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Far-Field Behavior of Waves in Non-Ideal Magnetogasdynamics

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Abstract: We have presented a study on the far-field behavior of weak nonlinear waves in magnetogasdynamics. An asymptotic analysis is carried out for the study. An evolution equation is obtained by using an asymptotic method which helps in learning the far-field behavior of a hyperbolic quasilinear system governing the propagation of nonlinear waves in a non-ideal gas. A numerical technique MVIM is employed to obtain the approximate solution of the evolution equation.
Keywords: far-field, magnetogasdynamics, non-ideal gas, evolution equation

1. Introduction

The present paper deals with the study of weak nonlinear wave motion described by the hyperbolic system in the far-field, i.e., the field far away from the piston location. As we know that in the far-field any nonlinear convection is associated with the low frequency characteristic, therefore, the study of wave motion in the low frequency domain becomes a topic of one's great interest. The plasma is taken to be of non-ideal gas with an infinite electrical conductivity in which the viscosity is neglected along with the effects of the heat conduction. It is also assumed to be permeated by the azimuthal magnetic field.

In the study of nonlinear physical phenomena, the evolutive or asymptotic equation, which is derived from the hyperbolic system of partial differential equations, plays an important role because it represents the character of its parent system, (for references see Sharma et al. [1], Germain [2], Radha and Sharma [3] and Hunter and Keller [4]). And, since, we do not have the liberty to always obtain an exact analytical solution for partial differential equations, we have to rely on numerical methods to obtain approximate solutions. Asymptotic analysis of nonlinear waves has been the subject of great interest from both physical and mathematical points of views to understand the behavior of nonlinearity.

Our analysis is based upon the work of Jena and Sharma [5], and Manickam et al. [6]. Jena and Sharma have studied the far-field behavior of waves in an inviscid relaxing gas by making use of the method of matched asymptotic expansions to find an approximate solution; whereas Manickam et al. incorporated certain modifications first and then used a different numerical scheme called as the semidiscrete collocation method, to find the numerical solutions. A number of problems relating to the wave propagation in MGD have been studied previously in which the works of Oliveri and Speciale [7], Bira and R. Shekhar [8], Sharma et al. [9], Gupta et al. [10], Kumar et al. [11] and Arora [12] are worth-mentioning.

In our work, we have first derived the asymptotic equation from the governing equations, which is found to be a generalized inviscid Burgers' equation, by considering a formal expansion procedure which slowly varies the solution through the stretched coordinates. The Burgers' equation is assumed to be one of the most important equations in studying the nonlinearity of the propagation of the planar and the cylindrical waves in the medium. This asymptotic equation describes the far-field behavior of the hyperbolic system of governing equations of magnetogasdynamics. And then we have applied the MVIM method Abassy [13, 14] on it to obtain its approximate solution. We have two different initial conditions to study the problem. The numerical solutions, absolute errors and the graphs are shown at the end of this paper, highlighting the effects of nonlinearity and non-ideal parameter b.

2. Derivation of asymptotic equation

Let us consider plasma of non-ideal gas with an infinite electrical conductivity in which the effects of heat conduction and viscosity are neglected. We assume that an azimuthal magnetic field exists initially in the plasma and this magnetic field in the plasma remains azimuthal due to the assumption of infinite conductivity. We consider the following

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hyperbolic system for an unsteady one-dimensional planar (m = 1) and cylindrically (m = 2) symmetric motion of magnetogasdynamics [15, 16]

$$\begin{aligned}
&\rho_t + \rho u_r + u\rho_r + \frac{(m-1)\rho u}{r} = 0, \\
&u_t + u u_r + \frac{1}{\rho}(p_r + h_r) = 0, \\
&p_t + u p_r - a^2(\rho_t + u\rho_r) = 0, \\
&h_t + u h_r + 2h u_r + 2h(m-1)\frac{u}{r} = 0,
\end{aligned} \tag{1}$$

where u, ρ, p and γ are the gas velocity, gas density, gas pressure and the constant specific heat ratio, respectively; t is the time, r is the spatial coordinate which is either axial for planar flows (m = 1) or radial for cylindrically symmetric flows (m = 2), a is the speed of sound in an equilibrium medium given by $a^2 = \gamma p / \rho(1 - b\rho)$, h is the magnetic pressure defined by $h = \mu H^2 / 2$ with μ being the magnetic permeability and H being the transverse magnetic field. We consider the following equation of state

$$p\left(1 - b\rho\right) = \rho RT,$$

where R is the gas constant and T is the translational temperature.

Here, we are interested in studying the behavior of the wave motion described by the hyperbolic system (1) in the far-field, i.e., far away from the piston location. So, we begin by considering a solution when a wave is propagating with a constant speed U into a homogeneous quiescent gas. We seek a progressive wave-type solution of the system (1) which is a function of t and variable ξ, given by $\xi = r - Ut$. When this new coordinate system (ξ, t) is introduced into the system (1), it changes into the following system:

$$\begin{aligned}
&\rho_t + (u-U)\rho_\xi + \rho u_\xi + \frac{(m-1)\rho u}{\xi + Ut} = 0, \\
&u_t + (u-U)u_\xi + \frac{1}{\rho}(p_\xi + h_\xi) = 0, \\
&p_t + (u-U)p_\xi - \frac{\gamma p}{\rho\left(1 - b\rho\right)}\left(\rho_t + (u-U)\rho_\xi\right) = 0, \\
&h_t + (u-U)h_\xi + 2h u_\xi + \frac{2h(m-1)u}{\xi + Ut} = 0.
\end{aligned} \tag{2}$$

Let us now introduce a small parameter ε which represents the ratio of the attenuation length of the medium to the characteristic length of the medium. The low-frequency wave process is determined when $\varepsilon^2 \ll 1$. We, now, consider a formal expansion procedure which slowly varies solution of system (2) through the stretched coordinates (ζ, η), where $\zeta = \varepsilon\xi$ and $\eta = \varepsilon^2 t$. On introducing the stretched coordinates (ζ, η) into the system (2), it is transformed into the following system

$$\varepsilon\rho_\eta + (u-U)\rho_\zeta + \rho u_\zeta + \frac{(m-1)\,\rho u\varepsilon}{(\zeta\varepsilon + U\eta)} = 0, \tag{3}$$

$$\varepsilon u_\eta + (u-U)u_\zeta + \frac{1}{\rho}(p_\zeta + h_\zeta) = 0, \tag{4}$$

$$\varepsilon p_\eta + (u-U)p_\zeta - \frac{\gamma p}{\rho(1-b\rho)}\left(\rho_\eta\varepsilon + (u-U)\rho_\zeta\right) = 0, \tag{5}$$

$$\varepsilon h_\eta + (u-U)h_\zeta + 2hu_\zeta + \frac{2h(m-1)u\varepsilon}{(\zeta\varepsilon + U\eta)} = 0. \tag{6}$$

We look for an asymptotic solution of this system which exhibits the property of a progressive wave, of the form:

$$\rho(\zeta,\eta) = \rho_0 + \varepsilon\rho^{(1)}(\zeta,\eta) + \varepsilon^2\rho^{(2)}(\zeta,\eta) + \dots, \tag{7}$$

$$u(\zeta,\eta) = \varepsilon u^{(1)}(\zeta,\eta) + \varepsilon^2 u^{(2)}(\zeta,\eta) + \dots, \tag{8}$$

$$p(\zeta,\eta) = p_0 + \varepsilon p^{(1)}(\zeta,\eta) + \varepsilon^2 p^{(2)}(\zeta,\eta) + \dots, \tag{9}$$

$$h(\zeta,\eta) = h_0 + \varepsilon h^{(1)}(\zeta,\eta) + \varepsilon^2 h^{(2)}(\zeta,\eta) + \dots, \tag{10}$$

where ρ_0, p_0 and h_0 are the values of ρ, p and h, respectively, in the undisturbed region. Now, we introduce the equations (7)-(10) into the system of equations (3)-(6) with the initial conditions $\rho = \rho_0$, $u = 0$, $p = p_0$ and $h = h_0$. We obtain equations in various powers of ε. Out of which we equate the coefficients of ε and ε^2 to zero, and obtain the following sets of partial differential equations for the first and second-order variables:

$$O(\varepsilon):\; -U\rho_\zeta^{(1)} + \rho_0 u_\zeta^{(1)} = 0,\; -Uu_\zeta^{(1)} + \frac{1}{\rho_0}(p_\zeta^{(1)} + h_\zeta^{(1)}) = 0, \tag{11}$$

$$-Up_\zeta^{(1)} + \frac{\gamma p_0}{\rho_0(1-b\rho_0)}U\rho_\zeta^{(1)} = 0,\; -Uh_\zeta^{(1)} + 2h_0 u_\zeta^{(1)} = 0, \tag{12}$$

$$O(\varepsilon^2):\; \rho_\eta^{(1)} + u^{(1)}\rho_\zeta^{(1)} - U\rho_\zeta^{(2)} + \rho^{(1)}u_\zeta^{(1)} + \rho_0 u_\zeta^{(2)} + \frac{(m-1)\rho_0 u^{(1)}}{U\eta} = 0, \tag{13}$$

$$u_\eta^{(1)} + u^{(1)}u_\zeta^{(1)} - Uu_\zeta^{(2)} - \frac{\rho^{(1)}}{\rho_0^2}(p_\zeta^{(1)} + h_\zeta^{(1)}) + \frac{1}{\rho_0}(p_\zeta^{(2)} + h_\zeta^{(2)}) = 0, \tag{14}$$

$$p_{\eta}^{(1)}+u^{(1)}\,p_{\zeta}^{(1)}-Up_{\zeta}^{(2)}-\frac{\gamma p_0}{\rho_0\left(1-b\rho_0\right)}\rho_{\eta}^{(1)}-\frac{\gamma p_0}{\rho_0\left(1-b\rho_0\right)}u^{(1)}\rho_{\zeta}^{(1)}+\frac{\gamma p_0\,\rho^{(1)}b}{\rho_0\left(1-b\rho_0\right)^2}U\rho_{\zeta}^{(1)}+$$

$$\frac{\gamma p_0}{\rho_0\left(1-b\rho_0\right)}U\rho_{\zeta}^{(2)}-\frac{\gamma p_0}{\rho_0{}^2}\frac{\rho^{(1)}}{\left(1-b\rho_0\right)^2}U\rho_{\zeta}^{(1)}+\frac{\gamma Up^{(1)}}{\rho_0\left(1-b\rho_0\right)}\rho_{\zeta}^{(1)}=0, \tag{15}$$

$$h_{\eta}^{(1)}+u^{(1)}\,h_{\zeta}^{(1)}-Uh_{\zeta}^{(2)}+2h^{(1)}u_{\zeta}^{(1)}+2h_0u_{\zeta}^{(2)}+\frac{2h_0(m-1)u^{(1)}}{U\eta}=0. \tag{16}$$

The first set of equations (11)-(12) yields a non-trivial solution provided

$$U=a_0\sqrt{\left[\frac{2h_0\left(1-b\rho_0\right)}{\gamma p_0}+1\right]},\ \text{where}\ a_0=\sqrt{\frac{\gamma p_0}{\rho_0\left(1-b\,\rho_0\right)}}, \tag{17}$$

and, thus, gives the following relations satisfied by the first order variables:

$$\begin{aligned}\rho^{(1)}&=\frac{p^{(1)}}{a_0^2},\\ u^{(1)}&=\frac{U\left(1-b\,\rho_0\right)p^{(1)}}{\gamma p_0},\\ h^{(1)}&=\frac{2h_0\left(1-b\,\rho_0\right)p^{(1)}}{\gamma p_0}.\end{aligned} \tag{18}$$

We obtain the values for second order variables $\rho_{\zeta}{}^{(2)}, p_{\zeta}{}^{(2)}$ and $h_{\zeta}{}^{(2)}$ from equations (13)-(16), which on using the equation (18), give us the following transport equation for $p^{(1)}$:

$$p_{\eta}^{(1)}\left\{\frac{U\left(1-b\,\rho_0\right)}{\gamma p_0}+\frac{1}{\rho_0U}+\frac{2h_0\left(1-b\,\rho_0\right)}{\rho_0U\gamma p_0}\right\}+p^{(1)}\left\{\frac{(m-1)}{\rho_0U\eta}+\frac{2h_0\left(1-b\,\rho_0\right)(m-1)}{\rho_0U\eta\gamma p_0}\right\}$$

$$+p^{(1)}p_{\zeta}^{(1)}\left\{\left(\frac{U\left(1-b\,\rho_0\right)}{\gamma p_0}\right)^2+\frac{b}{\gamma p_0}+\frac{1}{\rho_0 p_0}+\frac{4h_0\left(1-b\,\rho_0\right)^2}{\rho_0(\gamma p_0)^2}\right\}=0. \tag{19}$$

The equation (19), on simplification, yields the following equation:

$$p_{\eta}^{(1)}+\frac{(m-1)}{2\eta}p^{(1)}+\left\{\frac{\left(6h_0\left(1-b\,\rho_0\right)^2+(\gamma+1)\gamma p_0\right)}{2\gamma p_0\sqrt{\rho_0\left(1-b\,\rho_0\right)\left(2h_0\left(1-b\,\rho_0\right)+\gamma p_0\right)}}\right\}p^{(1)}p_{\zeta}^{(1)}=0. \tag{20}$$

The equation (20) is called a generalized inviscid Burgers' equation or evolution equation. This equation is considered to be one of the most important equations to study in detail the nonlinear behavior of the propagation of waves both in the planar (m = 1) and cylindrically (m = 2) symmetric waves in magnetogasdynamics.

We introduce the non-dimensional parameters as follows:

$$\eta^* = \frac{\eta a_c}{x_c},\ \zeta^* = \frac{\zeta}{x_c},\ P = \frac{p^{(1)}}{\rho_c a_c^2}, \tag{21}$$

where x_c, a_c and ρ_c are some appropriate constants, which do not contain any parameter b.

This transformation reduces the equation (20) into the following form as:

$$\frac{\partial P}{\partial \eta} + \mu P P_\zeta + \frac{(m-1)}{2\eta} P = 0, \tag{22}$$

where $\mu = \left\{ \frac{\left(6h_0\left(1-b\rho_0\right)^2 + (\gamma+1)\gamma p_0\right)}{2\left(1-b\rho_0\right)\sqrt{\gamma p_0 (2h_0\left(1-b\rho_0\right) + \gamma p_0)}} \right\}.$

The equation (22) is called the transport equation, which describes the far-field behavior of the propagation of wave. The second term in it represents the effects of nonlinear convection and the third term represents the geometrical spreading. It may be noticed, here, that the equation (22) does not possess an exact solution, so here we use the Modified Variational Iteration Method (MVIM), to study the nonlinear behavior of the waves.

3. Basic concept of MVIM

To illustrate the basic concepts of MVIM, let us consider the following general non-linear initial value problem:

$$\begin{aligned} &AV(x,t) + BV(x,t) + CV(x,t) = f(x,t), \\ &V(x,0) = g_0(x), \\ &\frac{\partial V(x,t)}{\partial t}\Big|_{t=0} = g_1(x), \\ &\vdots \\ &\frac{\partial^{s-1} V(x,t)}{\partial t^{s-1}}\Big|_{t=0} = g_{s-1}(x), \end{aligned} \tag{23}$$

where $A = \frac{\partial^s}{\partial t^s}$, $s = 1, 2, 3, ...$, is the highest order partial derivative with respect to t, B is a linear operator and $CV(x, t)$ is the nonlinear term. $BV(x, t)$ and $CV(x, t)$ are free of partial derivatives with respect to t and $f(x, t)$ is the non-homogeneous term.

To use the Modified Variational Iteration Method for solving the nonlinear partial differential equation (23), the following iteration formula is used

$$\begin{aligned} V_{n+1}\left(x,t\right) = V_n\left(x,t\right) + \int_0^t \lambda(\zeta)\{&B(V_n - V_{n-1}) + (F_n - F_{n-1}) \\ &-\left(f_{ns}(x)\zeta^{ns} + f_{ns+1}(x)\zeta^{ns+1} + \cdots + f_{s(n+1)-1}(x)\zeta^{s(n+1)-1}\right)\}d\zeta, \end{aligned} \tag{24}$$

where λ is a Lagrange multiplier [17] which can be optimally obtained by variational theory and given by equation (25); the subscript "n" denotes the n-th approximation, BV_n and CV_n are considered to have restricted variations, i.e.

$$\delta\left(BV_n\right)=0,\ \delta\left(CV_n\right)=0.$$

The Lagrange multiplier is given by

$$\lambda(\zeta)=\frac{-(t-\zeta)^{(s-1)}}{(s-1)!}. \tag{25}$$

$F_n(x, t)$ is a polynomial of degree $(s(n+1)-1)$ and is obtained from $CV_n(x, t) = F_n(x, t) + O(t^{s(n+1)})$ and $f_n(x)$ is obtained by Taylor's series expansion of $f(x, t)$ where $f(x,t)=\sum_{n=0}^{\infty} f_n(x)t^n$.

The iteration formula (24) can be solved iteratively using

$$V_{-1}=0,$$

$$V_0=g_0(x)+g_1(x)t+\cdots+\frac{g_{s-1}(x)}{(s-1)!}t^{(s-1)},$$

to obtain an approximate power series solution for equation (23).

The second term on the right-hand side of equation (24) is called the correction term, and the equation (24) is called as correction functional. This modification of variational iteration method yields approximate power series solutions which converge to closed form solution of the equation (23) in a neighbourhood of initial point.

4. Application of the MVIM to the problem

Now, we shall use two different initial conditions to solve equation (22) by using the Modified Variational Iteration Method (MVIM).

Case-I: Let us first consider the equation (22) with initial condition:

$$P(\zeta,\eta=1)=1-\cos\zeta.$$

To proceed further, we first use the transformation $\eta - 1 = \eta^*$ in above equations, which yields the following initial value problem on omitting the star sign,

$$\frac{\partial P}{\partial \eta}+\mu PP_{\zeta}+\frac{(m-1)}{2(\eta+1)}P=0, \tag{26}$$

with

$$P(\zeta,\eta=0)=1-\cos\zeta. \tag{27}$$

Here, $BV(\zeta, \eta) = \frac{(m-1)}{2(\eta+1)}P$, $CV(\zeta, \eta) = \mu PP_{\zeta}$, $f(\zeta, \eta) = 0$. Also, $s = 1$ which gives $\lambda(\zeta) = -1$. For solving (26) along with (27), the following iteration formula is used:

$$V_{n+1}=V_n-\int_0^t\left\{B(V_n-V_{n-1})+(F_n-F_{n-1})\right\}d\zeta, \tag{28}$$

where

$$V_{-1} = 0,\ V_0 = 1 - \cos\zeta,$$

and $F_n(\zeta, \eta)$ is the polynomial of degree n, which is obtained from the following formula:

$$\mu V_n (V_n)_\zeta = F_n(\zeta, \eta) + O\left(\eta^{(n+1)}\right).$$

Therefore, the Modified Variational Iteration Method yields the following iterative results:

$$P_1 = 1 - \cos\zeta + t\mu\left(-1 + \cos\zeta\right)\sin\zeta,$$

$$P_2 = 1 - \cos\zeta + t\mu\left(-1 + \cos\zeta\right)\sin\zeta + 2t^2\mu^2\left(2 + 3\cos\zeta\right)\sin\left(\zeta/2\right)^4, \tag{29}$$

$$P_3 = 1 - \cos\zeta + t\mu\left(-1 + \cos\zeta\right)\sin\zeta + 2t^2\mu^2\left(2 + 3\cos\zeta\right)\sin\left(\zeta/2\right)^4 + \frac{4}{3} t^3\mu^3\left(13\cos\left(\zeta/2\right) + 8\cos\left(3\zeta/2\right)\right)\sin\left(\zeta/2\right)^5.$$

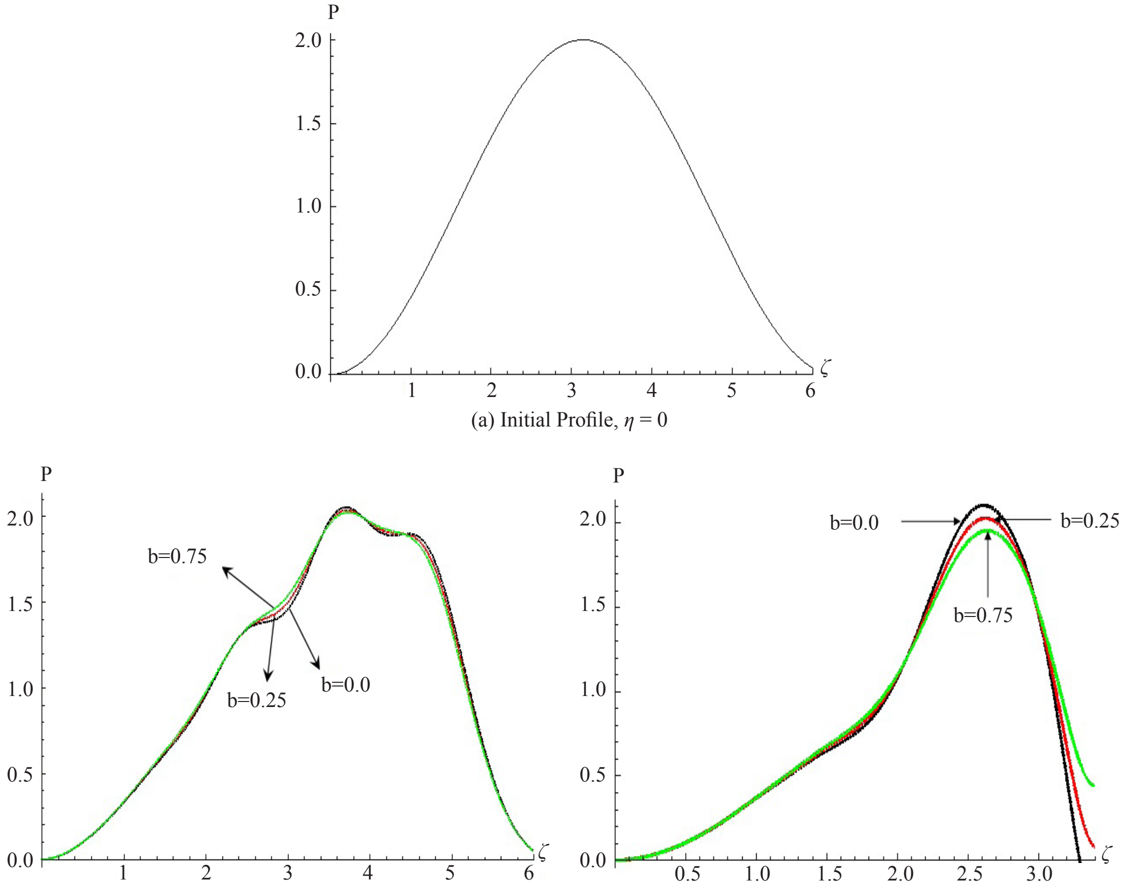
The higher approximations such as P_4, P_5 and P_6 are not shown here due to their large sized expressions. We computed the initial six iterations. The numerical results obtained by the MVIM $6th$-approximation is shown in Table 1.

Table 1. The 6th-approximation of *P* obtained by the MVIM for *m* = 1 for different values of ζ and η

η ζ	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0.1	0.004837	0.018722	0.040767	0.070155	0.106138	0.148036	0.195226
0.2	0.004691	0.017678	0.037614	0.063463	0.094503	0.130384	0.171142
0.3	0.004555	0.016767	0.035044	0.058677	0.088711	0.129422	0.188896
0.4	0.004428	0.015972	0.033157	0.057789	0.101355	0.194744	0.391873
0.5	0.004310	0.152972	0.032571	0.067674	0.171512	0.474369	1.200102

Table 2. The 6th-approximation of *P* obtained by the MVIM for *m* = 2 for different values of ζ and η

η ζ	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0.1	0.004635	0.018029	0.039488	0.068401	0.104240	0.146559	0.194920
0.2	0.004354	0.016676	0.036292	0.063514	0.099954	0.147375	0.205350
0.3	0.004129	0.015780	0.035737	0.068324	0.114980	0.168999	0.228682
0.4	0.003949	0.015528	0.038944	0.075924	0.118889	0.174043	0.210918
0.5	0.003813	0.016167	0.042506	0.077016	0.120099	0.120099	-2.212934



(a) Initial Profile, $\eta = 0$

(b) $m = 1$, $\eta = 0.15$

(c) $m = 2$, $\eta = 0.1$

Figure 1. Effects of non-ideal parameter *b* on the solutions of the Burgers' equation (22) for different values of "*b*" along with the initial profile

Case-II: Let us consider the transformations $t = \alpha\eta$ and $x = \beta\zeta$ in the equation (22), which change it to

$$\alpha \frac{\partial P}{\partial t} + \mu\beta P \frac{\partial P}{\partial x} + \frac{(m-1)\alpha}{2t} P = 0. \tag{30}$$

Let us, now, choose α and β as $\alpha = \mu^2$ and $\beta = \mu$ which on substituting in (30) yields us

$$\frac{\partial P}{\partial t} + P \frac{\partial P}{\partial x} + \frac{(m-1)}{2t} P = 0. \tag{31}$$

This equation (31) is called the inviscid Burgers' equation which has an exact solution, found by Sachdev et al. [18] and Oliveri [19], and given by

$$P(x, t) = \frac{3-m}{2t} x. \tag{32}$$

Now, we again use the transformation $t - 1 = t^*$ in (31) and (32), which on omitting the star sign changes to

$$\frac{\partial P}{\partial t} + P \frac{\partial P}{\partial x} + \frac{(m-1)}{2(t+1)} P = 0, \tag{33}$$

along with the exact solution:

$$P(x,t) = \frac{3-m}{2(t+1)} x. \tag{34}$$

We shall consider the planar case, i.e., when $m = 1$, then the equation (33) along with the initial condition obtained from the exact solution (34), is reduced to the following equation

$$\frac{\partial P}{\partial t} + P\frac{\partial P}{\partial x} = 0, \tag{35}$$

with the initial condition:

$$P(x,0) = x. \tag{36}$$

Here, the equation (35) is the reduced form of the equation (22).

Now, we shall apply the MVIM to solve the initial value problem given by equations (35) and (36).

Here, $BV(x, t) = 0$, $CV(x, t) = PP_x$, $f(x, t) = 0$. Also, $s = 1$ which gives $\lambda(\zeta) = -1$. For solving (35) along with (36) by the MVIM, the following iteration formula is used:

$$V_{n+1} = V_n - \int_0^t \left\{B(V_n - V_{n-1}) + (F_n - F_{n-1})\right\} d\zeta, \tag{37}$$

where

$$V_{-1} = 0,\ V_0 = x,$$

and $F_n(x, t)$ is the polynomial of degree n, which is obtained from the following formula:

$$V_n (V_n)_x = F_n(x,t) + O\left(t^{(n+1)}\right).$$

Therefore, the Modified Variational Iteration Method yields the following iterative results:

$$\begin{aligned}
P_1 &= x - tx,\\
P_2 &= x - tx + t^2x,\\
P_3 &= x - tx + t^2x - t^3x,\\
P_4 &= x - tx + t^2x - t^3x + t^4x,\\
P_5 &= x - tx + t^2x - t^3x + t^4x - t^5x,\\
P_6 &= x - tx + t^2x - t^3x + t^4x - t^5x + t^6x,\\
P_7 &= x - tx + t^2x - t^3x + t^4x - t^5x + t^6x - t^7x,\\
P_8 &= x - tx + t^2x - t^3x + t^4x - t^5x + t^6x - t^7x + t^8x,\\
P_9 &= x - tx + t^2x - t^3x + t^4x - t^5x + t^6x - t^7x + t^8x - t^9x,\\
P_{10} &= x - tx + t^2x - t^3x + t^4x - t^5x + t^6x - t^7x + t^8x - t^9x + t^{10}x.
\end{aligned} \tag{38}$$

We computed the iterations till the tenth-approximation. We then compare the results obtained by the MVIM with the exact solution given in (34) to illustrate the efficiency of the MVIM. The absolute errors between the $10th$-approximation value obtained by the MVIM and the exact solution, for $m = 1$ and $m = 2$, are shown in Tables 3 and 4, respectively.

Table 3. Absolute error between the tenth-approximation solution by the MVIM and the exact solution for *m* = 1

t x	-15	-10	1	5	12
0.05	7.105427 E-14	4.796163 E-14	4.551914 E-15	2.398082 E-14	5.684352 E-14
0.1	1.363638 E-10	9.091039 E-11	9.090950 E-12	4.545520 E-11	1.090914 E-10
0.15	1.128229 E-8	7.521525 E-9	7.521526 E-10	3.760762 E-9	9.025831 E-9
0.2	2.560000 E-7	1.706667 E-7	1.706667 E-8	8.533333 E-8	2.048000 E-7
0.25	2.861023 E-6	1.907349 E-6	1.907349 E-7	9.536743 E-7	2.288818 E-6

Table 4. Absolute error between the tenth-approximation solution by the MVIM and the exact solution for *m* = 2

t x	-15	-10	1	5	12
0.05	1.196231 E-2	7.974874 E-3	7.974874 E-4	3.987437 E-3	9.569850 E-3
0.1	4.125232 E-2	2.750155 E-2	2.750155 E-3	1.375077 E-2	3.300186 E-2
0.15	8.091142 E-2	5.394095 E-2	5.394095 E-3	2.697047 E-2	6.472914 E-2
0.2	1.265676 E-1	8.437850 E-2	8.437850 E-3	4.218925 E-2	1.012542 E-1
0.25	9.674374 E-1	1.169464 E-1	1.169464 E-2	5.847323 E-2	1.403357 E-1

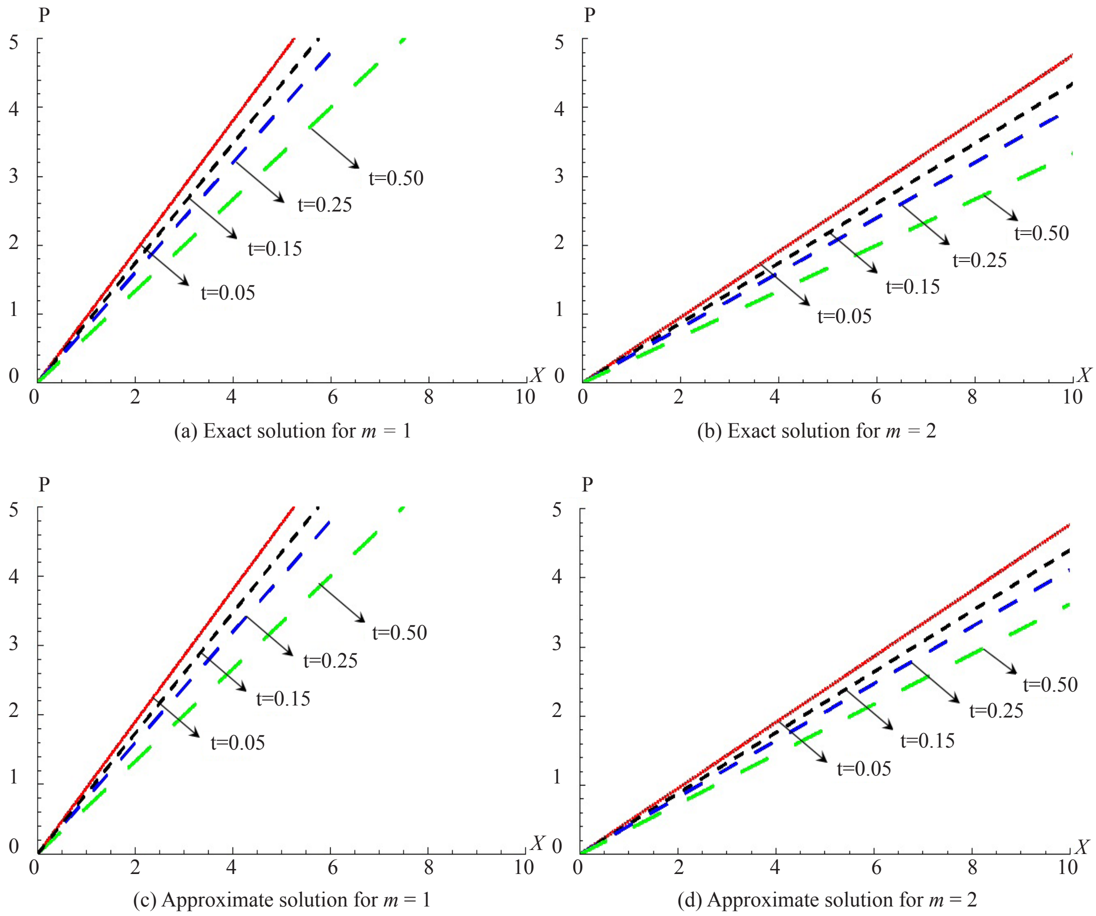


Figure 2. The solutions of the Burgers' equation (35) with initial condition (36) with varying "*t*" with values from 0.05 to 1.0

5. Results and conclusion

In the present paper we used an asymptotic method to derive a far-field evolution equation of the governing hyperbolic system of the equations describing planar and cylindrically symmetric flows in magnetogasdynamics. We studied the behavior of propagation of waves in the far-field of the original system with the help of the so-obtained asymptotic equation, i.e., Burgers' equation. This asymptotic equation describes the characteristics of its parent system. The MVIM numerical scheme is used to obtain the analytical solution for the asymptotic equation, and numerical results are also compared with the exact analytical solution for the symmetric flows. These absolute errors may be improved by increasing the number of iterations. It is observed that due to the geometrical spreading, the flattening in amplitude of the profile takes place. The effect of the parameter b is exhibited through the Figure 1.

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Conflict of interest

On behalf of both authors, the corresponding author states that there is no conflict of interest.

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