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Research Article

# Heuristic Incident Edge Path Algorithm for Interval-Valued Neutrosophic Transportation Network

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**Abstract:** Neutrosophic Set is an extension of Fuzzy set theory dealts with uncertain environments in Transportation, Decision-making etc. Finding the shortest path for an uncertain environment is one of the most challenging tasks, many heuristic algorithms play a vital role in releasing this task. This research article an heuristic algorithmic approach using graph theoretical methods helps the researcher to crack the challenge and which type of algorithm can use the type of score functions are discussed. Especially, the interval-valued Neutrosophic transportation problem (IVNTP) is considered for finding the shortest path which is a real-world decision-making problem in an ambiguous (uncertain) environment. Providing the shortest and finest way to address this ambiguity leads to the success of researcher to the organization. The algorithm used has been introduced for determination of interval-valued Neutrosophic shortest path (IVNSP) from the origin node to all other nodes by de-neutrosophicated edges using score functions. An illustrative Interval-valued neutrosophic transportation problem explains the algorithm's efficacy and authenticity to find optimum result; this research article discusses about the nature of score functions, and some score function were taken into consideration. Further, the heuristic algorithm used is applicable for both positive and negative weighted edges; it was elaborated and compared with existing algorithm results, and it also provided all pair of shortest path. Future study and brief study of this article was disclosed in conclusion.

***Keywords*:** score function, interval-valued neutrosophic number, ranking, de-neutrosophicated number, interval-valued neutrosophic shortest path problem

**MSC:** 05C72, 00A79, 49Q22, 05C85, 05C90

## Abbreviation

| | |
|---|---|
| FS | Fuzzy Set |
| DM | Decision Maker |
| TP | Trnasportation Problem |
| IFS | Intuitionistic Fuzzy Set |
| SP | Shortest Path |
| SPP | Shortest Path Problem |
| NS | Neutrosophic Set |


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| | |
|---|---|
| NSP | Neutrosophic Shortest Path |
| SVNS | Single-Valued Neutrosophic Set |
| IVNS | Interval-Valued Neutrosophic Set |
| IVNN | Interval-Valued Neutrosophic Number |
| IVNSP | Interval-Valued Neutrosophic Shortest Path |
| IVNPW | Interval-Valued Neutrosophic Path Weight |
| IVNMC | Interval-Valued Neutrosophic Minimum Cost |
| IVNEW | Interval-Valued Neutrosophic Edge Weight |
| IVNTP | Interval-VAlued Neutrosophic Transporation Problem |
| HIEPA | Heuristic Incident Edge Path Algorithm |

# 1. Introduction

## 1.1 *Transportation problem*

In the contemporary landscape, determining the overall transportation expenses poses challenges due to the intricate nature of various transportation system constraints that are impractical to incorporate into conventional mathematical models. Transportation costs stand as a significant component of expenditures for any supply-chain company, encompassing both supply-based transport costs and demand-related expenses. Additionally, transportation costs exhibit variability influenced by factors such as traffic conditions, diverse road surfaces along pathways, contributions to rural/urban communities during holidays, and imposed taxes on each vehicle.

This variability in transportation factors, including route and mode of travel, complicates the calculation of per-unit transportation costs, further compounded by fluctuating fixed charges. In situations where unit transport costs and other aspects of transportation delivery are ambiguous, the mathematical designs/models for total transportation system costs become excessively complex. The complexity arises from numerous constraints and criteria, including demand, weather conditions, distance, driver behavior, among others. Decision-makers (DMs) are tasked with selecting the appropriate criteria for evaluating total transport costs.

Transportation is intricately linked to the movement of goods from diverse sources to multiple destinations within the transportation system. For the transportation problem (TP), there is a plethora of potential effective implementations that researchers are actively pursuing to enhance TP development.

## 1.2 *Neutrosophic set theory*

Zadeh [1] recommended the use of fuzzy sets (FS) to address decision-making challenges in situations where data is inaccurate or ambiguous. Fuzzy sets involve a degree of membership ranging from 0 to 1. Atanasov [2] introduced the concept of intuitionistic fuzzy sets (IFS) to address the issue of lacking perception for non-membership degrees. IFS allows for the representation of uncertainty without specifying a precise membership value. While Fuzzy Set theory evolved into Intuitionistic Fuzzy Sets (IFS), it still falls short in handling other forms of ambiguity, such as incompatible and undefined information.

For instance, consider a statement declared as true with a possibility of 0.5, or if it appears false with a possibility of 0.6, or if it is uncertain with a possibility of 0.2. These challenges exceed the capabilities of fuzzy and intuitionistic fuzzy sets, necessitating the development of new theories. Various computational tools have been devised to address real-world problems and tackle impreciseness in different structural forms.

The neutrosophic theory emerged as a superior alternative when fuzzy sets and fuzzy logic could not handle false membership data, and intuitionistic fuzzy logic and intuitionistic fuzzy sets struggled with indeterminacy information. Neutrosophic theory accommodates ambiguous and imprecise content, including opposing and neutral information. The term ”neutrosophic” is derived from the philosophical term ”neutrosophy,” introduced by Smarandache in 1995 as a new variant of vague philosophy. Neutrosophy explores various additional aspects of the nature, origin, and scope of neutralities [3].

A Neutrosophic set is a generalization of both fuzzy sets and intuitionistic fuzzy sets. In neutrosophic logic, a proposition has degrees of membership functions for truth ($\mathscr{T}$), indeterminacy ($\mathscr{I}$), and falsity ($\mathscr{F}$). In 1965, Zadeh asserted that fuzzy set (FS) theory assigns a degree of membership function to each element, representing its level of involvement in an ambiguity problem. In 1989, K. Atanassov extended this idea with Intuitionistic Fuzzy Sets (IFS), introducing a degree of non-membership function alongside the degree of membership, ensuring $\mathscr{T}_{\overline{X}}(x)+\mathscr{F}_{\overline{X}}(x)\leq 1$ for each element $x\in\mathscr{U}$.

Smarandache further expanded this concept by introducing the neutrosophic set, which includes a degree of indeterminacy $\mathscr{I}_{\overline{X}}(x)\in[0, 1]$ in addition to degrees of membership and non-membership functions, addressing uncertainty problems. The neutrosophic set concept became a reality with the establishment of neutrosophy.

While fuzzy and intuitionistic fuzzy sets struggled with ambiguous relations, the neutrosophic set effectively handled indeterminate data. The neutrosophic set comprises three elements: truth ($\mathscr{T}$), indeterminate (whether untrue or true) ($\mathscr{I}$), and falsity ($\mathscr{F}$). Unlike $\mathscr{T}$, $\mathscr{I}$, and $\mathscr{F}$ having intervals, they can be any sets, including intersections or unions of the initial sets, discrete, continuous, open, closed, or partially opened half-closed intervals.

## 2. Literature review

Smarandache [3] and [4] originally conceived the notion of a Neutrosophic Set (NS) from a metaphysical perspective. This concept proved adept at handling uncertain, inadequate, incompatible, and inconclusive data more precisely [5]. The NS concept represented a broader scope than both fuzzy set theory [1] and [2] and intuitionistic fuzzy set theory [6], encapsulating only three membership degrees: truth, indeterminacy, and falsity, bounded within the objective standard or non-standard unit interval $[-0, 1+]$.

However, applying the principle of a Neutrosophic Set in real-world engineering and scientific domains posed considerable challenges. Recognizing this, Smarandache [3] introduced the concept of a Single Valued Neutrosophic Set (SVNS), where truth, indeterminacy, and falsity functions are restricted within [0, 1]. In such contexts, the precise definition of truth, indeterminacy, and falsity memberships regarding a given statement is often elusive, yet they can be expressed as a range of feasible interval values.

Wang et al. [7] and [8] expanded on this foundation by proposing the concept of an Interval-Valued Neutrosophic Set (IVNS), which offered greater specificity and flexibility compared to the SVNS. The IVNS generalizes the SVNS, with all three membership functions being interdependent, and their values falling within a unit interval [0, 1].

Further exploration of Neutrosophic sets, IVNS, and their applications across various disciplines is available in [9]. Additional insights on interval-valued neutrosophic sets and ranking methods are provided in [10] and [11].

### 2.1 *Application review of Neutrosophic Set*

The shortest path problem (SPP) is a widely recognized challenge in graph theory, with various algorithms designed to address it. This problem is prevalent in diverse fields, including transportation, road mapping, and other applications. SPP can be classified into three types according to Sengupta et al. [12]: determining the minimum (shortest) path from a single source, calculating the shortest path from the origin vertex to all other vertices in the graph, and solving the single-source shortest path problem, where the decision-maker's goal is to identify the shortest paths among all connected nodes in the network. Another variant involves determining the shortest path between any two vertices, aiming to compute the shortest paths for all pairs in a given transportation problem (TP).

SPP in a network entails finding the shortest path from a source vertex to another with the least weight, where the edge length of the network may represent real-world quantities such as time or cost. In traditional shortest path problems, decision-makers are presumed certain about parameters like time and distance among nodes. However, real-world scenarios introduce uncertainty regarding these parameters between various nodes. Numerous algorithms [13–18] have been developed to calculate the shortest path under various input element classifications, including fuzzy sets, interval-valued fuzzy sets, interval-valued intuitionistic fuzzy sets, and vague sets.

Dijkstra's Algorithm [18] is a well-known method for solving SPPs, particularly the single-source SPP, which computes the shortest path from the origin node to all other nodes in a network and [19]. Recent publications on Neutrosophic graph theory [20–28] focus on interval-valued Neutrosophic sets. Additionally, Broumi et al. [29–32] presented algorithms for finding the shortest path in a graph with arc weights represented as neutrosophic numbers, as well as Single Valued Neutrosophic Sets (SVNS) [33], Interval Valued Neutrosophic Numbers (IVNNs), and Bipolar Neutrosophic Numbers. Broumi et al. [34–36] pioneered a methodology for determining the shortest path (SP) within a triangular fuzzy and Single-valued trapezoidal Neutrosophic context [37]. They also introduced an alternative approach for solving the Shortest Path Problem (SPP) in a network using trapezoidal Neutrosophic numbers in [38]. The same authors devised an algorithmic method to find the SP in a setting involving bipolar Neutrosophic numbers [39]. Additionally, Broumi et al. [40] outlined an approach relying on a score function, employing an algorithm from a different paper. The Neutrosophic Shortest Path Problem (NSPP) was addressed for a graph with Interval Valued Neutrosophic Numbers (IVNN) as edges.

Liu et al. [41] proposed operators for the Interval Neutrosophic Muirhead average operator, implemented in group decision-making with multiple attributes. Consequently, numerous papers on Neutrosophic sets involving single, interval and bipolar neutrosophic sets with problems [42–55] have been published. Akash et al. [56] utilized the MAPE software to predict transportation costs, providing the minimum transportation cost through the fuzzy hierarchy process for an artificial neural network. They conducted an analysis using regression for approximately 50 trapezoidal Interval-valued Neutrosophic nodes with IVNNs. Smarandache [57] presented a paper introducing definitions and theorems related to neutrosophic sets and numbers, offering examples for better comprehension. Akram [58] authored a book chapter focusing on the application of Single-Valued Neutrosophic Sets in soft graphs for decision-making. In another work, Akram et al. [59] delved into the exploration of r-uniform single-valued neutrosophic soft hypergraphs, examining their direct sum, lexicographic product, and costrong product. The discussion extended to exploring applications in the human nervous system, involving artificial intelligence and decision support systems. Akram [60] explored concepts such as regular, totally regular, perfectly regular, full regular, perfectly irregular, as well as hyperedge regular, totally hyperedge regular, perfectly hyperedge regular, full hyperedge regular, and perfectly hyperedge irregular single-valued neutrosophic soft hypergraphs. The presented model, illustrated through an application showcasing the multilateral relationships among Asian countries through various regional organizations, demonstrates the versatility of the proposed hypergraphs in fields such as genetics, human activities, and applied sciences. Saba Siddique et al. [61] introduced a sophisticated neutrosophic set designed to address periodic nature, utilizing the generalized complex neutrosophic set of type 1 along with its properties. The authors illustrated the application of this set in decision-making through examples and conducted a comparative analysis of its results against existing models.

## 2.2 *Motivation to the article*

The main aim of this study was to introduce an iterative and alternative algorithmic method for discovering the shortest path (SP) in a network within an indeterminate domain. This approach is designed to be straightforward and efficient, particularly in handling both positive and negative weighted neutrosophic edges in real-world applications. Additionally, various score functions developed by different researchers were examined and analyzed to determine their applicability across all aspects of graphical algorithms.

## 2.3 *Main objective of the paper*

The paper focuses on the Neutrosophic Shortest Path (NSP) within a Neutrosophic network, where each link/arc length is assigned an Interval-Valued Neutrosophic Number (IVNN) instead of a real or fuzzy number. To determine the Shortest Path Problem (SPP) in an indeterminate domain and present the shortest path, the Incident Edge Path algorithm proposed by Kanchana et al. [62] was extended and applied within the realm of Neutrosophic set theory to discover the NSP. This algorithm is versatile, applicable to various types of neutrosophic sets, including Single-valued, Interval-valued, Bipolar, SuperHyper graph, Plithogenic neutrosophic sets, etc. For this study, Interval-valued neutrosophic sets were specifically considered. The direct sum was employed to add the IVNNs corresponding to the arc distances for a path, determining the path length between pairs of nodes/vertices. Throughout this work, our aim is to address the SPP by introducing a heuristic algorithm for a network where IVNNs represent arc distances between nodes, and the characteristics of score functions were analyzed using a numerical example.

The study unfolds as follows: Sections 1 and 2 provide an overview of existing work and a review of neutrosophic set theory. Section 3 includes basic definitions and considerations about neutrosophic sets, introducing the interval-valued Neutrosophic set and describing the algorithm. In Section 4, the algorithm is detailed in steps and flowcharts, accompanied by an analytical illustration through iterative steps, along with a comparative study of the heuristic algorithm. Section 5 concludes the study by employing existing score functions introduced by different authors, presenting them as a line graph, and evaluating the efficacy of these score functions for application in various algorithms. Finally, Section 6 offers conclusions and outlines potential future work.

## 3. Preliminaries

In this section, the basic definitions used and involved are given and refered from [3, 5, 7, 8, 27].

**Definition 1** [3]: Consider $\mathscr{U}$ to be a space of objects. The set X is Neutrosophic which has an object defined as $\overline{X} = \{< x\colon \mathscr{T}_{\overline{X}}(x),\ \mathscr{I}_{\overline{X}}(x),\ \mathscr{F}_{\overline{X}}(x) >,\ x \in \mathscr{U}\}$ where $\mathscr{T},\ \mathscr{I},\ \mathscr{F}\colon X \to ]^{-}0,\ 1^{+}[$ are defined as the functions of degree of Truth membership function, degree of indeterminacy membership function and degree of falsity membership function respectively, of $\forall x \in X$ to the set $X$ with the condition, $^{-}0 \leq \mathscr{T}_{\overline{X}}(x) + \mathscr{I}_{\overline{X}}(x) + \mathscr{F}_{\overline{X}}(x) \leq 3^{+}$.

Logically, the N.S. takes the points from the real non-standard or standard subset $]^{-}0,\ 1^{+}[$ . SoRather $]^{-}0,\ 1^{+}[$ one should consider the interval as [0, 1] for applications in a technological field, since $]^{-}0,\ 1^{+}[$ is strenuous to implement in real-life applications such as engineering and scientific network problems.

**Definition 2** [5]: Consider $\mathscr{U}$ as the universe of discourse with global objects noted by $x$. A single-valued Neutrosophic set (SVNS) is defined by $\overline{X}$and the global elements of set $\overline{X}$ are $\forall x \in \mathscr{U}$, $\mathscr{T}_{\overline{X}}(x),\ \mathscr{I}_{\overline{X}}(x),$
$\mathscr{F}_{\overline{X}}(x) \in [0,\ 1]$. The SVN set $\overline{X}$ is defined as

$$\overline{X} = \{x,\ \mathscr{T}_{\overline{X}}(x),\ \mathscr{I}_{\overline{X}}(x),\ \mathscr{F}_{\overline{X}}(x) > |x \in \mathscr{U}\}.$$

**Definition 3** [7]: Consider $\mathscr{U}$ as the universe of discourse with global objects noted by $x$. The set $\overline{X}$ is an IVNS Interval-valued Neutrosophic set and is represented by membership functions such as truth indeterminacy and falsity denoted by$\mathscr{T}_{\overline{X}}(x),\ \mathscr{I}_{\overline{X}}(x),\ \mathscr{F}_{\overline{X}}(x)$ respectively. $\{\forall x \in \mathscr{U},\ \mathscr{T}_{\overline{X}}(x),\ \mathscr{I}_{\overline{X}}(x),\ \mathscr{F}_{\overline{X}}(x) \in [0,\ 1]\}$ and IVNS $\overline{X}$ is defined by

$$\overline{X} = \{< [\mathscr{T}^{L}_{\overline{X}}(x),\ \mathscr{T}^{U}_{\overline{X}}(x)],\ [\mathscr{I}^{L}_{\overline{X}}(x),\ \mathscr{I}^{U}_{\overline{X}}(x)],\ [\mathscr{F}^{L}_{\overline{X}}(x),\ \mathscr{F}^{U}_{\overline{X}}(x)] > |x \in \mathscr{U}\}$$

Where the membership functions truth, Indeterminacy and falsity are defined as interval-valued Neutrosophic set by $\mathscr{T}_{\overline{X}}(x) = [\mathscr{T}_{\overline{\mathscr{X}}}{}^{L}(x),\ \mathscr{T}_{\overline{\mathscr{X}}}{}^{U}(x)]$, $\mathscr{I}_{\overline{\mathscr{X}}}(x) = [\mathscr{I}_{\overline{\mathscr{X}}}{}^{L}(x),\ \mathscr{I}_{\overline{\mathscr{X}}}{}^{U}(x)]$ and $\mathscr{F}_{\overline{\mathscr{X}}}(x) = [\mathscr{F}_{\overline{\mathscr{X}}}{}^{L}(x),\ \mathscr{F}_{\overline{\mathscr{X}}}{}^{U}(x)]$.

$$^{-}0 \leq \sup \mathscr{T}_{\overline{X}}(x) + \sup \mathscr{I}_{\overline{X}}(x) + \sup \mathscr{F}_{\overline{X}}(x) \leq 3^{+}$$

### 3.1 *Interval-valued Neutrosophic numbers (IVNN)-mathematical operations*

**Definition 4** [8]: Consider $\overline{X}_1$ and $\overline{X}_2$ as two inter-valued neutrosophic numbers defined by

$$\overline{X}_1 = \left\langle [\mathscr{T}^L_{\overline{X}_1}(x),\ \mathscr{T}^U_{\overline{X}_1}(x)],\ [\mathscr{I}^L_{\overline{X}_1}(x),\ \mathscr{I}^U_{\overline{X}_1}(x)],\ \ [\mathscr{F}^L_{\overline{X}_1}(x),\ \mathscr{F}^U_{\overline{X}_1}(x)] > |x \in \mathscr{U} \right\rangle \text{ and}$$

$$\overline{X}_2 = \left\langle [\mathscr{T}^L_{\overline{X}_2}(x),\ \mathscr{T}^U_{\overline{X}_2}(x)],\ [\mathscr{I}^L_{\overline{X}_2}(x),\ \mathscr{I}^U_{\overline{X}_2}(x)],\ \ [\mathscr{F}^L_{\overline{X}_2}(x),\ \mathscr{F}^U_{\overline{X}_2}(x)] > |x \in \mathscr{U} \right\rangle$$

then the following are the direct sum and direct product of IVNN,

$$\overline{X}_1 \oplus \overline{X}_2 = \left\langle \begin{array}{c} \left[\left(\mathscr{T}^L_{\overline{X}_1}(x) + \mathscr{T}^L_{\overline{X}_2}(x) - \mathscr{T}^L_{\overline{X}_1}(x) * \mathscr{T}^L_{\overline{X}_2}(x)\right),\ \left(\mathscr{T}^U_{\overline{X}_1}(x) + \mathscr{T}^U_{\overline{X}_2}(x) - \mathscr{T}^U_{\overline{X}_1}(x) * \mathscr{T}^U_{\overline{X}_2}(x)\right)\right], \\ \left[\mathscr{I}^L_{\overline{X}_1}(x) * \mathscr{I}^L_{\overline{X}_2}(x),\ \mathscr{I}^U_{\overline{X}_1}(x) * \mathscr{I}^U_{\overline{X}_2}(x)\right],\ \left[\mathscr{F}^L_{\overline{X}_1}(x) * \mathscr{F}^L_{\overline{X}_2}(x),\ \mathscr{F}^U_{\overline{X}_1}(x) * \mathscr{F}^U_{\overline{X}_2}(x)\right] \end{array} \right\rangle \tag{1}$$

$$\overline{X}_1 \otimes \overline{X}_2 = \left\langle \begin{array}{c} \left[\left(\mathscr{T}^L_{\overline{X}_1}(x) * \mathscr{T}^L_{\overline{X}_2}(x),\ \mathscr{T}^U_{\overline{X}_1}(x) * \mathscr{T}^U_{\overline{X}_2}(x)\right)\right],\ \left[\left(\mathscr{I}^L_{\overline{X}_1}(x) + \mathscr{I}^L_{\overline{X}_2}(x) - \mathscr{I}^L_{\overline{X}_1}(x) * \mathscr{I}^L_{\overline{X}_2}(x)\right),\right. \\ \left.\left(\mathscr{I}^U_{\overline{X}_1}(x) + \mathscr{I}^U_{\overline{X}_2}(x) - \mathscr{I}^U_{\overline{X}_1}(x) * \mathscr{I}^U_{\overline{X}_2}(x)\right)\right]\ \left[\left(\mathscr{F}^L_{\overline{X}_1}(x) + \mathscr{F}^L_{\overline{X}_2}(x) - \mathscr{F}^L_{\overline{X}_1}(x) * \mathscr{F}^L_{\overline{X}_2}(x)\right),\right. \\ \left.\left(\mathscr{F}^U_{\overline{X}_1}(x) + \mathscr{F}^U_{\overline{X}_2}(x) - \mathscr{F}^U_{\overline{X}_1}(x) * \mathscr{F}^U_{\overline{X}_2}(x)\right)\right] \end{array} \right\rangle \tag{2}$$

For $\gamma > 0$ of Neutrosophic number $\overline{X}$

$$\gamma\overline{X} = \left\langle \left[1 - \left(1 - \mathscr{T}^L_{\overline{X}}(x)\right)^\gamma,\ \ 1 - \left(1 - \mathscr{T}^U_{\overline{X}}(x)\right)^\gamma\right] \left[(\mathscr{I}^L_{\overline{X}}(x))^\gamma,\ (\mathscr{I}^U_{\overline{X}}(x))^\gamma\right],\ \left[(\mathscr{F}^L_{\overline{X}}(x))^\gamma,\ (\mathscr{F}^U_{\overline{X}}(x))^\gamma\right] \right\rangle \tag{3}$$

$$\overline{X}^\gamma = \left\langle \left[(\mathscr{T}^L_{\overline{X}}(x))^\gamma,\ (\mathscr{T}^U_{\overline{X}}(x))^\gamma\right],\ \left[1 - \left(1 - \mathscr{I}^L_{\overline{X}}(x)\right)^\gamma,\ 1 - \left(1 - \mathscr{I}^U_{\overline{X}}(x)\right)^\gamma\right] \left[1 - \left(1 - \mathscr{F}^L_{\overline{X}}(x)\right)^\gamma,\ 1 - \left(1 - \mathscr{F}^U_{\overline{X}}(x)\right)^\gamma\right] \right\rangle \tag{4}$$

**Definition 5** Let $\overline{X} = \left\langle [\mathscr{T}^L_{\overline{X}}(x),\ \mathscr{T}^U_{\overline{X}}(x)],\ [\mathscr{I}^L_{\overline{X}}(x),\ \mathscr{I}^U_{\overline{X}}(x)],\ [\mathscr{F}^L_{\overline{X}}(x),\ \mathscr{F}^U_{\overline{X}}(x)] \right\rangle$ be an interval-valued Neutrosophic number. Iff $\mathscr{T}^L_{\overline{X}}(x) = 0$, $\mathscr{T}^U_{\overline{X}}(x) = 0$, $\mathscr{I}^L_{\overline{X}}(x) = 1$, $\mathscr{I}^U_{\overline{X}}(x) = 1$ and $\mathscr{F}^L_{\overline{X}}(x) = 1$, $\mathscr{F}^U_{\overline{X}}(x) = 1$, then, IVNN $\overline{X}$ is known to be zero IVNN and denoted by

$$0_n = \{\langle x,\ \langle [0,\ 0],\ [1,\ 1],\ [1,\ 1] \rangle \rangle : x \in \mathscr{U}\}$$

## 3.2 *De-Neutrosophication of IVNN using score functions*

Comparison must be made to find the least IVNN among two IVNNs $\overline{X}_1$ and $\overline{X}_2$ . Such a comparison can be made by score functions. Some of the general de-Neutrosophication formulae according to authors [24, 33, 41, 42, 53, 57] are listed below from the literature review,

$$\mathscr{S}_{Boulturk}(\overline{X}) = \left\{ \frac{\mathscr{T}_{\overline{X}}^{L}(x) + \mathscr{T}_{\overline{X}}^{U}(x)}{2} + \left(1 - \frac{\mathscr{I}_{\overline{X}}^{L}(x) + \mathscr{I}_{\overline{X}}^{U}(x)}{2}\right) * \mathscr{I}_{\overline{X}}^{U}(x) - \frac{\mathscr{F}_{\overline{X}}^{L}(x) + \mathscr{F}_{\overline{X}}^{U}(x)}{2} * \left(1 - \mathscr{F}_{\overline{X}}^{U}(x)\right) \right\} \tag{5}$$

$$\mathscr{S}_{Ridvan}(\overline{X}) = \frac{1}{4}\left\{2 + \mathscr{T}_{\overline{X}}^{L}(x) + \mathscr{T}_{\overline{X}}^{U}(x) - 2\left(\mathscr{I}_{\overline{X}}^{L}(x) + \mathscr{I}_{\overline{X}}^{U}(x)\right) - \left(\mathscr{F}_{\overline{X}}^{L}(x) + \mathscr{F}_{\overline{X}}^{U}(x)\right)\right\} \tag{6}$$

$$\mathscr{S}_{Peng}(\overline{X}) = \left\{ \frac{2}{3} + \frac{\mathscr{T}_{\overline{X}}^{L}(x) + \mathscr{T}_{\overline{X}}^{U}(x)}{6} - \left(\frac{\mathscr{I}_{\overline{X}}^{L}(x) + \mathscr{I}_{\overline{X}}^{U}(x)}{6}\right) - \frac{\mathscr{F}_{\overline{X}}^{L}(x) + \mathscr{F}_{\overline{X}}^{U}(x)}{6} \right\} \tag{7}$$

$$\mathscr{S}_{Liu}(\overline{X}) = \left\{ 2 + \frac{\mathscr{T}_{\overline{X}}^{L}(x) + \mathscr{T}_{\overline{X}}^{U}(x)}{2} - \left(\frac{\mathscr{I}_{\overline{X}}^{L}(x) + \mathscr{I}_{\overline{X}}^{U}(x)}{2}\right) - \frac{\mathscr{F}_{\overline{X}}^{L}(x) + \mathscr{F}_{\overline{X}}^{U}(x)}{2} \right\} \tag{8}$$

$$\mathscr{S}_{harish}(\overline{X}) = \frac{1}{8}\left\{ \begin{array}{l} 4 + \left(\mathscr{T}_{\overline{X}}^{L}(x) + \mathscr{T}_{\overline{X}}^{U}(x) - 2\left(\mathscr{I}_{\overline{X}}^{L}(x) + \mathscr{I}_{\overline{X}}^{U}(x)\right)\right. \\ \left. - \left(\mathscr{F}_{\overline{X}}^{L}(x) + \mathscr{F}_{\overline{X}}^{U}(x)\right)\right) \\ * \left(4 - \mathscr{T}_{\overline{X}}^{L}(x) - \mathscr{T}_{\overline{X}}^{U}(x) - \mathscr{F}_{\overline{X}}^{L}(x) - \mathscr{F}_{\overline{X}}^{U}(x)\right) \end{array} \right\} \tag{9}$$

$$\mathscr{S}_{FSmaran}(\overline{X}) = \frac{1}{6}\left(4 + \mathscr{T}_{\overline{X}}^{L}(x) + \mathscr{T}_{\overline{X}}^{U}(x) - \left(\mathscr{I}_{\overline{X}}^{L}(x) + \mathscr{I}_{\overline{X}}^{U}(x)\right) - \left(\mathscr{F}_{\overline{X}}^{L}(x) + \mathscr{F}_{\overline{X}}^{U}(x)\right)\right) \tag{10}$$

$$\mathscr{S}_{N}(\overline{X}) = \frac{1}{2}\left[\mathscr{T}_{\overline{X}}^{L}(x) + \mathscr{T}_{\overline{X}}^{U}(x) - \left(\mathscr{I}_{\overline{X}}^{L}(x) * \mathscr{I}_{\overline{X}}^{U}(x)\right) + \left(\mathscr{I}_{\overline{X}}^{U}(x) - 1\right)^{2} + \mathscr{F}_{\overline{X}}^{U}(x)\right] \tag{11}$$

Hence, the ranking between $\overline{X}_1$ and $\overline{X}_2$ satisfies the following:

$$a)\ \overline{X}_1 < \overline{X}_2, \text{ if; } \mathscr{S}(\overline{X}_1) < \mathscr{S}(\overline{X}_2)$$

$$b)\ \overline{X}_1 > \overline{X}_2, \text{ if; } \mathscr{S}(\overline{X}_1) > \mathscr{S}(\overline{X}_2)$$

$$c)\ \overline{X}_1 = \overline{X}_2, \text{ if; } \mathscr{S}(\overline{X}_1) = \mathscr{S}(\overline{X}_2) \tag{12}$$

**Definition 6** [27] An Interval-valued Neutrosophic Network $\mathscr{N}(\mathscr{V}, \mathscr{E}, e_{ij})$ is defined with $|\mathscr{V}| = n$ number of vertices and $|\mathscr{E}| = m$ number of arcs. $e_{ij}$ be the interval-valued Neutrosophic edge weight from node $v_i$ to $v_j$.

**Definition 7** [27] Let a network $\mathcal{N}(V, E)$ with n number nodes/vertices and m number of arcs/links. Let the edge weight be an interval-valued neutrosophic number and it is denoted by

$$e_{ij} = \langle [\mathcal{T}_N^L(x), \mathcal{T}_N^U(x)], [\mathcal{I}_N^L(x), \mathcal{I}_N^U(x)] , [\mathcal{F}_N^L(x), \mathcal{F}_N^U(x)] \rangle \tag{13}$$

from node $i$ to $j$, where $j = 1, 2, 3, \ldots, m$ and $i = 1, 2, 3, \ldots, n$ for $i \neq j$.

**Definition 8** [27] Let $\mathcal{E}_i$ be the incident edge set of an interval-valued Neutrosophic network $\mathcal{N}(\mathcal{V}, \mathcal{E})$ defined by

$$\mathcal{E}_i = \{edges\, incident\, with\, node\, v_i\}, \; i = 1, 2, 3, \ldots, n.$$

# 4. Heuristic algorithm for finding IVNSP with IVNMC

## 4.1 *Algorithm description*

Construct a network with interval-valued Neutrosophic edge weights and convert IVNN to a crisp weight using the score function formulae. In this paper score function defined by Nagarajan abnd Harish et al. was used for further solution process. Consider $\mathcal{E}_i$ as the collection of incident edges of $v_i$ in a network. Find $\mathcal{E}_1$ = {edges incident with node $v_1$} and calculate the Interval-valued Neutrosophic path weight (IVNPW) $\mathcal{P}(v_j)$ for all edges of $\mathcal{E}_1$, initially there is no path so $\mathcal{P}(v_1) = 0$. Similarly, find the set of all edges incident with node $v_j$, and calculate the path weight. Proceed the process until $\mathcal{E}_i = \{\}$, i.e., there isn't any edges incident with the preceeding node; which says that the destination node has been reached.

## 4.2 *Heuristic Incident Edge Path Algorithm (HIEPA)*

Step 1: De-Neutrosophication of all Interval-valued neutrosophic edge weights (IVNEW) into a crisp weight using the score function and label it as $\mathcal{S}(e_{ij})$.

Step 2: Find $\mathcal{E}_1$ = {edges incident with node $v_1$} if $\mathcal{E}_1 = \{\}$ is empty go to step 6 otherwise, go to step 3.

Step 3: Calculate the INVMC for IVNSP using the direct sum of IVNS. Initially IVNPW $\mathcal{P}(v_1) = 0$ of source node $v_1$, find the Path weight (cost) $\mathcal{P}(v_j) = \{\mathcal{P}(v_1) + \mathcal{S}(e_{1j})\}$ of node $v_j$, $(j = 2, 3, \ldots, n)$ and select the respective shortest path.

Step 4: Find $\mathcal{E}_2$ = {edges incident with node $v_2$}, if $\mathcal{E}_2 = \{\}$ go to step 6 otherwise, go to step 5.

Step 5: Calculate the INVMC for IVNSP using the direct sum of IVNS. The corresponding IVNPW $\mathcal{P}(v_i)$ of node$v_i$, find the Path weight (cost) $\mathcal{P}(v_j) = \{\mathcal{P}(v_i) + \mathcal{S}(e_{ij})\}$ of node $v_i$, $(i \neq j, \; j = 2, 3, \ldots, n)$ and select the respective shortest path without negative weight with cycle.

Step 6: Proceed steps 2 to 5 untill $\mathcal{E}_i = \{\}$.

Step 7: Interval-valued nuetrosophic shortest path with minimum cost obtained, so stop the process.

Figure 1, briefly explains the working principle of the proposed heuristic Incident Algorithm (HIEPA).
The following illustration was used to demonstrate the proposed HIEPA and also analysis of various score functions for the same illustration help to understand about the nature of score function.

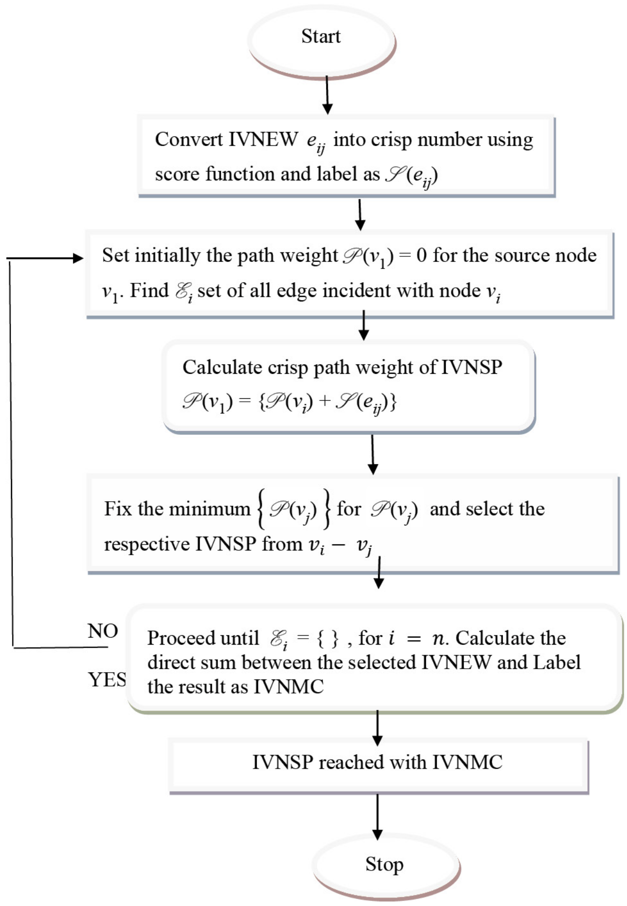


**Figure 1.** Flow chart of the proposed algorithm

## 5. Numerical example

The chosen numerical example for validating HIEPA involved a directed Interval-valued neutrosophic network sourced from Nayankumar et al. [55]. Subsections 5.2 and 5.3 subsequently detail the iterative process for various score values encompassing both positive and negative weighted edges. This is followed by outlining the algorithm steps and a comparison of the results obtained.

**Network**

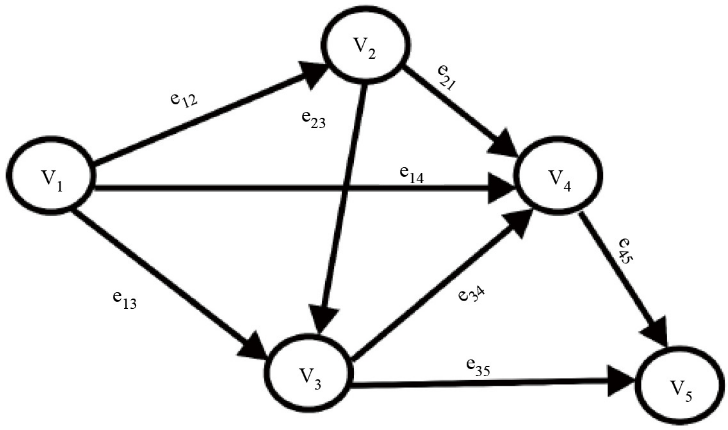


**Figure 2.** Neutrosophic network with edge label

Figure 2 represents the network diagram of the considered numerical example with edge label and edge weights as crisp numbers calculated using the score function.

The Table 1 consisting of interval-valued neutrosophic edge weight and its score function values $\mathscr{S}(e_{ij})$ used for the calculation of the proposed algorithm for Figure 2. Score function used in HIEPA was from equation (11) [53].

**Table 1.** Table shows the crisp number for IVNSP calculated using the score function for the taken IVN network

| Edge $e_{ij}$ | (IVNEW) Interval-Valued Neutrosophic Edge Weight | Crisp number ($\mathscr{S}_N$) |
|---|---|---|
| $(e_{12})$ | $\langle[0.5, 0.6], [0.1, 0.2], [0.2, 0.3]\rangle$ | 1.01 |
| $(e_{13})$ | $\langle[0.1, 0.2], [0.2, 0.3], [0.4, 0.5]\rangle$ | 0.615 |
| $(e_{14})$ | $\langle[0.3, 0.4], [0.2, 0.3], [0.1, 0.4]\rangle$ | 0.765 |
| $(e_{23})$ | $\langle[0.2, 0.3], [0.4, 0.5], [0.5, 0.6]\rangle$ | 0.575 |
| $(e_{24})$ | $\langle[0.3, 0.4], [0.2, 0.3], [0.1, 0.2]\rangle$ | 0.665 |
| $(e_{34})$ | $\langle[0.6, 0.7], [0.2, 0.3], [0.1, 0.2]\rangle$ | 0.965 |
| $(e_{35})$ | $\langle[0.3, 0.4], [0.5, 0.6], [0.6, 0.7]\rangle$ | 0.63 |
| $(e_{45})$ | $\langle[0.6, 0.7], [0.3, 0.4], [0.1, 0.2]\rangle$ | 0.87 |

## 5.1 *Analysis of Score function of IVN network*

The Table 2 provides different score function values (crisp numbers) for considered numerical example, the used score functions were introduced by various authors. The indication of author names with score function was displayed under each notation used along with the values [55].

Table 2. Crisp numbers of IVNEW for Considered IVNN by various score functions listed in section 3.2

| Score function by authors | $e_{12}$ | $e_{13}$ | $e_{14}$ | $e_{23}$ | $e_{24}$ | $e_{34}$ | $e_{35}$ | $e_{45}$ |
|---|---|---|---|---|---|---|---|---|
| Boulturk (S.B.) | 0.645 | 0.15 | 0.425 | 0.195 | 0.545 | 0.845 | 0.165 | 0.88 |
| Ridvan (S.R.) | 0.5 | 0.1 | 0.3 | −0.1 | 0.35 | 0.5 | −0.2 | 0.4 |
| Peng et al. (S.P.) | 0.72 | 0.48 | 0.62 | 0.42 | 0.65 | 0.75 | 0.32 | 0.72 |
| Liu et al. (S.L.) | 2.1 | 1.45 | 1.85 | 1.25 | 1.95 | 2.25 | 1.15 | 2.15 |
| Nagarajan et al. (S.N.) | 1.01 | 0.615 | 0.765 | 0.575 | 0.665 | 0.965 | 0.63 | 0.87 |
| Smarandache et al. (SFS) | 0.72 | 0.48 | 0.62 | 0.42 | 0.65 | 0.75 | 0.38 | 0.72 |
| Harish et al. (S.H.) | 0.5 | −0.06 | 0.4 | −0.22 | 0.275 | 0.5 | −0.2 | 0.38 |

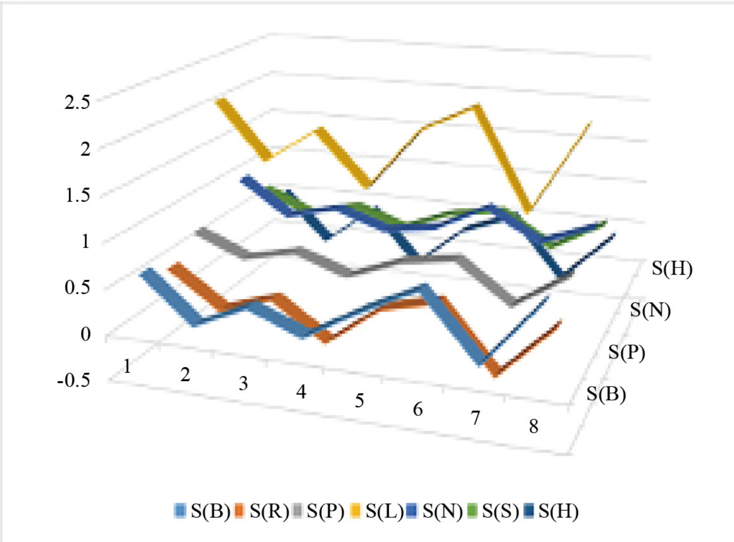


Figure 3. Line graph representation for each edge weights of considered network

Figure 3 was the line graph representing crisp numbers calculated by various score functions; and it helps to understand the minimum crisp weights for the considered IVN network.

## 5.2 *Calculation process by proposed HIEPA for IVNN by using $\mathscr{S}_{Nagarajan}$*

The follwoing iterations were made as per the algorithm and score funciton used for caculation was introduced by Nagarajan et al.

**Iteration 1:**

$\mathscr{E}_1 = \{e_{12},\ e_{13},\ e_{14}\}$ Initially, the IVNPW $\mathscr{P}(v_1) = 0,\ i = 1,\ j = 2, 3, 4$

Table 3. Iteration 1

| Edge $e_{ij}$ | $\mathscr{P}(v_j) = \{\mathscr{P}(v_1) + \mathscr{S}(e_{1j})\}$ | IVNMC | IVNSP |
|---|---|---|---|
| $(e_{12})$ | $\mathscr{P}(v_2) = \{\mathscr{P}(v_1) + \mathscr{S}(e_{12})\}, \mathscr{P}(v_2) = 0 + 1.01 = 1.01$ | $\langle[0.5, 0.6],\ [0.1, 0.2],\ [0.2, 0.3]\rangle$ | $v_1 - v_2$ |
| $(e_{13})$ | $\mathscr{P}(v_3) = \{\mathscr{P}(v_1) + \mathscr{S}(e_{13})\}, \mathscr{P}(v_3) = 0 + 0.615 = 0.615$ | $\langle[0.1, 0.2],\ [0.2, 0.3],\ [0.4, 0.5]\rangle$ | $v_1 - v_3$ |
| $(e_{14})$ | $\mathscr{P}(v_4) = \{\mathscr{P}(v_1) + \mathscr{S}(e_{14})\}, \mathscr{P}(v_4) = 0 + 0.765 = 0.765$ | $\langle[0.3, 0.4],\ [0.2, 0.3],\ [0.1, 0.4]\rangle$ | $v_1 - v_4$ |

Table 3, shows the IVNSP from initial node $v_1$ to succeeding nodes; it was calculated using the proposed algorithm. $\mathscr{P}(v_2)$, $\mathscr{P}(v_3)$ and $\mathscr{P}(v_4)$ are the crisp path weights and IVNMC was calculated using the direct sum between teh selected nodes.

**Iteration 2:**

Table 4 carries the edges incident with node $v_2$ to succeeding nodes. It was calculated as follows $\mathscr{E}_2 = \{e_{23},\ e_{24}\}$ was the incident edge set, IVNPW of $v_2$ was $\mathscr{P}(v_2) = 1.01,\ i = 2,\ j = 3, 4.$

**Table 4.** Iteration 2

| Edge $e_{ij}$ | $\mathscr{P}(v_j) = \{\mathscr{P}(v_2) + \mathscr{S}(e_{2j})\}$ | IVNMC | IVNSP |
|---|---|---|---|
| $(e_{23})$ | $\mathscr{P}(v_3) = \{\mathscr{P}(v_2) + \mathscr{S}(e_{23})\}$, $\mathscr{P}(v_3) = 1.01 + 0.575 = 1.585$ , $\mathscr{P}(v_3) = \min\{0.615,\ 1.585\} = 0.615$ | $\langle[0.6, 0.72],\ [0.04, 0.1],\ [0.1, 0.18]\rangle$ | $v_1 - v_3$ is shortest path than $v_1 - v_2 - v_3$ |
| $(e_{24})$ | $\mathscr{P}(v_4) = \{\mathscr{P}(v_2) + \mathscr{S}(e_{24})\}$, $\mathscr{P}(v_4) = 1.01 + 0.665 = 1.675$, $\mathscr{P}(v_4) = \min\{0.765,\ 1.675\} = 0.765$ | $\langle[0.65, 0.76],\ [0.02, 0.06],\ [0.02, 0.06]\rangle$ | $v_1 - v_4$ is shortest path than $v_1 - v_2 - v_4$ |

$\mathscr{P}(v_3)$ and $\mathscr{P}(v_4)$ are the crisp path weights and IVNMC was calculated using the direct sum between the selected nodes. From Table 3 and 4 we got two crisp path weights for $v_3$ and $v_4$. So, the minimum crisp weight and IVNMC of respective selected path was taken and the selected path has fixed as IVNSP.

**Iteration 3:**

Table 5, briefly provides the calculation process of succeeding node incident with $v_3$ and processed as follows $\mathscr{E}_3 = \{e_{34}, e_{35}\}$, IVNPW $\mathscr{P}(v_3) = 0.615,\ i = 3,\ j = 4, 5.$

**Table 5.** Iteration 3

| Edge $e_{ij}$ | $\mathscr{P}(v_j) = \{P(v_3) + \mathscr{S}(e_{3j})\}$ | IVNMC | IVNSP |
|---|---|---|---|
| $(e_{34})$ | $\mathscr{P}(v_4) = \{\mathscr{P}(v_3) + \mathscr{S}(e_{34})\}$, $\mathscr{P}(v_4) = 0.615 + 0.965 = 1.58$ , $\mathscr{P}(v_4) = \min\{0.765,\ 1.58\} = 0.765$ | $\langle[0.65, 0.76],\ [0.02, 0.06],\ [0.02, 0.06]\rangle$ | $v_1 - v_4$ is shortest path than $v_1 - v_3 - v_4$ |
| $(e_{35})$ | $\mathscr{P}(v_5) = \{\mathscr{P}(v_3) + \mathscr{S}(e_{35})\}$, $\mathscr{P}(v_5) = 0.615 + 0.63 = 1.245$ | $\langle[0.37, 0.52],\ [0.1, 0.18],\ [0.24, 0.35]\rangle$ | $v_1 - v_3 - v_5$ |

From the Table 4 and 5, path weight of node $v_4$ has to be calculated by taking the minimum amoung them as done in Table 5 and the minimum crisp weight, IVNMC and IVNSP ahs been fixed for node $v_4$.

**Iteration 4:**

Proceeding from iteration 3, Table 5; the crisp path weight was fixed and proceeds as follows for the succeeding nodes from $v_4$

$$\mathscr{E}_4 = e_{45},\ \text{IVNPW}\,\mathscr{P}(v_4) = 0.765,\ i = 4,\ j = 5.$$

**Table 6.** Iteration 4

| Edge $e_{ij}$ | $\mathscr{P}(v_j) = \{\mathscr{P}(v_4) + \mathscr{S}(e_{4j})\}$ | IVNMC | IVNSP |
|---|---|---|---|
| $(e_{45})$ | $\mathscr{P}(v_5) = \{\mathscr{P}(v_4) + \mathscr{S}(e_{45})\}$, $\mathscr{P}(v_5) = 0.765 + 0.87 = 1.635$ , $\mathscr{P}(v_5) = \min\{1.245,\ 1.635\} = 1.245$ | $\langle[0.37, 0.52],\ [0.1, 0.18],\ [0.24, 0.35]\rangle$ | $v_1 - v_3 - v_5$ is shortest path than $v_1 - v_2 - v_4 - v_5$ |

From Table 6, we could see that the destination node has reached and the crisp path weight, IVNMC and IVNSP has been cacluated by proposed algorithm. Here the node $v_5$ has more than one path weight so the minimum amoung them has selected for further calculation.

**Iteration 5:**

$\mathscr{E}_5 = \{\},\ i = n = 5$ reached hence the destination reached, the process has to stop. There are no incident interval-valued Neutrosophic edges. Hence, The Shortest path was obtained. Iterations 1 to 5 and Table 3 to 6 explains how the algorithm works for considered IVN network.

### 5.3 *Calculation process by proposed HIEPA for IVNN by using $\mathscr{S}_{Harish}$*

The score values were calculated and displayed in Table 2; the following iterations explains about the working principle of negative weighted edges by HIEPA for IVNN problem. The score fucntion introduced by harish et al. is defined by equation (9),

**Iteration 1:**

$\mathscr{E}_1 = \{e_{12},\ e_{13},\ e_{14}\}$ Initially, the IVNPW $\mathscr{P}(v_1) = 0,\ i = 1,\ j = 2, 3, 4$

**Table 7.** Iteration 1

| Edge $e_{ij}$ | $\mathscr{P}(v_j) = \{\mathscr{P}(v_1) + \mathscr{S}(e_{1j})\}$ | IVNMC | IVNSP |
|---|---|---|---|
| $(e_{12})$ | $\mathscr{P}(v_2) = \{\mathscr{P}(v_1) + \mathscr{S}(e_{12})\}$,$\mathscr{P}(v_2) = 0 + 0.5 = 0.5$ | $\langle[0.5, 0.6],\ [0.1, 0.2],\ [0.2, 0.3]\rangle$ | $v_1 - v_2$ |
| $(e_{13})$ | $\mathscr{P}(v_3) = \{\mathscr{P}(v_1) + \mathscr{S}(e_{13})\}$,$\mathscr{P}(v_3) = 0 - 0.06 = -0.06$ | $\langle[0.1, 0.2],\ [0.2, 0.3],\ [0.4, 0.5]\rangle$ | $v_1 - v_3$ |
| $(e_{14})$ | $\mathscr{P}(v_4) = \{\mathscr{P}(v_1) + \mathscr{S}(e_{14})\}$,$\mathscr{P}(v_4) = 0 + 0.4 = 0.4$ | $\langle[0.3, 0.4],\ [0.2, 0.3],\ [0.1, 0.4]\rangle$ | $v_1 - v_4$ |

Table 7, shows the IVNSP from initial node $v_1$ to succeeding nodes; it was calculated using the proposed algorithm. $\mathscr{P}(v_2)$, $\mathscr{P}(v_3)$ and $\mathscr{P}(v_4)$ are the crisp path weights and IVNMC was calculated using the direct sum between the selected nodes.

**Iteration 2:**

Table 8 carries the edges incident with node $v_2$ to succeeding nodes. It was calculated as follows $\mathscr{E}_2 = \{e_{23},\ e_{24}\}$ was the incident edge set, IVNPW of $v_2$ was $\mathscr{P}(v_2) = 0.5,\ i = 2,\ j = 3, 4$

**Table 8.** Iteration 2

| Edge $e_{ij}$ | $\mathscr{P}(v_j) = \{\mathscr{P}(v_2) + \mathscr{S}(e_{2j})\}$ | IVNMC | IVNSP |
|---|---|---|---|
| $(e_{23})$ | $\mathscr{P}(v_3) = \{\mathscr{P}(v_2) + \mathscr{S}(e_{23})\}$,<br>$\mathscr{P}(v_3) = 0.5 - 0.22 = 0.28$ ,<br>$\mathscr{P}(v_3) = \min\{0.28,\ -0.06\} = -0.06$ | $\langle[0.6, 0.72],\ [0.04, 0.1],\ [0.1, 0.18]\rangle$ | $v_1 - v_3$ is shortest path than $v_1 - v_2 - v_3$ |
| $(e_{24})$ | $\mathscr{P}(v_4) = \{\mathscr{P}(v_2) + \mathscr{S}(e_{24})\}$,<br>$\mathscr{P}(v_4) = 0.5 + 0.275 = 0.775$,<br>$\mathscr{P}(v_4) = \min\{0.4,\ 0.775\} = 0.4$ | $\langle[0.65, 0.76],\ [0.02, 0.06],\ [0.02, 0.06]\rangle$ | $v_1 - v_4$ is shortest path than $v_1 - v_2 - v_4$ |

$\mathscr{P}(v_3)$ and $\mathscr{P}(v_4)$ are the crisp path weights and IVNMC was calculated using the direct sum between the selected nodes. From table 7 and 8 we got two crisp path weights for $v_3$ and $v_4$. So, the minimum crisp weight and IVNMC of respective selected path was taken and the selected path has fixed as IVNSP.

**Iteration 3:**

Table 9, briefly provides the calculation process of succeeding node incident with $v_3$ and processed as follows $\mathscr{E}_3 = \{e_{34}, e_{35}\}$, IVNPW $\mathscr{P}(v_3) = -0.06,\ i = 3,\ j = 4, 5.$

**Table 9.** Iteration 3

| Edge $e_{ij}$ | $\mathscr{P}(v_j) = \{P(v_3) + \mathscr{S}(e_{3j})\}$ | IVNMC | IVNSP |
|---|---|---|---|
| $(e_{34})$ | $\mathscr{P}(v_4) = \{\mathscr{P}(v_3) + \mathscr{S}(e_{34})\}$, $\mathscr{P}(v_4) = -0.06 + 0.5 = 0.44$ , $\mathscr{P}(v_4) = \min\{0.44,\ 0.4\} = 0.4$ | $\langle[0.65,\ 0.76],\ [0.02,\ 0.06],\ [0.02,\ 0.06]\rangle$ | $v_1 - v_4$ is shortest path than $v_1 - v_3 - v_4$ |
| $(e_{35})$ | $\mathscr{P}(v_5) = \{\mathscr{P}(v_3) + \mathscr{S}(e_{35})\}$, $\mathscr{P}(v_5) = -0.06 - 0.2 = -0.26$ | $\langle[0.37,\ 0.52],\ [0.1,\ 0.18],\ [0.24,\ 0.35]\rangle$ | $v_1 - v_3 - v_5$ |

From the Table 7 and 8, path weight of node $v_4$ has to be calculated by taking the minimum amoung them as done in Table 5 and the minimum crisp weight, IVNMC and IVNSP ahs been fixed for node $v_4$.

**Iteration 4:**

Proceeding from iteration 3, Table 9; the crisp path weight was fixed and proceeds as follows for the succeeding nodes from $v_4$

$$\mathscr{E}_4 = e_{45},\ \text{IVNPW}\mathscr{P}(v_4) = 0.4,\ i = 4,\ j = 5.$$

**Table 10.** Iteration 4

| Edge $e_{ij}$ | $\mathscr{P}(v_j) = \{\mathscr{P}(v_4) + \mathscr{S}(e_{4j})\}$ | IVNMC | IVNSP |
|---|---|---|---|
| $(e_{45})$ | $\mathscr{P}(v_5) = \{\mathscr{P}(v_4) + \mathscr{S}(e_{45})\}$, $\mathscr{P}(v_5) = 0.4 + 0.38 = 0.78$ , $\mathscr{P}(v_5) = \min\{-0.26,\ 0.78\} = -0.26$ | $\langle[0.37,\ 0.52],\ [0.1,\ 0.18],\ [0.24,\ 0.35]\rangle$ | $v_1 - v_3 - v_5$ is shortest path than $v_1 - v_2 - v_4 - v_5$ |

From Table 10, we could see that the destination node has reached and the crisp path weight, IVNMC and IVNSP has been cacluated by proposed algorithm. here the node $v_5$ has more than one path weight so the minimum amoung them has selected for further calculation.

**Iteration 5:**

$\mathscr{E}_5 = \{\},\ i = n = 5$ reached hence the destination reached, the process has to stop. There are no incident interval-valued Neutrosophic edges. Hence, The Shortest path was obtained. Iterations 1 to 5 and Table 7 to 10 explains how the algorithm works for considered negative weighted IVNN.

### 5.3.1 *Interval-valued Neutrosophic Shortest path (IVNSP)*

The Table 11 provides crisp number calculated by score function, Interval valued neutrososphic minimum cost and its respective IVNSP from a origin node to all other nodes by HIEPA.

**Table 11.** Interval-valued Neutrosophic edge weight for the considered IVN network

| IVNSP from origin | IVNMC | $(\mathscr{S}_{Nagarajan})$ | $(\mathscr{S}_{Haris})$ |
|---|---|---|---|
| $v_1 - v_2$ | $\langle[0.5,\ 0.6],\ [0.1,\ 0.2],\ [0.2,\ 0.3]\rangle$ | 1.01 | 0.5 |
| $v_1 - v_3$ | $\langle[0.1,\ 0.2],\ [0.2,\ 0.3],\ [0.4,\ 0.5]\rangle$ | 0.615 | −0.06 |
| $v_1 - v_2 - v_4$ | $\langle[0.65,\ 0.76],\ [0.02,\ 0.06],\ [0.02,\ 0.06]\rangle$ | 0.765 | 0.4 |
| $v_1 - v_3 - v_5$ | $\langle[0.37,\ 0.52],\ [0.1,\ 0.18],\ [0.24,\ 0.35]\rangle$ | 0.947 | −0.26 |

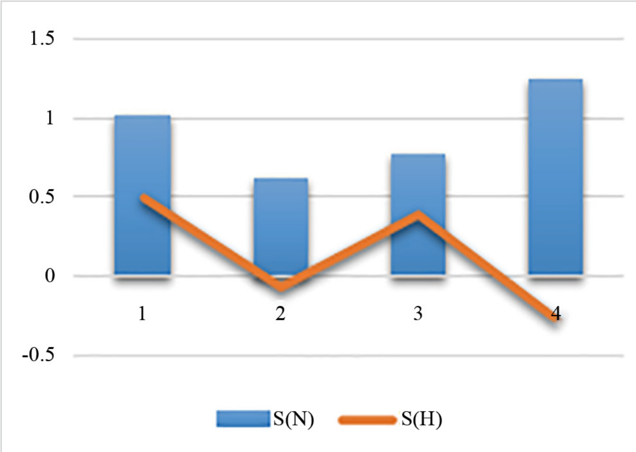


**Figure 4.** Graph of Score values between Nagarjan and Haris et al. for SP

The line graph in Figure 4 explains the value difference between the score functions from different authors. The path $v_1 - v_3 - v_5$ is IVNSP for the considered sample interval-valued neutrosophic network.

## 5.4 *Discussion of different crisp numbers by different score functions for IVNSP*

Score functions play a crucial role for decision-makers, field analysts, researchers, etc., enabling them to make informed decisions by comparing results and selecting the minimum value to achieve the optimum outcome. This article specifically addresses interval-valued neutrosophic networks, where the abundance of score functions proves valuable. In the context of neutrosophic set networks, score functions become essential for comparing neutrosophic numbers, particularly in graphical algorithms aimed at finding the shortest path. Some graphical algorithms encounter challenges in solving networks with negative edge weights. Hence, this section aids researchers in identifying useful score functions for algorithms, especially when comparing edge weights.

The example network examined in this paper involves IVNN, with a negative edge weight when applying score functions proposed by Ridvan et al. and Harish et al. Consequently, the well-known Dijkstra Algorithm cannot solve this numerical example network when using such score functions, as Dijkstra's algorithm is explicitly designed for positive edge-weighted network problems. The algorithm, which does not shy away from negative weights, accommodates score functions that produce negative crisp numbers, making it suitable for the considered example in this paper. Thus, the proposed algorithm is applicable to negative-weighted network problems, offering flexibility for utilizing any relevant score function to find the shortest path.

**Table 12.** Table shows the crisp number for IVNSP calculated using the score function for the taken IVN network

| Score function proposed by authors | Optimum Cost ($\mathscr{S}(e_{ij})$) for IVNSP $v_1 - v_3 - v_5$ |
|---|---|
| Boulturk et al. | 1.22805 |
| Ridvan et al. | 0.435 |
| Peng et al. | 0.67 |
| Liu et al | 2.01 |
| Harish et al. | 0.4181 |
| Smarandache et al. | 0.67 |
| Nagarajan et al. | 0.947 |

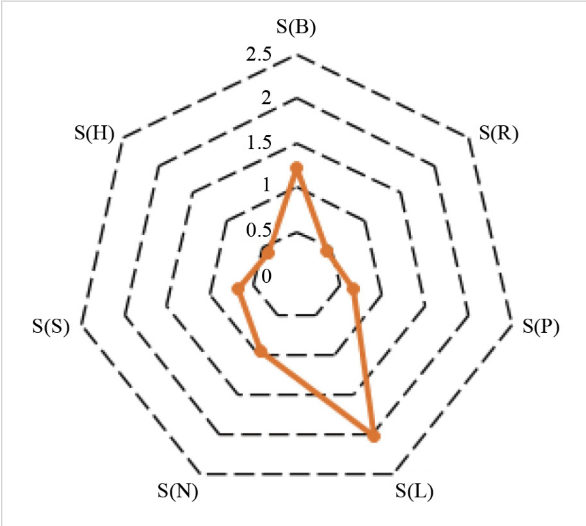


**Figure 5.** Graph representation for the table 7 value

Table 12 and Figure 5 elucidate the crisp weights of IVNSP obtained from the proposed algorithm. These weights are calculated using different score functions proposed by various authors. The article concludes that decision-makers have the freedom to select any score function for ranking and finding the SP. Several score functions are listed and tabulated, along with discussions on the types of graphical algorithms to which each score function can be applied. The numerical problem serves as an implementation example for the proposed algorithm, utilizing the score function from Nagarajan et al. [53]. The algorithm yields an IVNSP for all nodes from the source node, and the results, including the optimum cost and path for each iteration, are tabulated.

A line graph illustrating the crisp weights obtained for the considered network using various score functions provides insights into which type of score function is suitable for graph algorithms with both negative and positive edge-weighted directed graphs.

## 5.5 *Comparative study of the proposed algorithm*

The proposed algorithm facilitates the determination of SP for all nodes originating from a specified source node. The evaluation process is straightforward, and subsequent calculations yield all pairs of the shortest paths to each node from the origin. Each iteration provides clarity on the SP with the minimum cost for the entire network. In comparison, Nayankumar et al. [55] employed the transition path matrix in Floyd's Algorithm to identify all pairs of INVSP for the network. The proposed algorithm incorporates a score function by Nagarajan et al. to determine the minimum interval-valued cost of IVNSP. Table 13 in this paper presents all possible paths from a source node to a destination with IVNMC and its corresponding score value.

This article addresses both positive and negative weighted IVNTP using HIEPA, while Broumi et al. only discussed positive weighted IVNTP in their article.

Table 13. Table shows the crisp number for IVNSP calculated using the score function for the taken IVN network

| Multiple paths to reach destination node $v_5$ | IVNMC | $(\mathscr{S}_{Nagarajan})$ | ranking | $(\mathscr{S}_{Haris})$ | Ranking |
|---|---|---|---|---|---|
| $a$: $v_1-v_3-v_5$ | $\langle[0.37, 0.52], [0.1, 0.18], [0.24, 0.35]\rangle$, | 0.947 | 1 | 0.4181 | 1 |
| $b$: $v_1-v_2-v_3-v_5$ | $\langle[0.72, 0.83], [0.02, 0.06], [0.06, 0.126]\rangle$ | 1.2792 | 4 | 0.8407 | 5 |
| $c$: $v_1-v_2-v_4-v_5$ | $\langle[0.86, 0.52], [0.1, 0.18], [0.24, 0.35]\rangle$ | 1.1922 | 3 | 0.5583 | 2 |
| $d$: $v_1-v_3-v_4-v_5$ | $\langle[0.87, 0.95], [0.01, 0.04], [0.004, 0.02]\rangle$ | 1.3806 | 5 | 0.9570 | 6 |
| $e$: $v_1-v_2-v_3-v_4-v_5$ | $\langle[0.89, 0.98], [0.002, 0.012], [0.001, 0.72]\rangle$ | 1.78306 | 6 | 0.6974 | 3 |
| $f$: $v_1-v_4-v_5$ | $\langle[0.72, 0.82], [0.12, 0.24], [0.01, 0.08]\rangle$ | 1.0844 | 2 | 0.7162 | 4 |

Figure 6 illustrates all conceivable paths leading from the origin node to the destination. The nodes in the graph depict path weights of all possible routes to reach the destination, and the proposed algorithm calculates minimum path. This minimum path is then compared with all other potential paths, and the conclusion is drawn that the obtained path represents the Shortest Path.

Examining Figure 6, node $a$ stands out as the minimum compared to all other nodes. Consequently, the corresponding path for that node, $v_1-v_3-v_5$, is identified as the required IVNSP. The crisp weight for this path is 0.947 based on IVNMC $\langle[0.37, 0.52], [0.1, 0.18], [0.24, 0.35]\rangle$.

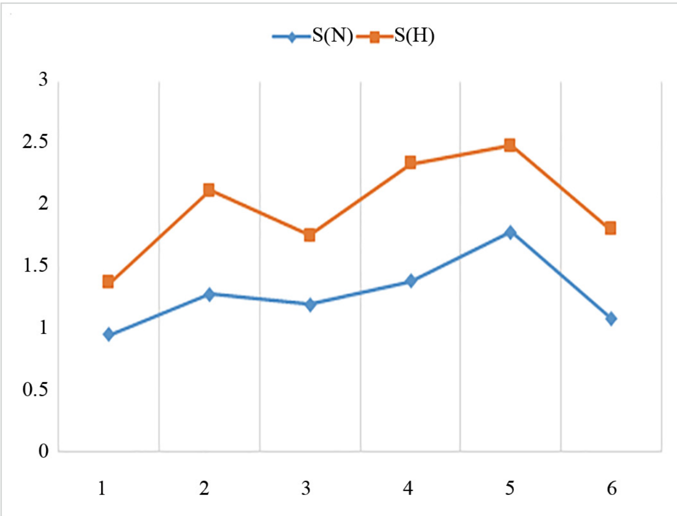


Figure 6. Line graph for all possible paths from origin node to destination node

Table 14. Comparison between existing method and proposed HIEPA

| Method | IVNMC | IVNSP | Score values |
|---|---|---|---|
| Broumi et al. [39] | $\langle[0.37, 0.52], [0.1, 0.18], [0.24, 0.35]\rangle$ | $v_1-v_3-v_5$ | 0.95 and 0.42 |
| Proposed HIEPA | $\langle[0.37, 0.52], [0.1, 0.18], [0.24, 0.35]\rangle$ | $v_1-v_3-v_5$ | 0.947 and 0.4181 |

Table 14 depicts the result obtained by HIEPA and Broummi et al., algorithm for finding the shortest path, it is verified by Table 13 data; also the score function used provided that the result obtained by HIEPA was minimum. This article gives the case study about score function with the help of HIEPA and its result is tabulated.

## 6. Conclusion

This article concludes that decision-makers have the flexibility to choose any score function for ranking purposes and determining the Shortest Path. Various score functions are listed and tabulated, and the discussion delves into which type of score function is suitable for specific graphical algorithms. The proposed algorithm is implemented using a numerical problem, employing the score function by Nagarajan et al. [53]. It yields an Interval-Valued Neutrosophic Shortest Path (IVNSP) for all nodes from the source, tabulated with the optimal cost and path for each iteration.

A line graph illustrating crisp weights for the considered network using different score functions clarifies which type of score function is applicable for graph algorithms involving both negative and positive edge-weighted directed graphs. The line graph detailing all possible paths provides comprehensive insights into the Shortest Path obtained by the proposed algorithm, elucidated through a numerical example network with interval-valued Neutrosophic weighted edges from Nayankumar et al. [55]. The algorithm doesn't solely focus on the IVNSP from the source to the destination node but also considers all other nodes.

This article introduces a novel algorithmic approach to solving Interval-Valued Neutrosophic network problems, analyzed through a numerical problem encompassing both negative and positive weighted edges. The HIEPA method proves valid for both Single-Valued and Interval-Valued neutrosophic transportation network problems, covering both negative and positive weighted networks. In contrast, Broumi et al. discuss positive weighted edge NS in their algorithm but not negative weighted ones, while the new HIEPA addresses both aspects of edge weights. This is briefly elaborated with examples and graphs.

Furthermore, the article provides a study on score functions, exploring which types can be applied to the nature of graphical algorithms. In the future, this algorithm will be implemented in plithogenic graphs, Neutrosophic hypergraphs, Neutrosophic soft graphs, etc.

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The authors declare there is nothing to disclose confidential for this article.

## Confilict of interest

The authors declare no competing financial interest.

## References

[1] Zadeh L. Fuzzy sets. *Information and Control*. 1965; 8(3): 338-353. Available from: https://doi.org/10.1016/S0019-9958(65)90241-X.
[2] Atanassov K. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*. 1986; 20: 87-96.
[3] Smarandache F. *A unifying field in logic. Neutrosophy: Neutrosophic probability, set and logic*. American Research Press, Rehoboth; 1998.
[4] Smarandache F. A geometric interpretation of the neutrosophic set-A generalisation of the intuitionistic fuzzy set. In: *2011 IEEE International Conference on Granular Computing*. Kaohsiung, Taiwan: IEEE; 2011. p.602-606. Available from: https://doi.org/10.1109/GRC.2011.6122665.
[5] Atanassov K, Gargov G. Interval valued intuitionstic fuzzy sets. *Fuzzy Sets and systems*. 1989; 31: 343-349.

[6] Turksen I. Interval valued fuzzy sets based on normal forms. *Fuzzy Sets and Systems*. 1986; 20: 191-120.
[7] Wang H, Smarandache F, Zhang Y, Sunderraman R. Single valued neutrosophic sets. *Multi Space and Multi Structure*. 2010; 4: 410-413.
[8] Wang H, Smarandache F, Zhang Y, Sunderraman R. *Interval Neutrosophic Sets and Logic: Theory and Application in Computing*. AZ. Hexis: Phoenix; 2005. Available from: http://fs.gallup.unm.edu/NSS.
[9] Şahin R, Liu PD. Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. *Neural Computing and Applications*. 2015; 27: 2017−2029. Available from: https://doi.org/10.1007/s00521-015-1995-8.
[10] Zhang HY, Wang JQ, Chen XH. Interval neutrosophic sets and their application in multicriteria decision making problems. *Scientific World Journal*. 2014; 2014: 645953. Available from: https://doi.org/10.1155/2014/645953.
[11] Gao Y. Shortest path problem with uncertain arc lengths. *Computers and Mathematics with Applications*. 2011; 62: 2591-2600.
[12] Sengupta A, Pal TK. Solving the shortest path problem with interval arcs. *Fuzzy Optimisation and Decision Making*. 2006; 5: 71-89.
[13] Jayagowri P, Ramani GG. Using trapezoidal intuitionistic fuzzy number to find optimised path in a network. *Advances in Fuzzy Systems*. 2014; 2014: 183607.
[14] Kumar A, Kaur M. A new algorithm for solving shortest path problem on a network with imprecise edge weight. *Applications and Applied Mathematics*. 2011; 6(2): 602-619.
[15] Kumar A, Kaur M. Solution of fuzzy maximal flow problems using fuzzy linear programming. *World Academy of Science and Technology*. 2011; 87: 28-31.
[16] Majumder S, Pal A. Shortest path problem on intuitionistic fuzzy network. *Annals of Pure and Applied Mathematics*. 2013; 5(1): 26-36.
[17] Okada S, Gen M. Fuzzy shortest path problems. *Computers and Industrial Engineering*. 1994; 27(1-4): 465-468.
[18] Dijikstra EW. A note on two problems in connexion with graphs. *Numerishe Mathematik*. 1959; 1: 269-271. Available from: https://doi.org/10.1007/BF01386390.
[19] Broumi S, Bakal A, Smarandache MTF, Vladareanu L. Applying Dijkstra algorithm for solving neutrosophic shortest path problem. *IEEE International Conference on Advanced Mechatronic Systems (ICAMechS)*. 2016; 37: 412-−416.
[20] Broumi S, Talea M, Bakali A, Smarandache F. Single valued neutrosophic graphs. *Journal of New Theory*. 2016; 10: 86-101.
[21] Broumi S, Talea M, Bakali A, Smarandache F. On bipolar single-valued neutrosophic graphs. *Journal of New Theory*. 2016; 11: 84-102.
[22] Harish G. An improved score function for ranking neutrosophic sets and its application to decision-making process. *International Journal for Uncertainty Quantification*. 2016; 6(5): 377-385. Available from: https://doi.org/10.1615/Int.J.UncertaintyQuantification.2016018441.
[23] Xindong P, Jingguo D. Algorithms for interval neutrosophic multiple attribute decision-making based on MABAC similarity measure and EDAS. *International Journal for Uncertainty Quantification*. 2017; 7(5): 395-421.
[24] Said B, Florentin S, Mohamed T, Assia B. Single valued neutrosophic graphs: Degree, order and size. *2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*. IEEE; 2016. p.2444-2451.
[25] Florentin S. *Symbolic Neutrosophic Theory*. Infinite Study; 2015.
[26] Smarandache F. Types of neutrosophic graphs and neutrosophic algebraic structures together with their applications in technology, seminar. *Transilvania University of Brasov Faculty of Product and Environmental Design Brasov Romania*. 2015; 6.
[27] Kandasamy V, Ilanthenral K, Smarandache F. *Neutrosophic graphs: A new dimension to graph theory*. Infinite Study; 2015.
[28] Broumi S, Bakali A, Talea M, Smarandache F. Computation of shortest path problem in a network with single valued neutrosophic number based on ranking method. In: *Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne*. UNM Digital Repository; 2016. p.417-422.
[29] Said B, Assia B, Mohamed T, Florentin S, Mumtaz A. Shortest path problem under bipolar neutrosphic setting. *Applied Mechanics and Materials*. 2017; 859: 59-66.
[30] Said B, Assia B, Mohamed T, Florentin S, Luige V. Applying Dijkstra algorithm for solving neutrosophic shortest path problem. In: *2016 International Conference on Advanced Mechatronic Systems (ICAMechS)*. IEEE; 2016. p.412-416.

[31] Said B, Mohamed T, Assia B, Florentin S. Application of Dijkstra algorithm for solving interval valued neutrosophic shortest path problem. In: *2016 IEEE Symposium Series on Computational Intelligence (SSCI)*. IEEE; 2016. p.1-6.

[32] Neha B, Guide KA. *Algorithms for some fuzzy network problems using ranking function*. Master Thesis; 2009.

[33] Said B, Assia B, Mohamed T, Florentin S. Isolated single valued neutrosophic graphs. *Infinite Study*. 2016; 11: 74-78.

[34] Said B, Assia B, Mohamed T, Florentin S, Luige V. Shortest path problem under triangular fuzzy neutrosophic information. In: *2016 10th International Conference on Software, Knowledge, Information Management & Applications (SKIMA)*. IEEE; 2016. p.169-174.

[35] Said B, Mohamed T, Assia B, Florentin S, Kishore KP. Shortest path problem on single valued neutrosophic graphs. In: *2017 International Symposium on Networks, Computers and Communications (ISNCC)*. IEEE; 2017. p.1-6.

[36] Said B, Mohamed T, Assia B, Florentin S. Shortest path problem under trapezoidal neutrosophic information *Computing Conference 2017*. London, UK; 2017. p.142-148.

[37] Said B, Assia B, Mohamed T, Florentin S, Luige V. Computation of shortest path problem in a network with SV-trapezoidal neutrosophic numbers. In: *2016 International Conference on Advanced Mechatronic Systems (ICAMechS)*. IEEE; 2016. p.417-422.

[38] Said B, Assia B, Mohamed T, Florentin S, Mumtaz A. Shortest path problem under bipolar neutrosphic setting. *Applied Mechanics and Materials*. 2017; 859: 59-66.

[39] Said B, Assia B, Mohamed T, Florentin S, Kishore KPK, Sahin R. Shortest path problem under Inter-valued neutrosphic setting. *International Journal of Advanced Trends in Computer Science and Engineering*. 2019; 8(1): 216-222. Available from: https://doi.org/10.30534/ijatcse/2019/4081.12019.

[40] Rıdvan Ş, Peide L. Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. *Neural Computing and Applications*. 2016; 27: 2017-2029.

[41] Peide L, Xinli Y. Interval neutrosophic Muirhead mean operators and their application in multiple attribute group decision-making. *International Journal for Uncertainty Quantification*. 2017; 7(4): 303-334.

[42] Abdel-Basset M, Gunasekaran M, Mohamed M, Chilamkurti N. A framework for risk assessment, management and evaluation: Economic tool for quantifying risks in supply chain. *Future Generation Computer Systems*. 2019; 90(1): 489-502.

[43] Abdel BM, Mai M, Kumar SA, Vipul J. An integrated neutrosophic AHP and SWOT method for strategic planning methodology selection. *Benchmarking: An International Journal*. 2018; 25(7): 2546-2564.

[44] Mohamed AB, Mai M, Florentin S. A hybrid neutrosophic group ANP-TOPSIS framework for supplier selection problems. *Symmetry*. 2018; 10(6): 226.

[45] Mohamed AB, Mai M, Florentin S. An extension of neutrosophic AHP-SWOT analysis for strategic planning and decision-making. *Symmetry*. 2018; 10(4): 116.

[46] Mohamed AB, Mai M, Florentin S, Victor C. Neutrosophic association rule mining algorithm for big data analysis. *Symmetry*. 2018; 10(4): 106.

[47] Mohamed AB, Mai M, Victor C. NMCDA: A framework for evaluating cloud computing services. *Future Generation Computer Systems*. 2018; 86: 12-29.

[48] Mohamed AB, Yongquan Z, Mai M, Victor C. A group decision making framework based on neutrosophic VIKOR approach for e-government website evaluation. *Journal of Intelligent & Fuzzy Systems*. 2018; 34(6): 4213-4224.

[49] Mohamed AB, Mai M, Yongquan Z, Ibrahim H. Multi-criteria group decision making based on neutrosophic analytic hierarchy process. *Journal of Intelligent & Fuzzy Systems*. 2017; 33(6): 4055-4066.

[50] Mohamed AB, Mai M. The role of single valued neutrosophic sets and rough sets in smart city: Imperfect and incomplete information systems. *Measurement*. 2018; 124: 47-55. Available from: https://doi.org/10.1016/j.measurement.2018.04.001.

[51] Mohamed AB, Gunasekaran M, Abduallah G, Florentin S. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*. 2018; 22: 257-278.

[52] Eda B, Cengiz K. A novel interval-valued neutrosophic AHP with cosine similarity measure. *Soft Computing*. 2018; 22(15): 4941-4958. Available from: https://doi.org/10.1007/s00500-018-3140-y.

[53] Said B, Deivanayagampillai N, Assia B, Mohamed T, Florentin S, Malayalan L. The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment. *Complex & Intelligent Systems*. 2019; 5: 391-402. Available from: https://soi.org/10.1007/s40747-019-0092-5.

[54] Said B, Assia B, Mohamed T, Florentin S, Krishnan KP, Şahin R. Shortest path problem under interval valued neutrosophic setting. *Advanced Trends in Computer Science and Engineering*. 2019; 8(1): 216-212. Available from: https://soi.org/10.30534/ijatcse/2019/4081.12019.

[55] Nayankumar P, Ritu S, Manoj S. Floyd's algorithm for all-pairs interval-valued neutrosophic shortest path problems. In: *Mathematical Modeling Computational Intelligence Techniques and Renewable Energy: Proceedings of the First International Conference MMCITRE 2020*. Springer; 2021. p.463-473. Available from: https://doi.org/10.1007/978-981-15-9953-8_40.

[56] Akash S, Amrit D, Kumar BU, Lee GM. Prediction of transportation costs using trapezoidal neutrosophic fuzzy analytic hierarchy process and artificial neural networks. *IEEE Access*. 2021; 9: 103497-103512.

[57] Florentin S. The score, accuracy, and certainty functions determine a total order on the set of neutrosophic triplets (T, I, F). *Neutrosophic Sets and Systems*. 2020; 38: 1-10. Available from: https://digitalrepository.unm.edu/nss_journal/vol38/iss1/1.

[58] Muhammad A. Single-valued neutrosophic graphs. In: *Infosys Science Foundation Series in Mathematical Sciences*. Springer, Singapore; 2018. p.2363-6157. Available from: https://doi.org/10.1007/978-981-13-3522-8.

[59] Muhammad A, Saba NH. Implementation of single-valued neutrosophic soft hypergraphs on human nervous system. *Artificial Intelligence Review*. 2023; 56(2): 1387-1425.

[60] Muhammad A, Saba NH. Algorithms for the computation of regular single-valued neutrosophic soft hypergraphs applied to supranational asian bodies. *Journal of Applied Mathematics and Computing*. 2022; 68(6): 4479-4506. Available from: https://doi.org/10.1007/s12190-022-01714-1.

[61] Saba S, Uzma A, Muhammad A. A study on generalized graphs representations of complex neutrosophic information. *Journal of Applied Mathematics and Computing*. 2021; 65: 481-514. Available from: https://doi.org/10.1007/s12190-020-01400-0.

[62] Kanchana M, Kavitha K. *A comparative study between incident edge path algorithm and dijkstra algorithm for finding shortest path for networks*. Nova Science Publishers; 2023. Available from: https://novapublishers.com/product-category/series/computer-science-.