^2 - W_2^2 \right)^3}{L_6} \right]^{\frac{1}{2}} \left[ \frac{1}{W_1 \cosh \left( \sqrt{-\tau} \xi \right) + W_2 \sinh \left( \sqrt{-\tau} \xi \right)} \right] \right\}^2 e^{[\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]},
\]
\[
r(x, t) = \mu \left\{ \frac{1}{2} \left[ - \frac{117855 \tau^3 (W_1^2 - W_2^2)}{L_6} \right] ^{\frac{1}{4}} \left[ \frac{1}{W_1 \cosh \left( \sqrt{-\tau \xi} \right) + W_2 \sinh \left( \sqrt{-\tau \xi} \right)} \right] \right\} ^{\frac{1}{2}}
\]

\[e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]}.
\]

The bright soliton solution is obtained when \(W_1 \neq 0, W_2 = 0\), we have:

\[
g(x, t) = \left\{ \frac{1}{2} \left[ - \frac{117855 \tau^3}{L_6} \right] ^{\frac{1}{4}} \text{sech} \left( \sqrt{-\tau \xi} \right) \right\} ^{\frac{1}{2}} e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]},
\]

\[
r(x, t) = \mu \left\{ \frac{1}{2} \left[ - \frac{117855 \tau^3}{L_6} \right] ^{\frac{1}{4}} \text{sech} \left( \sqrt{-\tau \xi} \right) \right\} ^{\frac{1}{2}} e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]},
\]

provided \(L_6 > 0\). The singular soliton solution is obtained when \(W_1 = 0, W_2 \neq 0\), we have:

\[
g(x, t) = \left\{ \frac{1}{2} \left[ - \frac{117855 \tau^3}{L_6} \right] ^{\frac{1}{4}} \text{csch} \left( \sqrt{-\tau \xi} \right) \right\} ^{\frac{1}{2}} e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]},
\]

\[
r(x, t) = \mu \left\{ \frac{1}{2} \left[ - \frac{117855 \tau^3}{L_6} \right] ^{\frac{1}{4}} \text{csch} \left( \sqrt{-\tau \xi} \right) \right\} ^{\frac{1}{2}} e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]},
\]

for \(L_6 < 0\).

In Figures 1 and 2, we can see 3D plots, contour plots, 2D plots of a bright soliton solution defined by Eq. (41). The parameters have specific values: \(V = 1.1, \tau = -1, \mu = 1.2, \xi_1 = 1.9, \eta_1 = 1.3, \zeta_1 = 1.4, \alpha_1 = 1.7, \kappa = 1.6, \Omega = 2.1, \theta = 2.4, \) and \(W(t) = t\).
Figure 1. Profile of a bright soliton solution
Figure 2. Profile of a bright soliton solution
4. Generalized Riccati equation mapping scheme

Eq. (33) obtains the formal solution [6–8]:

\[ Z(\xi) = \Delta_0 + \Delta_1 Q(\xi) + \frac{\Delta_{-1}}{Q(\xi)}, \tag{45} \]

and \( Q(\xi) \) satisfies the generalized Riccati equation:

\[ Q'(\xi) = h_0 + h_1 Q(\xi) + h_2 Q^2(\xi), \tag{46} \]

where \( \Delta_0, \Delta_1, \Delta_{-1}, h_0, h_1 \) and \( h_2 \) are arbitrary constants to be determined provided \( \Delta_1 \neq 0 \) or \( \Delta_{-1} \neq 0 \) and \( h_2 \neq 0 \). Plugging (45) together with (46) into Eq. (33), obtains the following results:

\[ \Delta_1 = \frac{h_2}{2} \left( -\frac{117855}{L_6} \right)^{\frac{1}{6}}, \Delta_0 = 0, \]

\[ \Delta_{-1} = 21 \frac{L_1}{6494 h_2} \left( -\frac{117855}{L_6} \right)^{\frac{1}{6}}, h_0 = \frac{21 L_1}{3247 h_2}, h_1 = 0, \tag{47} \]

and

\[ L_2 = \frac{2849259 L_1^3}{10543009}, L_3 = 0, L_4 = \frac{424829853 L_1^3}{34233150223}, L_5 = 0, \tag{48} \]

provided \( L_6 < 0 \). From (45) and (47), then we have the solutions:

\[ H_1(\xi) = \left\{ \frac{1}{2} \left( -\frac{117855}{L_6} \right)^{\frac{1}{2}} \left[ h_2 Q(\xi) + \frac{21 L_1}{3247 h_2} \left( \frac{1}{Q(\xi)} \right) \right] \right\}^{\frac{2}{3}}. \tag{49} \]

The following solutions are thus yielded.

If \( T = h_1^2 - 4 h_0 h_2 > 0 \) and \( h_1 h_2 \neq 0 \) or \( h_0 h_2 \neq 0 \), then, we have the straddled dark-singular soliton solutions:

\[ q(x, t) = \left\{ \left( -\frac{117855}{L_6} \right)^{\frac{1}{6}} \sqrt{\frac{21 L_1}{12988}} \left[ \tanh \left( \sqrt{\frac{21 L_1}{3247}} \xi \right) + \coth \left( \sqrt{\frac{21 L_1}{3247}} \xi \right) \right] \right\}^{\frac{3}{2}} e^{i [-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]}. \tag{50} \]
\[ r(x, t) = \mu \left\{ \left( -\frac{117855}{L_6} \right) \frac{1}{6} \sqrt{\frac{21 L_1}{12988}} \left[ \tanh \left( \frac{21 L_1}{3247} \xi \right) + \coth \left( \frac{21 L_1}{3247} \xi \right) \right] \right\}^{\frac{1}{2}} e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 \xi]}, \] (51)

\[ q(x, t) = e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 \xi]} \times \left\{ \left( -\frac{117855}{L_6} \right) \frac{1}{6} \sqrt{\frac{21 L_1}{51952}} \left[ \tanh \left( \frac{21 L_1}{12988} \xi \right) + \coth \left( \frac{21 L_1}{12988} \xi \right) \right] \right\}^{\frac{1}{2}} + \frac{4}{\tanh \left( \frac{21 L_1}{12988} \xi \right) + \coth \left( \frac{21 L_1}{12988} \xi \right)} \right\}, \] (52)

\[ r(x, t) = \mu \times e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 \xi]} \times \left\{ \left( -\frac{117855}{L_6} \right) \frac{1}{6} \sqrt{\frac{21 L_1}{51952}} \left[ \tanh \left( \frac{21 L_1}{12988} \xi \right) + \coth \left( \frac{21 L_1}{12988} \xi \right) \right] \right\}^{\frac{1}{2}} + \frac{4}{\tanh \left( \frac{21 L_1}{12988} \xi \right) + \coth \left( \frac{21 L_1}{12988} \xi \right)} \right\}. \] (53)

Also we obtain the straddled singular solitons
\[ q(x, t) = e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]} \times \]
\[
\left\{ -\sqrt{-\frac{21L_1}{12988} - \frac{117855}{L_6}} \pm \frac{1}{\coth \left( \sqrt{-\frac{84 L_1}{3247} \xi} \right) \pm \csc \left( \sqrt{-\frac{84 L_1}{3247} \xi} \right)} \right\}^\frac{1}{2},
\]
\[ (54) \]

\[ r(x, t) = e^{i[-\kappa x + \Omega t + \theta + \sigma W(t) - \sigma^2 t]} \times \]
\[
\mu \left\{ -\sqrt{-\frac{21L_1}{12988} - \frac{117855}{L_6}} \pm \frac{1}{\coth \left( \sqrt{-\frac{84 L_1}{3247} \xi} \right) \pm \csc \left( \sqrt{-\frac{84 L_1}{3247} \xi} \right)} \right\}^\frac{3}{2},
\]
\[ (55) \]

A few additional soliton solutions are structured as:

\[ q(x, t) = \left\{ -\frac{117855}{L_6} \pm \sqrt{-\frac{21L_1}{12988}} \right\} \left[ \frac{\sqrt{A^2 + B^2} - A \cosh \left( \sqrt{-\frac{84 L_1}{3247} \xi} \right)}{A \sinh \left( \sqrt{-\frac{84 L_1}{3247} \xi} \right) + B} \right] \left[ \frac{A \sinh \left( \sqrt{-\frac{84 L_1}{3247} \xi} \right) + B}{\sqrt{A^2 + B^2} - A \cosh \left( \sqrt{-\frac{84 L_1}{3247} \xi} \right)} \right]^\frac{3}{2},
\]
\[ (56) \]
\begin{equation}
\begin{aligned}
q(x, t) &= \left\{ \left( \frac{-117855}{L_6} \right) \frac{1}{6} \sqrt{-\frac{21 L_1}{12988}} \left[ \frac{\sqrt{A^2 + B^2} - A \cosh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)}{A \sinh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)} + B \right] + \frac{A \sinh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)}{\sqrt{A^2 + B^2} - A \cosh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)} \right\} \frac{1}{2} e^{\left[ -\kappa x + \Omega t + \Theta + \sigma W(t) - \sigma^2 t \right]},
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
r(x, t) &= \left\{ \left( \frac{-117855}{L_6} \right) \frac{1}{6} \sqrt{-\frac{21 L_1}{12988}} \left[ \frac{\sqrt{B^2 - A^2} + A \sinh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)}{A \cosh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)} + B \right] + \frac{A \cosh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)}{\sqrt{B^2 - A^2} + A \sinh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)} \right\} \frac{1}{2} e^{\left[ -\kappa x + \Omega t + \Theta + \sigma W(t) - \sigma^2 t \right]},
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
r(x, t) &= \left\{ \left( \frac{-117855}{L_6} \right) \frac{1}{6} \sqrt{-\frac{21 L_1}{12988}} \left[ \frac{\sqrt{B^2 - A^2} + A \sinh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)}{A \cosh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)} + B \right] + \frac{A \cosh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)}{\sqrt{B^2 - A^2} + A \sinh \left( \sqrt{-\frac{84 L_1}{3247}} \xi \right)} \right\} \frac{1}{2} e^{\left[ -\kappa x + \Omega t + \Theta + \sigma W(t) - \sigma^2 t \right]},
\end{aligned}
\end{equation}

provided \( L_1 < 0 \), \( A \) and \( B \) are two non-zero real constants satisfying \( B^2 - A^2 > 0 \).

5. Conclusions

The current paper conducts a detailed analysis and constitutes the retrieval of highly dispersive gap optical solitons that emerge from the Kundu-Eckhaus equation. To give the model a flavor of stochasticity, the effect of multiplicative white noise is included in the Itô sense for the first time. Two integration algorithms shed light on the model. They are the extended simplest equation approach and the generalized Riccati equation mapping scheme. These methods together yield bright and singular 1-soliton solutions as well as dark-singular and bright-singular straddled optical solitons. Both of the schemes have a severe shortcoming. They fail to recover dark 1-soliton solution to the model. Another feature that
is observed in this paper is that the effect of white noise stays confined to the phase component of the solitons and thus does not affect the amplitude component of any of the solitons.

The results are thus indeed promising to traverse along additional avenues to proceed with this model. It is imperative to consider more integration schemes that would reveal dark 1-soliton solutions to the model. The present paper thus stands incomplete in the sense that the two adopted algorithms fail to present a full spectrum of optical gap solitons to the model. This is one of the very many avenues to venture into in the upcoming days and the recovered results would be reported after aligning them with the various pre-existing ones [9–24].

**Conflict of interest**

The authors declare no competing financial interest.

**References**


