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Research Article

Impact of Local Thermal Non-Equilibrium and Gravity Fluctuations on the Onset of a Darcy-Brinkman Porous Convection

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Abstract: The numerical investigation of the Darcy-Brinkman convective problem with gravity fluctuations in a Local Thermal Nonequilibrium (LTNE) model is conducted. Utilizing linear stability analysis, the convective problem is explored, and the numerical values of the Rayleigh and wave numbers for onset convection are computed through the one-term Galerkin approach. Three distinct types of gravity fluctuations (linear, parabolic, and exponential) are considered. The results show the Darcy number and gravity parameter delay the onset of convection. The porosity-scald conductivity ratio and interface heat transfer coefficient have a significant effect on the stability of the configuration. Graphical representations depict the effects of various parameters, highlighting the significant impacts of incorporating gravity fluctuations and non-equilibrium conditions in determining convection stability thresholds.

***Keywords*:** Darcy-Brinkman model, porous convection, changeable gravity, local thermal nonequilibrium

MSC: 65Lxx, 76E06, 80A20

Nomenclature

d	Depth of the layer
β	Coefficient of expansion
μ	Fluid viscosity
$\vec{q}$	Velocity vector
θ	Fluid phase in the basic conduction state temperature
Da	Darcy number
ϖ	Stream function, dimensionless
R	Rayleigh number
Θ	Perturbation state temperature
p	Pressure
H	Interface heat transfer coefficient

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κ	Thermal diffusivity
δ	Gravity parameter
T	Temperature
ρ	Density
Ψ	Perturbation of the stream function
ε	Porosity
$G(y)$	Variable gravity function
χ	Porosity-scald conductivity ratio
k	Horizontal wave number
α	Diffusivity ratio
ϕ	Fluid phase in the conduction state temperature

1. Introduction

The Brinkman effect in LTNE porous convection is applied in geothermal systems and underground energy storage to model and optimize heat transfer and fluid flow through porous media, influencing energy extraction and storage efficiency. Scientific investigations into the fluctuations in gravity are focused on several aspects of geophysics and basic physics experiments related to gravity, including gravitational wave observations. These variations are often regarded in geophysics as a signal that conveys data about phenomena like fault ruptures and changes in air density. It manifests as ambient noise in investigations involving basic physics, which must be avoided or reduced (see Harms [1]). Vafai [2, 3], Straughan [4], Panfilov [5], Wu [6], Xu et al. [7], and Nield [8] provide excellent reviews on thermosolutal convection in porous channel. The LTNE model takes into account the fact that the fluid and the solid porous matrix are at different temperatures. Many researchers worked in this field because of the better agreement with the physical situation using the LTNE model: Kuznetsov [9] used the porous medium to investigate forced convection under a thermal non-equilibrium approach. Banu and Rees [10] used the Darcy model to investigate the outset of convection using an LTNE assumption. Assuming LTNE, Bhadauria and Agarwal [11] studied the nanofluid-saturated porous layer. The Brinkman model for Bénard convection and heat transfer in the presence of LTNE was further studied by Saravanan and Sivakumar [12], Celli et al. [13], Nield and Kuznetsov [14], Kuznetsov and Nield [15], and Siddheshwar and Siddabasappa [16]. While Gandomkar and Grey [17] studied the relationship between LTNE and heat conduction in a porous channel, Parhizi et al. [18] used an LTNE approach in a porous channel to find a non-constant Biot number under a fully developed flow.

By studying the porous bed arrangement's gravity fluctuation with internal heating, Alex and Patil [19] found that a delay in the gravity factor makes the structure more stable. Suma et al. [20] and Gangadharaiah et al. [21] studied the effects of linear gravity fluctuation with throughflow and internal heating in a porous bed design using the regular perturbation approach. For the porous channel, Nagarathnamma et al. [22] used the Galerkin method to study the effects of a gravity variation and Yadav [23] investigated the effects of a magnetic field and a throughflow. It is quite surprising that researchers have paid so little attention to the consequences of altering gravity in a fluid layer. When convective motion occurs due to non-uniformity in the thermophoresis parameter, Mahajan and Tripathi [24] investigated the effects of gravity fluctuation on the stability of a thermosolutal convective flow. Gangadharaiah et al. [25, 26] have examined the penetrative solutal convective motion in a fluid layer arrangement with dynamic gravity and throughflow and temperature-dependent viscosity with changing gravity about cross-diffusive terms. When a heat source and thermal profiles were present, Varalakshmi et al. [27] investigated the effects of LTNE on a two-layer structure. There are several studies available for composite systems without LTNE [28-30].

Examining the consequences of gravitational force fluctuations and the LTNE phenomenon at the start of Darcy-Brinkman porous convection was the primary goal of this study. This investigation encompassed an examination of three distinct categories of gravity fluctuation. The application of the one-tern Galerkin technique enables the numerical solution of the eigenvalue problem. The visual representation showcases the effect of the gravity parameter and LTNE on the Rayleigh number. A comprehensive explanation of the detailed conclusion is provided.

2. Problem statement

The simplified configuration of Darcy-Brinkman convection is depicted in Figure 1. The infinite porous bed bounded by the lower surface $y = 0$ and upper surface $y = d$ with a downward gravity fluctuation $g(y) = (1 + \delta h(y))g$. Under the assumption of substantial form-drag and boundary effects, with an isotropic porous medium and the exclusion of local thermal equilibrium, the governing equations consist of the continuity equation, a suitably expanded Darcy's law, and the energy equation incorporating the Boussinesq approximation.

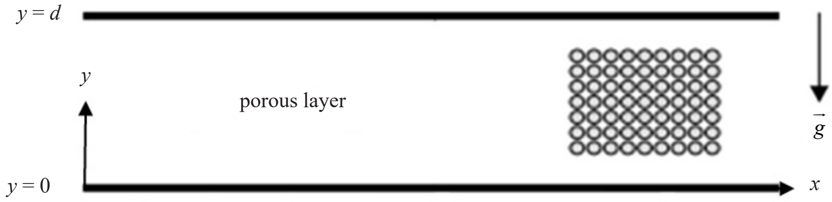

Figure 1. Physical configuration

The governing equations for the present model (see Postelnicu and Rees [31] and Postelnicu [32]) are

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{\rho_f}{\varepsilon^2}\left(\vec{q} \cdot \nabla \vec{q}\right) + \frac{\mu_f}{K}\vec{q} + \frac{\rho_f b}{\sqrt{K}}\vec{q}\left|\vec{q}\right| = -\nabla^* p^* + \mu_e \nabla^2 \vec{q} + \rho_f \beta\left(T - T_c\right) g(y)\,\hat{y} \tag{2}$$

$$\varepsilon(\rho c)_f \frac{\partial T_f}{\partial t} + (\rho c)_f \left(\vec{q} \cdot \nabla\right) T_f = \varepsilon \kappa_f \nabla^2 T_f + h\left(T_s - T_f\right) \tag{3}$$

$$(1-\varepsilon)(\rho c)_s \frac{\partial T_s}{\partial t} = (1-\varepsilon)\kappa_s \nabla^2 T_s - h\left(T_s - T_f\right) \tag{4}$$

With boundary conditions

$$\vec{q} = 0, \text{ at } y = 0,\ y = d \text{ and } T = T_h \text{ at } y = 0, T = T_c \text{ at } y = d \tag{5}$$

Introducing the non-dimensional

$$\text{Scheme: } \vec{x}^* = \frac{\vec{x}}{d},\ p^* = \frac{\mu \kappa_f p}{(\rho c)_f K},\ \theta = \frac{T_f - T_c}{T_h - T_c},\ \vec{q}^* = \frac{\varepsilon \kappa_f \vec{q}}{(\rho c)_f d},\ \phi = \frac{T_s - T_c}{T_s - T_c},\ t^* = \frac{(\rho c)_f d^2 t}{\kappa_f} \tag{6}$$

Introducing the above, (1)-(4) becomes:

$$\nabla \cdot \vec{q} = 0 \tag{7}$$

$$\varepsilon F_1 \frac{\partial \vec{q}}{\partial t} + F_1 \left(\vec{q} \cdot \nabla \vec{q} \right) + \frac{\mu_f}{K} \vec{q} + F_2 \vec{q} \left| \vec{q} \right| = -\varepsilon^2 F_1 \nabla p + Da \nabla^2 \vec{q} - \vec{q} + R\, G(y)\, \hat{y} \tag{8}$$

$$\frac{\partial \theta}{\partial t} + \left(\vec{q} \cdot \nabla \right) \theta = \nabla^2 \theta + H(\phi - \theta) \tag{9}$$

$$\alpha \frac{\partial \phi}{\partial t} = \nabla^2 \phi + \chi H \left(\phi - \theta \right) \tag{10}$$

The conditions are

$$\vec{q} = 0,\ \phi = 1,\ \theta = 1\ \ at\ \ y = 0 \text{ and } \vec{q} = 0,\ \phi = 0,\ \theta = 0\ \ at\ \ y = 1 \tag{11}$$

Where $R = \dfrac{\alpha_f g \beta \Delta T\, K d}{\varepsilon \mu_f \kappa_f}$, $Da = \dfrac{\mu_e}{\mu_f} \cdot \dfrac{K}{d^2}$, $\chi = \dfrac{\varepsilon \kappa_f}{\left(1-\varepsilon\right)\kappa_s}$, $H = \dfrac{hd^2}{\varepsilon \kappa_f}$ are the Darcy-Rayleigh number, Darcy number, porosity-scald conductivity ratio and interface heat transfer coefficient respectively. Also, $\alpha = \dfrac{(\rho c)_s}{(\rho c)_f} \cdot \dfrac{\kappa_f}{\kappa_s}$, $G(y) = (1 + \delta h(y))$, $F_1 = \dfrac{\kappa K \rho_f}{\varepsilon^2 d^2 \mu_f}$, $F_2 = \dfrac{\kappa \sqrt{K} \rho_f}{d\, \mu_f}$.

Examining the two-dimensional example, we delve into the fundamental conduction profile and explore its stability in this investigation.

$$\varpi = 0, \phi = 1 - y, \theta = 1 - y,\ \phi = 1 - y + \Phi(x, y), \varpi = \varpi(x, y), \theta = 1 - y + \Theta(x, y) \tag{12}$$

and partial differential equations are

$$\frac{\partial^2 \varpi}{\partial x^2} + \frac{\partial^2 \varpi}{\partial y^2} - Da\left(\frac{\partial^4 \varpi}{\partial x^4} + 2\frac{\partial^4 \varpi}{\partial x^2 \partial y^2} + \frac{\partial^4 \varpi}{\partial y^4} \right) = R\frac{\partial \Theta}{\partial x} G\left(y\right) \tag{13}$$

$$\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial \varpi}{\partial x} + H\left(\Phi - \Theta\right) = 0 \tag{14}$$

$$\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \chi H\left(\Theta - \Phi\right) = 0 \tag{15}$$

Considering the boundary settings

$$\varpi = 0,\ \frac{\partial \varpi}{\partial y} = 0,\ \Theta = 0,\ \Phi = 0 \text{ on } y = 0 \text{ and } y = 1 \tag{16}$$

Eqs. (13)-(15) admit the possible solutions can be found as

$$\varpi = f_1(y)\sin(kx),\ \Theta = g_1(y)\cos(kx),\ \Phi = h_1(y)\cos(kx) \tag{17}$$

$$Da\left(f_1^{iv} - 2k^2 f_1'' + k^4 f_1\right) - \left(f_1'' - k^2 f_1\right) = Rkg_1' G\left(y\right) \tag{18}$$

$$g_1'' - \left(k^2 + H\right) g_1 + kf_1 + Hh_1 = 0 \tag{19}$$

$$h_1'' - \left(k^2 + \chi H\right) h_1 + \chi H g_1 = 0 \tag{20}$$

The boundary conditions are leads to the form

$$f_1 = f_1' = g_1 = h_1 = 0 \text{ on } y = 0 \text{ and } y = 1 \tag{21}$$

3. Methodology

An approach based on Galerkin-type weighted residuals is used to find a numerical solution, in which three variables, f_1, g_1 and h_1 are considered as

$$f_1 = \sum_{i=1}^{N} \mathrm{A}_i f_{1i}, \; g_1 = \sum_{i=1}^{N} \mathrm{B}_i g_{1i} \text{ and } h_1 = \sum_{i=1}^{N} C_i h_{1i} \quad n = 1, 2, 3.... \tag{22}$$

where A_i, B_i and C_i are constants. The one-term Galerkin approach has been applied, the trial functions f_1, g_1 and h_1 are assumed as $f_2 = y^2(1-y)^2$, $g_2 = y(1-y)$, $h_2 = y(1-y)$ satisfying the boundary conditions (21) is mentioned above. Using trial functions f_1, g_1 and h_1 and integrating over [0, 1] to get system of homogeneous equations, solving which we obtain the expression for Rayleigh number for marginal stability

$$R = \frac{\left(\dfrac{12+k^2}{6}\right)\left(\dfrac{k^4}{20}+24\right)}{P_1 P_2} + \frac{P_3}{P_2} \tag{23}$$

Where

$$P_1 = \int_0^1 k^2 G\left(y\right) y^2 \left(1-y\right)^2 dy, \; P_2 = \int_0^1 y\left(1-y\right) dy, \; P_3 = \int_0^1 y\left(1-y\right) dy.$$

4. Results and discussion

A study examines how variations in gravity affect the onset of LTNE convection in a porous bed that is laid flat. To determine the onset convection, the one-term Galerkin method is used to precisely find the values of the Rayleigh and wave numbers values. The following types of gravity fluctuations are considered, the first model is linear $h(y) = -y$, the second model is parabolic $h(y) = -y^2$, and the third model is exponential $h(y) = -(e^y - 1)$.

Figure 2 displays neutral steadiness curves with the influence of gravity variations between the Rayleigh and wave numbers. Any kind of disturbance below this curve always results in a stable configuration; however, some specific wave number values above this curve cause the configuration to become unstable. Further, it is observed that the gravitational fluctuation of the exponential type is more stable than that of the linear and parabolic types.

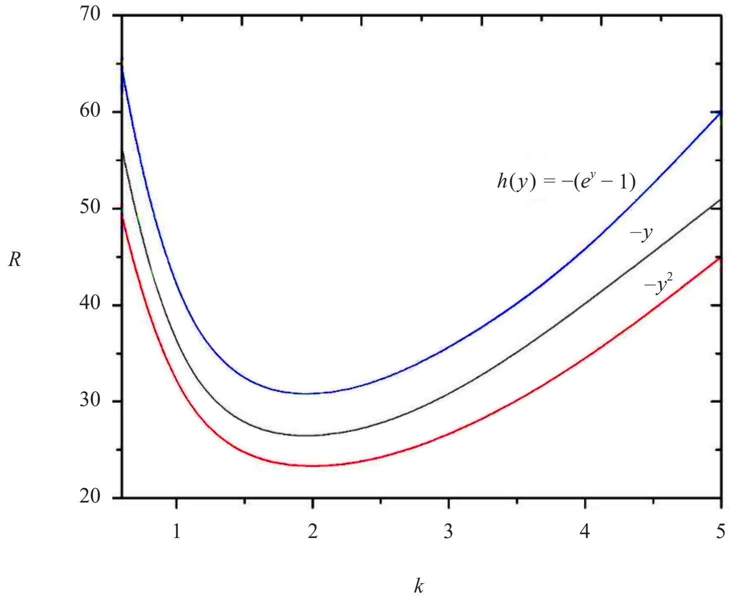

Figure 2. Impact of R verses k for all three types of gravity fluctuation with $Da = 0.001$

Figures 3, 4, and 5 depict the variation of Rayleigh number R against the wave number k for different values of the gravity parameter δ across all three types of gravity fluctuation functions. The trends show that an increase in the gravity parameter corresponds to a higher eigenvalue R, signifying a stabilizing impact on the system configuration. Additionally, it is observed that the stability of the system is more pronounced with exponential-type gravitational fluctuation compared to linear and parabolic fluctuations.

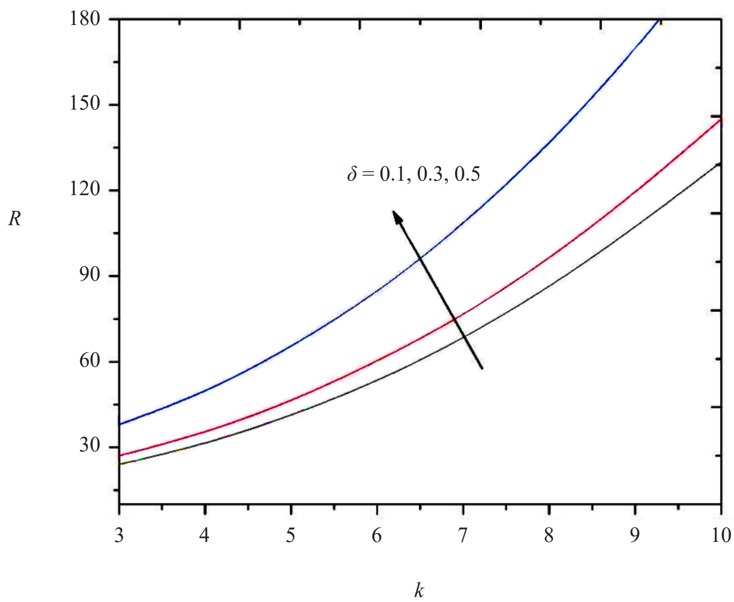

Figure 3. The effects of R verses k for different values of δ for linear model $h(y) = -y$ gravity fluctuation with $\chi = 0.3$

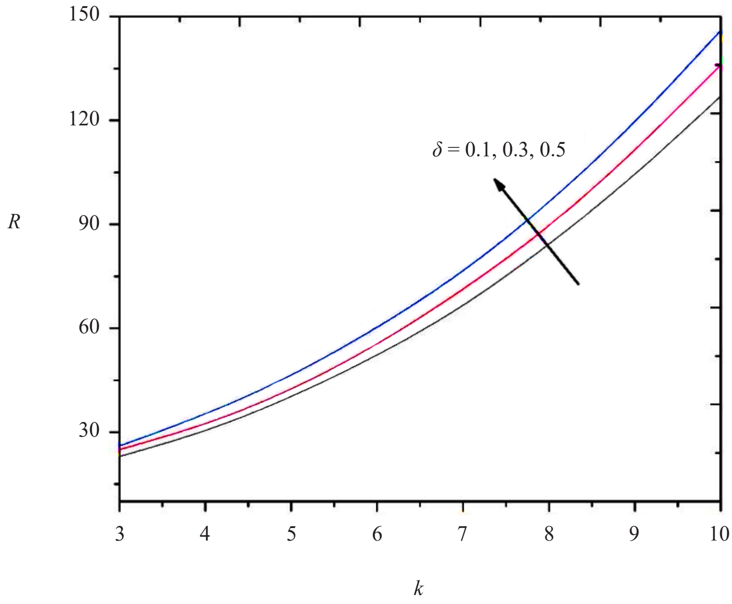

Figure 4. Effect of R verses k for several values of δ for parabolic model $h(y) = -y^2$ gravity fluctuation with $\chi = 0.3$

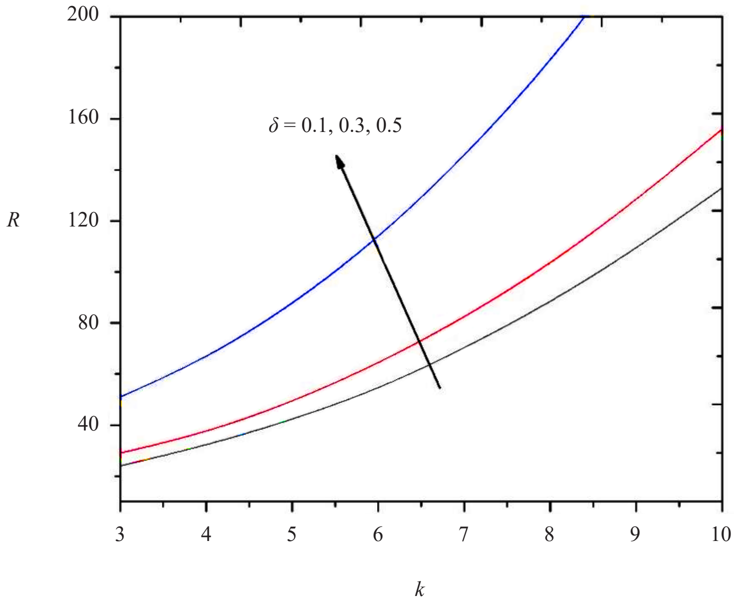

Figure 5. The variation of R verses k for various values of δ for exponential model $h(y) = -(e^y - 1)$ gravity fluctuation

Figure 6 portrays the effects of the R^c and $Log_{10}H$ for linear gravity field fluctuation. According to Bhadauria and Agarwal [11], as $H \rightarrow 0$ or ∞, the value of R^c is invariant, which means that an increase in R^c may be detected only for transitional values of $Log_{10}H$, which is the LTNE zone. When $H \rightarrow 0$, the heat transmission between the porous matrix and the fluid phase is practically insignificant, and they can be regarded in thermal equilibrium. As $H \rightarrow \infty$ has a comparable impact because the heat transfer between these two phases is too quick in this case, resulting in a thermal equilibrium situation. During the heat transmission process in the LTNE region, the solid phase absorbs heat from the fluid phase. Also, note that the R^c rises as the H rises and χ decreases. Figure 7 depicts the impacts of all three forms of gravity fluctuation functions; it can be seen from this figure that the same trend of exponential-type gravitational fluctuation is more stable than the other two types of gravity fluctuation.

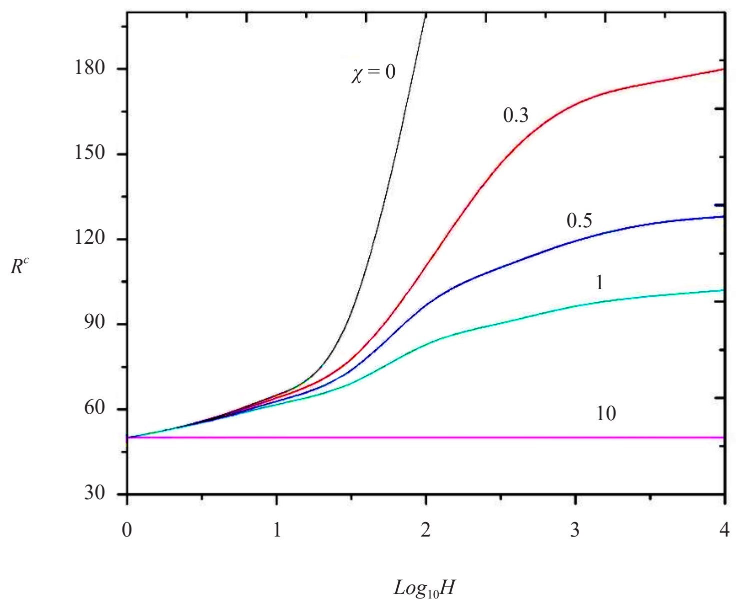

Figure 6. Impact of R^c verses $Log_{10}H$ for various values of χ for linear model $h(y) = -y$ gravity fluctuation

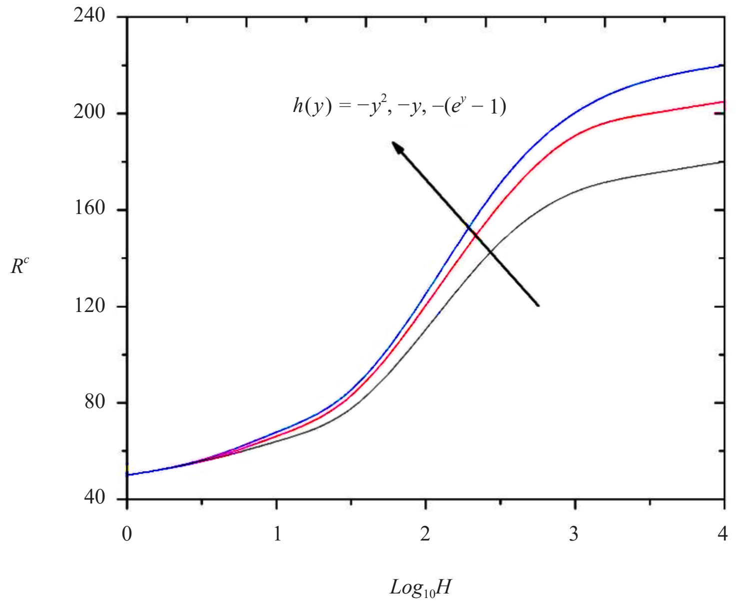

Figure 7. The effects of R^c verses $Log_{10}H$ for all three types of gravity fluctuation with $\chi = 0.3$

Figures 8, 9 and 10 show the effect of Darcy's number Da as a function of Rayleigh's number for all three types of gravity fluctuations. We can see from these graphs that as the R^c rises, correspondingly rises the Da. The quantity of Da is linked to the significance of viscous effects at the boundaries and decreases in Da result in a reduction of this impact. This facilitates the smoother flow of the fluid, ultimately leading to a decrease in R^c. Further, it is also noted that exponential-type gravitational fluctuation is more stable when compared to linear and parabolic-type fluctuation.

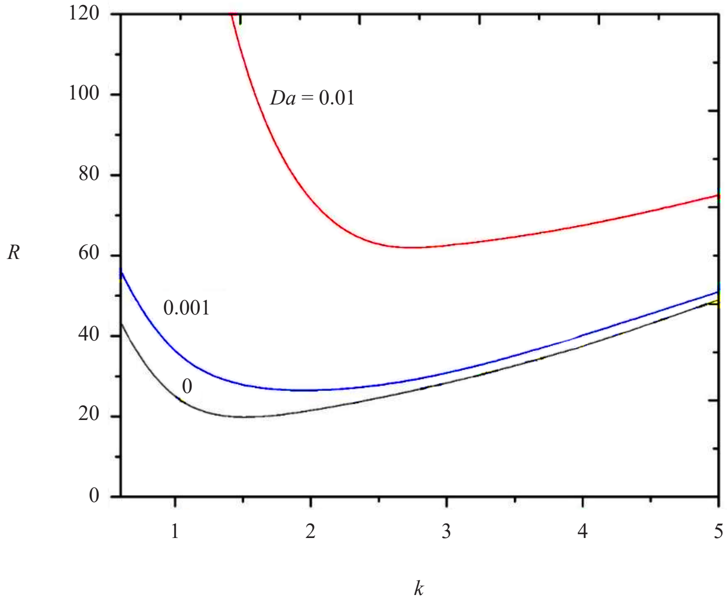

Figure 8. Effects of R verses k for various values of Da for linear model $h(y) = -y$ gravity fluctuation

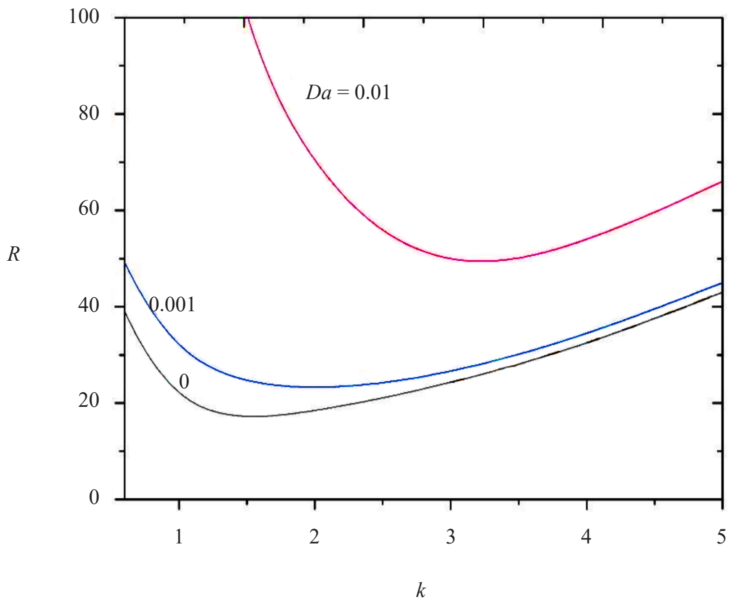

Figure 9. Variation of R verses k for various values of Da for parabolic model $h(y) = -y^2$ gravity fluctuation

5. Conclusion

This article explores the issue of LTNE onset convection in a porous bed, taking into account the influence of gravitational fluctuations. We conducted a comprehensive analysis of gravity functions and the effects of LTNE. Exploring the impact of different parameters on the system's stability is achieved through the utilization of linear instability. We utilized the one-term Galerkin approach to obtain numerical results. Based on the information provided, the following conclusions can be drawn:

• When Da is increased, the system configuration becomes more stable.

• At smaller amounts of R, convection begins when H drops or χ rises.

• A more stable system is the result of raising the value of the gravity parameter δ.

• The exponential-type gravitational fluctuation is more stable when compared to linear and parabolic-type fluctuation.

• This study's findings might be useful in several areas of geophysics and fundamental physics studies pertaining to gravity, such as gravitational wave observations, are the primary foci of scientific inquiries into the variations in gravity.

• In the future, it is important to consider the onset of convection with LTNE with other rheologies, including the gravity fluctuations, and Brinkman effect. For industrial applications as well as for crystals growing applications, it would also be important to investigate other types of boundary conditions. The approach developed here can be applied to those problems as well.

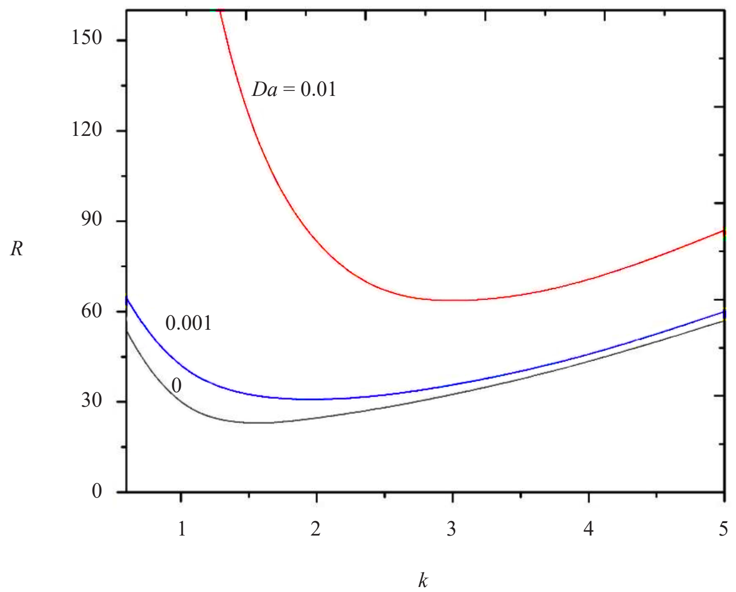

Figure 10. The impacts of R verses k for various values of Da for exponential model $h(y) = -(e^{y} - 1)$ gravity fluctuation

Conflict of interest

The authors do not have any competing interests to declare.

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