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Research Article

A Comparison of Fuzzy and Intuitionistic Fuzzy Frameworks for Individual and Group Replacement Approaches with Scrap Values Using Centroid-Based Ranking Method for Optimal Results

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Abstract: Replacement problems involve managing equipment or machines that degrade over time or with usage and those that fail after reaching specific thresholds. Large, expensive assets, such as machine tools and vehicles, have increased maintenance requirements and depreciation over time, raising the risk of obsolescence. The objective is to optimize the replacement and maintenance schedules. This optimization seeks to reduce total costs, which include operating, maintenance and investment expenses. In operations research, effective machine and equipment replacement strategies are critical for sustaining operational efficiency and reducing costs. Abrupt component failures can lead to system-wide disruptions, particularly in digital components like bulbs and resistors. To avoid sudden breakdowns, effective replacement techniques are required. This study compares environments of fuzzy and environments of intuitionistic fuzzy in group replacement and individual replacement approaches. Costs are modeled using triangular and triangular intuitionistic fuzzy numbers to capture uncertainty and vagueness. The research evaluates two strategies: immediate individual replacement and scheduled group replacement. Quantitative and analytical techniques are employed to explore cost uncertainties. Using a centroid-based ranking method, the study assesses outcomes from both fuzzy and intuitionistic fuzzy algorithms to solve complex decision-making scenarios. Results demonstrate that intuitionistic fuzzy approaches offer more effective and optimal outcomes compared to traditional fuzzy methods, enhancing decision-making precision in machine and equipment replacement strategies.

Keywords: fuzzy logic, logic of vagueness, decision theory, mathematical economics and fuzziness

MSC: 03B52, 91B06, 91B86

1. Introduction

Replacement problems are critical optimization issues in real life, encompassing the challenge of determining the optimal timing and strategy for replacing components that degrade over time or fail suddenly. Traditional approaches to these problems often rely on crisp numerical values for costs such as capital, running, and resale value. However, these

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methods are frequently imprecise due to market fluctuations, maintenance unpredictability, and subjective estimates. This imprecision can lead to suboptimal decisions, resulting in higher overall costs and reduced operational efficiency.

Fuzzy theory offers a way to accommodate the vagueness and uncertainty inherent in real-world data, thereby improving the precision and reliability of replacement strategies. Traditional fuzzy models, however, focus solely on membership degrees, which limits their ability to fully capture the dual nature of uncertainty. This is where Intuitionistic Fuzzy Environment (IFE) comes into play. IFE extends traditional fuzzy models by considering both membership and non-membership degrees, providing a more comprehensive framework for modeling uncertainty.

The research presented here aims to optimize replacement problems under IFE, developing strategies that better reflect real-world conditions and improve both operational efficiency and cost management. By addressing the limitations of traditional and fuzzy approaches, which often oversimplify cost components and fail to fully capture uncertainty, this study seeks to create more robust replacement strategies. Furthermore, the research bridges the gap between theory and practice by empirically validating these advanced models in industrial settings, demonstrating their effectiveness in enhancing operational decision-making.

A key aspect of this research is the incorporation of intuitionistic fuzzy logic to enhance multi-criteria decision-making. This approach balances cost, reliability, and downtime, while also developing dynamic models that adapt to changing situations, thereby offering flexible and responsive replacement strategies. The sudden failure of components, particularly digital parts like bulbs, resistors, and tube lamps, can cause entire systems to fail abruptly. To minimize the rate of yearly breakdowns, effective replacement strategies are essential. This study emphasizes the importance of developing and implementing such strategies to ensure the reliability and efficiency of systems that depend on these components.

Understanding the evolution of methodologies and identifying best practices for studying replacement problems is an important aspect. Liu et al. [1] proposed the most effective replacement strategy for a multistate structure with poor maintenance, attempting to develop an approach from a systems perspective. Barron [2] implemented group or bulk replacement techniques for a repairable cold manage framework with stipulated time frames for repair. Chiu et al. [3] discussed group or bulk replacement strategies for repairable N-component parallel systems. Liu [4] perceived conditionally large sums of money replacing the presumed structure of the discounted expense determination system in the production/service framework, transforming the originally proposed deductible expense algorithm into a relevant structure. Garg et al. [5] defined an ambiguous image/image linguistic set of interval values as a subset of intervals of units containing degrees of truth, abstention, and falsity. Van Staden et al. [6] explored a technique in which machine failure and maintenance records were used to predict the future machine failure rate, leading to developments in established preventive maintenance approaches. Forootani et al. [7] devised a randomly generated dynamic programming technique. The EDMO method was proposed by Hu et al. [8] to account for synergies between divergent division replacements and Decision Makers' (DMs') psychological research in an Interval 2-Tuple Linguistic (ITL) environment and Cumulative Prospect Theory (CPT). Qiao et al. [9] developed algorithms to help communities understand the effects of depreciation methods on equipment replacement decisions and the importance of precisely calculating depreciation of equipment to reduce equipment expenses over a specified study period. Before making a replacement decision, Haktanr et al. [10] used Picture Fuzzy Systems (PFSs) to determine the economic circumstances of the defender and challenger, two alternatives in replacement analysis. Estimates are generated by three PF experts and aggregated using PF aggregation operators. In the 1960s, Zadeh [11] introduced Fuzzy Sets, a further development of traditional set theory. Fuzzy set extensions and the theory of traditional set take different approaches to ambiguity. Atanassov [12] demonstrated IFN Sets as an uncertain set extrapolation in 1986. Fazli et al. [13] investigated a facility placement model with fuzzy value characteristics based on a hybrid meta-heuristic technique. Prakash et al. [14] introduced the notion of spherical fuzzy numbers as a new expansion to previous fuzzy set models. Because of their distinct function characteristics-positive, neutral, and negative membership degrees-the sum of their squared values is limited to one. Eryilmaz et al. [15] investigated age-based preventive replacement policies, focusing on discrete-time coherent systems composed of independent and identical components. The study aims to determine optimal replacement strategies to enhance system reliability and safety using statistical and probabilistic models to account for the age and performance of the components over time. Faizanbasha et al. [16] discussed optimal age replacement times for coherent systems under a Geometric Point Process, aiming to identify the best replacement times to maximize system reliability and efficiency using probabilistic methods to model and analyze the system's performance over time. Wu et al. [17] addressed optimal opportunity-based age replacement policies in discrete time, focusing on determining the best times to replace components based on opportunities rather than fixed schedules, aiming to enhance system reliability and cost-effectiveness by utilizing discrete-time models and optimization techniques.

Most existing research in this domain isolates the use of either fuzzy or intuitionistic fuzzy approaches, lacking direct comparative studies that elucidate the relative strengths and weaknesses of each method across various contexts. There is a substantial need for research that delves into the interaction and impact of these factors on decision-making processes, especially within complex replacement scenarios. Furthermore, empirical validation of these theoretical models in real-world replacement problems remains sparse. There is an acute need for case studies and practical applications that can validate these models, demonstrating their practical utility and effectiveness in industrial settings. This could bridge the gap between conceptual constructions and real execution, restoring real proof of the frameworks' applicability and effectiveness.

Additionally, most current models are static, lacking the capability to adapt to evolving conditions over time. Developing dynamic and adaptive models that can adjust replacement strategies based on real-time data and changing conditions is a critical area for future research. Such advancements would yield more responsive and flexible solutions, thereby enhancing the long-term efficiency and effectiveness of replacement strategies.

The objective of this comparative analysis is to determine the finest strategy between individual replacement and group or bulk replacement in the scenario of fuzzy and Intuitionistic Fuzzy Numbers (IFN). It showed that IFN provides more generalized conclusions than traditional fuzzy methods. A statistical instance is also provided to emphasize the advantages.

Replacement problems involve managing equipment or machines that degrade over time or with usage, and those that fail after reaching specific thresholds. Large, high-cost items like machine tools and trucks require escalating maintenance and depreciate over time, increasing the risk of obsolescence. The challenge is to optimize replacement timing and maintenance levels to minimize overall costs, including operating, maintenance, and investment expenditures. In operations research, effective machine and equipment replacement strategies are critical for sustaining operational efficiency and reducing costs. Abrupt component failures can lead to system-wide disruptions, particularly in digital components like bulbs and resistors. To mitigate breakdowns, efficient replacement strategies are essential.

This study compares fuzzy and intuitionistic fuzzy environments in group replacement and individual replacement approaches. Costs are modeled using triangular and triangular intuitionistic fuzzy numbers to capture uncertainty and vagueness. The research evaluates two strategies: immediate individual replacement and scheduled group replacement. Quantitative and analytical techniques are employed to explore cost uncertainties. Using a centroid-based ranking method, the study assesses outcomes from both fuzzy and intuitionistic fuzzy algorithms to solve complex decision-making scenarios. Results demonstrate that intuitionistic fuzzy approaches offer more effective and optimal outcomes compared to traditional fuzzy methods, enhancing decision-making precision in machine and equipment replacement strategies.

The structure of this work is as follows. Preliminary concepts are discussed in Section 2. In Section 3, centroid-based concepts for ranking algorithms are studied, and a deviation is developed. An application is worked out in Section 4, where all the explanations for computations are provided. In Section 5, the research findings of this work are discussed.

2. Preliminaries

This division's objective is to present fundamentally important definitions and the findings will apply in following computations.

Definition 1 A fuzzy number $\tilde{A}$ is 'defined on set of real numbers R is called a TFN if the membership function $\mu_{\tilde{A}} : R \to [0, 1]$ of $\tilde{A} = \{a_1, a_2, a_3\}$ has the' following conditions:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \le x \le a_2 \\ 1, & \text{for } x = a_2 \\ \frac{a_3-x}{a_3-a_2}, & \text{for } a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}$$

2.1 *Strategy of group and individual replacement*

Sometime the equipment expires within a certain time frame, it is replaced independently, or all components are replaced at an appropriate interval. As a result, we must determine the t-value for which the mean expense is lowest. Assuming that all failed equipment is replaced at the end of the time.

Let

$\tilde{E}_g$ = Expense or cost for group replacement.

$\tilde{E}_i$ = Expense of the individual replacement.

$\tilde{S}$ = Scrap fuzzy value.

$\tilde{E}(n) = \tilde{E}_g\tilde{N} + \tilde{E}_i[\tilde{N}(1) + \tilde{N}(2) + \cdots + \tilde{N}(t-1)] - \tilde{S} = \tilde{E}_g\tilde{N} + \tilde{E}_i \sum_{x=1}^{(t-1)} \tilde{N}(x) - \tilde{S}.$

The average or mean expense = $\tilde{M}(t) = (\tilde{E}(t) - \tilde{S})/t$

$\tilde{M}(t)$ is min

$[\Delta\tilde{M}(t-1)/1 < 0 < \Delta\tilde{M}(t)/1]$

$\Delta\tilde{M}(t) = \tilde{E}(t+1) - \tilde{S}/t + 1 - \tilde{E}(t) - \tilde{S}/t$

From $\tilde{E}(n)$, we get $\tilde{E}(1+t) = \tilde{E}(t) + \tilde{E}_i\tilde{N}(t)$

$\Delta\tilde{M}(t) = \dfrac{\tilde{E}(t) - \tilde{S}/1 + \tilde{E}_i\tilde{N}(t) - \tilde{S}/1}{(1+t)/1} - \dfrac{\tilde{E}(t) - \tilde{S}/1}{(t)/1}$

$\Delta\tilde{M}(t-1) = \dfrac{\tilde{E}_i\tilde{N}(t-1) - \tilde{S}/1 - \tilde{E}(t-1) - \tilde{S}/(t-1)}{(t)/1}.$

Similarly we get

$(\tilde{E}_i\tilde{N}(t-1) - \tilde{S})/(1 - \tilde{E}(t-1) - \tilde{S}/(t-1))$

$< 0 < (\tilde{E}_i\tilde{N}(t) - \tilde{S})/(1 - \tilde{E}(t) - \tilde{S}/(t))$

$(\tilde{E}_i\tilde{N}(t) - \tilde{S}/1) > (\tilde{E}(t) - \tilde{S}/(t))$

$(\tilde{E}_i\tilde{N}(t-1) - \tilde{S}/1) < (\tilde{E}(t-1) - \tilde{S}/(t-1)).$

According to the above results, when the average annual expense of individual replacement is low in comparison to the group replacement, the individual replacement is preferable. Similarly, when the average annual expense of group replacement is low in comparison to the individual replacement, the group replacement is preferable.

3. Centroid based concepts for ranking algorithms

The following section the centroid index classification method uses the geometrically derived mid-value of trapezoidal intuitionistic fuzzy numbers to derive the ranking algorithm. In intuitionistic nature, triangular and trapezoidal numbers are used to construct rankings. Consider a triangle or trapezoid with an intuitionistic fuzzy number and membership and non-membership operations such as

$$\mu_{\tilde{A}} = \begin{cases} 0, & l_1 > x \\ f_A^L(x), & l_1 \le x \le l_2 \\ 1, & l_2 \le x \le l_3 \\ f_A^R(x), & l_3 \le x \le l_4 \\ 0, & l_4 \le x \end{cases} \qquad \nu_{\tilde{A}} = \begin{cases} 0, & m_1 > x \\ g_{\tilde{A}}^L(x), & m_1 \le x \le m_2 \\ 0, & m_2 \le x \le m_3 \\ g_{\tilde{A}}^R(x), & m_3 \le x \le m_4 \\ 1, & m_4 < x \end{cases}$$

The trapezoidal intuitionistic fuzzy number's centroid point $\tilde{A} = (l_1,\ l_2,\ l_3,\ l_4;\ m_1,\ m_2,\ m_3,\ m_4)$ will written as

$$\tilde{z}_\mu(\tilde{A}) = \frac{\frac{1}{\alpha_\mu}\left[\frac{2(\alpha_\mu^3) + 6l_2l_1\alpha_\mu - 3\alpha_\mu(l_2l_1 + l_1^2)}{6}\right] + \frac{(l_3^2 - l_2^2)}{2} - \frac{1}{\beta_\mu}\left[\frac{2(\beta_\mu^3) + 6l_4l_3\beta_\mu - 3\beta_\mu(l_3l_4 + l_4^2)}{6}\right]}{\frac{1}{\alpha_\mu}\left[\frac{\alpha_\mu(l_2 + l_1)}{2} - \alpha_\mu l_1\right] + (l_3 - l_2) - \frac{1}{\beta_\mu}\left[\frac{\beta_\mu(l_4 + l_3)}{2} - \beta_\mu l_4\right]}$$

Afterwards integrating the values,

$$\tilde{z}_\mu(\tilde{A}) = \frac{1}{3}\left[\frac{2\alpha_\mu^2 + 3l_2l_1 - 3l_1^2 + 3(l_3^2 - l_2^2) - 2\beta_\mu^2 - 3l_4l_3 + 3l_4^2}{l_4 + l_3 - l_2 - l_1}\right]$$

In this case $\alpha_\mu = (l_1 - l_2)$, $\beta_\mu = l_4 - l_3$ are fuzziness of left as well as right for membership

$$\tilde{z}_\nu(\tilde{A}) = \frac{\int_{m_1}^{m_2} \frac{x^2 - xm_2}{-\alpha_\nu}dx + \int_{m_2}^{m_3} xdx + \int_{m_3}^{m_4} \frac{x^2 - xm_3}{-\beta_\nu}dx}{\int_{m_1}^{m_2} \frac{x^2 - m_2}{-\alpha_\nu}dx + \int_{m_2}^{m_3} dx + \int_{m_3}^{m_4} \frac{x^2 - m_3}{-\beta_\nu}dx}$$

$$\tilde{z}_\nu(\tilde{A}) = \frac{\frac{1}{\alpha_\nu}\left[\frac{2(\alpha_\nu^3) + 6m_2m_1\alpha_\nu - 3\alpha_\nu(m_2m_1 + m_1^2)}{6}\right] + \frac{(m_3^2 - m_2^2)}{2} + \frac{1}{\beta_\nu}\left[\frac{2(\beta_\nu^3) + 6m_4m_3\beta_\nu - 3\beta_\nu(m_3m_4 + d_4^2)}{6}\right]}{-\frac{1}{\alpha_\nu}\left[\frac{\alpha_\nu(m_2 + m_1)}{2} - \alpha_\nu m_2\right] + (m_3 - m_2) + \frac{1}{\beta_\nu}\left[\frac{\beta_\nu(m_4 + m_3)}{2} - \beta_\nu m_3\right]}$$

$$\tilde{z}_v(\tilde{A}) = \frac{1}{3}\left[\frac{-2\alpha_v^2 - 3m_2m_1 + 3m_1^2 + 3(m_3^2 - m_2^2) + 2\beta_v^2 + 3m_4m_3 - 3m_3^2}{m_4 + m_3 - m_2 - m_1}\right]$$

$$\tilde{w}_\mu(\tilde{A}) = \frac{\int_0^1 ((\alpha_\mu)y^2 + l_1 y)dy - \int_0^1 ((-\beta_\mu)y^2 + l_4 y)dy}{\int_0^1 ((\alpha_\mu)y + l_1 y)dy - \int_0^1 ((-\beta_\mu)y + l_4)dy},$$

after integrating $\tilde{w}_\mu(\tilde{A})$ will be

$$\tilde{w}_\mu = \frac{\dfrac{2\alpha_\mu + 3l_1 - 3l_4 + 2\beta_\mu}{6}}{\dfrac{\alpha_\mu}{2} + l_1 + \dfrac{\beta_\mu}{2} - l_4}$$

$$\tilde{w}_\mu = \frac{1}{3}\left[\frac{2\alpha_\mu + 3l_1 - 3l_4 + 2\beta_\mu}{\alpha_\mu + 2l_1 - 2l_4 + \beta_\mu}\right]$$

Where $\alpha_\mu =$ left fuzziness of IFN, $\beta_\mu =$ right fuzziness of IFN for MF

$$\tilde{w}_v(\tilde{A}) = \frac{\int_0^1 ((-\alpha_v)y^2 + m_2 y)dy - \int_0^1 ((\beta_v)y^2 + m_3 y)dy}{\int_0^1 ((-\alpha_v)y + m_2)dy - \int_0^1 ((\beta_v)y + m_3)dy}$$

$$\tilde{w}_v = \frac{\dfrac{1}{6}[-2\alpha_v + 3m_2 - 2\beta_v - 3m_3]}{-\dfrac{\alpha_v}{2} + m_2 - \dfrac{\beta_v}{2} - m_3}$$

$$\tilde{w}_v = \frac{1}{3}\left[\frac{2\alpha_v - 3m_2 + 3m_3 + 2\beta_v}{\alpha_v - 2m_2 + 2m_3 + \beta_v}\right]$$

Where $\alpha_v = -(m_1 - m_2)$, $\beta_v = -(m_3 - m_4)$ fuzziness of left as well as right for non membership function. Ranking can be defined by

$$R(\tilde{A}^{IFN}) = \sqrt{\frac{1}{2}\left(\left[\tilde{z}_\mu(\tilde{A}) - \tilde{w}_\mu(\tilde{A})\right]^2 + \left[\tilde{z}_v(\tilde{A}) - \tilde{w}_v(\tilde{A})\right]^2\right)}$$

Here the centroid points are defined by

$$\tilde{z}_\mu = \left[\frac{(l_3 + l_1 + l_2)}{3}\right]$$

$$\tilde{w}_\mu = \frac{1}{3}\left[\frac{(l_1 - l_3)}{(l_1 - l_3)}\right] = \frac{1}{3}$$

$$\tilde{z}_\nu = \left[\frac{(m_1 - (m_2 - m_1) + 2m_3)}{3}\right]$$

$$\tilde{w}_\nu = \frac{1}{3}\left[\frac{2(m_3 - m_1)}{(m_3 - m_1)}\right] = \frac{2}{3}$$

4. Computational explanations

4.1 *Comparative analysis of individual with group or bulk replacement in uncertain or fuzzy environment*

There are various LED lights that must be kept operational, with total quantities of (7,500, 8,500, and 9,500). When one of the lights ceases to work, it costs Rs. (500, 600, 700) to replace it. Alternatively, if all the lights are changed at a particular time, it costs Rs. (150, 250, 350) per light, with a scrap value of Rs. (50, 60, 70). Determine the optimal replacement time based on the proportion of lights that fail over consecutive time periods. The failure rates for the LED lights are as the follow Table 1 shows.

Table 1. Each year failure probability

Year	I	II	III	IV	V	VI
Possibility of failure	0.10	0.27	0.52	0.87	0.98	1.0

Let $\tilde{\eta}_n$ be the replacements in nth year end.

$\tilde{\eta}_0$ = (7,500, 8,500, 9,500)

The number of replacements for all the periods

$\tilde{\eta}_1$ = (7,500, 8,500, 9,500) $\times$ 0.10 = (850, 1,000, 1,000),

$\tilde{\eta}_2$ = ((8,500, 1,000, 1,000) $\times$ 0.17) + ((850, 1,000, 1,000) $\times$ 0.10) = (1,530, 1,000, 1,000),

$\tilde{\eta}_3$ = (2,422.5, 1,000, 1,000),

$\tilde{\eta}_4$ = (3,689.85, 1,000, 1,000),

$\tilde{\eta}_5$ = (2,395.81, 1,000, 1,000),

$\tilde{\eta}_6$ = (2,271.48, 1,000, 1,000),

Expected period of LED lights = $\sum_{i=1}^{6} x_i(P_{x_i}) = 3.26$.

Failures in every year $\tilde{\eta}$/mean age = (7,500, 8,500, 9,500)/3.26 = (2,608, 1,000, 1,000).

As a result, at Rs. (500, 600, 700) per light, the cost of individual replacement with scrap value is Rs. (1,407,320, 1,408,320, 1,409,320). Since replacing all (7,500, 8,500, 9,500) lights at the same time costs Rs. (150, 250, 350) per light, the mean cost for group or bulk replacement is indicated in the Table 2.

Table 2. Cumulative expense of group or bulk fuzzy replacement

Year	Cumulative expense of group or bulk IFN replacement	The average annual expense/year
1	((850, 1,000, 1,000) × (600 − 60)) + ((7,500, 8,500, 9,500) × (250 − 60))	(2,073,000, 2,074,000, 2,075,000)
2	((2,380, 1,000, 1,000) × (600 − 60)) + (7,500, 8,500, 9,500) × (250 − 60))	(1,449,100, 1,450,100, 1,451,100)
3	((4,802.5, 1,000, 1,000) × (600 − 60)) + ((7,500, 8,500, 9,500) × (250 − 60))	(1,401,783.3, 1,402,783.3, 1,403,783.3) Replace (min)
4	((8,492.35, 1,000, 1,000) × (600 − 60)) + ((7,500, 8,500, 9,500) × (250 − 60))	(1,549,217.3, 1,550,217.3, 1,551,217.3)
5	((10,888.16, 1,000, 1,000) × (600 − 60)) + ((7,500, 8,500, 9,500) × (250 − 60))	(1,497,921.3, 1,498,921.3, 149,921.3)
6	((13,159.64, 1,000, 1,000) × (600 − 60)) + ((7,500, 8,500, 9,500) × (250 − 60))	(1,452,534.3, 1,453,534.3, 1,454,534.3)

According to the suggested replacement policy, if the annual average expenditure of group replacement is less than the yearly average expense of individual replacement in the third year, group replacement is preferred and more profitable (from Figure 1) in fuzzy environment.

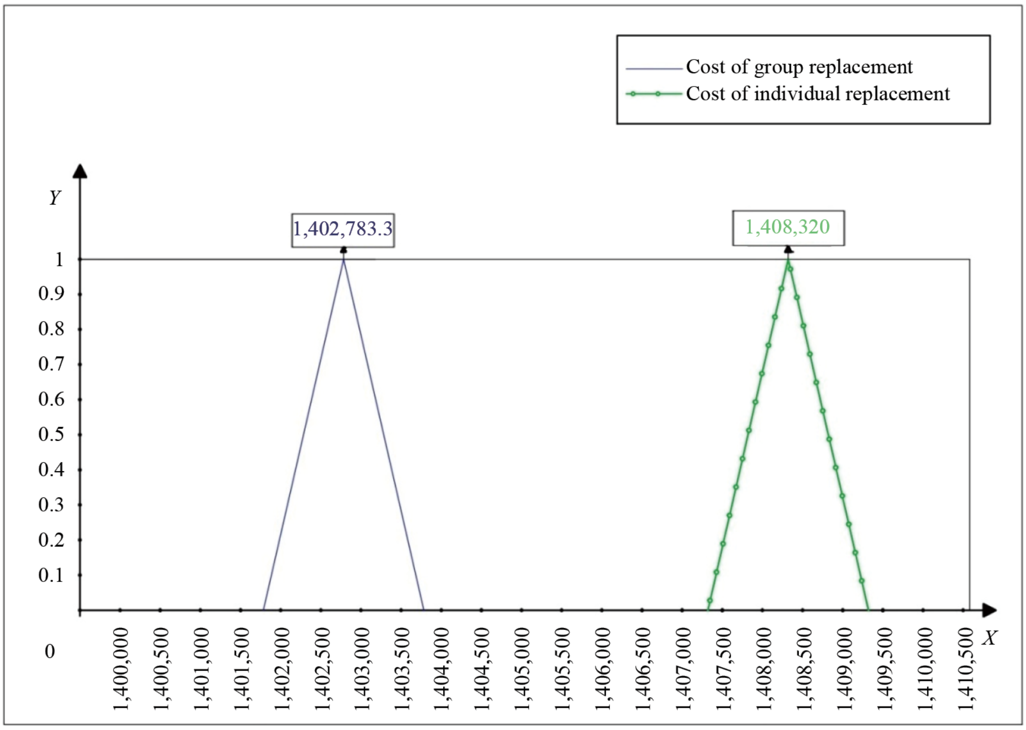

Figure 1. The figure depicted a fuzzy comparison study of group or bulk vs. individual replacement expenses

4.2 *Comparative analysis of individual with group or bulk replacement in intuitionistic fuzzy environment*

There are various LED lights that must be kept operational, with total quantities of (7,500, 8,500, 9,500; 6,500, 8,500, 10,500). When one of the lights ceases to work, it costs Rs. (500, 600, 700; 400, 600, 800) to replace it. Alternatively, if all the lights are changed at a particular time, it costs Rs. (150, 250, 350; 50, 250, 450) per light, with a scrap value of Rs. (50, 60, 70; 40, 60, 80). Determine the optimal replacement time based on the proportion of lights that fail over consecutive time periods. The failure rates for the LED lights are as the follow Table 3.

Table 3. Each year failure probability

Year	I	II	III	IV	V	VI
Possibility of failure	0.10	0.27	0.52	0.87	0.98	1.0

Let $\tilde{\eta}_n$ be the replacements in nth year end.

$\tilde{\eta}_0 = (7{,}500, 8{,}500, 9{,}500; 6{,}500, 8{,}500, 10{,}500)$

The number of replacements for all the periods

$\tilde{\eta}_1 = (7{,}500, 8{,}500, 9{,}500; 6{,}500, 8{,}500, 10{,}500) \times 0.10 = (850, 1{,}000, 1{,}000; 850, 2{,}000, 2{,}000)$,

$\tilde{\eta}_2 = ((8{,}500, 1{,}000, 1{,}000; 8{,}500, 2{,}000, 2{,}000) \times 0.17) + ((850, 1{,}000, 1{,}000; 850, 2{,}000, 2{,}000) \times 0.10)$
$= (1{,}530, 1{,}000, 1{,}000; 1{,}530, 2{,}000, 2{,}000)$,

$\tilde{\eta}_3 = (2{,}422.5, 1{,}000, 1{,}000; 2{,}422.5, 2{,}000, 2{,}000)$, $\tilde{\eta}_4 = (3{,}689.85, 1{,}000, 1{,}000; 3{,}689.85, 2{,}000, 2{,}000)$,

$\tilde{\eta}_5 = (2{,}395.81, 1{,}000, 1{,}000; 2{,}395.81, 2{,}000, 2{,}000)$, $\tilde{\eta}_6 = (2{,}271.48, 1{,}000, 1{,}000; 2{,}271.48, 2{,}000, 2{,}000)$,

Expected period of LED lights $= \sum_{i=1}^{6} x_i(P_{x_i}) = 3.26$.

Failures in every year

$\tilde{\eta}$/mean age = (7,500, 8,500, 9,500; 6,500, 8,500, 10,500)/3.26 = (2,608, 1,000, 1,000; 2,608, 2,000, 2,000).

As a result, at Rs. (500, 600, 700; 400, 600, 800) per light, the cost of individual replacement with scrap value is Rs. (1,407,320, 1,408,320, 1,409,320; 1,406,320, 1,408,320, 1,410,320). Since replacing all (7,500, 8,500, 9,500; 6,500, 8,500, 10,500) lights at the same time costs Rs. (150, 250, 350; 50, 250, 450) per light, the mean cost for group or bulk replacement is indicated in the Table 4.

Table 4. Cumulative expense of group or bulk fuzzy replacement

Year	Cumulative expense of group or bulk IFN replacement	The average annual expense/year
1	((850, 1,000, 1,000; 850, 2,000, 2,000) × (600 − 60)) + ((7,500, 8,500, 9,500; 6,500, 8,500, 10,500) × (250 − 60))	(2,073,000, 2,074,000, 2,075,000; 2,072,000, 2,074,000, 2,076,000)
2	((2,380, 1,000, 1,000; 2,380, 2,000, 2,000) × (600 − 60)) + ((7,500, 8,500, 9,500; 6,500, 8,500, 10,500) × (250 − 60))	(1,449,100, 1,450,100, 1,451,100; 1,448,100, 1,450,100, 1,452,100)
3	((4,802.5, 1,000, 1,000; 4,802.5, 2,000, 2,000)× (600 − 60)) + ((7,500, 8,500, 9,500; 6,500, 8,500, 10,500) × (250 − 60))	(1,401,783.3, 1,402,783.3, 1,403,783.3; 1,400,783.3, 1,402,783.3, 1,404,783.3) min
4	((8,492.35, 1,000, 1,000; 8,492.35, 2,000, 2,000) × (600 − 60)) + ((7,500, 8,500, 9,500; 6,500, 8,500, 10,500) × (250 − 60))	(1,549,217.3, 1,550,217.3, 1,551,217.3; 1,548,217.3, 1,550,217.3, 1,552,217.3)
5	((10,888.16, 1,000, 1,000; 10,888.16, 2,000, 2,000) × (600 − 60)) + ((7,500, 8,500, 9,500; 6,500, 8,500, 10,500) × (250 − 60))	(1,497,921.3, 1,498,921.3, 149,921.3; 1,496,921.3, 1,498,921.3, 1,500,921.3)
6	((13,159.64, 1,000, 1,000; 13,159.64, 2,000, 2,000) × (600 − 60)) + ((7,500, 8,500, 9,500; 6,500, 8,500, 10,500) × (250 − 60))	(1,452,534.3, 1,453,534.3, 1,454,534.3; 1,451,534.3, 1,453,534.3, 1,455,534.3)

According to the suggested replacement policy, if the annual average expenditure of group replacement is less than the yearly average expense of individual replacement in the third year, group replacement is preferred and more profitable (from Figure 2) in intuitionistic fuzzy environment.

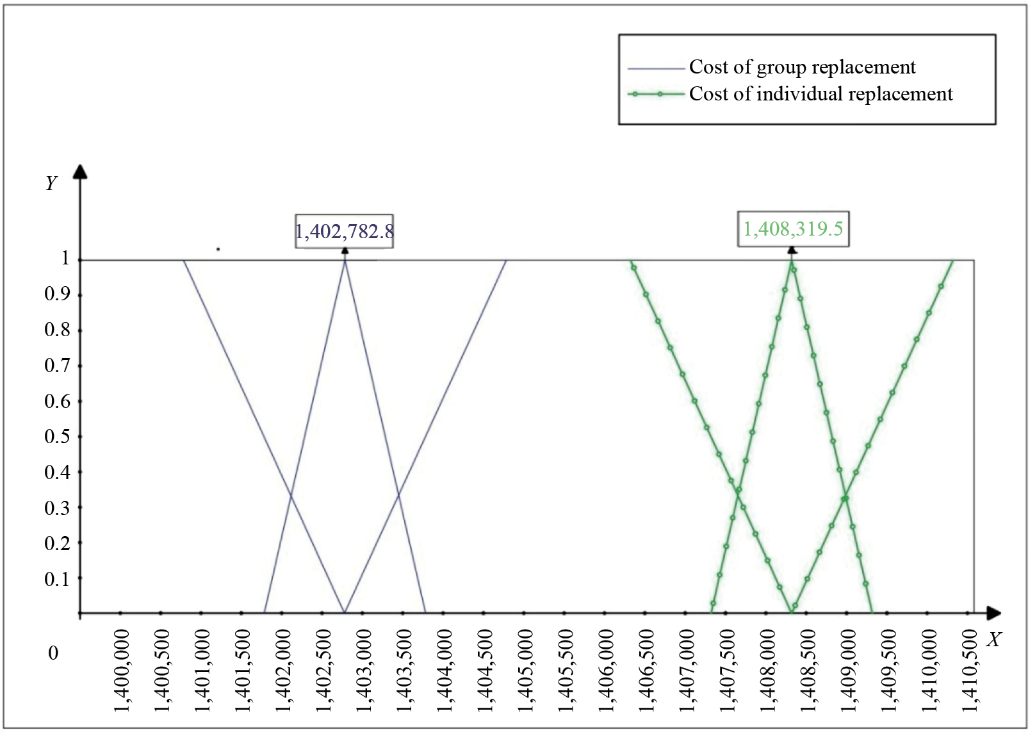

Figure 2. The figure depicted an intuitionistic fuzzy comparison study of group or bulk vs. individual replacement expenses

5. Discussion and research findings

The overall objective of this study is to provide accurate findings to industries, allowing them to find the optimal time to replace equipment or machines and develop the best substitution plan.

In a **fuzzy environment**, the cost of individual replacement is determined based on the expected period and cumulative number of LED lights. The individual replacement fuzzy expenses amount to 1,408,320, with membership values of 1,407,320 and 1,409,320.

To calculate the replacement time in a fuzzy environment, the cumulative fuzzy expense of LED lights is considered. From Table 2, the first year's group or bulk replacement expense of LED lights is 2,074,000, with membership values of 2,073,000 and 2,075,000. The second year's expense is 1,450,100, with membership values of 1,449,100 and 1,451,100. The third year's expense is 1,402,783.3, with membership values of 1,401,783.3 and 1,403,783.3. The fourth year's expense is 1,550,217.3, with membership values of 1,549,217.3 and 1,551,217.3. The fifth year's expense is 1,498,921.3, with membership values of 1,497,921.3 and 1,499,921.3. The sixth year's expense is 1,453,534.3, with membership values of 1,452,534.3 and 1,454,534.3. According to these results, the cost of individual replacement is 1,408,320, while the annual average cost of group replacement is 1,402,783.3. Therefore, replacing items as a group is more cost-effective than replacing them individually.

In an **intuitionistic fuzzy environment**, the cost of individual replacement is similarly determined based on the expected period of the LED lights. The individual replacement fuzzy expense is 1,408,319.5, nearly equivalent to 1,408,320, with membership values of 1,407,320 and 1,409,320, and non-membership values of 1,406,320 and 1,410,320.

For group or bulk replacement in an intuitionistic fuzzy environment, the expense is estimated using the cumulative fuzzy expense. From Table 4, the first year's group or bulk replacement expense is 2,073,999.5, nearly equivalent to 2,074,000, with membership values of 2,073,000 and 2,075,000, and non-membership values of 2,072,000 and 2,076,000. The second year's expense is 1,450,099.5, nearly equivalent to 1,450,100, with membership values of 1,449,100 and 1,451,100, and non-membership values of 1,448,100 and 1,452,100. The third year's expense is 1,402,782.8, nearly equivalent to 1,402,783, with membership values of 1,401,783.3 and 1,403,783.3, and non-membership values of 1,400,783.3 and 1,404,783.3. The fourth year's expense is 1,550,216.8, nearly equivalent to 1,550,217, with membership values of 1,549,217.3 and 1,551,217.3, and non-membership values of 1,548,217.3 and 1,552,217.3. The fifth year's expense is 1,498,920.8, nearly equivalent to 1,498,921.3, with membership values of 1,497,921.3 and 1,499,921.3, and non-membership values of 1,496,921.3 and 1,500,921.3. The sixth year's expense is 1,453,533.8, nearly equivalent to 1,453,534.3, with membership values of 1,452,534.3 and 1,454,534.3, and non-membership values of 1,451,534.3 and 1,455,534.3. Based on these findings, the cost of individual replacement in an intuitionistic fuzzy environment is 1,408,319.5, while the average yearly cost of group replacement is 1,402,782.8. Thus, group replacement proves to be significantly less expensive than individual replacement, making it the more advantageous and economically viable option in this scenario. By opting for group replacement, industries can achieve substantial cost savings and enhance their overall profitability. This approach not only reduces the financial burden associated with frequent individual replacements but also streamlines the maintenance process, leading to more efficient operations. Consequently, adopting group replacement as a strategy ensures better resource allocation and improved financial outcomes for the organization.

6. Conclusions

Based on the prior analysis, the findings in a fuzzy environment indicate that the individual replacement expense is 1,408,320, while the average annual expense of group replacement is 1,402,783.3. Similarly, in an intuitionistic fuzzy environment, the individual replacement expense is 1,408,319.5, and the average annual expense of group replacement is 1,402,782.8. Consequently, the group or bulk exchange technique emerges as the optimal approach. In this study, we conducted a thorough examination of individual replacement and bulk replacement techniques within both frameworks of fuzzy and frameworks of intuitionistic fuzzy, supported by a numerical example. Furthermore, our comparison of the algorithms in fuzzy and intuitionistic contexts revealed identical exchange durations and costs. Previous studies have often focused on traditional deterministic models for equipment replacement, which do not account for the inherent uncertainties in real-world conditions. These models typically rely on fixed parameters and fail to address the variability in replacement costs and the lifespan of equipment. In contrast, our study incorporates fuzzy and intuitionistic fuzzy logic, which better captures the ambiguity and uncertainty in replacement decision-making processes.

Earlier research has highlighted the limitations of deterministic approaches in providing precise and optimal solutions, especially in complex environments where equipment performance and costs are subject to fluctuation. Our findings corroborate these limitations, demonstrating that the fuzzy and intuitionistic fuzzy models offer more robust and reliable solutions by accommodating uncertainty. Our study revealed that intuitionistic replacement is one of the best strategies for determining score parameters due to the ease with which the evidence gained may be interpreted. The proposed method offers several advantages. Traditional algorithms have limitations when modeling real-world situations and fuzzy concepts can have multiple implications due to ambiguity. To compare the outcomes, we developed a new centroid-based ranking of intuitionistic fuzzy numbers.

The recommended replacement technique produces substantially more precise or optimal results and allows us to solve all problems without altering their nature. This paper addresses issues involving uncertainty as computational methods within the paradigm of ambiguous optimization and decision-making. We discuss the applicability, accuracy, and benefits of applying fuzzy technological advances to real-life problems, focusing on methods for solving problems and their computationally complicated nature.

To further enrich the research, we offer the following practical and detailed recommendations for industries considering equipment replacement strategies: develop customized replacement plans tailored to specific equipment types and operating conditions, using the average annual expense data as a baseline for decision-making, and adopt fuzzy and intuitionistic fuzzy models in replacement planning to account for uncertainties in equipment performance and costs. Future suggestions for researchers include incorporating external factors such as market conditions, technological advancements, and regulatory changes into replacement models to develop more comprehensive solutions, and exploring fuzzy neutrosophic environments in future studies to address and refine the challenges associated with replacement strategies.

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Conflict of interest

The authors declare that there is no conflict of interest.

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